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직교 및 준직교 시공간 블록 부호를 통한 2-사용자 X 채널에서의  
간섭정렬이슬람 모하이센<sup>1</sup> · 이셋별<sup>2</sup> · 마나르 모하이센<sup>2\*</sup> · 하템 엘아이디<sup>1</sup>Interference Alignment in 2-user X Channel System with Orthogonal and  
quasi-orthogonal Space-time Block CodesIslam Mohaisen<sup>1</sup> · Saet-byeeol Lee<sup>2</sup> · Manar Mohaisen<sup>2\*</sup> · Hatem Elaydi<sup>1</sup><sup>1</sup>Department of Electrical Engineering, Islamic University of Gaza, Gaza, Palestine<sup>2</sup>Department of EEC, Korea University of Technology and Education, Cheonan 330-708, Korea

## 요 약

본 논문에서는 각 단말에 2개 이상의 안테나의 간섭 정렬을 이용한 X채널에서 직교 및 준직교 시공간 블록 부호를 통하여 더 높은 다이버시티와 전력 이득을 달성하고자 했다. 제안한 방법으로 다이버시티 차수는 직교 시공간 블록 부호에서 최대에 도달한 반면, 준직교 시공간 블록 부호에서는 유효 채널 행렬의 비 직교성에 의해 약간의 성능 저하가 나타났다. 수신기의 유효 채널 행렬에서의 유리한 구조에 의해 단순 제로 포싱 수신기는 최대 다이버시티 차수를 달성하는 반면, 간섭 제거 수신기는 성능이 저하되었다. 기존의 방법과 비교했을 때, 시뮬레이션 결과는 제안된 방법이 같은 스펙트럼 효율을 얻으면서, 3-4개의 안테나의 각 단의 직교 시공간 블록 부호의 경우 각각 목표 비트 에러율  $10^{-4}$ 에서 14dB와 16.5dB의 이득을 얻는 것을 증명하였다. 또한 4개의 안테나의 각 단의 준직교 시공간 블록 부호의 경우 같은 목표 비트 에러율에서 10dB의 이득을 얻었다.

## ABSTRACT

In this paper, we investigate achieving the full diversity order and power gains in case of using OSTBCs and quasi-OSTBCs in the x channel system with interference alignment with more than 2 antennas at each terminal. A slight degradation is remarked in the case of quasi-OSTBCs. In terms of receiver structure, we show that due to the favorable structure of the channel matrices, the simple zero-forcing receiver achieves the full diversity order, while the interference cancellation receiver leads to degradations in performance. As compared to the conventional scheme, simulation results demonstrate that our proposed schemes achieve 14dB and 16.5dB of gain at a target bit error rate (BER) of  $10^{-4}$  in the case of OSTBCs with 3 and 4 antennas at each terminal, respectively, while achieving the same spectral efficiency. Also, a gain of 10dB is achieved at the same target BER in the case of quasi-OSTBC with 4 antennas at each terminal.

키워드 : 간섭 정렬, X 채널, MIMO 검출, 직교 시공간 블록 부호, 준직교 시공간 블록 부호

Key word : Interference alignment, x channel, MIMO detection, orthogonal STBC, quasi-orthogonal STBC

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## I. Introduction

In wireless communications systems, interference plays a major role in defining the achievable performance and capacity [1, 2]. In conventional receivers, in multi-user scenarios, the interference is either ignored, hence considered as an additional noise, or jointly decoded via employing successive interference cancellation (SIC) detectors [3-5]. In both cases, the dimension of the interference remains the same, leading to degraded performance and diversity gain in the first case, while powerful algorithms should be employed in the case of SIC so as to avoid degradation in the performance.

Interference alignment is a transmission technique used to reduce the dimensions of the interference while maintaining the useful signals discernible at the intended receivers. This is achievable by precoding the transmit signals such that the interference is aligned at unintended receivers [6]. As such, interference is removed at the intended receivers using simple mathematical operations leading to an interference-free system, where appropriate decoding algorithms can then be used to decode the useful signals. In [6], Jafar and Shamai proposed a linear alignment algorithm for the two-user X channel, that achieves the maximum data rate of  $(4/3 \times n_T)$  symbols/channel use and a diversity gain of 1, with  $n_T$  denoting the number of antennas at each terminal.

In addition to the multiplexing gain, quantified by symbols/channel use, the diversity gain is an important measure of the system performance. When the channel is in deep fading, systems with unity diversity gain suffer from low signal-to-noise ratio (SNR) at the receiver side, leading to degradation in the bit-error rate (BER). Several diversity techniques have been proposed in the literature to explore further diversity gain [7-9]. In [10], a technique that combines interference alignment in X channel and Alamouti diversity scheme with two transmit antennas has been proposed to achieve the maximum multiplexing gain of  $(n_T \times 4/3 = 8/3)$  and the full diversity gain of 2, which is equal to the number of

antennas at each of the four nodes. Furthermore, the proposed scheme inherits the space-time orthogonality of the Alamouti algorithm, hence a simple linear receiver, that avoids computationally complex matrix inversion, is required to achieve the aforementioned gains.

In this paper, we propose several transmitter and receiver structures for the case of more than 2 transmit antennas aiming to increase the diversity and power gains. First, we examine the combination of orthogonal space-time block codes (OSTBC) for  $n_T = 3$  and 4 with code rate  $R_c = 1/2$  built on the 2-user X channel system with interference alignment. Due to the orthogonality of the codes, a modified interference cancellation receiver (ICR) is used to decouple the symbols transmitted from the two transmitters. Surprisingly enough, due to the special structure of the effective channel matrices, we show that the simple linear zero-forcing (ZF) receiver achieves the full diversity without requiring explicit matrix inversion. We extend this scenario to the quasi-orthogonal STBCs (quasi-OSTBCs) with  $n_T = 4$ , where in addition to proposing the interference alignment structure, we introduce a modified version of the ICR that takes into consideration the non-orthogonal nature of the codes. Note that the extension to the case of  $n_T = 2^{(n+2)}$  for any positive integer  $n$  is possible. Furthermore, we show once again that the simple ZF receiver is superior to the ICR in terms of power and diversity gains. Due to the non-orthogonality of the effective channel matrices, the achieved diversity in this later case slightly lacks the optimum value. Further research will be conducted to detail the reasons behind such a degradation and investigate methods to minimize it. It is fundamental to mention that in this paper we don't claim that our proposed techniques achieve the maximum multiplexing gain of  $(n_T \times 4/3)$ . However, at the same spectrum efficiency, quantified by bits/Hz/channel use, our proposed schemes achieve far better performance in terms of power and diversity gains.

The rest of the paper is as follows. In Section II we introduce the system model and review the LJJ scheme.

In Sections III and IV, we introduce the OSTBCs and quasi-OSTBCs built on the 2-user X channel with interference alignment, respectively. In Section V we present the simulation results and in Section VI we draw the final conclusions.

