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Frequency Dependent FDTD Formulation for Debye Medium

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Abstract

According to the rapidly growing use of finite difference time domain method (FDTD) in numerical electromagnetic, this technique can be used to solve problems involving complex permittivity such as a dispersive materials. Since the dielectric constant for such media vary with frequency. In this paper we simulate and study the electromagnetic waves with different frequencies through Debye medium using (FDTD) method, Debye model describes a single pole frequency dependence. The results shows a simulation of a pulse going into frequency dependent dielectric material. Also we evaluate electric field, magnetic field and power density in dispersive medium with different frequencies. Noticed that at higher relative dielectric constant with lower frequency produced smaller amplitude, and the amplitude attenuated rapidly at high frequency. It is the most widely used time domain numerical method largely due to the simplicity of the solution algorithm.

Keywords:

Finite difference time domain method,
Debye medium,
Electromagnetic Simulation.

1. Introduction:

The finite difference time domain (FDTD) method is investigated by (Yee, 1966). This method is an important technique for solving complex electromagnetic problems (Chen & Wang, 2009) and (Taflov & Hagness, 1995). Also several techniques have been introduced recently to incorporate frequency dispersion into FDTD models (Sullivan, 2013). Dispersive FDTD method is an important for simulate optical pulse propagation. FDTD is a time domain algorithm but the constitutive relation of dispersive medium is given by a frequency domain expression. Hence (Kashiwa, Yoshida, & Fukai, 1990), it is difficult to use FDTD method directly in analyzing electromagnetic characteristics of dispersive medium (Sullivan, 2013). According to

the different treatments of constitutive relation, the FDTD methods used in dispersive medium can be classified into many methods method SO method ADE which we used method. In general the relationship of a dispersive medium is in frequency-domain and can be transformed to new form as in time-domain (Hemmi et al., 2014), since the dielectric properties of the dispersive medium is frequency-dependent. Therefore, it is difficult to study and add FDTD directly. Using Z-transform definition to translate the relationships between the time-domain, frequency-domain and dispersive time-domain. The recursive relationship between D and E in a dispersive medium has been deduced in references (Wei, Zhang, Dong, & Wang, 2009).

Bia et al.(Bia et al., 2015) studied a novel finite difference time domain method which based on fractional derivatives for dispersive media. He used numerical method based on Fourier transformation over a wide frequency range. Another research study An unconditionally stable frequency-dependent FDTD formulation for non-magnetized materials using Associated Hermite expansion, the dielectric constant which is complex is convert into AH domain and represented considered as a matrix shape as (Huang, Shi, Zhou, & Chen, 2015). Furthermore, a Stable and Efficient Direct Time Integration of the Vector Wave Equation in the FDTD formula for dispersive media has been studied by (Akbarzadeh-Sharbat & Giannacopoulos, 2015) and in addition, Akbarzadeh-Sharbat et al. evaluated a high order FDTD methods for transverse magnetic modes TM with dispersive material interfaces(Nguyen & Zhao, 2014). He analysis the finit difference at high order convergences in two dimension, then he achieved a simulations of dispersive inhomogeneous media., the von Neumann method with the Routh–Hurwitz criterion has been used by(Cho, Ha, Park, Kim, & Jung, 2014), the numerical stability conditions of the quadratic complex rational function–finite-difference time-domain are studied. He has been shown that the numerical stability conditions of the quadratic complex rational function–of FDTD methods are not same as those of the conventional finite-difference time-domain schemes, which studied by(Liu, 2013), he investigated an unified methodology for deriving new difference schemes. It is based on certain modifications of the characteristic equation that accompanies any given discretized version of the wave equation. In this study, the electromagnetic waves with different frequencies through Debye medium evaluated, and as human muscles tissue which we considered we FDTD method to simulate the electric and magnetic field in medium, Debye model describes a single pole frequency dependence.

2. Theory and model:

The dielectric properties of many material are difference by changing frequency. When we simulate the dispersive material with electromagnetic waves frequencies, we choose Gaussian pulse or sinusoidal waves. in this work the pulse using is sinusoidal waves at global system mobile frequency with difference frequency as 915 MHz and 500MHz as in Table1 below,also we consider a Debye medium as muscles

human tissue. The dielectric properties of a life tissue are different according changing frequency.

