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1. INTRODUCTION

The cracking in the NPP piping system remains a main concern and deterministic structural integrity assessment need to be combined with probabilistic approaches in order to consider uncertainties in material, environment and loading properties. The deterministic fracture mechanics approach for a defined problem can be transformed to a probabilistic approach by considering some of the inputs to be random variables. Candidates as random variables include initial crack locations and size (depth and length), fracture toughness, subcritical crack growth characteristics, stress levels, environmental effects, etc. In addition, the effects of inspection can be included through their influence on crack detection, sizing and repair. The characterization of the statistical distribution of fracture mechanics random variables consists of collecting information from testing and literature and, also, characterizing its scatter by selection the type of distribution and the parameters of the distribution. The analytical developments could be sustained by finite element analyses (FEA) performed to provide input stress loadings.

Thermal fatigue has caused through-wall cracking and leakage in BWR and PWR plants from low-cycle and high-cycle thermal fatigue. Experience [1], [2] suggests that service-induced fatigue in operating plants are caused primarily by thermal stratification or hot-cold water mixing conditions, such as thermal striping, and cyclic turbulent mixing not analyzed in the original plant designs. The problem of thermal fatigue in mixing areas arises in pipes where turbulent mixing or vortices produce rapid fluid temperature fluctuations with random frequencies. These loadings consist of many cycles with the cyclic boundary conditions changing quickly, causing cracks to initiate and grow at multiple locations on the inside surface of the pipe. The large nonlinear gradient stress profiles associated with these service conditions yield to cause growing of the cracks in the length direction where the highest surface stresses appear. Under thermal transient conditions typically associated with component failures, the high-cycle loadings attributed to thermal striping, turbulence penetration and thermal mixing can lead to mainly surface cracking with shallow flaw depths. Also, the crack growth in thermal fatigue is strongly dependent on the aspect ratio of the surface crack and on cyclic stress gradients.

A methodology for fatigue crack growth assessment in a pipe subjected to sin-wave thermal loading has been developed and implemented in a MATLAB software environment based on the analytical solutions for thermal and associated stress field, developed in previous works [3],[4],[5],[6]. A beneficial aspect of this type of methodology, based on analytical solutions in case of thermal stripping modeling is the fact that can be easily modified in their definition to check the influence of various parameters (component geometry, thermal loading, thermal stress components, critical frequencies, crack shape ratios, crack geometry, etc.) in a systematic way.

To reasonably account for uncertainties, scatter and randomness of the thermal loading data and material properties the present work deals with a prospective analysis to define a probabilistic approach for thermal fatigue crack growth in mixing areas using the computed elastic stress distribution through the pipe wall. Because the initial crack size was known to be a critical input to the fatigue crack growth calculation, a reliability model taking into account initial crack size distribution would be useful to evaluate the cumulative probability of the crack arrest/penetration during a specified reference period. Probabilistic assessment of thermal fatigue crack growth in high-cycle loadings, under the large nonlinear gradient stress profiles through wall-thickness is an on-going assessment approach in many research programs [7], [8], [9], [10], [11].

The present work constitutes a prospective study on probabilistic approach of thermal fatigue in mixing tees (Civaux 1 damage case) by means of the limit state function (or failure function) and using Monte Carlo Simulation (MCS).

2. BACKGROUND ON THE STRUCTURAL RELIABILITY METHOD

Probabilistic structural analysis may be defined as the art of formulating a mathematical model which is able to give the probability that a structure behaves in a specific way, given one or more of its material properties to be of random or incompletely known nature, or that the actions on the structure in some respects have random or incompletely known properties [12].

2.1 Limit States

A limit state is generally defined as a state of a structure or part of the structure that no longer meets the requirements laid down for its performance or operation. In other way, the limit states can be defined as a specific set of states that separate a desired state from an undesirable state which fails to meet the design requirements. Also, more generally, we can say that a *Limit State* is a mathematical criterion that categorises any set of values of the relevant structural variables (loads, material and geometrical variables) into one of two categories: a “safe” set and a “failure” set [12].

A component or system may fail a limit state in any of a number of failure modes. Modes of failure (at both component and system levels) may include mechanisms such as: yielding, bursting, ovality, bending, buckling (local or large scale), creep, ratcheting, fatigue, fracture, corrosion (internal or external), erosion, environmental cracking, excessive displacement, excessive vibration.

The internationally approved format for general principles is to categorise limit states as:

- ultimate limit state (ULS), which corresponds to the maximum load carrying capacity, and include all types of collapse behavior;
- serviceability limit states (SLS), which concerns normal functional use and all aspects of structure at working loads.

A number of definitions for limit states for operating pipelines have been proposed. Most use the concepts of ULS and SLS and many of these are confined to these two limit states only. Examples of ULS include leaks and ruptures and examples of SLS include permanent deformation due to yielding or denting.

A Fatigue Limit State (FLS) is a ULS condition accounting for accumulated cyclic load effects (crack initiation and crack growth).

2.2 Probabilistic approach and assessment procedure

Probabilistic analysis, based on structural reliability analysis methods, is an extension of deterministic analysis since deterministic quantities can be interpreted as random variables of a particularly nature in which their density functions are concentrated to spikes and in which their standard deviations tend to zero. Variations in the values of the basic engineering parameters occur because of: natural physical variability, poor information and accidental events involving human error. In addition to the uncertainties associated with individual load and strength parameters, also both the methods of global analysis and the equations used for assessing the strength of individual components are not exact.

In the case of global structural analysis, the true properties of the materials and components often deviate from the idealizations on which the methods are based. Without exception, all practical structural systems exhibit behavior that is nonlinear and dynamic, and have properties that are time-dependent, strain-rate dependent and spatially non-uniform. Moreover, the practical structures contain some levels of residual stresses resulting from a particular fabrication and installation sequence adopted. In

addition they can often contain non-structural components which are normally ignored in the analysis, but which often contribute in a significant way, particularly to stiffness. These differences between real and predicted behavior can be termed global analysis model uncertainty.
The variability in load and strength parameters (including model uncertainty) arising from physical variability and inadequacies in modeling are allowed for in deterministic design and assessment procedures by an appropriate choice of safety factors and by an appropriate degree of bias in the Code design equations. In probabilistic methods the variability in the basic design variables, including model uncertainty, is taken into account directly in the probabilistic modeling of the quantities.

2.3 Objective of structural reliability analysis

The risk analysis approaches often used in the safety assessment of process or plant operations are generally referred to as quantitative risk assessment (QRA). QRA can be defined as the formal and systematic approach of identifying potentially hazardous events, and estimating the likelihood and consequences to people, the environment and resources due to accidents developing from these events. A definition of risk is based on a function of the probability of failure and the consequences of failure. Thus,

$$Risk = function(P_f, Consequences) = P_f \cdot Consequences \qquad (1)$$

The probability of failure or frequency of an event is expressed as “event per time”, usually per year. Consequences can be expressed as number of people affected, amount of leak (or area affected, money lost, etc.). Consequences are expresses “per event”. For pressure vessels, pipeline systems, etc. the probability of partial or complete failure of structural integrity during the life is one input, often the key input into a risk assessment.
The structural reliability analysis (SRA) techniques to determine structural failure probabilities differ from the other techniques and input to typical QRAs. Primarily, structural reliability analysis uses probability distributions to model the uncertainty in the basic engineering variables influencing the problem in order to synthesize the probability of component or system failure. Statistics and probability can be confused. Probability applies to events that have happened, may be occurring, or may yet occur. Statistics, on the other hand, applies only to events that have happened.

The results of structural reliability analysis are often combined with failure rates for process operations in order to assess the failure probability of complete systems; structural failure is therefore only part of the total failure probability.

The important point is that the risk analysis and structural reliability analysis are not fundamentally different and it is important that in future these techniques become fully integrated.

The objective of structural reliability analysis is to determine the probability of an ***event*** *occurring during a* ***specified reference period****.*

There are two following points to remark.

a) The first one is that the probability refers to the occurrence of an event. Normally, the events are defined in terms of the overtaken of a criterion or limit state, that means the failure of a component or system, P_f. They may also be defined in terms of non-failure or safety, i.e. $1\text{-}P_f$. For structural analysis, limit states are generally defined for ultimate failure, but also other limit states may also be defined, including serviceability or operability criteria. A failure event may refer to the:

- Failure of a component in a particular failure mode (component may be a single structural item, but also a complete pipeline system or pressure vessel may be treated a single component);
- Failure of a component from any of number of specified failure modes;
- Failure a group or system of components in a particular failure modes;
- Sequential failure of a number of components;
- Failure of a complete structural systems (a system in reliability terminology is the combination of a number of individual failure events for components and/or failure modes; failure events may be combined in series or parallel).

b) The second point is that the failure probability relates to an event occurring within a specified reference period. Where comparisons with failure rates for other types of events or hazard are being considered the failure probability may be evaluated on an annual basis. It is particularly important to ascertain and understand the significance of the reference period when comparing evaluated probabilities with targets. Important to note is that probabilities evaluated using structural reliability techniques are often referred to as notional.

2.4 Structural reliability theory

A rigorous structural reliability assessment involves modeling all of the sources of uncertainty that may affect failure of the component or system [12]. This means modeling the fundamental quantities entering in problem, and also the uncertainties that arise from lack of knowledge and idealized modeling (terms referred to as basic variables). Basic variables include common engineering quantities: diameter, wall thickness, material and contents density, yield stress, maximum operating pressure, maximum operating temperature, corrosion rate, fatigue crack growth rate, etc.