## II. System model and related work

### 2.1. System model

Consider a two-user X channel system as depicted in Figure 1, with each of the two transmitters equipped with  $n_T$  antennas and each of the receivers equipped with  $n_R$  antennas, where in the deployed scenario  $n_T = n_R$ . Each transmitter has independent symbols intended for each of the receivers. These symbols are drawn independently from a finite modulation set  $\Omega$ . Transmitter 1 has  $s_{11} = [s_{11}^1, \dots, s_{11}^K]^t$  and  $s_{12} = [s_{12}^1, \dots, s_{12}^K]^t$  intended for receiver 1 and receiver 2, respectively. In  $s_{ij}^k$ , the superscript  $k$  denotes the index of the symbol, the first subscript denotes the index of the transmitter, and the second subscript denotes the index of the intended receiver. Likewise, transmitter 2 has  $s_{21} = [s_{21}^1, \dots, s_{21}^K]^t$  and  $s_{22} = [s_{22}^1, \dots, s_{22}^K]^t$  intended for receiver 1 and receiver 2, respectively. Vectors  $s_{ij}$ , for  $i, j = 1, 2$ , are encoded by the STBC block to generate the  $T \times n_T$  matrices  $s_{ij}$ , for  $i, j = 1, 2$ , with  $T$  denoting the number of channel uses. Finally, encoded symbols are beamformed and linearly combined to generate the  $T \times n_T$  block codes  $X_i$ , for  $i = 1, 2$ . To denote the channels between the transmitters' and the receivers' antennas, we use H, G, A and B to denote the  $n_T \times n_R$  matrices coupling transmitter 1 and receiver 1, transmitter 2 and receiver 1, transmitter 1 and receiver 2, transmitter 2 and receiver 2, respectively. Each entry in the channel matrices is i.i.d.  $\text{CN}(0, 1)$ , where  $\text{CN}(\mu, \sigma^2)$  denotes a complex Gaussian random variable with mean and variance of  $\mu$  and  $\sigma^2$ . The  $T \times n_T$  signal matrices received at the antennas of receiver 1 and receiver 2,

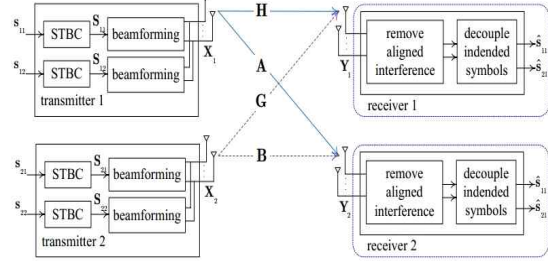


그림 1. 채널 모델과 계통도

Fig. 1 Channel model and system diagram

respectively, are given by:

$$\begin{aligned} Y_1 &= X_1 H + X_2 G + W_1, \\ Y_2 &= X_1 A + X_2 B + W_2. \end{aligned} \quad (1)$$

The elements of the noise matrices  $W_1$  and  $W_2$  are i.i.d.  $\text{CN}(0, \sigma_n^2)$ . The block codes are given as following:

$$\begin{aligned} X_1 &= S_{11} V_{11} + S_{12} V_{12}, \\ X_2 &= S_{21} V_{21} + S_{22} V_{22}. \end{aligned} \quad (2)$$

The beamforming matrices  $V_{ij}$  for  $i, j = 1, 2$  are designed to align the interference at the unintended receivers. That is,  $s_{11}^k$  and  $s_{21}^k$  are aligned at receiver 2, whereas  $s_{12}^k$  and  $s_{22}^k$  are aligned at receiver 1. The zero-forcing precoding is a suitable choice to design the beamforming matrices  $V_{ij}$  for  $i, j = 1, 2$ . As such, they are given by:

$$\begin{aligned} V_{11} &= \alpha_A A^{-1}, \quad V_{12} = \alpha_B H^{-1}, \\ V_{21} &= \alpha_C B^{-1}, \quad V_{22} = \alpha_G G^{-1}. \end{aligned} \quad (3)$$

where the real-valued scalars  $\alpha_A, \alpha_B, \alpha_C$ , and  $\alpha_G$  satisfy the power constraint  $\text{tr}(V_{ij} V_{ij}^H) = 1$ , where  $\text{tr}(\cdot)$  denotes the transpose operator. Hence,  $\alpha_R = \sqrt{1/\text{tr}(R^{-1} R^{-1H})}$  for any matrix R.

### 2.2. Review of the LJJ scheme

In the LJJ scheme [10], each node is equipped with 2 antennas where each transmitter has  $K=2$  symbols intended for each of the two receivers. As such, at each channel use a total of  $4 \times K=8$  symbols are

transmitted. These symbols are encoded using Alamouti scheme as following:

$$S_{i1} = \begin{bmatrix} s_{i1}^1 & s_{i1}^2 \\ -s_{i1}^{2*} & s_{i1}^{1*} \\ 0 & 0 \end{bmatrix}, S_{i2} = \begin{bmatrix} 0 & 0 \\ -s_{i2}^{2*} & s_{i2}^{1*} \\ s_{i2}^1 & s_{i2}^2 \end{bmatrix} \quad (4)$$

for  $i = 1, 2$ , where  $i$  refers to the index of the transmitter. Note that the conjugated symbols are present in the same row of  $S_{i1}$  and  $S_{i2}$ . At the receivers, the corresponding rows in the receive block codes are conjugated so that the system can be re-written as a function of conjugate-free symbols. The details of the decoding process which is partially similar to that depicted in[10].

### III. Orthogonal STBC with $R_c = 1/2$ top of the x-channel With Interference Alignment

It is proven that Alamouti scheme is the only OSTBC with rate  $R_c = 1$ . For more than 2 transmit antennas, several orthogonal codes have been derived with lower rates (i.e.,  $R_c < 1$ ). In the following section, we investigate the integration of OSTBC with  $R_c < 1$  in the system depicted in Figure 1. Our goal is to increase the diversity gain [12], while maintaining the same capacity, quantified by bits/channel use, as compared to the LJJ algorithm.