Table 1 Dielectric properties of muscles tissue at 915MHz and 500MHz

Tissue name	Conductivity σ [S/m]by 915MHz	Relative permittivity by 900MHz	Conductivity σ [S/m]by 500MHz	Relative permittivity by 500MHz
Air	0	1	0	1
Muscle	1.6	51	1.34	53

Material which dielectric properties considered as frequency dependent medium is supposes in this work. We consider Debye formulation as

$$\epsilon_r^*(\omega) = \epsilon_r + \frac{\sigma}{j\omega_0\epsilon} + \frac{\chi}{1 + j\omega t_0} \quad (1)$$

This formula represent Debye one, that is a dielectric permittivity and conductivity are dependent on frequency, also , the third term in debye formula is dependent on changing frequency. We use technique to simulate this supposed medium in FDTD, that the equation (1) rewrite in another form as

$$S(\omega) = \frac{\chi}{1 + j\omega t_0} E(\omega) \quad (2)$$

Apply inverse Fourier transform of the third term in Debye form, also we use time domain, then the approximate form becomes as

$$S^n = \chi \cdot \frac{\Delta t}{t_0} \sum_{i=0}^n e^{-\Delta t(n-i)/t_0} \cdot E^i \quad (3)$$

$$S^n = \chi \cdot \frac{\Delta t}{t_0} (E^n + \sum_{i=0}^{n-1} e^{-\Delta t(n-i)/t_0} \cdot E^i)$$

and

$$S^{n-1} = \chi \cdot \frac{\Delta t}{t_0} \sum_{i=0}^{n-1} e^{-\Delta t(n-1-i)/t_0} \cdot E^i \quad (4)$$

$$S^{n-1} = \chi \cdot \frac{\Delta t}{t_0} e^{\Delta t/t_0} \sum_{i=0}^{n-1} e^{-\Delta t(n-i)/t_0} \cdot E^i$$

Substitute equation (3) in equation(4), we have

$$S^n = \chi \cdot \frac{\Delta t}{t_0} \cdot E^n + e^{-\Delta t/t_0} S^{n-1} \quad (5)$$

Also we can write the electric flux density as

$$D^n = \epsilon_r \cdot E^n + I^n + S^n$$

$$D^n = \epsilon_r \cdot E^n + \left[\frac{\sigma \cdot \Delta t}{\epsilon_o} \cdot E^n + I^{n-t} \right] + \left[\chi \cdot \frac{\Delta t}{t_o} \cdot E^n + e^{-\frac{\Delta t}{t_o}} \cdot S^{n-1} \right] \quad (6)$$

And finally we denote the equation in term of E

$$E^n = \frac{D^n - I^{n-1} - e^{-\frac{\Delta t}{t_o}} S^{n-1}}{\epsilon_r + \frac{\sigma \cdot \Delta t}{\epsilon_o} + \chi \frac{\Delta t}{t_o}} \quad (7)$$

$$I^n = I^{n-1} + \frac{\sigma \cdot \Delta t}{\epsilon_o} \cdot E^n \quad (8)$$

$$S^n = e^{-\frac{\Delta t}{t_o}} S^{n-1} + \chi \cdot \frac{\Delta t}{t_o} \cdot E^n \quad (9)$$

Equations 7,8,9 are implemented to solve it numerically by software program and found a lot of relation and curves illustrated the simulation of electromagnetic waves with different frequencies in Debye medium

3. Results and discussion:

The electromagnetic waves with different frequencies through Debye medium using (FDTD) method have been evaluated in this work. We plot many curves with different frequency, the curves shows electric and magnetic field through Debye medium such as muscles human tissue, this material dielectric properties are change with different frequency. The results show a simulation of a pulse going into frequency dependent dielectric material. Also we evaluate electric and magnetic field in dispersive medium with different frequencies. Notice that at higher relative dielectric constant with lower frequency produced smaller amplitude, and the amplitude attenuated rapidly at high frequency. In Figure 1 Electric field simulation dispersive medium after 1500time steps and frequency =900MHz, where the Relative permittivity is 51 at frequency 900MHz and 53 when put the frequency equal 500 MHz. Also, by continue the simulation through assumed cell and increased the time steps to 1500 as shown in Figure 2. The first beak is at point-0.6. furthermore, Figures 3,4 and 5, we changed the time steps as 2000,3000 and 5000 respectively in our software program and study the simulation of electromagnetic waves in medium. The dispersive

