Original scientific paper

PASSIVE ATMOSPHERIC WATER HARVESTING UTILIZING AN ANCIENT CHINESE INK SLAB

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Abstract. *Extraction of atmospheric water using a passive mechanism instead of a complex and advanced equipment has become an emerging subject. There is a clear record in MengxiBitan by Shen Kuo(1031~1095) that an ink slab has the ability to collect water from the air. Its mechanism is exactly similar to the Fangzhu [1], a recently investigated device for atmospheric water harvesting (AWH). Based on the Fangzhu device, a mathematical model for the AWH mechanism in ink slab-like materials is suggested. Using He's frequency formulation and two-scale fractal derivatives the possible working mechanism of ink slab-like materials is investigated. The potential applications of ink slab-like structures for AWH in interior and exterior architecture are also presented and discussed. It is revealed that efficiency of the slabs highly depends on velocity and temperature of the flowing air and also its low-frequency characteristics.*

Key Words: *Nanotechnology, Chinese Civilization, MengxiBitan, Fangzhu, Fractal Oscillator, Two-scale Fractal Derivative*

1. INTRODUCTION

The freshwater inadequacy has become an alarming issue in the contemporary lives of humankind as well as flora and fauna. This issue escalates especially in the lands with low or none natural water accessibility. Furthermore, responding to an increasing demand for consumable water due to climate change and population growth has turned into a difficult task based on polluted water sources. Several water purification methods employed to overcome this matter, filtration [2], solar water purification [3], and reverse osmosis [4], are examples of wastewater and seawater refinement solutions. However, the mentioned strategies highly depend, firstly, on natural water sources such as lakes and rivers [5], and, secondly, on advanced equipment. As a promising alternative, researchers turned toward atmospheric water harvesting (AWH) methods to overcome this problem in arid regions. Earth's atmosphere contains approximately 50,000 cubic kilometers of water [6], this phenomenon introduced AWH as a sustainable water source. Fog/dew collection [7] mechanisms accelerate the growth of droplets, based on wettability engineering. However, dependency on saturated local humidity significantly restricts its applications in areas with dry environment. Moisture harvesting, on the other hand, has become a suitable alternative in AHW because of its independence of climate conditions since moisture is water vapor and is widespread across the earth [5]. The goal of this method is to liquefy the trapped moisture in the atmosphere by cooling the air below its dew point and subsequently condense the air [3]. However, this cooling procedure is intense energy costly. Based on the interaction between nanostructure and water molecules, passive AWH materials emerged as a novel alternative to address this problem. The capability of harvesting water in low and high relative humidity and low energy demand are the main advantages of these materials. Zhao and et al. [6] improved these materials by suggesting a super moisture-absorbent gel. However, the fundamental design principles of these materials are still mostly unknown [6]. The nanostructure of these materials can be obtained by carefully studying the water harvesting mechanism among desert animals and plants. Gurera and Bhushan [8] studied passive water harvesting mechanisms among desert species. Comanns [9] studied passive water collection in animal's integuments.

On the other hand, with the development of solar panels, off-grid and self-sufficient architecture has become an attractive subject to engineers. However, few studies focused on freshwater problems in rural and arid areas and potential solutions in architecture. Passive AWH materials can provide novel ideas in architectural engineering. This paper introduces an ancient ink slab, which was recorded in MengxiBitan (梦溪笔谈) by Shen Kuo(沈括)(1031~1095), for water collection from air. Shen wrote:

孙之翰，人尝与一砚，直三十千。孙曰："砚有何异而如此之价也？"客曰："砚以石润为贤, 此石呵之则水流。"孙曰："一日呵得一担水才直三钱，买此何用？"竟不受。

Literally, someone gave Sun Zhihan an ink slab as a gift, which was worth 30,000 dollars. "Has this ink slab anything special? Why is this gift so expensive?" Sun asked. "The ink slab is special for its stone moist; when you blow it, flowing water can be

collected." The giver replied. "Though the ink slab can collect a pail of water per day, it is only worthy of 3 dollars, so it is not worthy buying it." Sun said, and rejected it.