#### 3.1. Alignment design for orthogonal STBC with $n_T = n_R = 3$

For the case of  $n_T = 3$ , the matrices  $S_{i1}$  and  $S_{i2}$  for  $i = 1, 2$  are designed as follows.

$$\begin{bmatrix} s_{i1}^1 & 0 & -s_{i1}^2 & 0 & -s_{i1}^3 & 0 & -s_{i1}^4 & 0 & s_{i1}^{1*} & -s_{i1}^{2*} & -s_{i1}^{3*} & -s_{i1}^{4*} \\ s_{i1}^2 & 0 & s_{i1}^1 & 0 & s_{i1}^4 & 0 & -s_{i1}^3 & 0 & s_{i1}^{2*} & s_{i1}^{1*} & s_{i1}^{4*} & -s_{i1}^{3*} \\ s_{i1}^3 & 0 & -s_{i1}^4 & 0 & s_{i1}^1 & 0 & s_{i1}^2 & 0 & s_{i1}^{3*} & -s_{i1}^{4*} & s_{i1}^{1*} & s_{i1}^{2*} \end{bmatrix}^T \quad (5)$$

and

$$\begin{bmatrix} 0 & s_{i1}^1 & 0 & -s_{i1}^2 & 0 & -s_{i1}^3 & 0 & -s_{i1}^4 & s_{i1}^{1*} & -s_{i1}^{2*} & -s_{i1}^{3*} & -s_{i1}^{4*} \\ 0 & s_{i1}^2 & 0 & s_{i1}^1 & 0 & s_{i1}^4 & 0 & -s_{i1}^3 & s_{i1}^{2*} & s_{i1}^{1*} & s_{i1}^{4*} & -s_{i1}^{3*} \\ 0 & s_{i1}^3 & 0 & -s_{i1}^4 & 0 & s_{i1}^1 & 0 & s_{i1}^2 & s_{i1}^{3*} & -s_{i1}^{4*} & s_{i1}^{1*} & s_{i1}^{2*} \end{bmatrix}^T \quad (6)$$

with  $(\cdot)^T$  denoting the transpose operator. The received signal matrices at receiver 1 and 2 are given in (1), where  $X_1$  and  $X_2$  are given in (2) with  $H, G, A, B$  and  $V_{ij}$  for  $i, j = 1, 2 \in C^{3 \times 3}$ , whereas  $Y_1$  and  $Y_2 \in C^{12 \times 3}$ . Due to system symmetry, we focus in the sequel on the decoding at receiver 1. Let  $y_i$ , whose  $j$ -th element denoted by  $y_{i,j}$ , be the  $i$ -th column of  $Y_1$  with conjugate operator applied to rows 9 to 12, then

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} \begin{bmatrix} s_{11}^1 \\ s_{11}^2 \\ s_{11}^3 \\ s_{11}^4 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} \begin{bmatrix} s_{21}^1 \\ s_{21}^2 \\ s_{21}^3 \\ s_{21}^4 \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \quad (7)$$

where  $H_i$  and  $G_i$  have the same structure. Due to space limit, we give only  $H_i$  as follows:

$$\begin{bmatrix} h_{1i} & 0 & h_{2i} & 0 & h_{3i} & 0 & 0 & 0 & h_{1i}^* & h_{2i}^* & h_{3i}^* & 0 \\ h_{2i} & 0 & -h_{1i} & 0 & 0 & 0 & h_{3i} & 0 & h_{2i}^* & -h_{1i}^* & 0 & h_{3i}^* \\ h_{3i} & 0 & 0 & 0 & -h_{1i} & 0 & -h_{2i} & 0 & h_{3i}^* & 0 & -h_{1i}^* & -h_{2i}^* \\ 0 & 0 & -h_{3i} & 0 & h_{2i} & 0 & -h_{1i} & 0 & -h_{3i}^* & h_{2i}^* & -h_{1i}^* \end{bmatrix} \quad (8)$$

The aligned interference coefficient matrices are given by:

$$C_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}^T \quad (9)$$

$$C_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T \quad (10)$$

$$C_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}^T \quad (11)$$

where  $I_i = \alpha_H s_{12}^i + \alpha_G s_{22}^i$ , for  $i = 1, \dots, 4$ .

The aligned interference can be simply removed by constructing the set of vectors.

$$\tilde{y}_i = \begin{bmatrix} y_{i,1} \\ y_{i,3} \\ y_{i,5} \\ y_{i,7} \\ y_{i,9} - y_{i,2} \\ y_{i,10} - y_{i,4} \\ y_{i,11} - y_{i,6} \\ y_{i,12} - y_{i,8} \end{bmatrix} \quad \text{for } i = 1, \dots, 3 \quad (12)$$

where the system becomes interference-free and is represented by the following equation.

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \end{bmatrix} = \begin{bmatrix} \tilde{H}_1 \\ \tilde{H}_2 \\ \tilde{H}_3 \end{bmatrix} \begin{bmatrix} s_{11}^1 \\ s_{11}^2 \\ s_{11}^3 \\ s_{11}^4 \\ s_{11}^4 \end{bmatrix} + \begin{bmatrix} \tilde{G}_1 \\ \tilde{G}_2 \\ \tilde{G}_3 \end{bmatrix} \begin{bmatrix} s_{21}^1 \\ s_{21}^2 \\ s_{21}^3 \\ s_{21}^4 \\ s_{21}^4 \end{bmatrix} + \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \tilde{w}_3 \end{bmatrix} \quad (13)$$

with the  $8 \times 4$  matrices  $\tilde{H}_i$  and  $\tilde{G}_i$  being derived from  $H_i$  and  $G_i$  by removing their zero rows.

### 3.2. Alignment design for orthogonal STBC with $n_T = n_R = 4$

For the case of  $n_T = 4$ , the matrices  $S_{i1}$  and  $S_{i2}$  for  $i = 1, 2$  are designed as follows.

$$\begin{bmatrix} s_{i1}^1 0 - s_{i1}^2 0 - s_{i1}^3 0 - s_{i1}^4 0 & s_{i1}^{1*} - s_{i1}^{2*} - s_{i1}^{3*} - s_{i1}^{4*} \\ s_{i1}^2 0 s_{i1}^1 0 s_{i1}^4 0 - s_{i1}^3 0 & s_{i1}^{1*} s_{i1}^{4*} - s_{i1}^{3*} \\ s_{i1}^3 0 - s_{i1}^4 0 s_{i1}^1 0 s_{i1}^2 0 & s_{i1}^{3*} - s_{i1}^{4*} s_{i1}^{1*} s_{i1}^{2*} \\ s_{i1}^4 0 s_{i1}^3 0 - s_{i1}^2 0 s_{i1}^1 0 & s_{i1}^{4*} s_{i1}^{3*} - s_{i1}^{2*} s_{i1}^{1*} \end{bmatrix}^T \quad (14)$$

and

$$\begin{bmatrix} 0 s_{i2}^1 0 - s_{i2}^2 0 - s_{i2}^3 0 - s_{i2}^4 0 & s_{i2}^{1*} - s_{i2}^{2*} - s_{i2}^{3*} - s_{i2}^{4*} \\ 0 s_{i2}^2 0 s_{i2}^1 0 s_{i2}^4 0 - s_{i2}^3 0 & s_{i2}^{1*} s_{i2}^{4*} - s_{i2}^{3*} \\ 0 s_{i2}^3 0 - s_{i2}^4 0 s_{i2}^1 0 s_{i2}^2 0 & s_{i2}^{3*} - s_{i2}^{4*} s_{i2}^{1*} s_{i2}^{2*} \\ 0 s_{i2}^4 0 s_{i2}^3 0 - s_{i2}^2 0 s_{i2}^1 0 & s_{i2}^{4*} s_{i2}^{3*} - s_{i2}^{2*} s_{i2}^{1*} \end{bmatrix}^T \quad (15)$$

The  $12 \times 4$  received matrices  $Y_1$  and  $Y_2$  are given as in (1). Rows 9 to 12 are then conjugated and received matrices are remodeled as in (7) as follows.