The sources of uncertainty that are relevant to structural reliability analysis can be classified into two categories: those that are a function of the physical uncertainty or randomness (aleatoric uncertainties), and those that are a function of understanding or knowledge (epistemic uncertainties). These can be subdivided further, but this fact is not mentioned here.

Reliability is defined as:

$$reliability = 1.0 - P_f \tag{2}$$

where P_f is the probability of failure of an event.

Usually reliability is expressed in terms of a reliability index (or safety index):

$$\beta = \Phi^{-1}(1.0 - P_f) = -\Phi^{-1}(P_f) \tag{3}$$

where $\Phi^{-1}(z)$ is the inverse of the standard normal distribution function. The standard normal function has zero mean and unit standard deviation.

The probability of failure is defined mathematically as a multi-dimensional integral:

$$P_f = P[g \leq 0] = \int_{g \leq 0} f_x(x)dx \tag{4}$$

where g is the failure criterion or failure function (sometimes termed limit state function or performance function) for the event, and $f_x(x)$ is the probability density function for the basic variables, X. A particular realization of the failure function, g, for a particular structure, is termed the margin of safety.

The simplest failure function is of the form:

$$g = rezist. - load \tag{5}$$

In this case the failure occurs when $g \leq 0$.

If the uncertainty in the resistance is modeled by a single variable ***R***, and the load by a single variable ***S***, and if the two variables are independent (uncorrelated), then the joint probability density function of the basic variables can be expressed as:

$$f_x(x) = f_{R,S}(r,s) = f_R(r) f_S(s) \tag{6}$$

In some reliability texts and papers the probability density function for the load and resistance are illustrated together on a single axis (Figure 1).
Since the variables *r* and *s* are independent, Equation 4 can be written as a double integral.

$$P_f = \int_{g \le 0} f_x(x)dx = \int_{-\infty}^{\infty} \int_{-\infty}^{S \ge 0} f_R(r) f_S(s) dr ds \quad (7)$$

Considering that the cumulative distribution function for a variable is given by:

$$F_X(x) = P[X \le x] = \int_{-\infty}^{x} f_x(y)dy \quad (8)$$

then Equation 7 can be expressed as a single integral, or convolution integral:

$$P_f = P[R - S \le 0] = \int_{-\infty}^{\infty} F_R(y) f_S(y) dy \quad (9)$$

or

$$P_f = P[R - S \le 0] = \int_{-\infty}^{\infty} f_R(y)[1 - F_S(y)]dy \quad (10)$$

The integrand in Equation 9 or 10 is illustrated in Figure 1 and the shaded area below the curve, represents the failure probability. The mean safety margin in this simple case is the difference between the mean resistance and the mean load. The design safety margin is less than the mean safety margin because of the use of characteristic or nominal resistance parameters (based on lower fractiles) and characteristic or nominal loading parameters (based on upper fractiles), and by the use of partial factors which usually reduce the nominal resistance and increase the nominal load. The ratio of the mean safety margin to the design safety margin is the factor of safety of the design.
Figure 2 gives an alternative representation of Figure 1, in which each variables is plotted on a separate axis. The two basic variables are represented on the horizontal axes and the probability density is represented as the vertical axis. The joint probability density function is shown as a “hill”. The distributions (the uncertainty in basic variables) are shown on the “walls” of the plot. The probability of failure is represented as the volume of the “hill”, which is within the failure region defined by the failure function less than zero.
Generally, when there are more than two basic random variables, the figure should be considered in multi-dimensional hyperspace.

3. PROBABILITY ANALYSIS

As already was mentioned the structural reliability and failure probability should always be defined for a specified reference period of time. There are two classes of time dependent problems which are generally considered:

- overload failure;
- cumulative failure.

The analysis of overload failure can be greatly simplified if time-varying resistance effects (i.e. fatigue and corrosion) are being ignored. In this case the failure of structure or structural component is going to occur under the maximum load effect during the reference period or period of exposure. By treating the loading in this way the analysis is termed *time invariant reliability analysis*.

In the case of cumulative failure, i.e. due to fatigue, corrosion, etc. , the total history of the load up to the point in question is of importance. Failure may occur solely as the results of cumulative loading, e.g. the formatting of a through-thickness crack due to cyclic failure.

The structural reliability analysis for various failure modes requires an accurate model to predict the failure. In present application the failure model is derived from the existing deterministic model for crack growth rate (Paris law), and thermal fatigue damage due to sin-wave temperature loading and non-uniform stress gradient through wall-thickness [6].

As already discussed above, the probability of failure is defined:

$$P_f = P[g \le 0] = \int_{g \le 0} f_x(x)dx \qquad (4')$$

In some cases this equation can be integrated analytically. In principle, the probability of failure or reliability can be evaluated using numerical integration (trapezoidal rule, Simpson’s rule, etc.). In practice, this is not generally practical in structural reliability because of the number of dimensions of the problem – one dimension for each basic variable –, and because the area of interest is generally located in the tails of the distributions. There are a number of other more commonly used methods available for estimating the failure probability, and four are more often used:

- Monte Carlo simulation methods;
- Mean value estimates;

- First-order second-moment methods (FORM);
- Second-order methods (SORM).

In the present work we use the crude Monte Carlo simulation (MCS), that uses the fact that the failure probability integral can be interpreted as a mean value in a stochastic experiment [13], [14]. An estimate is therefore given by averaging a suitable large number of independent outcomes (simulations) of this experiment. MCS offers a direct method for estimating the failure probability. In essence, the technique involves sampling randomly a set of values of the basic variables from the probability density function, and evaluating the failure function for these values to see if failure occurs. By generating a large number of samples, or trials, the probability density function for failure is simulated, and the ratio of the number or trials leading to failure to the total number of trials tends to the exact probability of failure.

The drawback with crude Monte Carlo simulation is often the computational effort involved. To produce a reasonably accurate estimate of the failure probability at least $100/P_f$ trials are required. A number of techniques have been developed to reduce the number of samples required (variance reduction techniques), and in favorable circumstances they can be very efficient. These techniques include:

- importance sampling – modifying the sampling density function to ‘important regions” of the failure space;
- directional sampling – sampling along random vectors;
- adaptive sampling – successive updating of the sampling density function;
- axis orthogonal simulation – a semi-analytic technique.

These techniques can also be combined together. Knowledge of the failure region (for example from first-order methods) can be exploited to significantly improve the efficiency of Monte Carlo simulation by tailoring the sampling scheme to the particular situation.

Monte Carlo simulation rely on the use of the random numbers. A number of types of random number generator are available, including multiplicative congruence types, Fibonacci series, etc. It is important to realize that these types of generator produce pseudo-random numbers that form a long sequence of numbers which, although may be expected to pass all standard test for randomness, will eventually repeat. For most applications standard random number generators, often available as functions in software libraries, are acceptable. In present application we use function ***randn*** from the library of MATLAB Statistics Toolbox (which generates numbers from normal standard distribution).

The Monte Carlo methods are an easily applied tool. They can be used to produce ‘exact” answers to problems, and can be used to provide answers to problems that cannot be accurately modelled using first - or second-order methods. Such problems include load combinations problems and time-varying problems (as fatigue).

4. PROSPECTIVE STUDY FOR PROBABILISTIC ANALYSIS APPLIED TO CIVAUX 1 CASE

In the following sections an example of the application of probability analysis on a case study - Civaux 1 thermal fatigue damage – is illustrated. A description of the Civaux I thermal fatigue damage case is given in the reference [15] and a short summary is given below.

4.1 Case description

In 1998 a longitudinal crack was discovered at outer edge of an elbow in a mixing zone of the Residual Heat Removal System (RHRS) of the Civaux NPP unit 1. It is worth mentioning that the time for crack development to a significant depth through the wall was only about ≈1500 hours. The system operated at a pressure of 36 bar, the hot leg contains water at 180^o C and the cold leg contains water at 20^{o}C. In the damage zone of interest the pipe inner radius was $r_i \cong$120 mm and outer radius was r_o =129 mm. Thermal and mechanical properties of austenitic steel (304L) at room temperature were referred as: specific heat coefficient c_{heat}=480 J/kg·C; thermal conductivity λ=14.7 W/m·C; mass density ρ=7800 kg/m^3; mean thermal expansion α=16.4·10^{-6} C^{-1}; Young’s modulus E=177·10^9 Pa; Poison’s coefficient ν=0.3; thermal diffusivity coefficient κ=3.93·10^{-6} m^2/s.

The temperature fluctuation was reported to be in the range 20-180^o C, and at the inner surface of the pipe the maximum temperature fluctuation range was estimated to be 120^{o}C. In the scope of the present work we consider a simple model for pipe as a hollow cylinder having geometrical and material characteristics similar to those mentioned before for Civaux 1 case. The transient temperature loading is considered in the sin-wave form time dependence, which acts at inner surface of the cylinder, of amplitude θ_0=60 °C.

A deterministic study based on SIN-method has been developed [4], [6] and analytical solutions for thermal and associated thermal stress were used to assess the crack initiation and crack growth in a hollow cylinder. The main parameters in deterministic analysis, which we also use in the probabilistic approach, are: pipe geometry, thermal loading, critical frequencies, Paris law.