MengxiBitan was a collection of the most advanced technology and natural phenomena at his times; it is still perhaps the most prestigious and influential book on ancient science and technology, and natural phenomena in China. Sun Zhihan (998 ~ 1057) was an official in Huazhou, which is adjacent to Xi'an city, China.

The ink slab is a traditional tool for painting and writing, which plays an important role in Chinese civilization. The recorded ink slab was special for its water collection by blowing its surface. The fabrication technology has been lost for a long time; no one can re-build the ink slab so far. There is a similar ancient device for water collection from the air; it is called as Fangzhu, which was widely recorded in many ancient Chinese classics. A detailed discussion is given in ref. [1] and refs. [10-12]. Wang found that the nanoscale surface morphology of Fangzhu plays an important role in its water collection [10]. This paper shows that the material of Fangzhu has some similar properties as the modern metamaterials, which are famous for attenuation of sound and vibration [13]. The Fangzhu material has the low frequency property favorable for water transmission [14, 15]. First we suggested a working mechanism for ink slab material by explaining Fangzhu working principles. In the second and third sections, a mathematical model for water molecule vibration behavior is suggested and solved. Finally, potential applications of ink slab-like material in modern architecture are discussed in the last section.

2. INK SLAB AND FANGZHU MATERIAL

Fangzhu was an ancient device famous for its water-harvesting ability. If it is the fact that the recorded ink slab is actually a Fangzhu-like device, its water collection property can be easily revealed. According to ref. [1], Fangzhu is easy to collect water from the air when it is placed against a moving air flow. Fig. 1-a, illustrates a traditional Chinese ink slab under the warm air flow.

A key factor in attracting water molecules in these materials is a hydrophilic-hydrophobic surface. The Convex surface (super hydrophobic) generates surface potential duo to its curvature, thus easily attracting water molecules in the flowing air to its surface; the attracted water molecule then integrates into water droplets (Fig.1-b). On the contrary, the concave surface (super hydrophilic) merges water droplet and stores the extracted water. The recorded ink slab might be Fangzhu in an ink slab form (see Fig. 1-c). The record emphasizes the water collection property by blowing to its surface. According to ref. [1], Water Harvesting of Fangzhu highly depends on the flowing air velocity and temperature; higher temperature results in higher efficiency for water collection. When we blow on the surface of the ink slab, a higher velocity and a higher temperature enable the ink slab to work much more efficiently.
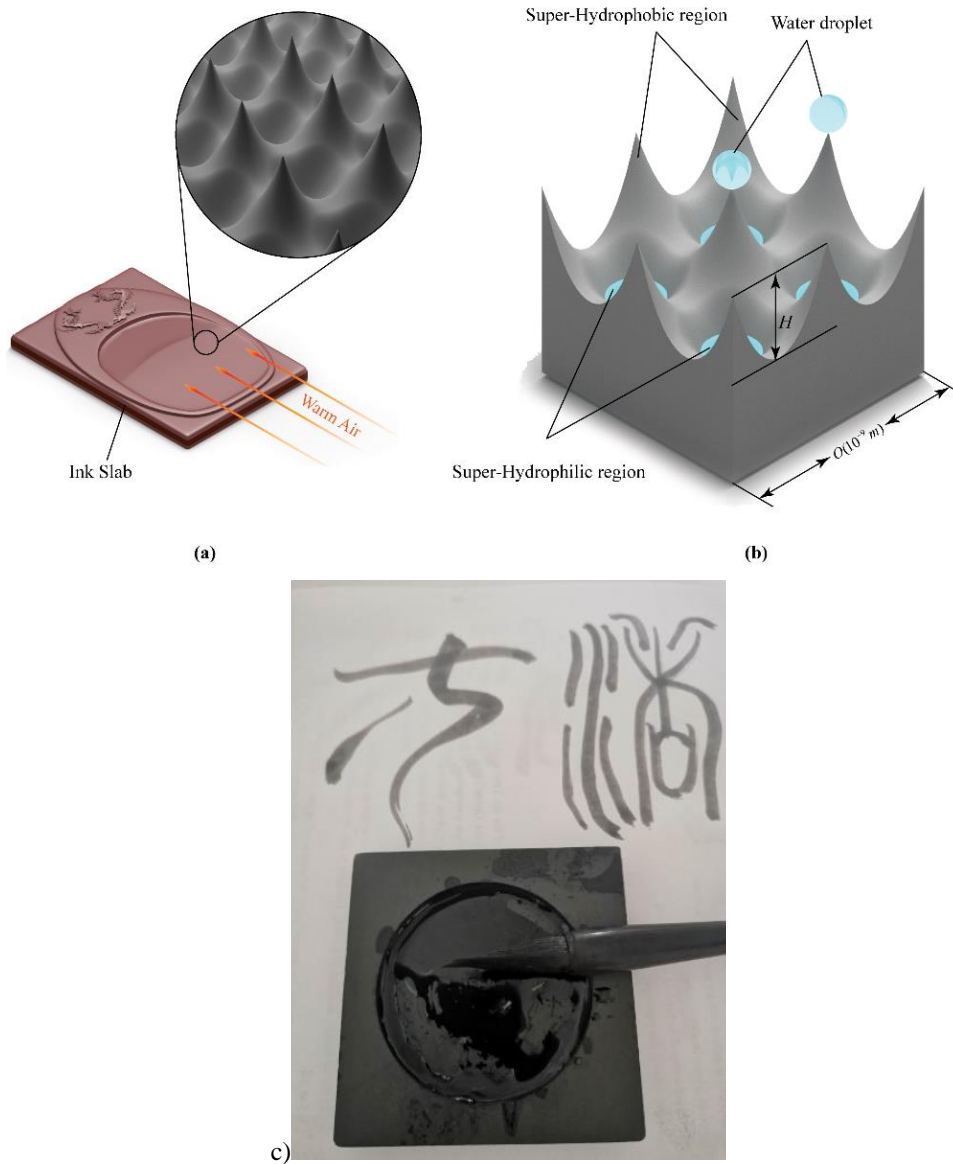
Fig. 1(a) Traditional Chinese ink slab and its potential nanostructure, (b) nanoscale unit cell of the slab(c) a real ink slab in the modern time and ancient Chinese characters for Fangzhu(方诸)