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} \begin{bmatrix} s_{11}^1 \\ s_{11}^2 \\ s_{11}^3 \\ s_{11}^4 \\ s_{11}^4 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{bmatrix} \begin{bmatrix} s_{21}^1 \\ s_{21}^2 \\ s_{21}^3 \\ s_{21}^4 \\ s_{21}^4 \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} \quad (16)$$

where  $H_i$  and  $G_i$  have the same structure.  $H_i$  is given by:

$$\begin{bmatrix} h_{1i} 0 h_{2i} 0 h_{3i} 0 h_{4i} 0 & h_{1i}^* h_{2i}^* h_{3i}^* h_{4i}^* \\ h_{2i} 0 -h_{1i} 0 -h_{4i} 0 h_{3i} 0 & h_{2i}^* -h_{1i}^* -h_{4i}^* h_{3i}^* \\ h_{3i} 0 h_{4i} 0 -h_{1i} 0 -h_{2i} 0 & h_{3i}^* h_{4i}^* -h_{1i}^* -h_{2i}^* \\ h_{4i} 0 -h_{3i} 0 h_{2i} 0 -h_{1i} 0 & h_{4i}^* -h_{3i}^* h_{2i}^* -h_{1i}^* \end{bmatrix} \quad (17)$$

In (16),  $C_1$  to  $C_3$  are given in (9) to (10), respectively, while  $C_4$  is given by

$$C_4 = \begin{bmatrix} 00000 & 0 & 0100 & 0 & 1 \\ 00000 & -1 & 0000 & -1 & 0 \\ 00010 & 0 & 0001 & 0 & 0 \\ 01000 & 0 & 0010 & 0 & 0 \end{bmatrix}^T \quad (18)$$

Now, let the vectors  $\tilde{y}_i$ , for  $i = 1, \dots, 4$ , be constructed as in (12), then the interference-free system is given by

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \\ \tilde{y}_4 \end{bmatrix} = \begin{bmatrix} \tilde{H}_1 \\ \tilde{H}_2 \\ \tilde{H}_3 \\ \tilde{H}_4 \end{bmatrix} \begin{bmatrix} s_{11}^1 \\ s_{11}^2 \\ s_{11}^3 \\ s_{11}^4 \\ s_{11}^4 \end{bmatrix} + \begin{bmatrix} \tilde{G}_1 \\ \tilde{G}_2 \\ \tilde{G}_3 \\ \tilde{G}_4 \end{bmatrix} \begin{bmatrix} s_{21}^1 \\ s_{21}^2 \\ s_{21}^3 \\ s_{21}^4 \\ s_{21}^4 \end{bmatrix} + \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \tilde{w}_3 \\ \tilde{w}_4 \end{bmatrix} \quad (19)$$

with the  $8 \times 4$  matrices  $\tilde{H}_i$  and  $\tilde{G}_i$  being derived from  $H_i$  and  $G_i$  by removing their zero rows.

### 3.3. Decoupling Symbols from Different Transmitters

In the following, we introduce two decoupling algorithms; the first is an interference cancellation receiver (ICR) based on the work given in [10] and [11], whereas the second algorithm is the linear zero-forcing (ZF), where we shed light on the simplicity of the channel inversion due to the structure of the resulting interference-free system.

#### 3.3.1. Interference cancellation receiver

Let the interference-free system be represented by the following two equations.

$$\begin{aligned} \tilde{y}_1 &= \tilde{H}_1 s_{11} + \tilde{G}_1 s_{21} + \tilde{w}_1, \\ \tilde{y}_2 &= \tilde{H}_2 s_{11} + \tilde{G}_2 s_{21} + \tilde{w}_2. \end{aligned} \quad (20)$$

where in the case of  $n_T = 3$ .

$$\begin{aligned} \hat{y}_1 &= \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix}, \hat{H}_1 = \begin{bmatrix} \tilde{H}_1 \\ \tilde{H}_2 \end{bmatrix}, \hat{G}_1 = \begin{bmatrix} \tilde{G}_1 \\ \tilde{G}_2 \end{bmatrix}, \hat{w}_1 = \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \end{bmatrix} \\ \hat{y}_2 &= \tilde{y}_3, \hat{H}_2 = \tilde{H}_3, \hat{G}_2 = \tilde{G}_3, \hat{w}_2 = \tilde{w}_3. \end{aligned} \quad (21)$$

In the case of  $n_T = 4$ , the following holds.

$$\begin{aligned} \hat{y}_1 &= \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix}, \hat{H}_1 = \begin{bmatrix} \tilde{H}_1 \\ \tilde{H}_2 \end{bmatrix}, \hat{G}_1 = \begin{bmatrix} \tilde{G}_1 \\ \tilde{G}_2 \end{bmatrix}, \hat{w}_1 = \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \end{bmatrix} \\ \hat{y}_2 &= \begin{bmatrix} \tilde{y}_3 \\ \tilde{y}_4 \end{bmatrix}, \hat{H}_2 = \begin{bmatrix} \tilde{H}_3 \\ \tilde{H}_4 \end{bmatrix}, \hat{G}_2 = \begin{bmatrix} \tilde{G}_3 \\ \tilde{G}_4 \end{bmatrix}, \hat{w}_2 = \begin{bmatrix} \tilde{w}_3 \\ \tilde{w}_4 \end{bmatrix}. \end{aligned} \quad (22)$$

To decouple the vector  $s_{11}$ ,  $s_{21}$  is decoupled similarly, we construct the following two equations.

$$\begin{aligned} \frac{\hat{G}_1^H}{\|\hat{G}_1\|_F} \hat{y}_1 &= \frac{\hat{G}_1^H}{\|\hat{G}_1\|_F} \hat{H}_1 s_{11} \\ &+ \frac{\hat{G}_1^H}{\|\hat{G}_1\|_F} \hat{G}_1 s_{21} + \frac{\hat{G}_1^H}{\|\hat{G}_1\|_F} \hat{w}_1 \end{aligned} \quad (23)$$

$$\begin{aligned} \frac{\hat{G}_2^H}{\|\hat{G}_2\|_F} \hat{y}_2 &= \frac{\hat{G}_2^H}{\|\hat{G}_2\|_F} \hat{H}_2 s_{11} \\ &+ \frac{\hat{G}_2^H}{\|\hat{G}_2\|_F} \hat{G}_2 s_{21} + \frac{\hat{G}_2^H}{\|\hat{G}_2\|_F} \hat{w}_2 \end{aligned} \quad (24)$$

where  $\|G\|_F$  is the Frobenius norm of  $G$ . Since the matrices in (20) are orthogonal, then subtracting (24) from (23) results in

$$\acute{y} = \acute{H} s_{11} + \acute{w} \quad (25)$$

Let

$$\Phi = \begin{bmatrix} \frac{\hat{G}_1^H}{\|\hat{G}_1\|_F} & -\frac{\hat{G}_2^H}{\|\hat{G}_2\|_F} \end{bmatrix},$$

then

$$\acute{y} = \Phi \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix}, \acute{H} = \Phi \begin{bmatrix} \hat{H}_1 \\ \hat{H}_2 \end{bmatrix}, \acute{w} = \Phi \begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \end{bmatrix}. \quad (26)$$