Pipe geometry: inner radius r_i=0.120 m; outer radius r_o=0.129 m

Thermal loading:

Inner surface: $q(t) = 60 \cdot \sin(2\pi \cdot f \cdot t)$. (11)

Outer surface: adiabatic

Critical frequency (in terms of shortest life for fatigue crack growth**):**

- f=0.4 *Hz*, for long axial crack;
- f=0.*2 Hz,* for fully circumferential crack.

Paris law: Due to the only sinusoidal thermal loading applied at the inner surface of the pipe, the fluctuation of K_I is symmetric in positive and negative variation, and the fluctuation of K increases with the increasing of the crack depth (*a)*, [4],[6]. The crack growth rate is given by the Paris law relation which is generally of form

$$\frac{da}{dN} = C \cdot (\Delta K_I)^m \tag{12}$$

where m is the slope of the *log(da/dN)* versus *log(*ΔK_I*)* and C is a scaling parameter.

For crack growth assessment, ΔK_I has been replaced with the maximum stress intensity factor range, $\Delta K_{max}=K_{Imax}-K_{Imin}$, and further when K_{Imin} was less than zero, the value $K_{Imin}=0$ was chosen. By replacing ΔK_I with $\Delta K_{max}=K_{Imax}$, we consider a function which describes the envelope of the maximum values for K_I on crack depth, a, from 0 to 80% of wall-thickness (l):

$$K_{I.\max} = K_{I.\max}(a). \tag{13}$$

Finally, the form of the Paris law used in present approach is:

$$\frac{da}{dN} = C \cdot (K_{I.\max}(a))^m . \tag{14}$$

The number of cycles N_{cycles} required for a crack to advance between a_i (initial crack depth) and a_f (final crack depth) is

$$N_{cycles} = \int_{a_i}^{a_f} \frac{da}{C \cdot (K_{I.\max}(a))^m} . \tag{15}$$

The fatigue crack growth rates are in the units of mm/cycle with K_I in units of MPa√m. The threshold stress intensity factor range for this steel was assumed to be ΔK_{th} =5.0 MPa√m.

In previous works [4],[6], the $K_{I.\max}(a)$ dependence for two types of cracks have been determined as follow:

a) long axial crack at the inner surface of the cylinder under hoop thermal stress (f=0.4 Hz)

$$K_{I.\max}(a) = 5.17\cdot 10^7 \cdot a^3 - 1.07\cdot 10^5 \cdot a^2 + 3.71\cdot 10^2 \cdot a + 9.73 \quad (16)$$

b) fully circumferential crack at the inner surface of the cylinder under axial thermal stress (f=0.2 Hz, fixed end boundary for the cylinder)

$$K_{I.\max}(a) = 3.22\cdot 10^7 \cdot a^3 - 4.43\cdot 10^5 \cdot a^2 + 3.68\cdot 10^3 \cdot a + 8.35\,. \quad (17)$$

The information provided above are used in each Monte Carlo trial, in a manner which are specified in the following sections.

4.2 Failure mode and function

The present work on probabilistic approach of thermal fatigue crack growth depends on the following basic elements:

- establishment of the limit states to be considered;
- identification of the failure modes that could lead to the limit states;
- construction of the limit state functions;
- data analysis and the construction of appropriate probability density functions;
- evaluation of failure probabilities;

The damage due to thermal fatigue in mixing tees is characterized by a high gradient of thermal stress through the wall-thickness and as a consequence "long" shallow cracks will appear at the pipe inner surface. During a thermal loading at the inner surface, some of these cracks could penetrate the wall-thickness during the thermal loading, or also can arrest. Leakage occurs with through-wall crack formation, most probable after coalescence of long cracks. Therefore, a possibility is to perform fatigue crack assessment considering a single equivalent crack with large crack shape ratio [8]. In the present work we consider two type of cracks: long axial crack and fully circumferential crack at the inner surface of the pipe.

This probabilistic approach considers as limit state of thermal fatigue damage due to sinusoidal thermal loading a crack penetration depth of 80% wall thickness. Thus, it is possible to define the failure function or limit state function as function of number of thermal fatigue cycles, N_{cycles}, as

$$g\left(N_{cycles}\right) = a_{cr} - a_f\left(N_{cycles}\right) \tag{18}$$

or equivalent

$$g(N_{cycles}) = 1 - \frac{a_f(N_{cycles})}{a_{cr}} \tag{19}$$

where a_{cr} is a critical depth of the fatigue crack, corresponding to 80% of the wall-thickness, and $a_f(N_{cycles})$ is the final crack depth after N cycles of thermal loading. The failure is predicted when the number of cycles N_{cycles}, will produce the following condition

$$g(N_{cycles}) \leq 0 \tag{20}$$

which means $a_f > a_{cr}$., failure condition. We should note that in the present study, we consider long cracks; as a consequence the limit state function is referred just to the crack depth.

During the Monte Carlo simulation (MCS), the trials which satisfy condition (20) are accounted as n_{fail} and the probability of failure for a certain period of time is given by

$$P_f = \frac{n_{fail}}{N_{trials}} \tag{21}$$

where N_{trials} is the total number of trials simulation.

The limit state function, defined in Equations 18 or 19 contains the final crack depth, $a_f(N_{cycles})$, which is obtained using the Paris law crack growth rate, in connection with a specified period of time (*Time*). To specify that N cycle, time is fixed in this analysis. The number of fatigue cycles (N_{cycles}) required to reach the final crack depth, $a_f(N_{cycles})$, depends also on the loading frequency *(f)*, as :

$$N_{cycles} = Time \times f \ . \tag{22}$$

The frequency is fixed in this case by the sinusoidal load applied with 0.4Hz and 0.2Hz respectively.

The crack growth could be calculated on a cycle-by-cycle basis or as a group of *N'* identical cycles. The crack size after the block of *N'* cycles is given by

$$a_{after} = a_{before} + N' \cdot \left[C \cdot \left(K_{I.\max}(a_{before}) \right)^m \right]. \tag{23}$$

N' is also referred to as the "blocking factor", and the case of cycle-by-cycle crack growth corresponding to *N'*=1 is used in the present work. As already mentioned the crack length has not been considered, because just long axial crack and fully circumferential crack (equations 16 and 17) are considered in the present prospective study. Figure 3 shows a flow chart of probabilistic approach used in the present study based on crude MCS and limit state function from Equation (19).

The probabilistic variables used in this study were: the starting crack depth, a_i, the Paris law parameter *C* for a fixed m, while the deterministic variables were the thermal load at the inner surface given a certain loading frequency, the correlated stress distributions across the wall thickness, the K solutions for varying crack depth and a fixed number of cycle or life time for evaluating the failure criteria.

4.3 Input parameters and distributions

The distributions used in the paper to model parameters involved in thermal fatigue crack growth are given in Appendix, see also [16].

During the Monte Carlo simulation it is necessary to model non-standard distributions in a standard manner via a transformation and involving a standard normal distribution in the following way as in Ref. [17].

Considering the cumulative form *F(x)* of the distribution concerned in the following identity:

$$F(x) = \Phi(u) \, , \tag{24}$$

where x is the variable concerned (i.e. initial crack depth a_i or scaling parameter *C* from Paris law), and $\Phi(u)$ is CDF (cumulative distribution function) for standard normal distribution. The inverse of this is found:

$$x = F^{-1}(\Phi(u)) \; . \tag{25}$$

and *u* is standard normally distributed variable [18] or uniform random number [19]. This has the effect of forcing the variable *x* to adopt the required probability density function.

Crack size distribution

The initial crack size distribution has a very strong influence on the deterministic and also probabilistic assessment of a component lifetime. Usually, the initial crack distribution involves three kinds of distributions:

- crack depth distribution;
- crack aspect ratio distribution;

- crack existence frequencies.

The present approach considers only cracks that start out as long inner surface cracks, characterized by initial crack depth distribution as an exponential distribution [20]. The corresponding probability density function (pdf) is

$$f(x;\mu)=\frac{1}{\mu}e^{-\frac{x}{\mu}}, \qquad x\geq 0;\ \mu>0 \tag{26}$$

In present work we adopted the pdf from equation (26) in the following form

$$p(a;\mu_a)=\frac{1}{\mu_a}e^{-\frac{a}{\mu_a}} \qquad 0\leq a\leq a_0 \tag{27}$$

For a pipe thickness l=9 mm as in the present work, we consider a mean value of the crack depth as

$$\mu_a=1\ \text{mm} \tag{28}$$

and the coefficient of variation in this case is CoV=1. The mean value of the crack depth is small, but in deterministic assessments this value is generally assumed as a started depth for fatigue crack growth. The proposed value to be used for a_0 in Eq.(27), which usually is considered ∞ in the case of thick pipe-wall, can be seen as cracks detected by ISI, before assessment. Therefore, in this application, a value of 3 mm is chosen, which means about 30% of the wall pipe. In this way the cracks with depth bigger than 3 mm, generated by MC sampling from exponential distribution, are not accounted for crack growing.

The exponential distribution is used to produce random value for initial crack depth a_i, with the following sequence in MATLAB Statistics Toolbox [21]:

$$a_i=F^{-1}(\Phi(u);\mu_a);\mu_a=1mm; \tag{29}$$

where $F^{-1}(\Phi(u);\mu_a)$ is the MATLAB inverse function of exponential cumulative distribution function (CDF) and $\Phi(u)$ is CDF for standard normal distribution.

