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# Multirate Ripple-Free Deadbeat Control

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To my parents who have been

a constant source of motivation, and support.

To my wife

To my Brother

To my sisters

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# Abstract

The design of multirate ripple-free deadbeat controllers is a complex and difficult task. The ripple-free deadbeat control problem can be solved using two approaches, the time domain approach and the polynomial approach. The time domain approach depends on a minimum energy solution and solves the problem in a state space setting. The polynomial approach depends on the solution of the Diophantine equation and solves the problem in a transfer function setting.

One approach which has shown promise for solving multirate ripple-free deadbeat control (MRFDC) problems is the use of Diophantine equation parameters. This thesis proposes a hybrid two degree of freedom controller for the fixed-order constrained optimization problem addressing performance and robustness specifications utilizing the parameters of Diophantine equation to build a multirate ripple-free deadbeat control (MRFDC). The salient feature of the proposed approach is that it combines the concept of multirate input which was demonstrated by Salgado and Oyarzun and use this concept in the single rate which was demonstrated by Paz. This research discusses the single rate input, then it proposes the multirate input using the parameterization of the Diophantine equations. Simulation results show that the output signal tracks the input sinusoidal signal in short settling time either in single rate or multirate ripple – free deadbeat control. The time domain specification for the output signal, control signal, error signal and the output of the filter signal are computed and satisfied that it was guaranteed the requirement and constraint.

## ملخص البحث

تصميم نظام التحكم المرهق الخالي من التموجات هو مهمة معقدة وصعبة. ونظام التحكم هذا بإمكاننا إيجاد الحلول له باستخدام طريقتين. إحداهما هي طريقة المجال الزمني والأخرى باستخدام مجال المعادلات. طريقة المجال الزمني تعتمد على الحل الذي يستهلك طاقة أقل ويحل المشكلة باستخدام معادلات فضاء الحالة (State Space). أما طريقة مجال المعادلات فانها تعتمد على حل معادلة الدايفنتين (Diophantine) ويحل المشكلة باستخدام معادلة النقل (Transfer Function) .

أحد هذين المجالات والذي يعرض لحل نظام التحكم المرهق الخالي من التموجات هو استخدام معادلة الدايفنتين. هذه الرسالة تعرض نظاما هجينا لنظام ثنائي حر الحركة من التحكم. السمة الظاهرة من المجال المقترح يجمع مابين مفهوم تعدد المعدلات والذي عرضه سلقادو وأويارزين واستخدام النظام الوحيد المعدلات المعروض من قبل باز. هذا البحث يناقش الدخل الوحيد المعدل ومن ثم يعرض الدخل متعدد المعدلات لمعادلة الدايفنتين. برنامج العرض والمحاكاة سيعرض نتائج الدخل والخرج في زمن قليل جدا سواء أكان هذا في الدخل وحيد أو متعدد المعدلات. خصائص المجال الزمني لإشارة الخرج وإشارة التحكم وإشارة الخطأ والخرج أيضا هو نظام محكوم بمتغيرات وثوابت ستعرض خلال الرسالة.

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# CHAPTER 1

# Introduction

## 1.1 Introduction

In recent years, control systems have assumed an increasingly important role in the development and advancement of modern civilization and technology. Practically every aspect of our day-to-day activities is affected by some type of control systems. Control systems are found in abundance in all sectors of industry, such as quality control of manufactured products, automatic assembly line, machine-tool control, computer control and many others [1].

A *control system* by definition consists of the system to be controlled – called the *plant* – as well as the system which exercises control over the plant, called the *controller*. A controller could be either human, or an artificial device. The controller is said to supply a signal to the plant, called the *input* to *the plant*, in order to produce a desired response from the plant, called the *output from the plant* [2].

The basic ingredients of a control system can be described by:

1- Objectives of control.
2- Control-system components.
3- Results or outputs.

We have two control systems categories, open loop and closed loop control systems. A control system, in which the control input is applied without the knowledge of the plant output, is called an open – loop control system as shown in Figure 1.1. The controller does not change according to the output.

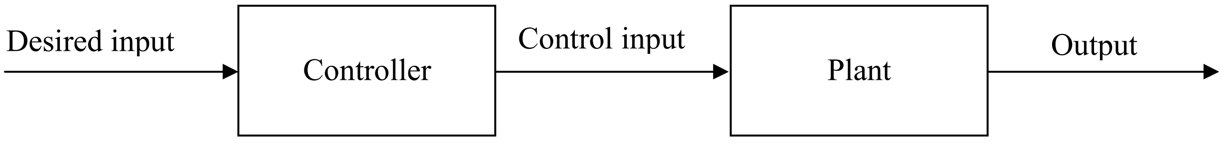


Figure 1.1: Open – loop control system

A control system in which the control input is a function of the plant's output is called a *closed- loop system* as shown in Figure 1.2.