material absorbed the waves for along time. It is explain the effect of electromagnetic waves in human body tissue, where there are a specific absorption rate. It appeared through the human tissue exposure to the mobile phone radiation at global system mobile with frequency 900MHz. At low frequency Figure 6shows the simulating electric field through Debye medium f=500MHz, X =2, to=1*10⁻¹¹, σ =53, ε_r= 1.34, the curve start to appear after 10000 time steps and 0.007 eclectic field. It is very small compare for high value of frequency. The electric and magnetic field, also plotted in Figure 7, where the simulation through Debye medium at frequency 500MHz. The positive maximum peak of magnetic field is more than electric field at frequency 500MHz as obvious in Figure 8, this results confirm many results in published paper(Elwasife, 2012)(Elwasife, 2012). Figure 9 and Figure 10 Illustrated the power density through the Debye medium which we considered as human muscles tissue according to the dielectric constant permittivity and permeability σ =53, ε_r= 1.34 in case of 500MHz, and σ =51, ε_r= 1.6.

When the frequency is 900MHz as shown in Table 1.

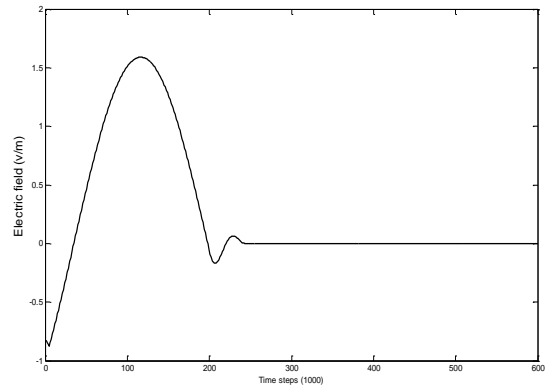


Figure 1 Electric field simulation dispersive medium after 1000time steps and frequency =900MHz

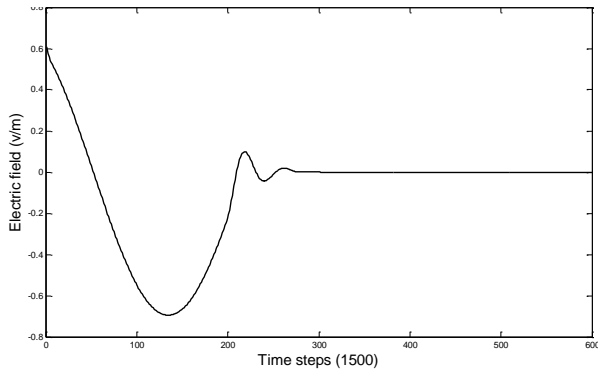


Figure 2 Electric field simulation dispersive medium after 1500 time steps and frequency =900MHz

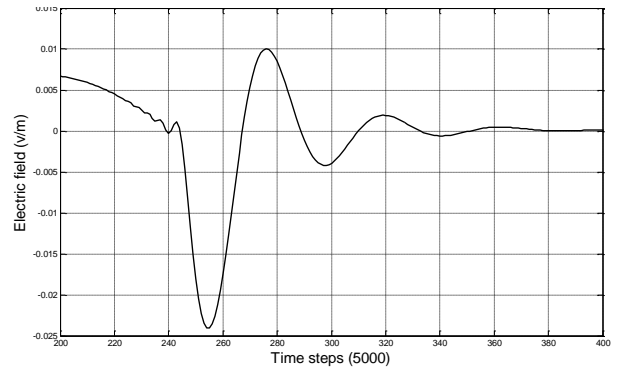


Figure 5 Electric field simulation dispersive medium after 5000 time steps and frequency =900MHz

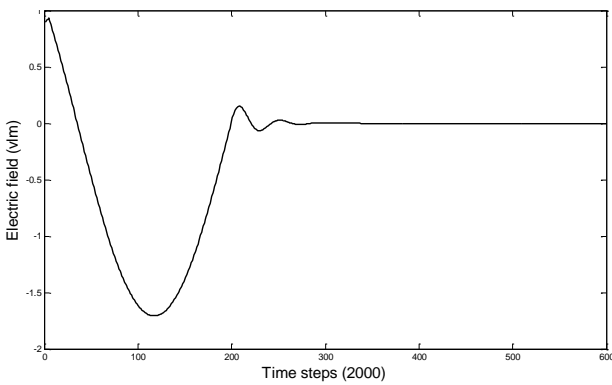


Figure 3 Electric field simulation dispersive medium after 2000 time steps and frequency =900MHz