Paris law parameters distribution

For this study, the fatigue crack growth rates were calculated using stainless steel crack growth law given in ASME [22]:

$$\frac{da}{dN}=C\cdot(\Delta K_I)^m \tag{30}$$

where m=3.3 is the slope of the *log(da/dN)* versus *log(ΔK_I)* and C is a scaling parameter. Slopes (m) and intercepts (C) for all fatigue data are usually highly correlated. Ignoring this correlation can give misleading results in a simulation.

An alternate method to account for this correlation is to use a constant slope and put all of variability into the intercept. For a constant slope, the variability in fatigue lives will be directly related to variability in the material constant C. The scatter in the experimental fatigue data is represented by a lognormal distribution for C scaling parameter, with the following pdf

$$f(x;\mu_0,\sigma_0)=\frac{1}{x\cdot\sigma_0\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln x-\mu_0}{\sigma_0}\right)^2}, \tag{31}$$

where μ_0, σ_0, are parameters, calculated from the median/mean value and standard deviation as shown in Appendix.

From reference [8] we consider the following parameters:

Median: C_{median}=10.04·10^{-12} (m/cycle/ MPa√m) , (32)

Standard deviation: σ_C=2.2·10^{-11} . (33)

With these parameters a mean value is derived as C_{mean}= 1.664·10^{-11} and coefficient of variation by *CoV*=1.32. The log-normal distribution is used to produce random values for C constant in Paris law with the following sequence in MATLAB Statistics Toolbox:

$C = F^{-1}(\Phi(u);\mu_0,\sigma_0)$; μ_0= - 25.3244; σ_0=1.0053. (34)

Here $F^{-1}(\Phi(u);\mu_0,\sigma_0)$ is the MATLAB function for inverse function of log-normal CDF. The parameters μ_0 and σ_0 were derived with relationships from Appendix.

4.4 Analysis and discussion

The probabilistic approach of thermal fatigue crack growth by means of crude Monte Carlo simulation and based on the limit state function (failure function) follows the main steps from flow chart shown in Figure 3. To develop a prospective study for probabilistic fatigue approach, the Civaux 1 fatigue damage case is used, where a crack growth fatigue life of 1000 hours was estimated [6].

Procedure is summarized below.

Geometry:

- a pipe model with main parameters from Civaux 1 fatigue damage case and thermal loading as sin-wave time dependence with fixed frequency at the inner pipe surface is assumed;

Defects:

- long shallow defects exist at the inner surface of the pipe (axial or circumferential), and after growing through the wall-thickness they are not coalesced with the other;
- the defects originated at the inner surface grow only due to the thermal stresses arising from sinusoidal thermal loading applied at the inner surface;
- it is assumed that the initial crack depth distribution follows the exponential distribution; none crack shape distributions are considered;

Crack growth:

- the crack growth rate is given by Paris law, and dependence on crack depth, $\Delta K_{max} = K_{Imax}(a)$, have been obtained for long axial and circumferential cracks under associated elastic thermal stresses in previous works;
- the scatter in the experimental fatigue data is represented by a lognormal distribution for value of scaling parameter *C*; the slope *m* has a specified fixed value;

Method:

- the crude Monte Carlo method is used in simulation (see flow chart in Figure 3);
- the procedure uses the limit state function approach to obtain probability of failure, P_f, for a specified fixed period of time (or for a defined fatigue life);
- the empirical cumulative density function approach is used to benchmark results for P_f.

The specific routines were implemented to perform the trials in concordance with the sequences from the flaw chart shown in Figure 3, using Statistics Toolbox from MATLAB software.

For a MC sampling simulation of 10^4-10^5 trials, a typical pdf histogram for initial crack depth distribution is shown in Figure 4, which is based on the hypotheses of a mean value of the crack μ_a= 1mm, and that flaws grater then 3 mm don't exist. Note that these hypotheses are not based on the real ISI measurements, but they are useful for a prospective analysis. As can be seen, it approximates quite well the shape of known theoretical exponential pdf. In the case of Paris law *C* scaling parameter the log-normal pdf histogram is shown in Figure 5, with modeling parameters already mentioned.

The result of MC simulations gives the probabilities associated with crack growth lives for specified periods of time (fatigue lives), for both axial and circumferential crack. In order to check the probabilities failure predictions obtained by means of limit state function, we use the empirical cumulative distribution (ECD) (see Appendix) to represent an output from the Monte Carlo simulation in sense of cumulative probability of failure, P_f. Thus, it is firstly derived P_f and Figure 6 compares these ECDs for axial and circumferential crack as function of fatigue lives in hours. Dependences have typical shape for both directions and slightly higher P_f predictions are obtained for circumferential crack up to fatigue lives of 200 hours.

In the case of probabilistic fatigue assessment for axial crack by means of failure function and MCS, the sinusoidal thermal loading with frequency f=0.4 Hz has been considered. We have to recall that this frequency was found to be the critical frequency in term of shortest life for long axial crack [6]. By using Paris law within MCS trials with $K_{I.\max}(a)$ dependence from equation (16) and failure function $g(N_{cycles})$ given by equation (19) the results are plotted in Figure 7. A good agreement is found with predictions from ECD for whole range of fatigue lives.

In the case of probabilistic fatigue assessment for circumferential crack, a similar estimation as before, but with f=0.2 Hz and $K_{I.\max}(a)$ dependence from equation (17), has been done. The comparison with predictions from ECD is shown in Figure 8.

Figures 7 and 8 show that the crack growth lifetime of 1000 hours has a very high probability to occur, although only thermal stresses have been used in the assessment.

An important task is to estimate cumulative distribution functions for fatigue lives after finding probabilities failure associated with corresponding fatigue lives. The following characteristics of fatigue life distributions were estimated, by means of the specific MATLAB functions from Statistics Toolbox: probability density function, associated mean value of life and coefficient of variation (CoV). The log-normal distribution has been found to best fit the results from failure function approach with MCS, for both cases. The results of fitting are summarized in the next.

Long axial crack growth (f=0.4 Hz):

- pdf: $f(x;\mu_0,\sigma_0)=\dfrac{1}{x\cdot\sigma_0\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln x-\mu_0}{\sigma_0}\right)^2}$; μ_0=4.63; σ_0=1.20; (35)
- mean=212 hours;
- CoV=1.83.

Fully circumferential crack growth (f=0.2 Hz):

- pdf: $f(x;\mu_0,\sigma_0)=\frac{1}{x\cdot\sigma_0\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln x-\mu_0}{\sigma_0}\right)^2}$; μ_0=4.52; σ_0=1.30; (36)

- mean=223 hours;

- CoV=2.13.

Figures 9 and 10 show a comparison between cumulative distribution functions estimated with parameters from Equations (35) and (36), and results from approach of limit state function. The agreement is quite good for both cases and we can conclude that fatigue lives in this case of fatigue crack growth follow a log-normal distribution. However, the parameters which define the mentioned pdf (Equations 35 and 36) were derived assuming some hypothesis (initial mean value of the initial crack depth, a constant slope *m*, sin-wave thermal loading, only thermal stresses, etc.). More refined probabilistic analysis needs to be performed in order to account for a realistic thermal loading spectrum, and also the variability of more specific parameters.

5. CONCLUSIONS

The present work performs a prospective study on probabilistic approach of thermal fatigue in mixing tees (Civaux 1 damage case) by means of the limit state function and Monte Carlo simulation. It is based on previous work where a deterministic assessment for thermal fatigue crack growth in high-cycle loadings, under the large nonlinear gradient stress profiles through wall-thickness due to sinusoidal thermal loading, has been done.

The probabilistic approach considers variability in initial crack depth and Paris law *C* scaling parameter by means of specific probability density distributions. A given sinusoidal load has been considered as first step approach, even though the real load represents the main variable, being the most difficult to be determined. In the case of first variable a hypothetic mean value of initial crack depth is adopted, and for second one values from literature are used. The crude Monte Carlo Simulations are performed using specific routines implemented in MATLAB software with Statistics Toolbox, and probabilities of failure are derived using the failure function which is defined based on a limit state given by the critical crack depth. The results were checked against predictions from empirical cumulative distribution and good agreements was found. An important task was to estimate distribution function for fatigue lives after finding probabilities failure associated with corresponding fatigue lives by means of the failure function approach. Using specific MATLAB functions from Statistics Toolbox, for both axial and circumferential cracks, then the pdf, associated mean value of the fatigue life and CoV have been estimated. The log-normal distribution has been found to best fit the results from failure function approach with MCS, for both cases.

The prospective study results will be used for a probabilistic approach of thermal fatigue in mixing tees (initiation and crack growth), based on a realistic thermal loading spectrum and mechanical loading, and also for the variability of more specific parameters.

GLOSSARY

Basic variables: A set of variables entering the failure function equation to define failure. They may include basic engineering parameters, such as wall thickness, yield stress, allowed crack depth etc., as well as model uncertainty in the failure function itself.

Beta-point: The point with maximum probability density, and values of the basic variables at this point represent the most probable values to cause failure.

