Propagation of surface waves at the interface between nonlinear MTMs and anisotropic materials

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Propagation of surface waves at the interface between nonlinear MTMs and anisotropic materials

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Abstract Metamaterials (MTMs), which have both negative permeability and negative permittivity, have potential applications in optoelectronics and communications. These materials are fabricated in laboratories which is an added advantage. The focus of this work is on the propagation of surface waves at the interface between nonlinear MTMs and anisotropic materials in the optical range. The dispersion equation is derived from Maxwell's equations. The dispersion equation is solved numerically to study the characteristics of the propagated wave. Only TE modes are considered. The results display the dependence of the propagating waves on the characteristics of the structure composite materials.

1 Introduction

The importance of anisotropic materials is due to the fact that they can be used as directional means for different optical equipment like fibers and converters [1–4]. A dramatic research on wave propagations in anisotropic medium has been done [5–7]. It is shown that the propagation is coupled and depends on polarization.

Metamaterials are labeled Left-Handed materials as they satisfy the characteristic laws for left-handed substances. These materials were first theoretically investigated by Veselago [8]. Pendry et al. [9] presented a periodic group of nonmagnetic directing units having an effective permeability to broaden the electromagnetic properties range of effective media. The effective permeability is enhanced by making the basic units resonant. This leads to large positive effective permeability close to the low frequency side of the resonance and negative effective permeability near the high frequency side of the resonance. The materials of negative effective permeability attracted attention because they can be united with negative permittivity materials to produce metamaterials. Split-ring resonators (SSRs) can be used to create a material with negative permeability which can be used to build metamaterials.

Smith et al. [10] treated metamaterial as a square matrix of periodic arrays. An array encompasses split-ring resonators and conducting wires. This combined structure shows a frequency in the microwave region with negative permeability and negative permittivity. These MTMs have several engineering applications because their permeabilities and permittivities can be adapted according to particular necessities. MTMs have various applications in sensors [11–13] and isolators [14–20].

The term negative index of refraction is an essential development of metamaterials. To get negative refraction, both permeability and permittivity should be negative. This can be done by using metamaterials where the frequency of negative permittivity can be reduced using wires whereas the negative permeability can be attained with resonant particles [21–24]. Furthermore, theoretical [25] and experimental [26–28] researches demonstrate that the properties of negative refraction occur in some anisotropic media labeled anisotropic-MTMs media.

Nonlinear electromagnetic responses in MTMs is established by the addition of nonlinear elements inside the

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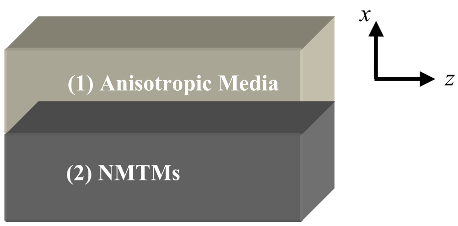

Fig. 1 The proposed configuration

MTMs, for example by inserting the SRRs in a Kerr-type dielectric [29], or by introducing certain nonlinear elements like diodes in the split-ring resonators [30]. When ultrashort pulses propagate in nonlinear electric polarization MTMs, the permeability combined with the nonlinear polarization leading to a self-steepening effect and a higher order nonlinear term [31–34].

The purpose of this work is to study surface wave propagation at the interface between nonlinear MTMs (NMTMs) and anisotropic materials. The next section is dedicated to introduce the proposed structure and the theory of the problem. In Sect. 2, results are presented followed by a discussion in Sect. 3. The conclusion is given in Sect. 4.

2 Structure of the problem and dispersion equation

In this work, we propose to study the effect of nonlinear metamaterials (NMTMs) on the propagation of the surface waves at the interface of the anisotropic media. Figure 1 shows the proposed structure. The structure consists of two semi-infinite planes. The lower half of the space $x < 0$ is filled by NMTMs and the upper half $x > 0$ is filled by anisotropic media. We considered two cases of anisotropy: anisotropic dielectric with positive permittivity and permeability and the second case is by replacing dielectric anisotropic media with anisotropic media with negative permittivity and negative permeability (anisotropic-MTMs).

The nonlinear MTMs have characteristic parameters defined as follows:

$$\varepsilon_2^{NL}(\omega) = \varepsilon_2(\omega) + \alpha I, \tag{1}$$

$$\mu_2^{NL}(\omega) = \mu_2(\omega) + \alpha I, \tag{2}$$

where I is the field intensity which equals $|E|^2$, α is the strength of the nonlinear term taken to be equal 2×10^{-9}.