3. Fangzhu Oscillator and its Low Frequency Property

Fangzhu represents a long lost device characterized by some incomparable properties which cannot be found in both natural and artificial materials. The main property of

Fangzhu material is to absorb water molecules which vibrate with an extremely low frequency [1]. It was reported that the low frequency property is beneficial for a long transmission of air/moist permeability through a microscale capillary tube [14, 15]. This property is used for sound and vibration attenuation in modern metamaterials [13].

The Fangzhu oscillator, which describes the motion of a water molecule in Fangzhu's surface, can be written in the form [1]:

$$\frac{d^2 x}{dt^2} + \frac{\varepsilon_1}{x^{2p+1}} - \frac{\varepsilon_2}{(x+H)^{2q+1}} = 0, \varepsilon_1 > 0, \varepsilon_2 > 0 \tag{1}$$

with the initial conditions:

$$x(0) = A, x'(0) = 0 \tag{2}$$

where ε_1 and ε_2 are surface factors for the nanoscale concave and convex areas in Fangzhu's surface, respectively. Likewise, p and q are also surface-dependent constants while H is the distance between the concave and the convex, x is the distance between an absorbed water molecule and the convex area. Eq. (1) with the initial conditions of Eq. (2) is difficult to be solved analytically; the homotopy perturbation method [11, 16], the variational iteration method [17], the Taylor series method [18], and the reproducing kernel method [12] can effectively solve Fangzhu's oscillator.

In this section, we will apply He's frequency formulation [19, 20] to reveal the frequency property of the Fangzhu oscillator. The variational principle can be established by the semi-inverse method [21], which is:

$$J(x) = \int_0^T \left\{ \frac{1}{2}(\frac{dx}{dt})^2 + \frac{\varepsilon_1}{2p} x^{-2p} - \frac{\varepsilon_2}{2q}(x+H)^{-2q} \right\} dt \tag{3}$$