Due the operation completeness of the matrices in 20,  $\acute{H}$  is also orthogonal. Then, the estimate of  $s_{11}$  is given by

$$\hat{s}_{11} = Q\left(\frac{4 \cdot \acute{H}^H \acute{y}}{\|\acute{H}\|_F^2}\right) \quad (27)$$

where  $Q(\cdot)$  denoting the vector demodulation operator.

### 3.3.2. Linear receiver

Let the system in (20) be reformulated as following.

$$\underbrace{\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix}}_y = \underbrace{\begin{bmatrix} \hat{H}_1 & \hat{G}_1 \\ \hat{H}_2 & \hat{G}_2 \end{bmatrix}}_H \begin{bmatrix} s_{11} \\ s_{21} \end{bmatrix} + \underbrace{\begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \end{bmatrix}}_w \quad (28)$$

The estimate of the symbols from the transmitters are then obtained using the linear ZF detector as follows.

$$\begin{bmatrix} \tilde{s}_{11} \\ \tilde{s}_{21} \end{bmatrix} = (\tilde{H}^H \tilde{H})^{-1} \tilde{H}^H \tilde{y} \quad (29)$$

with the estimate of  $s_{i1}$  is given by  $\hat{s}_{i1} = Q(\tilde{s}_{i1})$ . Due to the structure of the matrices  $\tilde{H}_i$  and  $\tilde{H}_i$ , for  $i = 1, \dots, n_R$ , the following holds.

$$\tilde{H}^H \tilde{H} = \begin{bmatrix} C & E \\ E^H & D \end{bmatrix} \quad (30)$$

where the matrices C, D and E are real-valued with C and D being diagonal with diagonal elements equal to

$$2 \cdot \sum_{i=1}^{n_R} \sum_{j=1}^{n_R} |h_{ij}|^2 \quad \text{and} \quad 2 \cdot \sum_{i=1}^{n_R} \sum_{j=1}^{n_R} |g_{ij}|^2, \quad \text{respectively.}$$

Based on the block matrix inverse lemma,

$$\begin{aligned} (\tilde{H}^H \tilde{H})^{-1} &= \begin{bmatrix} I_4 & -C^{-1}E \\ -D^{-1}E^H & I_4 \end{bmatrix} \\ &\cdot \begin{bmatrix} (C - ED^{-1}E^H)^{-1} & 0_4 \\ 0_4 & (D - E^H C^{-1}E)^{-1} \end{bmatrix} \end{aligned} \quad (31)$$

Note that the matrix  $(C - ED^{-1}E^H)$  is a scaled identity matrix, and hence calculating its inverse requires a single real division operation. The same holds for the inversion of the matrices C and D. As such, the complexity of the ZF detection reduces to simple real operations rather than fully computing the inverse of an  $8 \times 8$  complex-valued matrix.

#### IV. Full-rate Quasi-orthogonal STBC on top of the x-channel with interference alignment system

To achieve higher diversity gains employing a single receive antenna, the conventional Alamouti code [7] with two transmit antennas is extended to the  $2^m$  transmit antennas case with  $m \geq 2$  [13-15]. In the following, we introduce alignment design and receiver structures for the case of the extended Alamouti with  $n_T = n_R = 4$  built on the system depicted in Figure 1.

##### 4.1. Alignment design

In the current design, (2) and (3) still hold with the difference that the matrices  $V_{ij}$  for  $i = 1, 2, A, B, G$ , and H are of size  $4 \times 4$ , and  $S_{i1}$  and  $S_{i2}$ , for  $i = 1, 2$ , are respectively designed as

$$S_{i1} = \begin{bmatrix} s_{i1}^1 & s_{i1}^2 & s_{i1}^3 & s_{i1}^4 \\ 0 & 0 & 0 & 0 \\ s_{i1}^{2*} - s_{i1}^{1*} & s_{i1}^{4*} - s_{i1}^{3*} & 0 & 0 \\ s_{i1}^{3*} & s_{i1}^{4*} - s_{i1}^{1*} - s_{i1}^{2*} & 0 & 0 \\ s_{i1}^4 - s_{i1}^3 - s_{i1}^2 & s_{i1}^1 & 0 & 0 \end{bmatrix}, \quad (32)$$

$$S_{i2} = \begin{bmatrix} s_{i2}^1 & s_{i2}^2 & s_{i2}^3 & s_{i2}^4 \\ s_{i2}^{2*} - s_{i2}^{1*} & s_{i2}^{4*} - s_{i2}^{3*} & 0 & 0 \\ s_{i2}^{3*} & s_{i2}^{4*} - s_{i2}^{1*} - s_{i2}^{2*} & 0 & 0 \\ s_{i2}^4 - s_{i2}^3 - s_{i2}^2 & s_{i2}^1 & 0 & 0 \end{bmatrix}.$$

Based on (1), the  $6 \times 4$  received signal matrices at receiver 1 and 2 are respectively written as

$$Y_1 = S_{11} \underbrace{V_{11}H}_{\hat{H}} + S_{21} \underbrace{V_{21}H}_{\hat{G}} + W_1$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ I_1 & I_2 & I_3 & I_4 \\ I_2^* - I_1^* & I_4^* - I_3^* & 0 & 0 \\ I_3^* & I_4^* - I_1^* - I_2^* & 0 & 0 \\ I_4 - I_3 - I_2 & I_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (33)$$

$$Y_2 = S_{12} \underbrace{V_{12}A}_{\hat{A}} + S_{22} \underbrace{V_{22}B}_{\hat{B}} + W_2$$

$$+ \begin{bmatrix} I_5 & I_6 & I_7 & I_8 \\ 0 & 0 & 0 & 0 \\ I_6^* - I_5^* & I_8^* - I_7^* & 0 & 0 \\ I_7^* & I_8^* - I_5^* - I_6^* & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_8 - I_7 - I_6 & I_5 & 0 & 0 \end{bmatrix} \quad (34)$$

where  $I_i = (\alpha_H s_{12}^i + \alpha_G s_{22}^i)$  and  $I_{(i+4)} = (\alpha_A s_{11}^i + \alpha_B s_{21}^i)$ , for  $i = 1, \dots, 4$ . Without loss of generality, we will focus on the decoding at receiver 1, where receiver 2 has the same decoding and performance due to system symmetry. The system in (33) is converted to the equivalent  $24 \times 1$  vector  $\tilde{y}_1$ , where the elements of  $\tilde{y}_1$  are filled in row-wise. That is, the first 6 elements of  $\tilde{y}_1$  are the first row of  $Y_1$ , elements 7 to 12 are the second row of  $Y_1$ . The conjugate operator is applied to rows 3 and 4 before being inserted in  $\tilde{y}_1$ .