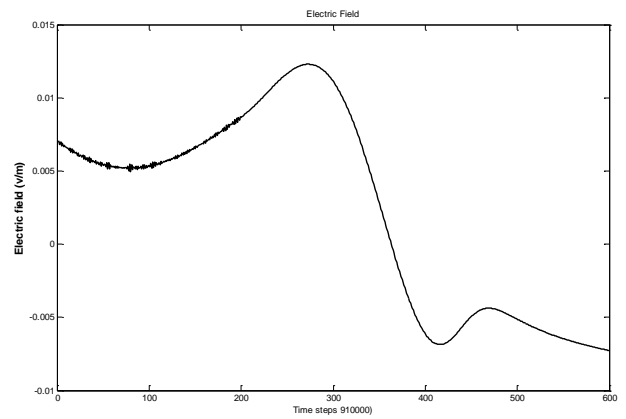


Figure 6 Simulating electric field through Debye medium $f=500\text{MHz}$, $X=2$, $t_0=1 \cdot 10^{-11}$, $\sigma=53$, $\epsilon_r=1.34$, 10000 time steps

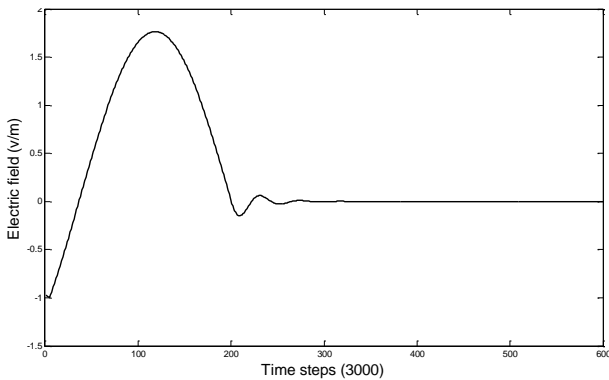


Figure 4 Electric field simulation dispersive medium after 3000 time steps and frequency =900MHz

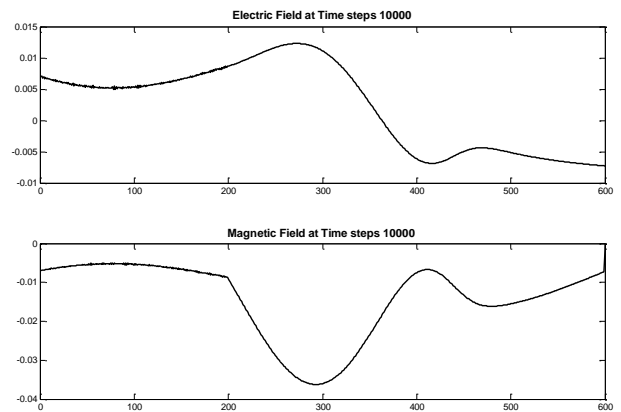


Figure 7 Simulating electric and magnetic fields through Debye medium $f=500\text{MHz}$, $X=2$, $t_0=1 \cdot 10^{-11}$, $\sigma=53$, $\epsilon_r=1.34$, 10000 time steps

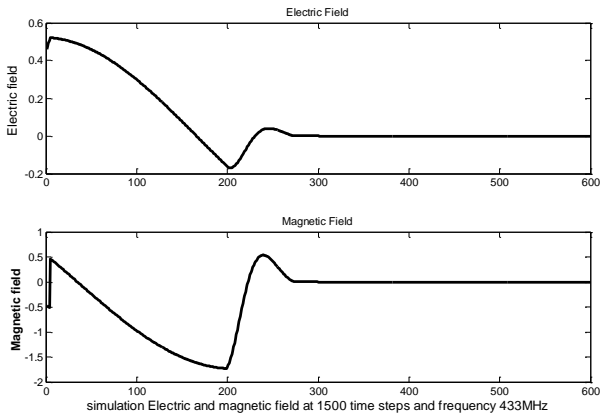


Figure 8 Simulating electric and magnetic fields through Debye medium $f= 500\text{MHz}$, $X =2$, $t_0=1 \times 10^{-11}$, $\sigma =53$, $\epsilon_r= 1.34$, 1500 time step