CoV (Coefficient of Variation): The ratio of standard deviation to mean value of a variable:

$$CoV = \frac{\sigma}{\mu_X}.$$

Expected value, E[X]): The mean value of a variable. It is defined as first moment of the distribution function of a variable, and is evaluated from distribution function $f_X(x)$:

$$E[X] = \mu_X = \int_{-\infty}^{\infty} f_X(x)dx.$$

Failure function, g: The failure function (or limit state function) in reliability analysis is a mathematical function used to predict the failure event for a component, part of a structure, or a structural system. The failure function is expressed in terms of basic variables, and is defined such that $g \leq 0$ correspond to failure.

Model uncertainty: The inherent uncertainty associated with the mathematical models used to predict resistance (and loading).

Probability of failure, P_f: The probability of failure of an event is the probability that the limit state criterion or failure function defining the event will be exceeded in a specified reference period.

Probability density function, pdf: The probability that a random variable X shall appear in the interval $[x, x+dx]$ is $f_X(x)dx$ where $f_X(x)$ is the probability density.

Reference period: Reliabilities and probabilities of failure should be defined in terms of a reference period, which may typically be one year of the design life.

Reliability: The probability that a component will fulfill its design purposes. Defined as $1-P_f$.

Reliability index, β: A useful measure to compare P_f*s.* it is defined using the standard normal distribution function *$\Phi(u)$,*

$$\beta = \Phi^{-1}\left(1 - P_f\right) = -\Phi^{-1}\left(P_f\right).$$

Standard deviation, Sd[X] or σ: The standard deviation is defined as square root of the Variance of a variable:

$$\sigma = \sqrt{Var(X)}$$

Standard normal space, U-space: A space of independent normally distributed random variable with zero mean and unit standard deviation. Basic variable space is transformed into standard normal space in some reliability analysis procedures (FORM, SORM).

Variance, Var[X]: The variance of a variable is defined as the second central moment of the distribution function of a variable, and is evaluated from the distribution function $f_X(x)$:

$$Var\left[X\right] = \int_{-\infty}^{\infty} \left(x - \mu_X(x)\right)^2 f_X(x)dx\ ,$$

where μ_X is the mean or expected value.

APPENDIX: THE CDFS AND PDFS USED FOR PROBABILISTIC FATIGUE APPROACH

a) Normal distribution (Gauss distribution)

Probability density function (pdf):

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \qquad -\infty<x<\infty; \quad -\infty<\mu<\infty; \; \sigma^2>0. \tag{A1}$$

Cumulative distribution function (CDF):

$$F(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right]dt \tag{A2}$$

Note: For $\mu=0$ and $\sigma=1$ we refer to this distribution as standard normal distribution

$$f(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2} \tag{A3}$$

which has the cumulative distribution (CFD)

$$\Phi(u)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u}\exp\left[-\frac{t^2}{2}\right]dt \tag{A4}$$

Moments:

mean (expected value):

$$E(X)=\mu_X=\mu\,. \tag{A5}$$

variance:

$$Var(X)=\sigma^2 \tag{A6}$$

standard deviation:

$$\sigma=\sqrt{Var(X)} \tag{A7}$$

b) Exponential distribution

Probability density function (pdf):

$$f(x;\mu)=\frac{1}{\mu}e^{-\frac{x}{\mu}} \qquad x\geq 0; \; \mu>0 \tag{A8}$$

Cumulative distribution function (CDF):

$$F(x;\mu)=1-e^{-\frac{x}{\mu}}\,. \tag{A9}$$

Moments:

mean (expected value):

$$E(X) = \mu_X = \mu \ . \tag{A10}$$

variance:

$$Var(X) = \mu^2 \tag{A11}$$

standard deviation:

$$\sigma = \sqrt{Var(X)} = \mu \ . \tag{A12}$$

The exponential (Marshal) distribution is used to produce random value for initial crack depth a_i.

c) Log-normal distribution

Probability density function (pdf):

$$f(x;\mu_0,\sigma_0) = \frac{1}{x \cdot \sigma_0 \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln x - \mu_0}{\sigma_0}\right)^2} \tag{A13}$$

Cumulative distribution function (CDF):

$$F(x;\mu_0,\sigma_0) = \frac{1}{\sigma_0\sqrt{2\pi}} \int_0^x \frac{1}{t} \cdot \exp\left[- \frac{(\ln(t) - \mu_0)^2}{2\sigma_0^2} \right] dt \tag{A14}$$

with μ_0, σ_0, parameters.

Moments:

mean (expected value):

$$E(X) = \mu_X = e^{\mu_0 + \frac{\sigma_0^2}{2}} \tag{A15}$$

variance:

$$Var(X) = e^{2\mu_0 + \sigma_0^2}\left(e^{\sigma_0^2} - 1\right) \tag{A16}$$

standard deviation:

$$\sigma = \sqrt{Var(X)} \tag{A17}$$

median:

$$med = e^{\mu_0} \tag{A18}$$

The log-normal distribution is used to produce random value for C constant in Paris law and to fit fatigue lives distribution.

d) Empirical cumulative distribution

The cumulative distribution function is given by

$$F(x) = \int_{-\infty}^{x} f(t)dt \tag{A19}$$

for a continuum variable and by

$$F(x) = \sum_{y_i \le x} f(y_i) \tag{A20}$$

for a discrete random variable.

When is not suitable to assume a distribution for a random variable, then we can use a cumulative distribution function called the empirical distribution function, as an estimate of the underlying distribution. One can call this a nonparametric estimate of a distribution function, because it is not assuming a specific parametric form for the distribution that generates the random phenomena. In a parametric setting we would assume a particular distribution generated the sample and estimate the cumulative distribution function by estimating the appropriate parameters.

The empirical distribution function is based on the order statistics. The order statistics for a sample are obtained by putting the data in ascending order. Thus, for a random sample of size n, the order statistics are defined as

$$X_{(1)} \le X_{(2)} \le X_{(3)} \le X_{(4)} \le \ldots X_{(n)}, \tag{A21}$$

with $X_{(i)}$ denoting the i^{th} order statistic. The empirical distribution function $F_n(x)$ is defined as the number of data points less than or equal to x $(\#(X_i \le x))$ divided by the sample size n. It can be expressed in terms of the order statistics as follows

$$F_n(x) = \begin{cases} 0; & x < X_{(1)} \\ \dfrac{j}{n}; & X_{(j)} \le x \le X_{(j+1)} \\ 1; & x \ge X_{(n)} \end{cases} \tag{A22}$$

We use the empirical cumulative distribution (ECD) to represent an output from the Monte Carlo simulation in sense of cumulative probability of failure for estimated fatigue life derived by means of Paris law fatigue crack growth.