Feedback control systems may be classified in a number of ways, depending upon the purpose of the classification. For instance, according to the method of analysis and design, control systems are classified as **linear** or **nonlinear**, and **time-varying** or **time-invariant.**

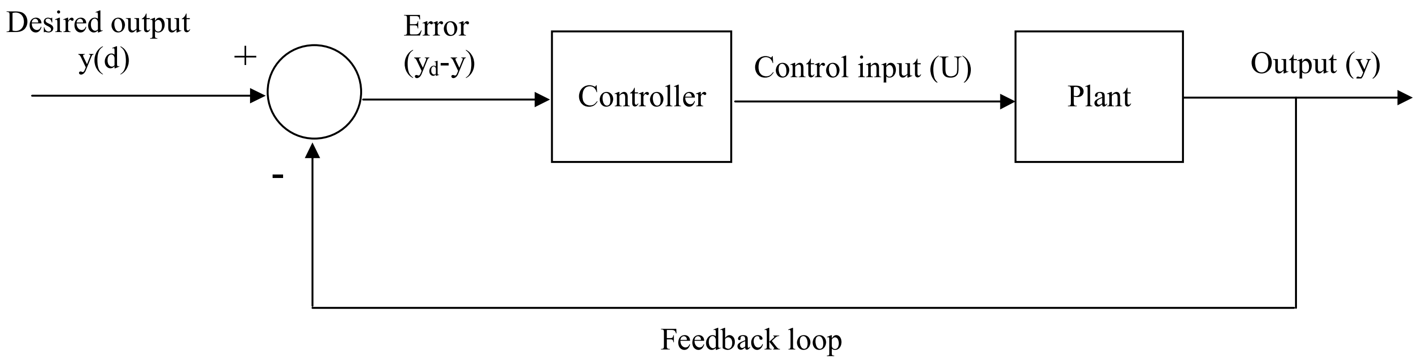


Figure 1.2: Closed – loop control system

According to the types of signal, found in the system, reference is often made to **continuous-data** or **discrete-data** systems.

A **continuous-data system** is one in which the signal at various parts of the system are all functions of the continuous time variable t. Examples of continuous-time signals include periodic, positive time transient, sinusoidal and random signals.

A continuous-time system is a mapping or an assignment of a continuous-time output signal for every continuous-time input signal.

Classically, a system has been illustrated by the block diagram shown in Figure 1.3, where x(t) represent the input signal, $T\,[\cdot]$ represents the mapping, and y(t) represents the output.

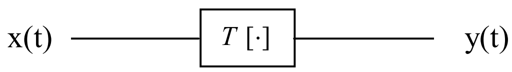


Figure 1.3: Block diagram of continuous-time system

A **discrete-data control systems** differ from the continuous-data systems in that the signals at all points of the system are in the form of either a pulse train or a digital code. A discrete-time signal is a sequence, that is, a function defined on the positive and negative integers. A discrete-time system is a mapping from the set of acceptable discrete-time signals called the input set, to a set of discrete-time signals called the output set. A discrete-time signal whose values are from a finite set is called a digital signal. A digital system is a mapping which assigns a digital output signal to every acceptable digital input signal [3].

*In this study*, **hybrid (continuous and discrete) system** is used. Also the system is closed loop, linear and time-invariant.

Today, almost all controllers are computer implemented. Consequently, the theory which is used to design digital controllers and explains the phenomena that occur is utmost importance. The usual configuration of computer controlled closed loop systems is given in Figure 1.4.

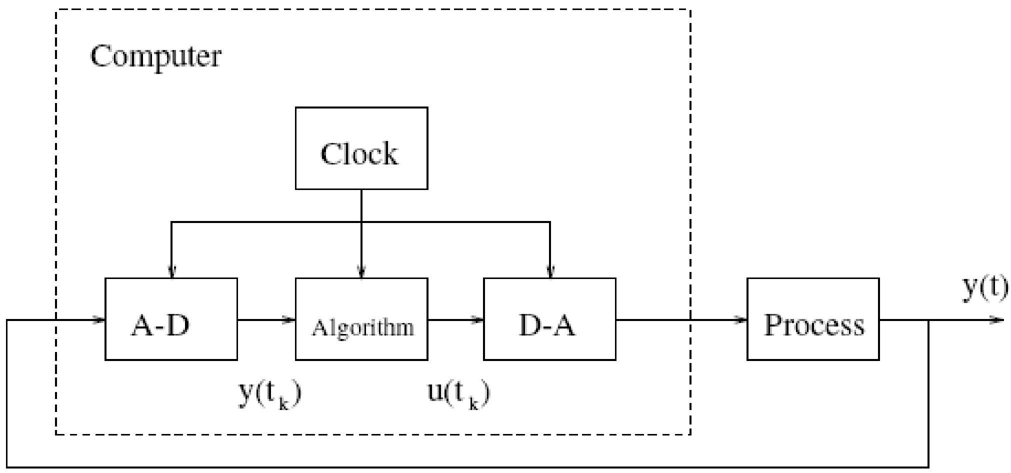


Figure 1.4: A computer controlled process

The output of the process y(t) is a continuous time signal. The measurements of the output signal into a digital signal - a sequence of measurements at sampling times $t_k$ . If a measurement device is itself digital, the measurements are taken at sampling times only and there is no need for an A-D converter. The sequence of numbers y($t_k$) is used by the control algorithm in order to compute a sequence of controls u($t_k$) - the digital control signal. The sequence is converted into a continuous time signal by a digital-to-analog (D-A) converter. Between the sampling instants the system is in open loop mode. Consequently, the inter-sample behavior is very often an issue and should not be disregarded. The system is synchronized by a real time clock in the computer.

One could develop a theory in a continuous time setting that takes account of the specific properties of the sampling process. From an applications point of view, it is often sufficient to understand the system's behavior at sampling instants only. The response between the sampling instants, being dictated by the open loop response of the system, can then be described in a secondary analysis to obtain a rather complete picture. This approach leads to a simpler analysis. Although it neglects to a certain degree the interaction between the continuous time response and the digital control design process, it often suffices to come to a good engineering control design.

The above given approach gives rise to discrete-time models, which are used to model the properties of the system at sampling instants $t_k$. Discrete-time models are described by sets of difference equations, which play the same role in discrete-time as differential equations in continuous time. Modeling of a sampled process given in Figure 1.4 is the main source of discrete-time models.