The aligned interference at receiver 1, *i.e.*,  $I_i$  terms, is removed by performing simple addition and subtraction operations on the elements of  $\tilde{y}_1$ . To this end, let us define the following  $4 \times 1$  vectors

$$\hat{y}_1 = \begin{bmatrix} \tilde{y}_{1,1} \\ \tilde{y}_{1,9} - \tilde{y}_{1,6} \\ \tilde{y}_{1,13} + \tilde{y}_{1,18} \\ \tilde{y}_{1,21} \end{bmatrix}, \quad \hat{y}_2 = \begin{bmatrix} \tilde{y}_{1,2} \\ \tilde{y}_{1,10} - \tilde{y}_{1,5} \\ \tilde{y}_{1,14} + \tilde{y}_{1,17} \\ \tilde{y}_{1,22} \end{bmatrix}, \quad (35)$$

$$\hat{y}_3 = \begin{bmatrix} \tilde{y}_{1,3} \\ \tilde{y}_{1,11} - \tilde{y}_{1,8} \\ \tilde{y}_{1,15} + \tilde{y}_{1,20} \\ \tilde{y}_{1,23} \end{bmatrix}, \quad \hat{y}_4 = \begin{bmatrix} \tilde{y}_{1,4} \\ \tilde{y}_{1,12} - \tilde{y}_{1,7} \\ \tilde{y}_{1,16} + \tilde{y}_{1,19} \\ \tilde{y}_{1,24} \end{bmatrix}.$$

where the vectors  $\hat{w}_i$  for  $i = 1, \dots, 4$  are derived similarly from  $\tilde{w}_i$ . Then,  $\hat{y}_i = \hat{H}_i s_{11} + \hat{G}_i s_{21} + \hat{w}_i$  with

$$\hat{H}_i = \begin{bmatrix} \tilde{h}_{1i} & \tilde{h}_{2i} & \tilde{h}_{3i} & \tilde{h}_{4i} \\ -\tilde{h}_{2i}^* & \tilde{h}_{1i}^* & -\tilde{h}_{4i}^* & \tilde{h}_{3i}^* \\ -\tilde{h}_{3i}^* & -\tilde{h}_{4i}^* & \tilde{h}_{1i}^* & \tilde{h}_{2i}^* \\ \tilde{h}_{4i} & -\tilde{h}_{3i} & -\tilde{h}_{2i} & \tilde{h}_{1i} \end{bmatrix},$$



$$\hat{G}_i = \begin{bmatrix} \tilde{g}_{1i} & \tilde{g}_{2i} & \tilde{g}_{3i} & \tilde{g}_{4i} \\ -\tilde{g}_{2i}^* & \tilde{g}_{1i} & -\tilde{g}_{4i}^* & \tilde{g}_{3i} \\ -\tilde{g}_{3i}^* & -\tilde{g}_{4i} & \tilde{g}_{1i} & \tilde{g}_{2i} \\ \tilde{g}_{4i} & -\tilde{g}_{3i} & -\tilde{g}_{2i} & \tilde{g}_{1i} \end{bmatrix} \quad (36)$$

Note that  $\hat{H}_i$  and  $\hat{G}_i$ , for  $i=1, \dots, 4$ , have the same structure of the extended Alamouti matrices, which implies that they have the same properties. After aligned interference cancellation, the system can be rewritten as

$$\underbrace{\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \end{bmatrix}}_y = \underbrace{\begin{bmatrix} \hat{H}_1 & \hat{G}_1 \\ \hat{H}_2 & \hat{G}_2 \\ \hat{H}_3 & \hat{G}_3 \\ \hat{H}_4 & \hat{G}_4 \end{bmatrix}}_H \begin{bmatrix} s_{11} \\ s_{21} \end{bmatrix} + \underbrace{\begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \\ \hat{w}_3 \\ \hat{w}_4 \end{bmatrix}}_w \quad (37)$$

where the noise is still white with a covariance matrix  $R_w = I_4 \otimes \text{diag}(\sigma_n^2, 2\sigma_n^2, 2\sigma_n^2, \sigma_n^2)$ , where  $\otimes$  denotes the Kronecker product and  $I_4$  is the 4x4 identity matrix.

Before introducing the decoupling schemes, it is worthy to shed some light on the properties of the matrices given in (36), where these properties are essential in the design of the receiver. Let  $F_i$  for  $i=1, \dots, n$  have the same structure of the matrices given in (36), then

- 1) The matrix  $F_i^H F_i$  is real-valued, having the form

$$f_i^2 \begin{bmatrix} 1 & 0 & 0 & X_i \\ 0 & 1 & -X_i & 0 \\ 0 & -X_i & 1 & 0 \\ X_i & 0 & 0 & 1 \end{bmatrix} \quad (38)$$

where

$$f_i^2 = \sum_{j=1}^4 |f_{i,j}|^2 \text{ and } X_i = 2\text{Re}(f_{i,11}f_{i,14}^* - f_{i,12}f_{i,13}^*)/f_i^2;$$

- 2) These matrices are complete in terms of matrix addition, matrix subtraction, and matrix multiplication. That is, the Gram matrix of  $F_i F_j$ , for  $j \neq i$ , has the same structure explained in Property I. This can be simply proven using the block form of each matrix and perform each of the aforementioned operations.

- 3) The inverse of  $F_i^H F_i$ , given in Property I, has the

following form

$$h_i^2 \begin{bmatrix} 1 & 0 & 0 & Y_i \\ 0 & 1 & -Y_i & 0 \\ 0 & -Y_i & 1 & 0 \\ Y_i & 0 & 0 & 1 \end{bmatrix} \quad (39)$$

with  $h_i^2 = \frac{1}{f_i^2(1-X_i^2)}$  and  $Y_i = -X_i$

- 4) The sum of Gramians of the matrices defined in (36) is given by

$$\sum_{i=1}^n F_i^H F_i = f^2 \begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & -X & 0 \\ 0 & -X & 1 & 0 \\ X & 0 & 0 & 1 \end{bmatrix} \quad (40)$$

with  $f^2 = \sum_{i=1}^n f_i^2$  and  $X = \frac{1}{f^2} \sum_{i=1}^n X_i f_i^2$

Having that been said about the properties of the matrices given in (36), we introduce the properties of the Gramian matrix  $H_f = \tilde{H}^H \tilde{H}$  which will be used in the decoupling algorithms to be introduced in the sequel. The (8x8) Gramian matrix is given by:

$$H_f = \begin{bmatrix} C & E \\ E^H & D \end{bmatrix} \quad (41)$$

where

$$C = \sum_{i=1}^4 (\hat{H}_i^H \hat{H}_i), D = \sum_{i=1}^4 (\hat{G}_i^H \hat{G}_i), E = \sum_{i=1}^4 (\hat{H}_i^H \hat{G}_i).$$

Note that the 4x4 matrices C and D have the structure explained in Property 4, while the 4x4 matrix E has the structure given in (36). As such, it comes with no surprise that the inverse of  $H_f$  has the same structure given (41), as will be explained later on.