FIGURES

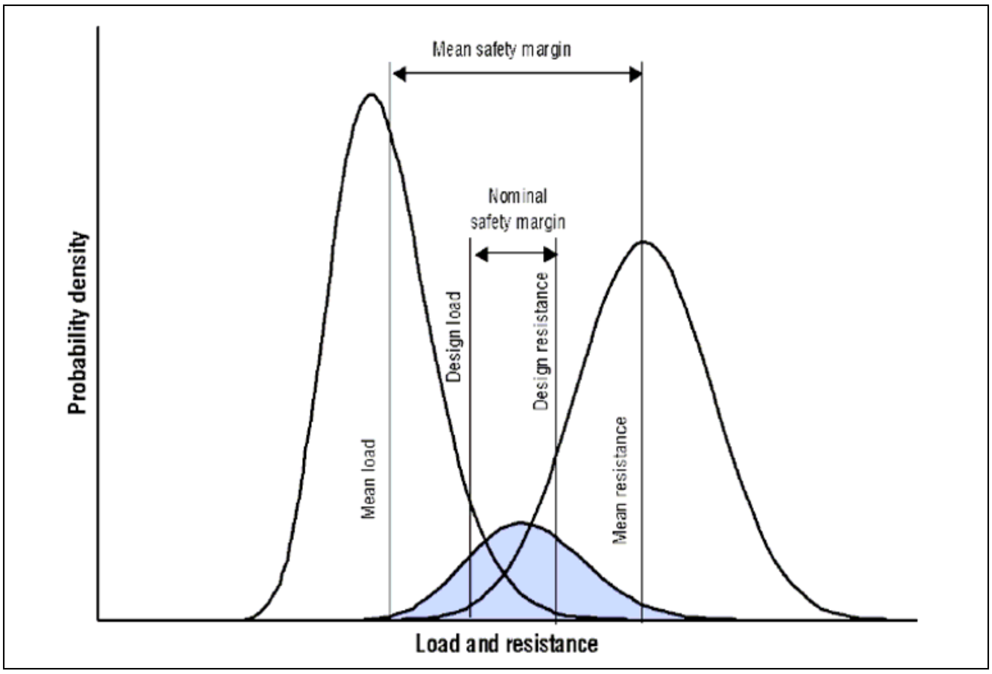

Figure 1. Conventional illustration of probability of failure

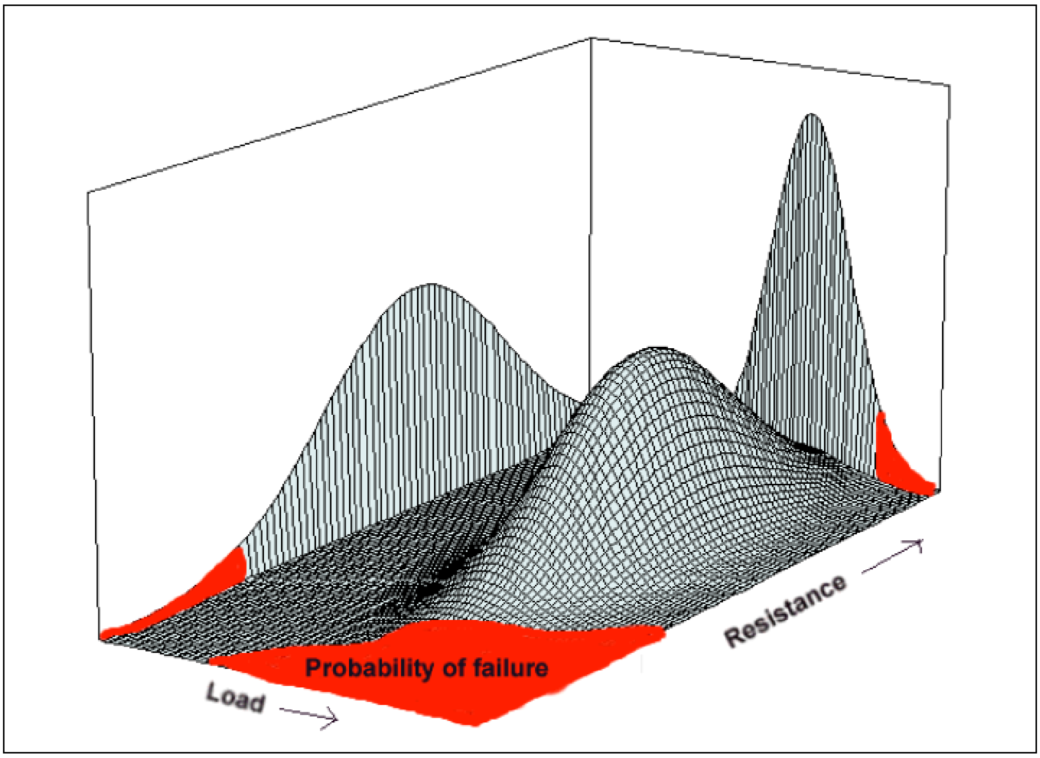

Figure 2. Conventional 3D geometric illustration of probability of failure

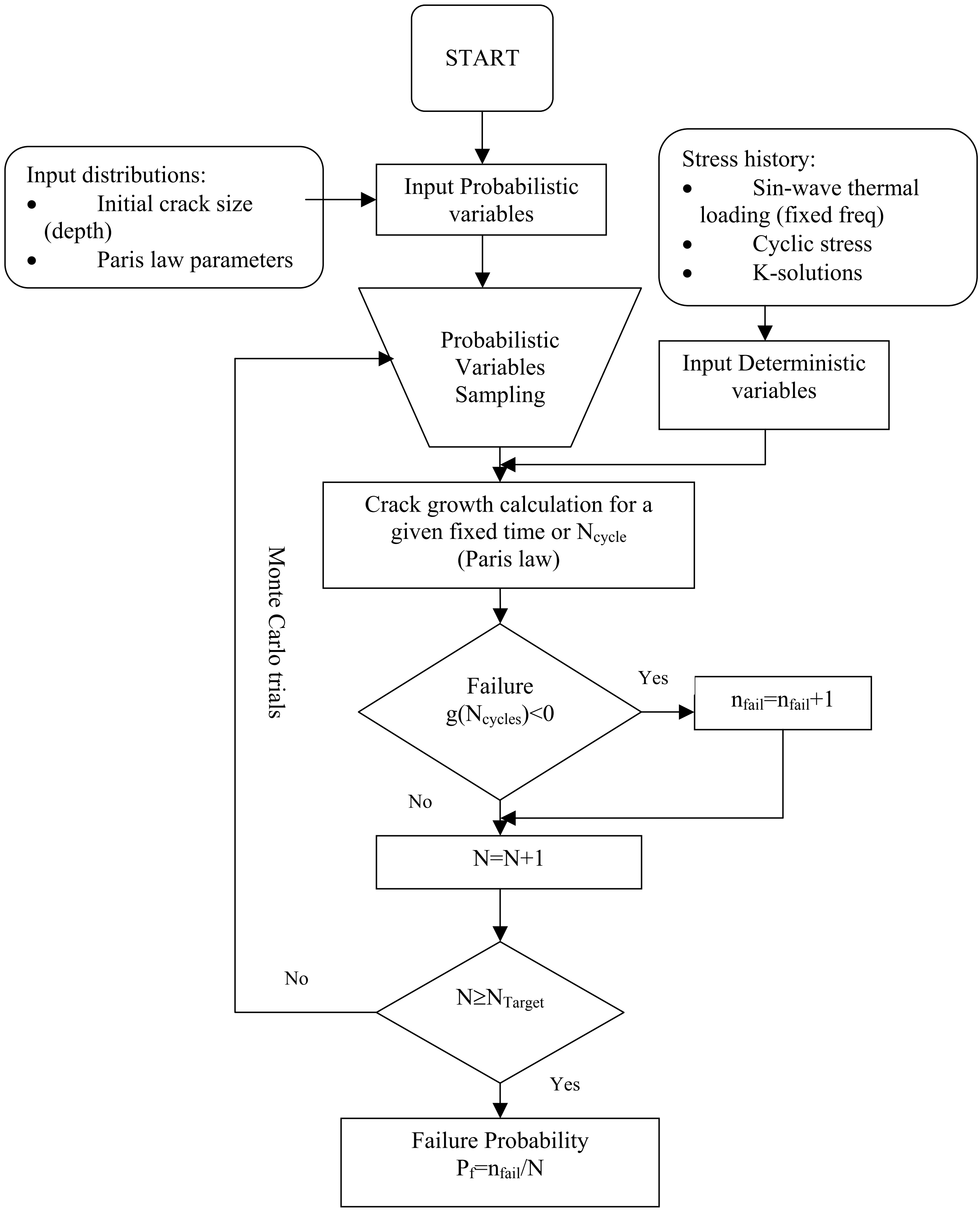

Figure 3. Flow chart of probabilistic approach of thermal fatigue crack growth by means of limit state function and MCS.

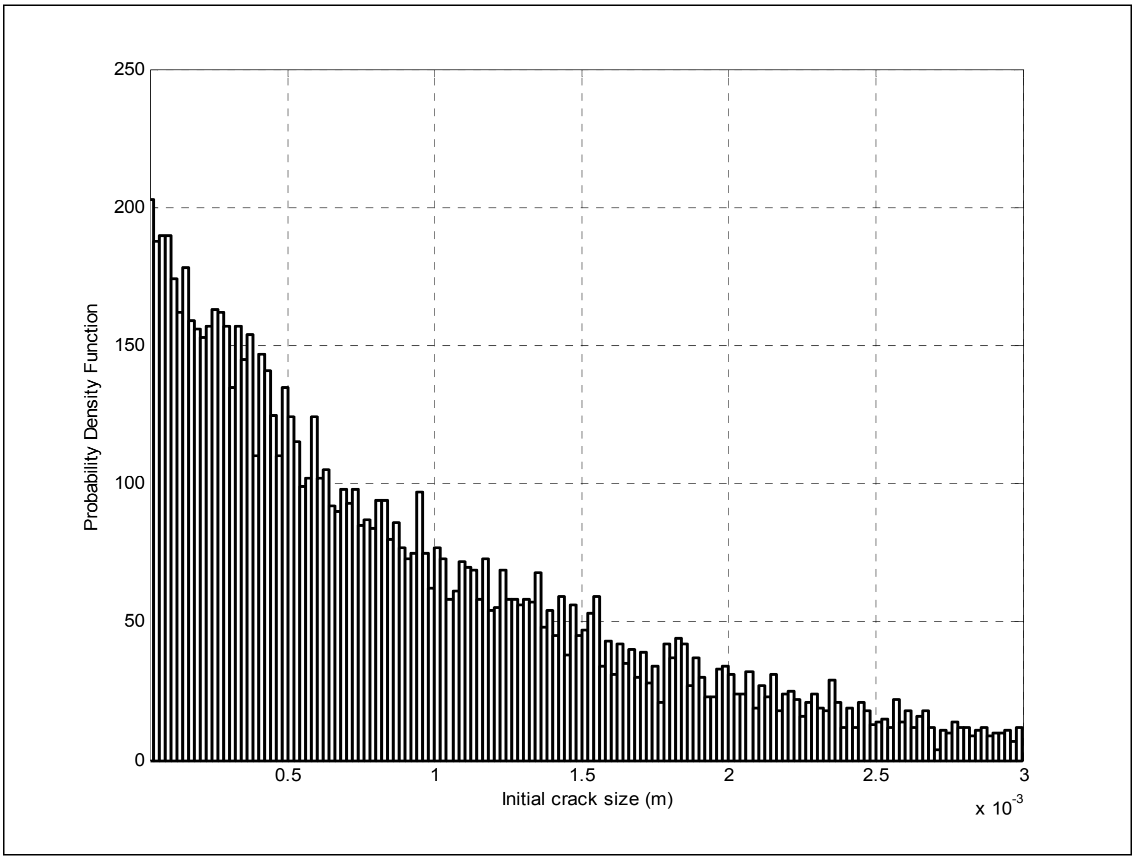

Figure 4. A typical pdf of initial crack depth on the inner surface generated by means of MCS