These models may also arise from identification, where we identify a model of a sampled plant. This method of obtaining discrete-time models is also motivated by computer controlled systems. We also point out that a number of processes, such as economic and biologic systems, radars, internal combustion engines, etc. are inherently discrete in time.

In practice, we have that all plants and processes are nonlinear. The most typical nonlinearity is saturation. It is present in every system, since it is never possible to deliver an infinite amount of energy to any real-world system. Computer implemented controllers are today a standard configuration. Basing the controller design on a linearized model may not yield desired performance or even not be possible at all. Indeed, we can not use linear control theory in cases where: large dynamic range of process variables is possible, multiple operating points are required, the process is operating close to its limits, small actuators cause saturation, etc [4] .

The characterization of linear control systems in the time domain are: Maximum Overshoot, Delay Time, Rise Time, Settling Time and Steady State Error.
*In this study*, **the settling time** will be optimized while satisfying other constraints.

As compared to analog control, digital control suffers from a reduced control loop bandwidth due to the presence of time delays inherent to the control structure. The improvement of digital control performance is an issue that needs to be assessed and solved in order to make of digital control a viable technological option.
Control systems are often designed with the objective that the output response should reach the desired reference value as quickly as possible and without any overshoot. This type of response is generally referred to as a deadbeat response.
Deadbeat control problem consists of finding what input signal must be applied to a system in order to bring the output to the step reference in the smallest number of time steps as shown in Figure 1.5.

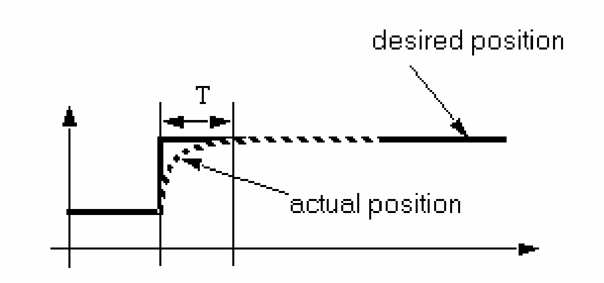


Figure 1.5: Deadbeat control problem

We can optimize digital control performance by using the deadbeat concept where the control variable is calculated ahead of time and in such a way that the error is canceled out after a fixed number of steps when the error can be represented as a polynomial. Typically, this technique relies on the model of the process, which obviously makes it also sensitive to model uncertainties. In addition, deadbeat algorithms can be computationally intensive and thus require extensive processor resources. Nevertheless, deadbeat control offers a much faster dynamic response than conventional control [5].

It is possible (for a first order process) to match the output function to the input function in one step. Trying to eliminate the error in one step can require an extremely high (power) gain. When this gain is excessive, we may use a less powerful controller [6].

The deadbeat-response design is characterized by the following design criteria:

1- The system must have zero steady-state error at the sampling instants for the specified input signal.
2- The time for the output to reach the steady state should be finite and minimum
3- The digital controller D(z) must be physically realizable; i.e., it must not have more zeros than poles

Note that poles and zeros of the plant are all inside the unit circle.

## 1.2 Motivation

Advancement in control systems theory has progressed at enormous rates over the past four decades. Not long ago; issues such as stability and performance were the topics of the hour. Issues such as multirate deadbeat, time delays and disturbance rejection could barely be addressed.

A huge knowledge in control research, intelligent control, mechanical and electrical engineering would be achieved.

It is well recognized that digital control provides various advantages over the usual time-invariant feedback controls. Deadbeat control makes a type of stabilization possible that cannot be achieved with continuous-time linear time-invariant feedback. We can also implement a much more complex logic in control actions making use of the recent advances in computer technology. For example, it is recognized that multirate sampling and generalized hold functions providing much greater capability in control [7]. Issues such as complexity, ease of implantation, economic cost, reliability play an important role in designing and selecting new controllers. The deadbeat controller which will be used in this thesis is efficient and powerful to get the best and optimization results.

The control of systems with time delay is a long standing problem which has important practical implications – especially for systems that have large time delays with respect to the plant dynamics. Digital controllers offer a natural and easy solution for controlling systems with time delay without restrictions on the complexity of the plant dynamics.

## 1.3 Literature Review

Multirate ripple-free deadbeat control had been discussed in many issues. H. Elaydi and R. A. PAZ [8] demonstrated an Optimal Ripple-Free Deadbeat Controllers for Systems with Time Delays. A ripple free deadbeat controller for a system with time delays was proposed. Matrix parameterization of the Diophantine equation was the approach which used to solve this problem. Based on this parameterization, LMI conditions were provided for optimal or constrained controllers with design quantities such as overshoot, undershoot, control amplitude, "slew rate" as well as for norm bounds such as $\ell_1, \ell_2$ *and* $\ell_\infty$. However, they didn't tack the multirate problem.

L. Jetto and S. Longhi [9] parameterized solution of the deadbeat ripple-free control problem for multirate sampled data systems. The purpose of this paper was to provide a parameterization of all causal feedback periodic controllers which guaranteed the deadbeat ripple-free behavior of the output of a linear time-invariant plant with a general multirate control scheme. However, they didn't tack the time delay problem.

H. Ito [10] improved performance of deadbeat servomechanism by means of multirate input control. A state-space approach to deadbeat servomechanism design was proposed using multirate input control. Multirate input mechanism yielded shorter settling time than single-rate control using the same frequency of sampling. However, multirate control often exhibited intersample ripple. Furthermore, the paper proposed a design method for multirate ripple-free deadbeat control which guaranteed robustness against continuous-time model uncertainty and disturbance. However, they didn't deal with various reference input signals and the input signal was a step signal only.