In general, anisotropic media have the following dielectric tensor:

$$\varepsilon = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} \tag{3}$$

The material is chosen such that only diagonal elements are not zero. We assumed the fields are oscillatory and have the following TE polarization:

$$\vec{E} = (0, E_y, 0)\exp\big[i(\omega t - \beta z)\big], \tag{4}$$

$$\vec{H} = (H_x, 0, H_z)\exp\big[i(\omega t - \beta z)\big], \tag{5}$$

where $\beta = n_{\text{eff}} k$ denotes the propagation constant, n_{eff} the effective refractive index, $\omega = ck$ the frequency, k is the propagation constant in free space, and c is the speed of light.

Applying Eqs. (4) and (5) into Maxwell's equations, we get the field equations and their solutions in the two media.

For $x < 0$, the field equation and its solution are written as follows:

$$\frac{d^2 E_y^{(2)}}{dx^2} + \big(-\beta^2 + k^2\big(\varepsilon_2\mu_2 + \alpha\mu_2\big|E_y^{(2)}\big|^2\big)\big)E_y^{(2)}, \tag{6}$$

$$E_y^{(2)} = \frac{q_2}{k}\sqrt{\frac{2}{\alpha\mu_2}}\,\sec h\big(q_2(x - x_0)\big), \tag{7}$$

where $q_2 = \sqrt{\beta^2 - k\mu_2\varepsilon_2}$, and x_0 is the integration constant and indicates the location of the maximum power. For $x > 0$, the field equation and its solution are expressed as

$$\frac{d^2 E_y^{(1)}}{dx^2} - \big(-\beta^2 + k^2(\varepsilon_1\mu_1)\big)E_y^{(1)} = 0, \tag{8}$$

$$E_y^{(1)} = A\,\mathrm{e}^{-q_1 x}, \tag{9}$$

where A is the constant of integration can be found from boundary conditions and $q_1 = \sqrt{\beta^2 - k\mu_1\varepsilon_1}$.

The dispersion equation can be easily derived by imposing the boundary condition at $x = 0$ into the field equations.

$$\tanh(q_2 x_0) = \frac{q_1}{q_2}\frac{\mu_2}{\mu_1} \tag{10}$$

Equation (11) shows the relation between the characteristic constants of the two media and gives the needed information about the propagated surface waves between the two media.

The time averaged power flowing in the z direction per unit beam width is calculated:

$$P = -\frac{1}{2}\,\mathrm{Re}\int_{-\infty}^{\infty}\big(HxE^*\big)_z dx \tag{11}$$

The dispersion equation (11) is solved numerically for the range of $\tanh(q_2 x_0)$ between -1 to $+1$. The value of n_{eff} is used to obtain the power of the propagated waves.

The fields (Eqs. (7) and (9)) in the two media are also solved numerically. In all the calculations, the linear parts of Eqs. (1) and (2) values are taken in the optical range at $\lambda = 812$ nm [35] where the permittivity ε and permeability μ are simultaneously negative.

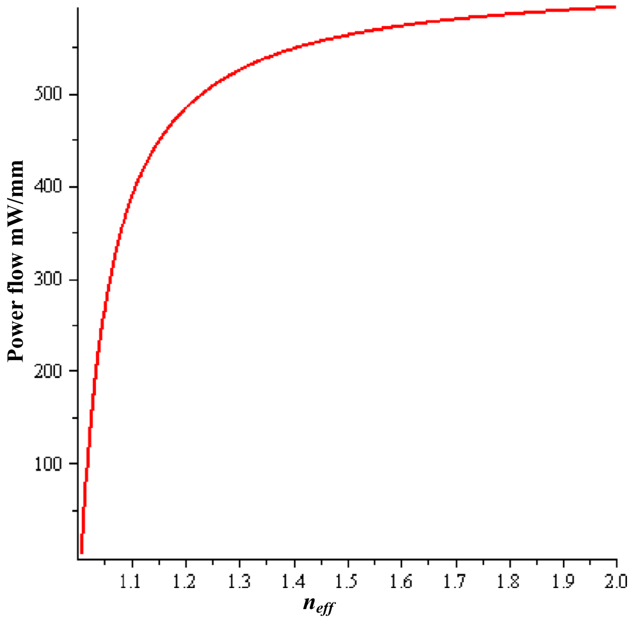

Fig. 2 The variation of the total power flow at the NMTMs/anisotropic dielectric interface with n_{eff}