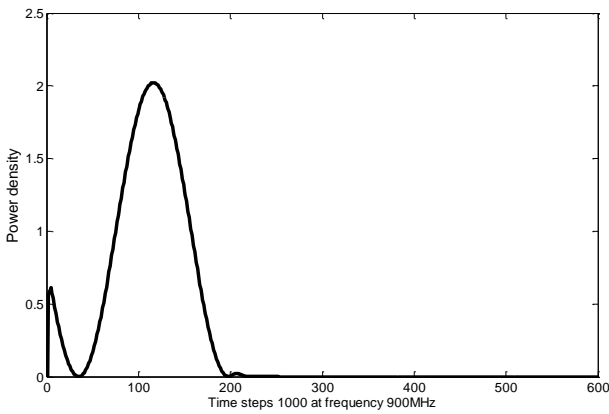


Figure 9 Simulating of power density through Debye medium $f= 900\text{MHz}$, at 1000 time step, $\sigma =51$, $\epsilon_r= 1.6$

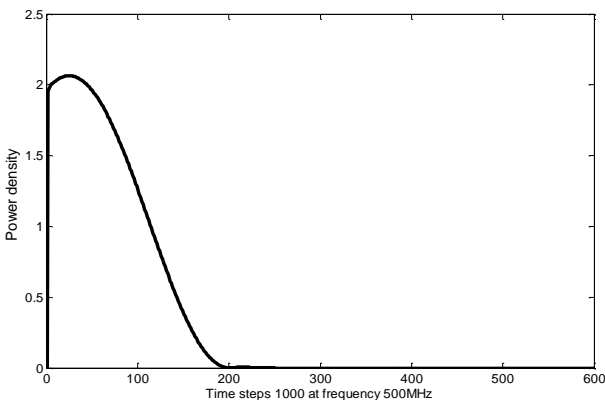


Figure 10 Simulating of power density through Debye medium $f= 500\text{MHz}$, at 1000 time step, $\sigma =51$, $\epsilon_r= 1.6$

Conclusion:

The finite-difference-time-domain (FDTD), recognized as a flexible methods and simple to implement method for solving complex electromagnetic problems. This technique also can be used to solve problems involving complex permittivity such as a dispersive materials. we simulate and study the electromagnetic waves with different frequencies through Debye medium using (FDTD) method, The results. confirm many results in previous research. Noticed that at higher relative dielectric constant with lower frequency produced smaller amplitude, and the amplitude attenuated rapidly at high frequency. It is the most widely used time domain numerical method largely due to the simplicity of the solution algorithm.

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استخدام طريقة المجال الزمني الفرقي المحدود في وسط ديبي المعتمد على التردد

كلمات مفتاحية:

طريقة المجال الزمني الفرقي
المحدود
وسط ديبي
موجات كهرومغناطيسية
محاكاة،

وفقا للاستخدام المتزايد لطريقة المجال الزمني الفرقي المحدود في الكهرومغناطيسية العددية، هذه التقنية يمكن استخدامها في حل المشاكل التي تحتوي على السماحية المعقدة مثل المواد المشتتة. وحيث أن ثابت العزل الكهربائي لهذه الوسائط تختلف مع التردد. فإنه في هذا البحث تم عمل محاكاة ودراسة الموجات الكهرومغناطيسية مع ترددات مختلفة في وسط ديبي باستخدام طريقة النطاق المحدود بفارق الزمن. في هذه الطريقة، نموذج ديبي يصف حد واحد معتمد على التردد. وتظهر نتائج محاكاة لإشارة موجة تخترق وسط يعتمد على الزمن، كما تم ايجاد المجال الكهربائي والمغناطيسي وكثافة القدرة في وسط مشتت ومتغير التردد، ونلاحظ ان النفاذية النسبية ارتفعت عند استخدام تردد منخفض منتجة سعة صغيرة. إضافة إلى ملاحظة اضمحلال مفاجئ في السعة عند ضغط مرتفع. تستخدم طريقة طريقة المجال الزمني الفرقي المحدود على نطاق واسع في الحلول اللوغارتمية