R. A. PAZ [11] proposed a ripple free tracking approach with robustness. A hybrid two-degree-of freedom (2DOF) controller for the fixed-order constrained optimization problem addressing performance and robustness specifications was proposed. This controller was given in terms of the solution of two Diophantine equations. However, he didn't tack the multirate problem.

M. E. Salgado and D. A. Oyarzun [12] proposed two objective optimal multivariables ripple free deadbeat control. A simple parameterization of all stabilizing ripple-free deadbeat controller of a given order was given. The free parameter is then optimized in the sense that a quadratic index is kept minimal. However, they didn't tack the time delay problem and they didn't deal with various reference input signals and the input signal was a step signal only and never dealt with robustness issue.

## 1.4 Contribution

This thesis presents methodologies for designing single rate and multirate ripple – free deadbeat controllers to solve the tracking of an arbitrary reference signal and the attenuation of general disturbances. Ripple – free deadbeat tracking is formulated based on the solution of the Diophantine equation. The ripple – free deadbeat tracking formulation is based on Paz results [11]. Moreover, I study the two objective optimal multivariable ripple-free deadbeat controls which are presented by Salgado and Oyarzun [12]. I combined the approach of multirate which demonstrated by Salgado and Oyarzun with Paz approach. The multirate ripple – free deadbeat control based on the solution of the Diophantine equation is proposed. The approach presented in this thesis can handle systems with time delays, where the time delay is not an integer multiple of the sampling time. A discretizing form of the system with the modeled time delay is obtained and used in the controller design.

## 1.5 Outline of the Thesis

This thesis contains five chapters: the first one talks about introduction and motivation of the project. The second chapter presents the Deadbeat Control concept. Third chapter shows the methodology and approach to solve Multirate Ripple-Free Deadbeat Control Problem. Chapter four is Simulation and Result. The final chapter concludes this thesis.

# CHAPTER 2

# Deadbeat Control

The purpose of this chapter is to emphasize the importance of the concept of deadbeat control. The main topic of this thesis, Multirate Ripple-Free Deadbeat Control (MRFDC), is introduced and motivated. Hybrid systems are discussed. A general discussion on multirate deadbeat control is provided. Also a delayed system will be studied. An overview of the existing literature dealing with deadbeat control is provided.

## 2.1 Deadbeat Control

The study of deadbeat control of discrete systems dates back to the early 1950's. Deadbeat control makes the output of the systems coincide with the reference input signal in a finite period of time. Deadbeat control achieves exact settling after a finite number of discrete sampling instants; however, there may exist ripple (non – zero deviation between the output response and the reference input signal) in the continuous plant between the discrete sampling instants. This intersample ripple is undesirable. There are two sources of inter – sample ripple: The first source is due to the failure of the design to cause the control signal to settle. This problem is due to the design allowing cancellation of the plant zeros. The second source of inter – sample ripple is due to the system being unable to track a moving reference between samples (lack of a continuous internal model). To obtain a ripple – free deadbeat design, a continuous internal model and cancellation of no plant zeros are required, so that the response has the zero ripple property [13].

In deadbeat control, the most important design parameter is the sampling period or the settling time; the settling time, the sampling period and the number of discrete steps that to settle are related. Designing a deadbeat controller with constraints requires some trade – off between these constrained parameters. A rough definition of minimum time deadbeat problem is the design of a controller which takes the system from any initial state ( output ) to the origin ( zero output ) in a minimum number of time steps.

Deadbeat control has been actively studied as an interesting area by many researches. One of the most attractive features is that the tracking error settles down to exactly zero in a finite number of control steps. The error between the plant output and the reference signal is made to decay to zero in a finite number of sampling intervals. The appropriate controller, in general, involves cancellation of both plant poles and zeros; extensions to handle open-loop-unstable and non minimum-phase plants are available. However, with this type of controller, the control signal may not attain its steady-state form in finite time; hence, ripples may appear in the plant output between sampling instants. This happens because cancellation of plant zeros by controller poles results in controller modes that may be excited by the reference signal but are unaffected by feedback. This problem is overcome by the so-called ripple-free deadbeat controller, which aims at finite settling times for both the error and control signals. Cancellation of only plant poles is involved; hence, non minimum phase plants are immediately accounted for.

On the other hand, a deadbeat control has three drawbacks in the practical use; the robustness, the disturbance rejection and the tracking performance in the transient response. Deadbeat controllers tackling the robustness of systems and robustness index was minimized based on the $L_2$ -norm of the sensitivity function [14] [15].

The deadbeat control is well known and widely used technique in high speed and high precision control. It is proven theoretically that the conventional single-rate deadbeat control could not guarantee zero tracking error for arbitrary reference signals. The reason is that the feed forward controller which is realized by inverse of the closed-loop system becomes unstable, because the discrete-time plant with zero-order hold has unstable zeros when relative degree of plant is greater than two [16] .

Conventional deadbeat controllers deal with fixed desired trajectories such as step or ramp function and followed them within several sampling time [17]. Deadbeat controller provides benchmark controller regarding settling time but has drawback such as big control signal which may lead to saturation and require too much power. Also, there may be ripples between sampling instances.

Conventional deadbeat current controllers show large sensitivity to plant uncertainties. In a trade-off between the bandwidth requirements and the stability, the conventional deadbeat controller should be designed with a lower equivalent feedback gains; this negates the deadbeat control performance; therefore, lower control bandwidth is yielded [18].

The terminology of "perfect tracking control (PTC)" means the plant output perfectly tracks the desired trajectory with zero tracking error at every sampling point. In the perfect tracking control, the tracking error of plant state becomes completely zero at every sampling period of reference input for nominal plant without disturbance. Moreover, by combining the proposed feed forward controller with a robust feedback controller, high tracking performance is preserved even if the plant has modeling error and disturbance [16].