#### 4.2. Decoupling symbols from different transmitters

In the literature, several detection techniques have been proposed [10], which can be used to decouple the symbols from different receivers in (37). However, the effective channel matrix, or matrices, in (37) has a special structure that motivates the proposal of modified receivers that take into account the particular structure of these matrices. In the following, we introduce the

details of two receiver structures.

#### 4.2.1. Interference cancellation receiver

The idea of the ICR is based on the work introduced in [10] and [11], with the particularity that the system discussed here is not orthogonal. That is, the Gram matrix of  $\hat{G}_i$  and  $\hat{H}_i$  in (37) include both diagonal and skew-diagonal elements. As such, to decouple  $s_{11}$  the terms which include  $s_{21}$  should be canceled, where two IC stages are required rather than a single stage as in [10].

A) Cancellation of the diagonal elements: Starting with (38), we construct the following four vectors.

$$\begin{aligned} \frac{\hat{G}_i^H}{\hat{g}_i} \hat{y}_i &= \frac{\hat{G}_i^H}{\hat{g}_i} \hat{H}_i s_{11} + \frac{\hat{G}_i^H}{\hat{g}_i} \hat{G}_i s_{21} \\ &+ \frac{\hat{G}_i^H}{\hat{g}_i} \hat{w}_i, \text{ f or } i = 1, \dots, 4 \end{aligned} \quad (42)$$

Where based on property 1

$$\frac{\hat{G}_i^H}{\hat{g}_i} \hat{G}_i = I_4 + \text{diag}(X_{\hat{G}_i}, -X_{\hat{G}_i}, -X_{\hat{G}_i}, X_{\hat{G}_i}).$$

where  $\text{diag}()$  is a skew-diagonal matrix. Subtracting the second equation, i.e.,  $i = 2$  in (42), from the first one, and the fourth equation from the third one, cancels the diagonal elements of the channel matrix associated with  $s_{21}$ , resulting in the following equations

$$\begin{aligned} \Phi_1 \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix} &= \Phi_1 \begin{bmatrix} \hat{H}_1 \\ \hat{H}_2 \end{bmatrix} s_{11} + \Phi_1 \begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \end{bmatrix} \\ &+ \theta_1 \cdot \text{diag}(1, -1, -1, 1) s_{21}, \end{aligned} \quad (43)$$

$$\begin{aligned} \Phi_2 \begin{bmatrix} \hat{y}_3 \\ \hat{y}_4 \end{bmatrix} &= \Phi_2 \begin{bmatrix} \hat{H}_3 \\ \hat{H}_4 \end{bmatrix} s_{11} + \Phi_2 \begin{bmatrix} \hat{w}_3 \\ \hat{w}_4 \end{bmatrix} \\ &+ \theta_2 \cdot \text{diag}(1, -1, -1, 1) s_{21}, \end{aligned} \quad (44)$$

where

$$\begin{aligned} \theta_1 &= (X_{\hat{G}_1} - X_{\hat{G}_2}), \quad \theta_2 = (X_{\hat{G}_3} - X_{\hat{G}_4}), \\ \Phi_1 &= \begin{bmatrix} \hat{G}_1^H & \hat{G}_2^H \\ \hat{g}_1 & \hat{g}_2 \end{bmatrix}, \quad \Phi_2 = \begin{bmatrix} \hat{G}_3^H & \hat{G}_4^H \\ \hat{g}_3 & \hat{g}_4 \end{bmatrix}. \end{aligned}$$

B) Cancellation of the off-diagonal elements: Let (43) and (44) be divided by  $\theta_1$  and  $\theta_2$ , respectively, then subtract the second equation from the first one, results in

$$\underbrace{\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \end{bmatrix}}_{y_{icr}} = \underbrace{\begin{bmatrix} \hat{H}_1 \\ \hat{H}_2 \\ \hat{H}_3 \\ \hat{H}_4 \end{bmatrix}}_{H_{icr}} s_{11} + \underbrace{\begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \\ \hat{w}_3 \\ \hat{w}_4 \end{bmatrix}}_{w_{icr}} \quad (45)$$

where  $\Psi = \begin{bmatrix} \Phi_1 & -\Phi_2 \\ \theta_1 & \theta_2 \end{bmatrix}$ . It is worth mentioning that based on Property II, the structure of  $H_{icr}^H H_{icr}$  is defined by Property I.

Finally, the ZF decoder can be used to recover the transmitted vector  $s_{11}$  as following

$$\begin{aligned} \tilde{s}_{11} &= (H_{icr}^H H_{icr})^{-1} H_{icr}^H y_{icr} \\ &= s_{11} + (H_{icr}^H H_{icr})^{-1} H_{icr}^H w_{icr} \end{aligned} \quad (46)$$

where the estimate  $\tilde{s}_{11} = Q(\tilde{s}_{11})$ . Note that  $(H_{icr}^H H_{icr})^{-1}$  can be computed using Property III to reduce the computational complexity.

#### 4.2.2 Linear zero-forcing receiver

The system modeled in (37) is first filtered using  $\tilde{H}^H$  resulting in

$$\underbrace{\tilde{H}^H y}_{y_f} = \underbrace{\begin{bmatrix} C & E \\ E^H & D \end{bmatrix}}_{H_f} \begin{bmatrix} s_{11} \\ s_{21} \end{bmatrix} + \underbrace{\tilde{H}^H w}_{w_f} \quad (47)$$

where  $C = \sum_{i=1}^4 (\tilde{H}_i^H \hat{H}_i)$ ,  $D = \sum_{i=1}^4 (\hat{G}_i^H \hat{G}_i)$ ,  $E = \sum_{i=1}^4 (\hat{H}_i^H \hat{G}_i)$ , and the subscript f refers to filtered. As such, C and D have the structure given in Property 1, as well as satisfying Property 4, while E satisfies Property II. The transmitted vectors are recovered as follows

$$\begin{bmatrix} \tilde{s}_{11} \\ \tilde{s}_{21} \end{bmatrix} = H_f^{-1} y_f = \begin{bmatrix} s_{11} \\ s_{21} \end{bmatrix} + H_f^{-1} w_f \quad (48)$$

To avoid the inversion of the  $8 \times 8$  matrix  $H_f$ , we introduce a cost-effective method to compute the inverse without explicitly inverting  $H_f$ . Based on the block matrix inverse lemma, we make the following remarks:

1) C, D,  $(C - ED^{-1}E^H)$ , and  $(D - E^H C^{-1}E)$  are real-valued and have the structure given in Property I, which implies that their inverses are simply computed

using Property III. The inverse of each of these matrices requires only three operations - two real multiplication and one division operations.