The Hamiltonian invariant is:

$$\frac{1}{2}(\frac{dx}{dt})^2 - \frac{\varepsilon_1}{2p} x^{-2p} + \frac{\varepsilon_2}{2q}(x+H)^{-2q} = h \tag{4}$$

where h is the Hamilton constant. By the initial conditions of Eq. (4), the Hamilton constant can be identified, and Eq. (4) becomes

$$\frac{1}{2}(\frac{dx}{dt})^2 - \frac{\varepsilon_1}{2p} x^{-2p} + \frac{\varepsilon_2}{2q}(x+H)^{-2q} = -\frac{\varepsilon_1}{2p} A^{-2p} + \frac{\varepsilon_2}{2q}(A+H)^{-2q} \tag{5}$$

Differentiating Eq. (5) with respect to t results in:

$$\frac{dx}{dt}\frac{d^2 x}{dt^2} + \varepsilon_1 x^{-2p-1}\frac{dx}{dt} - \varepsilon_2 (x+H)^{-2q-1}\frac{dx}{dt} = 0 \tag{6}$$

which is equivalent to Eq. (1). Eq. (4) implies that the total energy during the oscillation keeps unchanged. We use He's frequency formulation to elucidate the frequency property of the Fangzhu oscillator. Consider a nonlinear oscillator in the form

$$\frac{d^2 x}{dt^2} + f(x) = 0, f(x)/x > 0 \tag{7}$$

He's frequency formulation is [19, 20]

$$\omega^2 = \frac{f(\theta A)}{\theta A} \tag{8}$$

where θ is a constant, satisfying $0<\theta<1$. As an example, we consider the following oscillator [22]:

$$\frac{d^2 x}{dt^2} + \frac{1}{x} = 0 , \ x(0) = A, x'(0) = 0 \tag{9}$$

We choose $\theta=0.8$, and Eq. (8) leads to the following result:

$$\omega = \frac{1}{0.8A} = \frac{1.25}{A} \tag{10}$$

The exact frequency can be obtained as [22]

$$\omega_{\text{exact}} = \frac{1.2533}{A} \tag{11}$$

The relative error is as small as 0.26%. Now He's frequency formulation and its various modifications have been widely applied to various nonlinear oscillators [23-29]. We predict the Fangzhu's frequency property by Eq. (8) as follows:

$$\omega = \sqrt{\frac{\varepsilon_1}{(0.8A)^{2p+2}} - \frac{\varepsilon_2}{(0.8A+H)^{2q+2}}} \tag{12}$$

A water molecule is absorbed on the convex and then is transferred to the concave; it requires that the frequency is as low as practically possible:

$$\frac{\varepsilon_1}{(0.8A)^{2p+2}} - \frac{\varepsilon_2}{(0.8A+H)^{2q+2}} > 0 \tag{13}$$

and

$$\frac{\varepsilon_1}{(0.8A)^{2p+2}} - \frac{\varepsilon_2}{(0.8A+H)^{2q+2}} \to 0 \tag{14}$$

For the sake of practical design of these materials Eq. (14) can be used to optimize the surface geometrical properties and materials to achieve water harvesting effects.

4. FRACTAL OSCILLATOR

Abro et al. revealed that nano fluid can be modeled by fractional differential equations [30]. Wang et al. pointed out that the water molecule's vibration along the Fangzhu's

surface should consider the unsmooth morphology, and a fractal modification is suggested, which is [10]:

$$\frac{d^2x}{dt^{2\alpha}} + \frac{\varepsilon_1}{x^{2p+1}} - \frac{\varepsilon_2}{(x+H)^{2q+1}} = 0 \tag{15}$$

with the initial conditions

$$x(0^{\alpha}) = A, x'(0^{\alpha}) = 0 \tag{16}$$

where dx/dt^{α} is a two-scale fractal derivative [31-33]. Its approximate solution can be expressed as

$$x = a + (A-a)\cos(\omega t^{\alpha}) \tag{17}$$

where a is a positive constant, ω is defined by Eq. (12). The frequency property of the fractal oscillator is important. We consider a special case of Eq. (17):

$$x = 1 + 0.5\cos(\omega t^{\alpha}) \tag{18}$$

Fig. 2 shows a low frequency property of the fractal oscillator when time tends to infinity. Based on capillary effect in transporting water molecule mass, the low frequency can effectively guarantee both safe water transmission and an extremely low loss of water [14].