## 2.2 Hybrid Systems

The use of digital microprocessors to compute a control action for continuous time dynamic systems requires the fundamental operation of sampling. Continuous time physical signals, such as position, velocity, temperature, etc., are sampled, then the control action is computed based on these samples in a closed – loop setting. Such systems are called hybrid systems, where discrete signals appear in some parts of the systems and continuous signals appear in some other parts. Since some continuous data is sampled before being used such systems may be referred to as sampled – data systems. These type of systems have been investigated since the early 1950's.

The sampled – data (hybrid) system usually has five parts: the continuous plant, the analog to digital converter (A/D), the digital to analog converter (D/A), the discrete controller, and a clock for synchronization. The analysis of a sampled – data systems is more complicated than the analysis of a purely discrete system or a purely continuous system. Therefore, the role of sampling and the conversion from continuous to discrete and back from discrete to continuous is very important in understanding digital control.

In 1950, Porter and Stoneman showed that it is possible to reconstruct a continuous signal from discrete one with relatively small error. The time between successive sampling is called the sampling period. The choice of the sampling period is very important to the behavior of sampled – data systems. Nyquist and Shannon made great contributions to theory of sampled signals. The development of the z – transform theory in the 1940's and

1950's by Hurewicz, Lawden, Barker, and Ragazzini and Zadeh [19] [20], contributed greatly to the development of analysis and design tools.

Hold devices are necessary part in the reconstruction of continuous signals. Ragazzini and Zadeh [21] presented a clear discussion of hold devices and demonstrated the zero – order hold. The zero – order hold is the most common reconstruction device, and is considered for use in this thesis.

Jury [22] contributed greatly to sampled data systems theory. He credited Barker for the development of the modified Z – transforms. He presented the stability analysis of closed loop systems and discussed time root locus.

## 2.3 Multirate Digital Control

Multirate digital control is a significant area of current research and application that is motivated by practical implementation needs. The motivation for multirate control has traditionally been in aerospace applications where guidance and control laws must be designed to accommodate multiple rates of sensor measurements and finite throughput capabilities of on board computers. Multirate design techniques should soon find further utility in control applications for highly distributed systems, such as communication networks, and power-plant or power-distribution networks where the characteristic frequencies and time-constants of a local station's dynamics may differ significantly from those of the network as a whole.

We refer to multirate digital control as digital systems in which the output sampling rate differs from the input sampling and/or internal processing rate. Multirate digital control also have coefficients which can be implemented using fewer bits than are required for single rate control , thus making them more attractive for minimum microcomputers, where word length is limited.

## Historical Background

A historical overview of digital control development is presented in Figure 2.l. The field of digital control, or more precisely, the sampled data control, originated in radar applications during World War II. Because the rotating antenna of a radar system illuminates a target only intermittently, early radar-aided tracking and fire-control systems had to bed designed to utilize data in sampled form. Methods for effective design of control systems using sampled data were under initial development during the later 1940's, and multirate systems theory followed these efforts in the early 1950's.

Initially, researchers developed multirate techniques as a method of evaluating more conventional types of controllers such as continuous systems and single-rate sampled data systems. For example, one could study the intersample behavior of a signal or output of a single rate control system by introducing a "phantom sampler" (i.e., a fictitious sampler that operates at a rate some integer ratio higher than that of the controller). A significant early contribution to this general method of analysis, known as frequency decomposition, was made by Sklansky and Ragazzini who described the use of this technique in error-sampled control system development [23]. In the late 1950's, Ragazzini and Franklin published a textbook that described both this technique and the closely related switch decomposition technique [24]. Friedland later related the frequency decomposition technique to periodically varying control structures, followed by contributions of Coffey and Williams and Boykin and Frazier which dealt with the analysis of multi loop, multirate control structures (multi loop) referring to a feedback control structure having nested single-input/single-output compensating elements)[24].

Shortly following the origin of the frequency decomposition technique, a similar frequency domain technique known as switch decomposition was developed. Researchers had begun to see the potential value of multirate systems beyond being a technique for analyzing single-rate systems; switch decomposition seemed a "natural" approach to developing such systems. The switch-decomposition technique attributed to Kranc[25], provided a means of representing a multirate control structure as an equivalent single-rate controller; this representation accomplished, the controller could be designed and analyzed using existing single rate techniques. In the late 1960's, Jury showed an equivalence of the switch decomposition technique and the frequency decomposition technique [26].

Recently, Whitbeck has developed a vector form of the switch decomposition technique and applied it to various problems in flight control. Time-domain methods of multirate stability analysis and design were initiated by Kalman and Bertram with the publication of