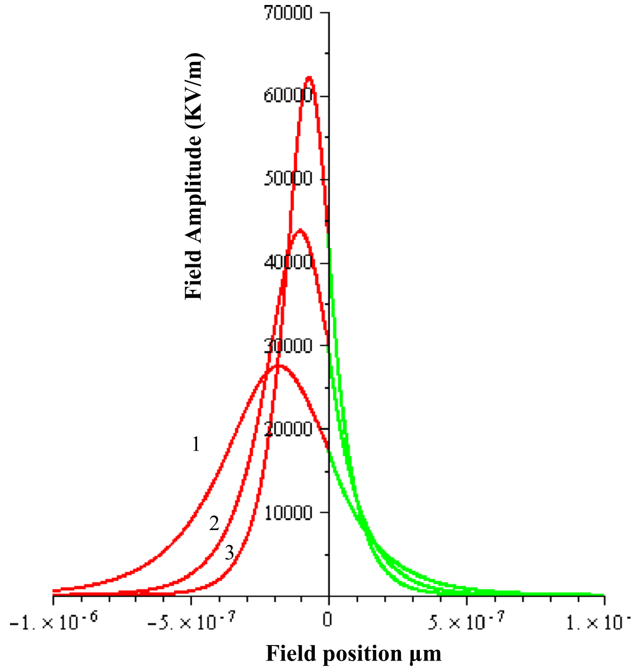

Fig. 3 The field amplitude as a function of position for different values of n_{eff}: 1.2 for *curve* 1, 1.5 for *curve* 2, and 1.9 for *curve* 3

3 Discussion

Figure 2 illustrates the propagated power flow at the interface between the NMTMs and the dielectric anisotropic media where we notice that as the wave goes from the nonlinear media to the anisotropic media the power increases until it saturates around the cutoff at $n_{\mathrm{eff}} = 2$. Moreover, the power has a definite power threshold where the surface waves start to propagate. The threshold value is determined by the minimum value of n_{eff} at which both q_1 and q_2 have real values. In this case, the threshold value occurs at $n_{\mathrm{eff}} = \sqrt{\varepsilon_2\mu_2} = 0.95$. The field distribution at different values of n_{eff} at different positions for the proposed structure is plotted in Fig. 3. It is noticeable that increasing the wave index increases the field maximum, which explains the increase in the power as the index increases.

Replacing the dielectric anisotropic material with anisotropic-MTMs media we get the power distribution for the proposed structure as shown in Fig. 4. We noticed that the power has no saturation value and the power threshold appears at $n_{\mathrm{eff}} = \sqrt{\varepsilon_1\mu_1} = 1.41$.

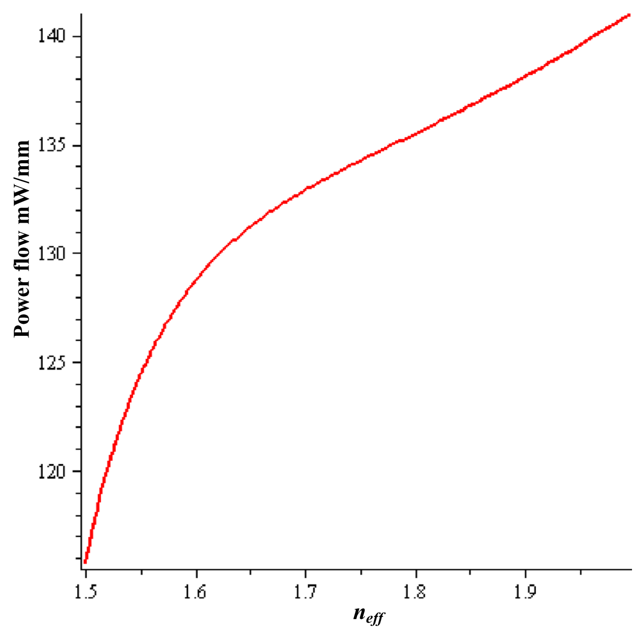

Fig. 4 The dependence of the total power flow at the NMTMs/anisotropic-MTMs interface on n_{eff}

4 Conclusions

In this work we studied surface wave propagation at the interface between nonlinear-MTMs and anisotropic media. We studied two cases of anisotropic media: one case in which the anisotropic media has positive permittivity and permeability and the second case both permittivity and permeability is negative. We found that the dispersion equations which explain the behavior of the surface waves. In the first case we noticed that the power increases as the wave index increases and has a saturation limit. In the second case we found that the power increase as the wave index increases without saturation limit. The field amplitude distribution at different wave index for different positions is plotted. The plot shows that as the wave index increases the maximum of the amplitude increases.

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