The equivalent frequency can be approximately written as

$$\omega_{eq} = \omega t^{\alpha-1} \tag{19}$$

When $\alpha < 1$, we have the low equivalent frequency when $t >> 1$. The frequency-amplitude relationship given in Eq. (12) reveals that a lower frequency results in larger amplitude, which implies that the vibrating water molecule can be transmitted to a longer distance.
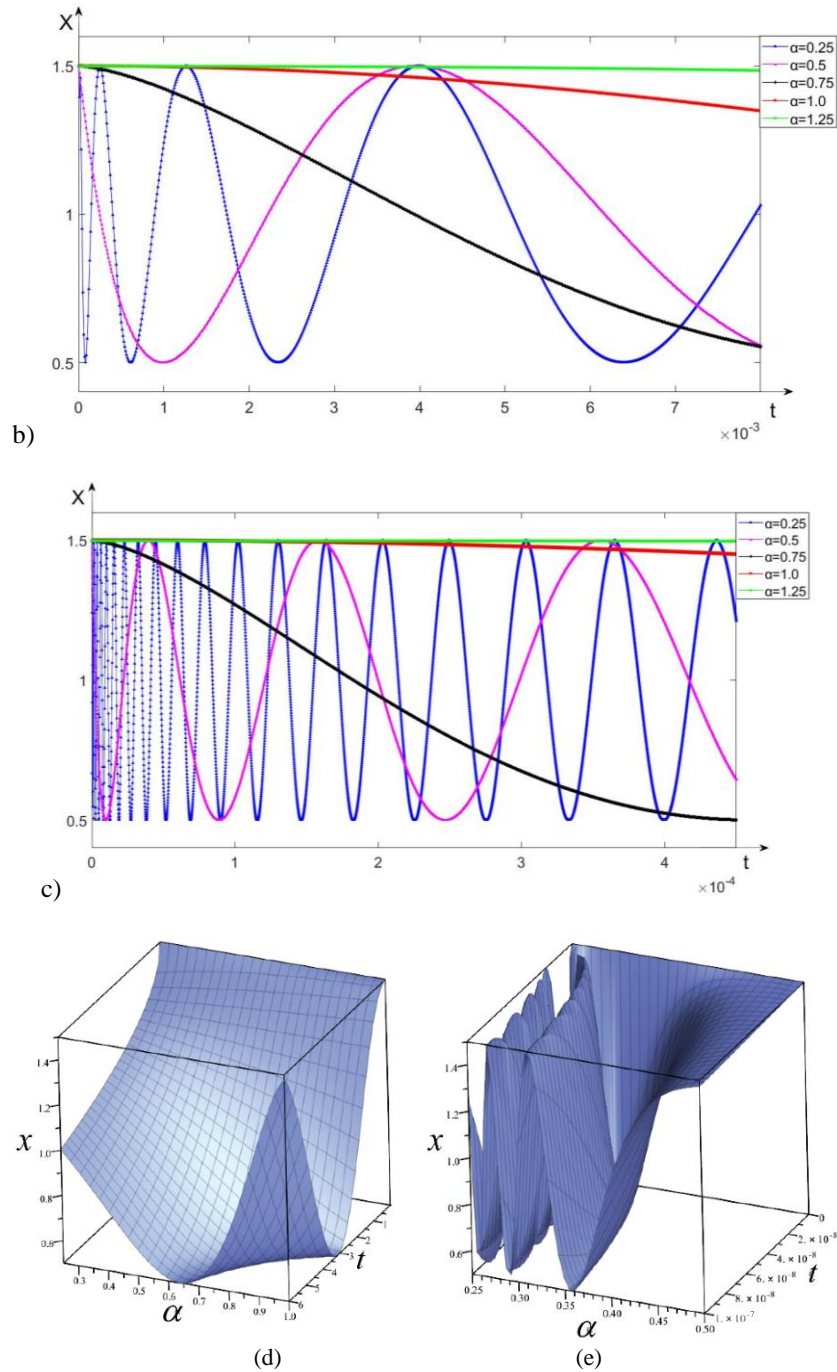

a)

b)

c)