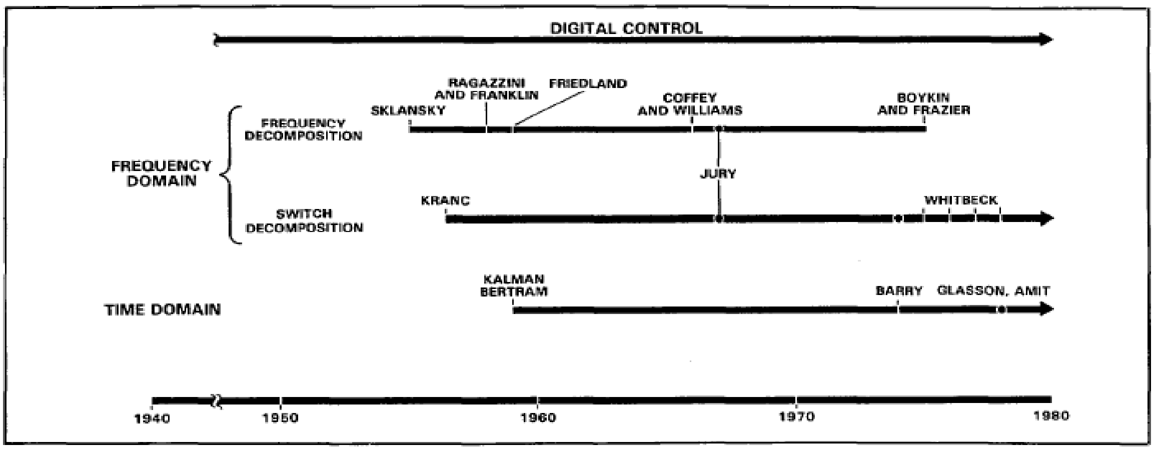


Figure 2.1: Development of multirate digital control

their state space stability analysis technique in 1959. This paper made a major contribution in showing the power and flexibility of state space techniques in characterizing many types of sampled data control systems, including time-varying systems. Apparently, little significant work was initiated to build on this work for nearly fifteen years. Barry published a paper in 1975 in which he described the design of a multirate regulator and showed that its performance was superior to a single rate regulator having the same base (slow) sample rate [27]. During 1979-81, researchers at The Analytic Sciences Corporation (TASC) developed a new multirate control design technique based on an optimal estimation and control formulation. These researches included mathematical formulation of the design problem, development of computational design techniques, and applications of these techniques to flight control examples [24].

To obtain a high performance controller for a multirate system, it is necessary to consider the continuous-time, i.e. hybrid nature of the problem. That is, the design should be a direct sampled data design, where a discrete-time controller is to be designed to satisfy performance objectives in terms of continuous-time closed loop mappings [28].

The solution of several control problems for multirate sampled data systems can be derived from the theory developed for linear periodic discrete time systems. If the original continuous-time plant is linear and time invariant, the corresponding multirate sampled data system has a linear, periodic, discrete-time state space form.

For single-rate sampled data systems, state and output deadbeat control problems were extensively studied in the framework of discrete-time systems. However, in the case of multi-rate sampled data systems, an exact output tracking between sampling instants is required, since the original system originates from a continuous time one. In fact, even if an exact tracking at sampling instants is obtained, undesired intersampling ripples may exist on the continuous-time output. A parameterization of all causal output feedback periodic controllers guarantees the deadbeat ripple free behaviors of the output of a multirate sampled data plant. A polynomial approach is adopted and the parameterization is provided in terms of the general solution of a suitably defined Diophantine equation [9].

## 2.4 Time-scales of multirate systems

The time-domain analysis of multirate (MR) sampled-data systems is governed by two periods of particular significance [29], which will be referred to as the ‘short time interval (STI) and the repetitive time interval (RTI). Representing the shortest possible interval between successive sampling operations, the STI is the time frame with respect to which system models are discredited initially, whereas the RTI constitutes one cycle of the overall sampling sequence.

The MIMO plant to be considered is depicted in Figure 2.2, in which the respective sampling intervals of the control and output variables are $T/L_j \, s, L_j \in \mathbb{Z}, j = 1,2.....,l$, and $T/M_i \, s, M_i \in \mathbb{Z}, i = 1,2.....,m$, and where it is assumed that:

$(i)$ $\qquad L_j \le L_{j+1}$ (2.1)

and

$M_i \le M_{i+1}$ (2.2)

( ii ) each control signal is applied via a zero – order hold (ZOH) with transfer function

$$H^{(L_j)}(s) = \frac{1-e^{-sT/L_j}}{s} \tag{2.3}$$

The aforementioned time – scale then are specified as:

$$STI = \frac{T}{lcm(L_1, L_2, ...., L_l, M_1, M_2, ...., M_m)} = \frac{T}{N} \tag{2.4}$$

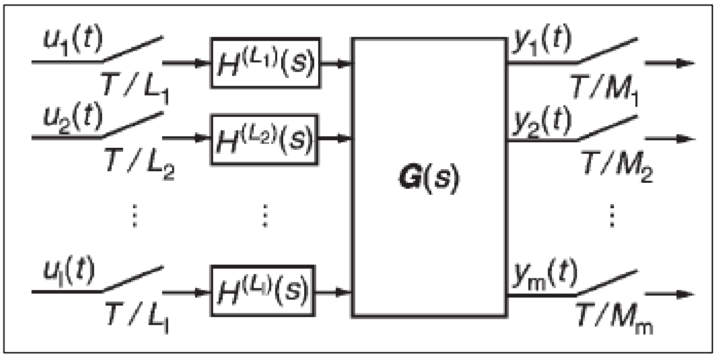


Figure 2.2: Multirate – sampled MIMO plant

and

$$RTl = \frac{T}{\gcd(L_1, L_2, \ldots.., L_l, M_1, M_2, \ldots.., M_m)} = T \tag{2.5}$$

It is also appropriate at this juncture to designate the parameter $\bar{L}_j$ *and* $\bar{M}_i$ , which are related to each $L_j$ *and* $M_i$ *thus* :

$$\bar{L}_j = N / L_j \tag{2.6}$$

and

$$\bar{M}_i = N / M_i \tag{2.7}$$

# CHAPTER 3

# Methodology and Approach to solve Multirate Ripple-Free Deadbeat Control Problem

## 3.1 Introduction

The problem of tracking a general reference signal in a deadbeat fashion for continuous, LTI, SISO Multirate ripple-free deadbeat control systems with time delays is considered. We give a design procedure for a controller under which the output of the closed-loop system exactly coincides with the reference signal after a fixed (finite) time. The design provided here allows for constraints on control magnitude as well as on many time domain properties such as overshoot, undershoot, slew rate, and also on such system norm quantities as $\ell_1, \ell_2, \ell_\infty$ *and* $H_\infty$ norms.