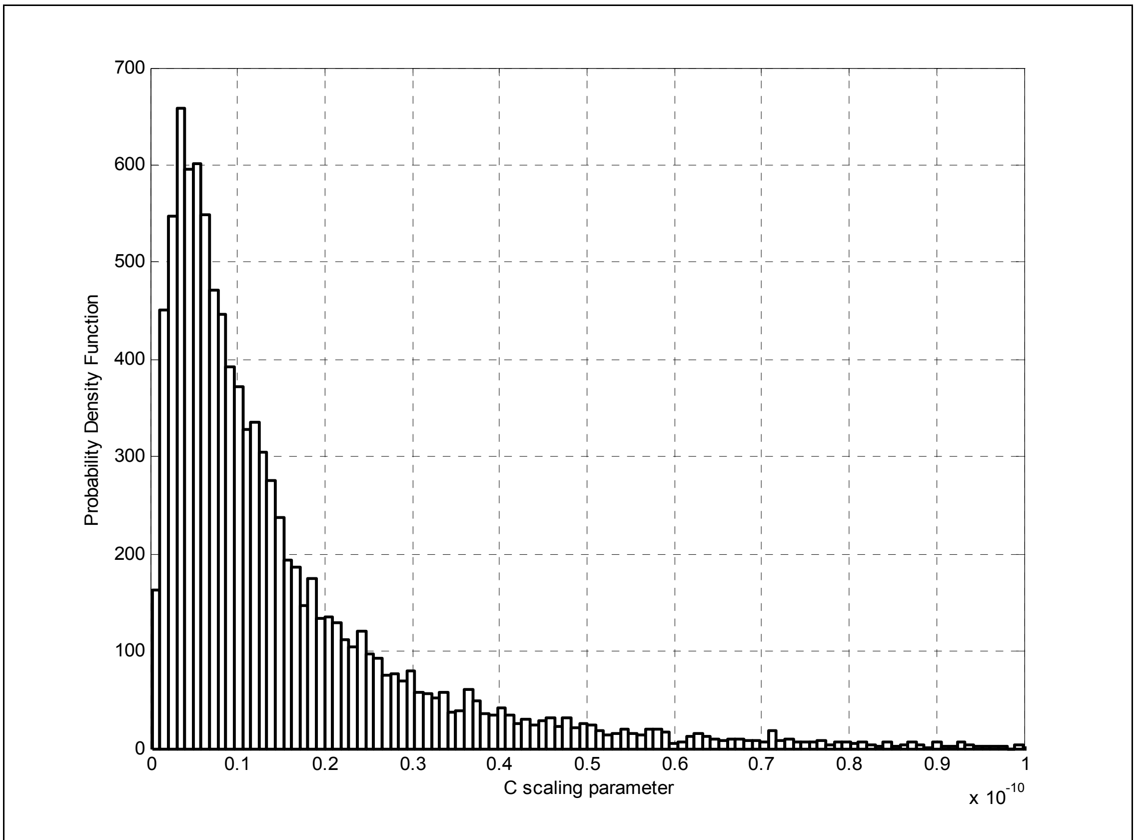

Figure 5. A typical pdf for Paris law *C* scaling parameter generated by means of MCS

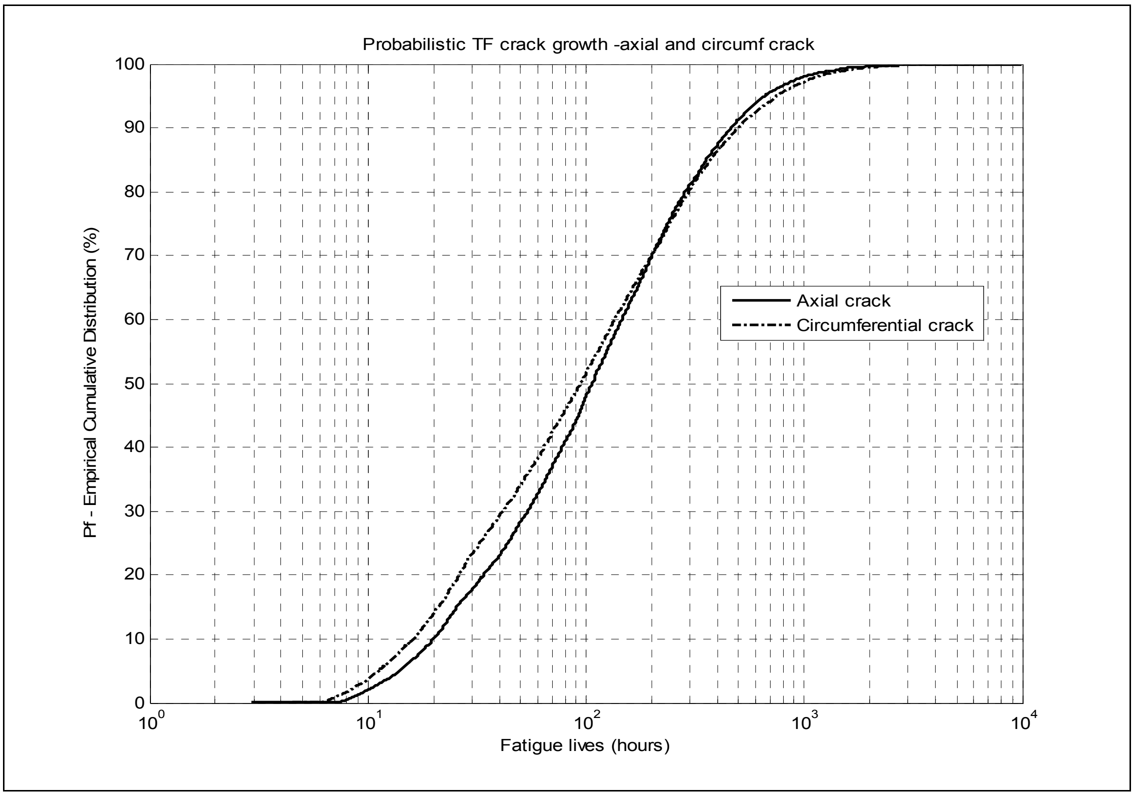

Figure 6. Empirical Cumulative Distribution (P_f) for thermal fatigue crack growth in case of long axial and fully circumferential cracks; sinusoidal thermal loading (Civaux 1 case)

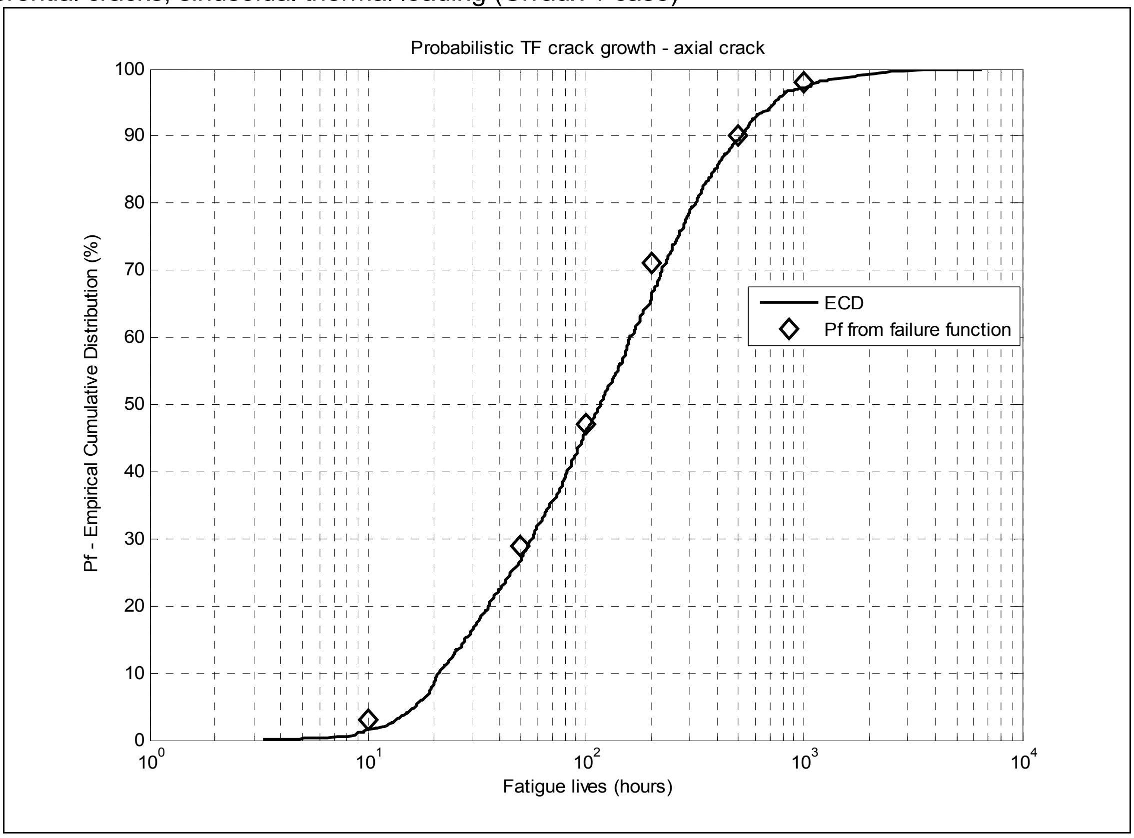

Figure 7. Comparison between P_f from ECD and those resulted from MCS with limit state function approach for long axial crack; sin-wave thermal loading (f=0.4 Hz)