(d)   (e)
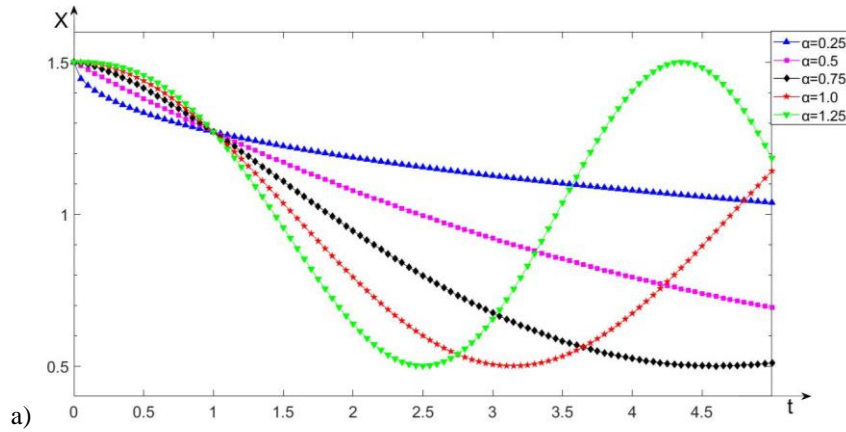
Fig. 2 Frequency property of a fractal oscillator at the initial stage for different values of α. (a) $\omega = 1$; (b) $\omega = 100$; (c) $\omega = 1000$; three-dimensional illustrations for $\omega=1$ (d) and $\omega=100$ (e)

4. POTENTIAL APPLICATIONS OF INK SLAB-LIKE MATERIALS

Many architects have been recently attracted to the idea of self-reliance houses and complexes. In the case of water harvesting, these novel materials can play an influential role in the development of this idea. In this manner, the ink slab-like materials can be modified into tile panels for the exterior of houses (Fig. 3-a). These panels can be used to cover building facades or rooftops. The extracted water will be collected through pipelines and ducts toward the water tank. The functionality of this system relies on gravitational force; therefore, very low external energy is needed. In interior design, a similar mechanism can be used to collect water into air condition vents. Extracted water can be stored *via* outlet piping toward water tank. (Fig. 3-b) can be used to collect water into air condition vents. Extracted water can be stored *via* outlet piping toward water tank. (Fig. 3-b).
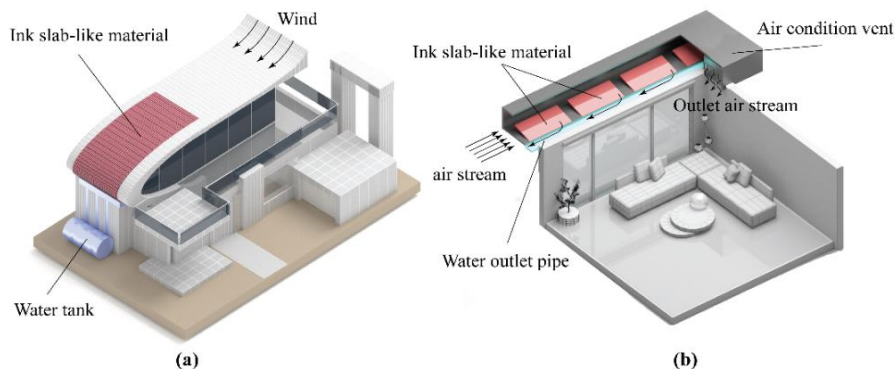

Fig. 3 (a) Exterior roof platform made of ink slab-like materials, (b) interior ink slab-like material usage in air condition vent

Few pieces of equipment and a simple collecting system allow these materials to be used in various scenarios. Fig. 4 shows the flexibility of these materials by maintaining the visual beauty in modern architectural elements and also harvesting fresh atmospheric water at the same time.

Fig. 4 Using Ink slab-like material in architectural elements and maintaining visual beauty

5. CONCLUSION

According to the record in the *MengxiBitan,* the device was extremely rare, and the ink slab implies that Fangzhu could still be seen about 1,000 years ago. When we blow on the Fangzhu-like ink slab, the hot air, high air velocity and high humidity are all favorable for water collection as discussed in ref. [1]. It is shown how these materials can significantly improve the idea of self-reliance houses through various examples. The mechanism of the atmospheric water harvesting (AWH) revealed in this paper is extremely helpful for re-designing the long-lost device for water collection from the air, and there are new promises and future challenges for modern applications in architectural engineering and other fields.

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