The solution to the Ripple Free Deadbeat problem with constraints on various time domain properties was introduced in terms of the Diophantine equations considering time delayed systems and general reference signals.

The approach considered here provides a unified framework for optimization of the internally stable ripple free deadbeat controller. Using the solution of the Diophantine equations, the optimization is flexible and efficient, unlike previous approaches at optimizing deadbeat systems. Joint minimization and constraints are readily considered [8] [20].

## 3.2 Diophantine Equation Parameterization

The Diophantine equation plays an important role in the design and synthesis of controllers in the frequency domain. The Diophantine equation has an infinite number of solutions that all provide an internally stabilizing controller. However, the Diophantine equation in a polynomial form masks its design freedom. A parameterization of the Diophantine equation is obtained, allowing simple access to the degrees of freedom. Polynomial multiplication and division is given as matrix multiplication. The parameterization of the Diophantine equation is based on obtaining a matrix equation with the two unknown expressed in matrix form [15].

## Methods for solving the Diophantine equation

Given the two polynomials, A(q) and B(q), with

$$A(q) = a_o q^n + a_1 q^{n-1} + a_n, \text{ and } a_o \neq 0 \tag{3.1}$$

$$B(q) = b_o q^m + b_1 q^{m-1} + b_m, \text{ and } m < n \tag{3.2}$$

and given two more polynomials $Q_n(q)$ *and* $Q_d(q)$, $C(q)$ is defined as

$$C(q) = A(q)Q_n(q) + B(q)Q_d(q) \tag{3.3}$$

The polynomial equation (3.3) is called the Diophantine equation. This equation is linear in terms of the polynomials $Q_n(q)$ *and* $Q_d(q)$ for the known polynomials $A(q), B(q)$ *and* $C(q)$, where $A(q)$ *and* $B(q)$ are coprime. The coprimeness of $A(q)$ *and* $B(q)$ guarantees the existence of a solution to the above equation for any arbitrary $C(q)$. There are several methods for solving the Diophantine equation such as: Euclidean algorithm, Sylvester's resultant, Bezout's resultant, and MacDuffee's resultant [20] [30].

## 3.3 A Matrix Parameterization of the Diophantine Equation

The solution of the Diophantine equation, using the resolving matrix is not compact for optimization purposes. As a result, searching for optimal solution is very complicated. The former parameterization does not lend itself well characterizing control constraint conditions that arise in practice. Since the Diophantine equation has two polynomial products, the vectorization is a convenient method to express the equation. The Diophantine equation of the tracking problem is parameterized, and then the Diophantine equation of the disturbance rejection problem is also parameterized.

## A Matrix Representation of Polynomial Products

Suppose the polynomials A(q) and B(q) are given such that [31][20]

$$A(q) = a_0 + a_1 q + a_2 q^2 + ...... + a_m q^m \tag{3.4}$$

$$B(q) = b_0 + b_1 q + b_2 q^2 + ...... + b_n q^n \tag{3.5}$$

The vectorized form of the polynomials is defined as

$$\vec{A} = \begin{bmatrix} a_0 \\ \vdots \\ a_m \end{bmatrix}, \vec{B} = \begin{bmatrix} b_0 \\ \vdots \\ b_n \end{bmatrix} \tag{3.6}$$

Indeed, any polynomial may be vectorized this way. This vectorization may be expressed as an operator $(\vec{\bullet})$. The expanded matrix form is also defined as

$$\overline{A}_p = \begin{bmatrix} \vec{A} & & 0 \\ & \ddots & \\ 0 & & \vec{A} \end{bmatrix} \in \mathbb{R}^{m+p+1\times p}, \ \overline{B}_p = \begin{bmatrix} \vec{B} & & 0 \\ & \ddots & \\ 0 & & \vec{B} \end{bmatrix} \in \mathbb{R}^{n+p+1\times p} \tag{3.7}$$

The following result is given, the proof of which is by simply multiplying the polynomials and gathering the coefficients of each power of $q$.

**Lemma 3.1:** The following hold

$$\left(\overrightarrow{AB}\right) = \overline{A}_{n+1}\vec{B} = \overline{B}_{m+1}\vec{A} \tag{3.8}$$

Lemma 3.1 illustrates the fact that the vectorization of a polynomial product may be written in terms of a matrix product. In a comparable way, a polynomial division may also be written as a matrix equation

Assuming now (without loss of generality) that B(0)=1, the polynomial division can be considered now

$$C(q) = \frac{A(q)}{B(q)} = c_0 + c_1 q + c_2 q^2 + ..... \qquad (3.9)$$

The right hand side of equation (3.9) is the Maclauren series expansion of the function and thus has an infinite number of terms. Noting that equation (3.9) may also be written as

$$A(q) = B(q)C(q) \qquad (3.10)$$

Noting that the left hand side of equation (3.10) has at most $m+1$ non zero terms, and thus the right hand side must also. Thus, restricting the attention to a finite version of the sequence, the approach now considers the first N coefficients of C. The truncated version of this can be written as

$$C_N(q) = c_0 + c_1 q + .... + c_{N-1} q^{N-1} \qquad (3.11)$$

and the following result is stated.