2)  $E^H C^{-1} E = C^{-1} E^H E$ , where  $E^H E$  has the structure given in Property I. The same applies for  $ED^{-1}E^H$ . Using Property II,  $E^H E$  can be obtained where 9 multiplication and 6 addition operations are required. Multiplying the result by  $C^{-1}$  requires 4 multiplication and 2 addition operations.

3)  $H_f$  is a Hermitian matrix, which implies that only the elements of an upper triangular matrix are computed.

Based on these remarks, it stems out that the computational complexity of the linear ZF detector is low due to the special structure of the effective channel matrices.

### V. Simulation Results and Discussion

In this section, we consider that each transmitter has full knowledge of *only* the channels coupling his transmit antennas with those of the two receivers. The elements in the channel matrices are independent and follow the circular-symmetric complex Gaussian distribution with mean and variance of zero and unity, respectively.

Figure 2 depicts the performance of OSTBC with code rate of 1/2 and  $n_T = 3$ , referred to as  $X_3$ , built on the system depicted in Figure 1.

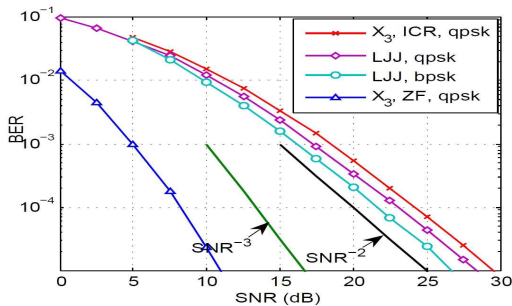


그림 2.  $n_T = n_R = 3$ 를 이용한 직교 STBC로 제안된 시스템의 BER 성능  
Fig. 2 BER performance of the proposed system with  $n_T = n_R = 3$  employing orthogonal STBC.

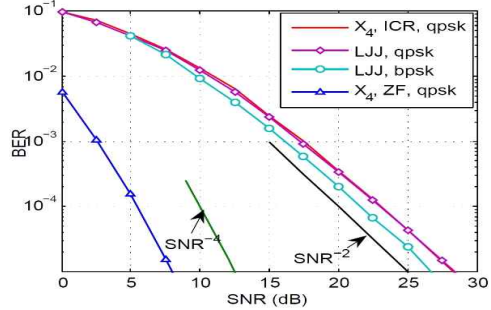


그림 3.  $n_T = n_R = 4$ 를 이용한 직교 STBC로 제안된 시스템의 BER 성능  
Fig. 3 BER performance of the proposed system with  $n_T = n_R = 4$  employing orthogonal STBC.

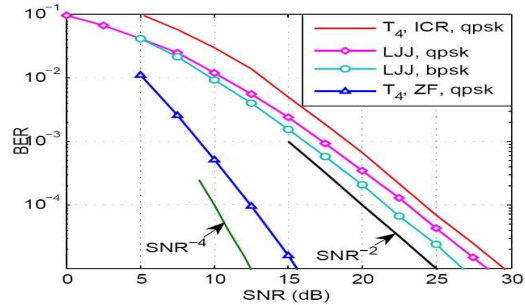


그림 4.  $n_T = n_R = 4$ 를 이용한 준직교 STBC로 제안된 시스템의 BER 성능  
Fig. 4 BER performance of the proposed system with  $n_T = n_R = 4$  employing quasi-orthogonal STBC.

To hold a fair comparison between the proposed scheme and the LJJ scheme, we fix the spectral efficiency to 8/3 bits/s/Hz, implying that the modulation schemes used for and LJJ schemes are QPSK and BPSK, respectively. Starting with the ICR, due the increased dimensionality of the interference, this receiver fails to achieve the full diversity gain of  $n_R = n_T = 3$ . Due to the special structure of the effective channel matrix, as explained in Section III, the simple linear ZF receiver achieves the full diversity order of  $n_T = n_R = 3$  while attaining high gain in the bit error rate. At a target BER of  $10^{-4}$ , the proposed  $X_3$  outperforms the conventional LJJ schemes by 14dB.

Figure 3 depicts the performance of OSTBC with code rate of 1/2 and  $n_T = 4$ , referred to as  $X_4$ , built on

the system depicted in Fig. 1. Again, for the same spectral efficiency of 8/3 bits/s/Hz, we employ QPSK and BPSK bit-to-symbol mapping for the proposed  $X_4$  scheme and the conventional LJJ scheme, respectively. The ICR shows similar performance as in the case of the  $X_3$  scheme. The simple linear ZF receiver achieves the full diversity order of  $n_T = n_R = 4$  with a gain of about 16.5dB at a target BER of  $10^{-4}$ .

Finally, Figure 4 depicts the performance of the quasi-OSTBC with  $n_T = 4$ , referred to as  $T_4$ , built on the system depicted in Figure 1. Since the ICR employed a two-stage SIC scheme to cancel the diagonal and skew-diagonal interference, the performance is therefore even worsen as compared to that in the cases of the  $X_3$  and  $X_4$ . Despite the quasi-orthogonality of the effective channel matrices, the linear ZF receiver for the proposed  $X_4$  scheme achieves a superior performance of about 10dB at a target BER of  $10^{-4}$  as compared to the LJJ algorithm both employing QPSK modulation, resulting in an equal spectral efficiency of 8/3 bits/s/Hz. Our proposed scheme performs close to the optimum diversity order of  $n_T = n_R = 4$ . The small degradation in the diversity order is due to the quasi-orthogonality of the effective channel matrices. A further research will be conducted to optimize the code matrices so as to achieve the full diversity order, or at least to reduce the degradation.

## VI. Conclusion and Future work

In this paper, we introduced three schemes that achieve superior performances as compared to the conventional LJJ scheme in terms of BER and diversity order all at the same spectral efficiency. The first two proposed schemes, namely  $X_3$  and  $X_4$ , employ orthogonal STBCs to generate the code matrices, resulting in orthogonal effective matrices, hence achieving the optimum diversity order of 3 and 4, respectively. The third proposed scheme, referred to as

$T_4$ , is superior to the conventional LJJ algorithm in terms of BER and diversity order, however, due to the quasi-orthogonality of the effective channel matrices, the diversity order of this scheme slightly lacks the optimum one.

As a future work, we plan to investigate the implementation of several quasi-OSTBCs with  $n_T > 4$  built on the described system in this paper. Besides, a careful analysis of the diversity gain will be carried out and methods to achieve the optimum diversity order will be introduced.

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