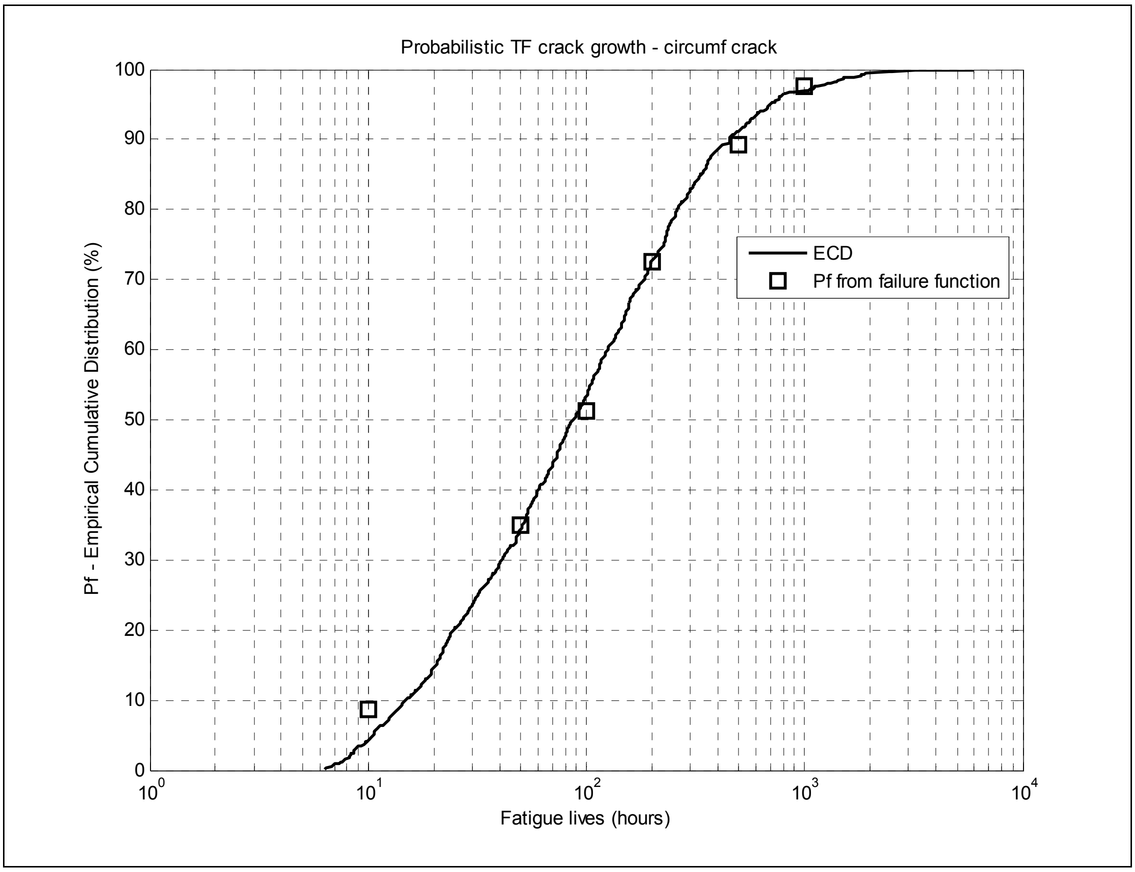

Figure 8. Comparison between P_f from ECD and those resulted from MCS with limit state function approach for fully circumferential crack; sin-wave thermal loading (f=0.2 Hz)

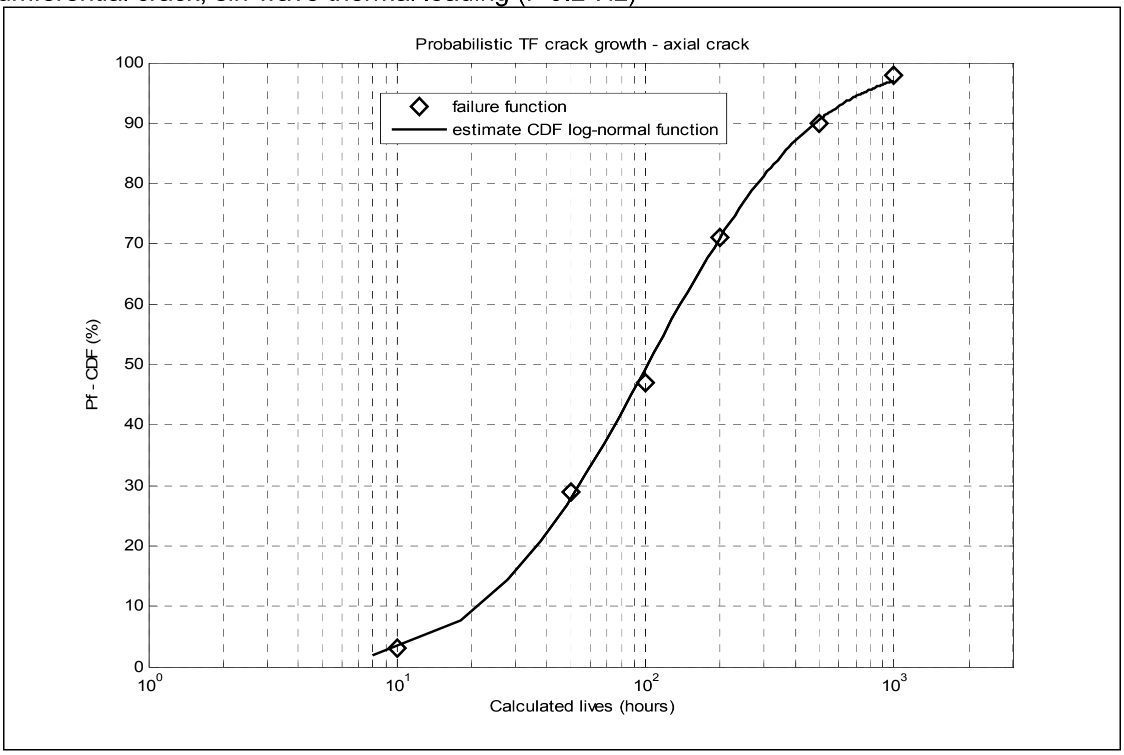

Figure 9. Comparison between P_f resulted from MCS with limit state function approach and estimated CDF as log-normal distribution for fatigue lives; long axial crack, sin-wave thermal loading (f=0.4 Hz)

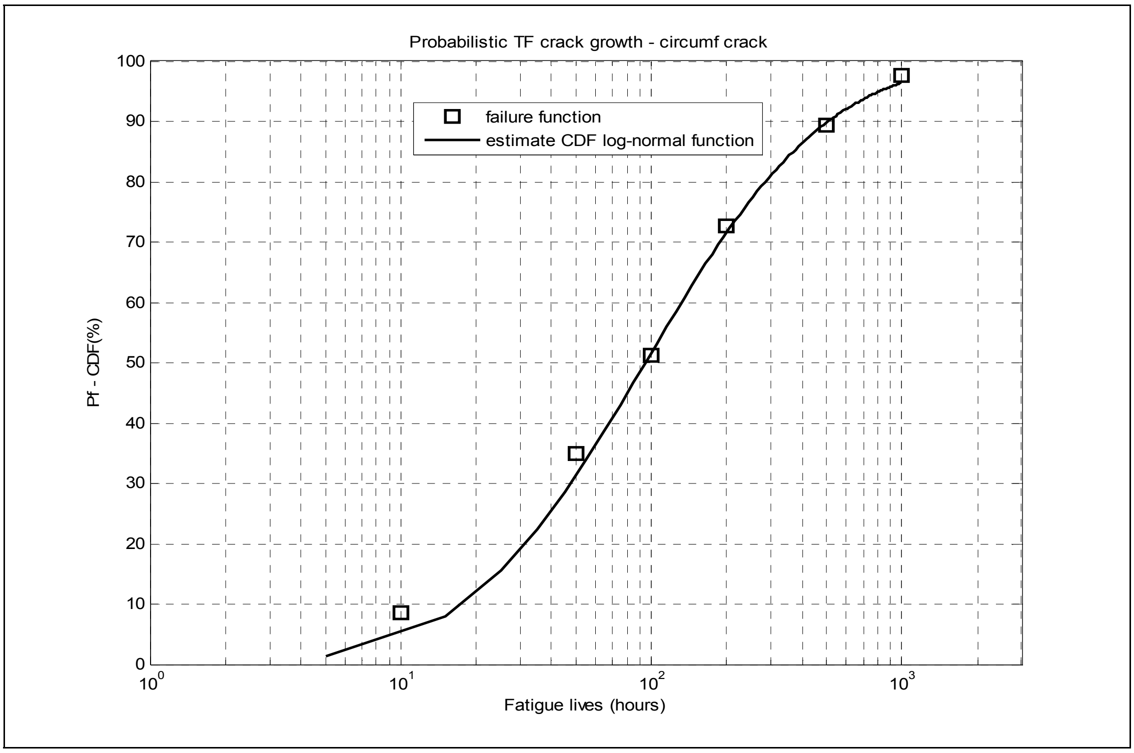

Figure 10. Comparison between P_f resulted from MCS with limit state function approach and estimated CDF as log-normal distribution for fatigue lives; fully circumferential crack, sin-wave thermal loading (f=0.2 Hz)

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Abstract
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