**Lemma 3.2:** For the polynomial equation, $A(q) = B(q)C(q)$, the first N coefficients of C may be computed by

$$\vec{C}_N = B_x^{-1} \begin{bmatrix} \vec{A} \\ 0_{N-m-1\times 1} \end{bmatrix} \qquad (3.12)$$

where $B_x$ is obtained from the decomposition

$$\begin{bmatrix} B_x \\ B_y \end{bmatrix} = \overline{B}_N, B_x \in \mathbb{R}^{N \times N}, B_y \in \mathbb{R}^{n+1 \times N} \tag{3.13}$$

## 3.4 Solving Ripple-free control problem with meaning of finding Diophantine equations Parameters

### 3.4.1 System demonstration:

The 2DOF hybrid system as shown in Figure 3.1 with the hybrid control system shows the continuous-time plant $G(s)$ and the 2DOF discrete time controller $C(q)$ [where $(q = z^{-1})$ ] and z is the Z-transform variable along with a tracking model $M(s)$ [32].

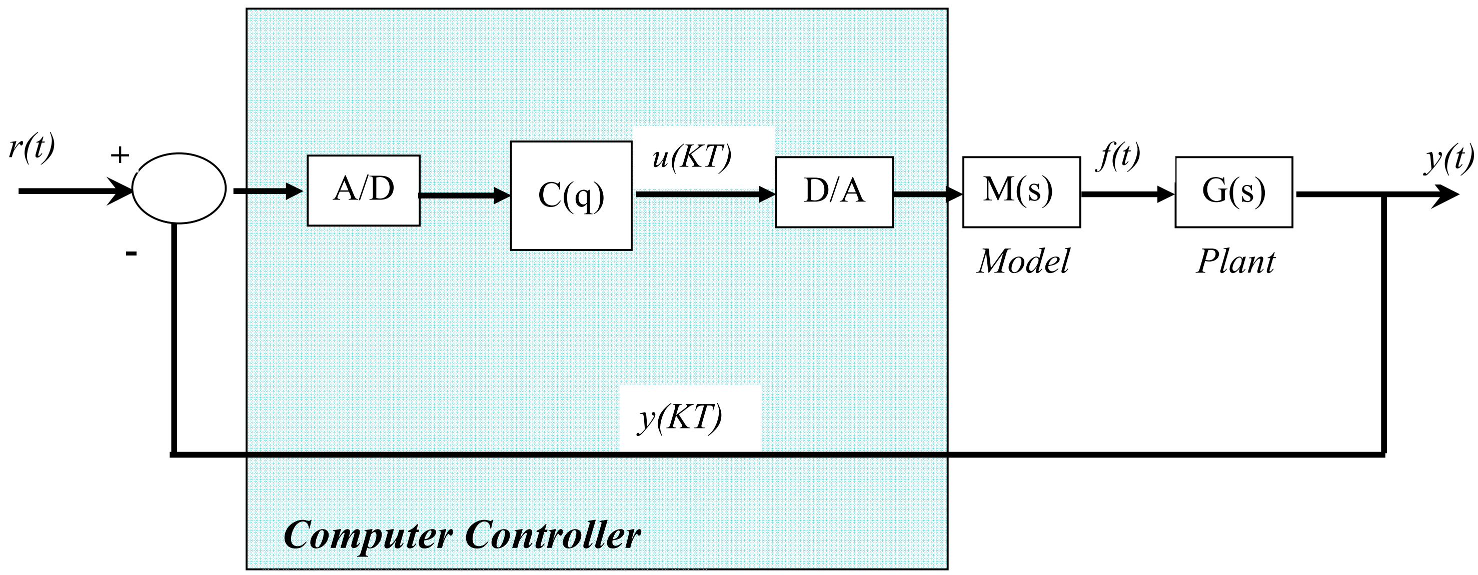


Figure 3.1: Hybrid 2DOF control configuration.

We assume a plant of the form

$$G(s) = e^{-s\tau_d} \frac{N_{gc}(s)}{D_{gc}(s)} \tag{3.14}$$

where $\tau_d \geq 0$ is the time-delay of the system and the rational function $\frac{N_{gc}(s)}{D_{gc}(s)}$ is strictly proper, and $N_{gc}$ and $D_{gc}$ are coprime.

The system is designed to track the reference signal, $r(t)$, that has a real, rational Laplace transform

$$R(s) = \frac{N_{rc}(s)}{D_{rc}(s)} \tag{3.15}$$

The signal which to be tracked is periodic, so the signal has a pole on the imaginary axis. The continuous-time filter, $M(s)$ (the tracking model filter), is a requirement that ensures the exact tracking of the reference signal $r(t)$ with no ripple after the transient period [33]. This filter

$$M(s) = \frac{N_{mc}(s)}{D_{mc}(s)} \tag{3.16}$$

is intended to match the dynamics of the reference signal. This system is to be hybrid since the inclusion of both $M(s)$ which an analog filter is and $C(q)$ which a digital controller is. In a hybrid setting, with sampling period T, we obtain the discrete model of the plant transfer function, $P(q)$, using zero-order hold

$$P(q) = Z\left\{\frac{1-e^{-sT}}{s} G(s) M(s)\right\}\Big|_{q=z^{-1}} = \frac{N_p(q)}{D_p(q)} \tag{3.17}$$

with $\deg(N_p) = m$ and $\deg(D_p) = n$. Note that because of the presence of a time delay in $G(s)$, it may be necessary to employ modified transforms in obtaining this model. For analysis purposes, we assume that the noise $w = 0$.

### 3.4.2 The ripple-free deadbeat control problem definition

The ripple-free control problem (RFCP) has several goals [15].

1. The closed-loop system is internally stable.
2. The error of the system $e(t) = r(t) - y(t) = 0$ for all $t \geq N_s T$ , where $N_s$ is the number of steps to settle.
3. The control signal $u(KT)$ settles down after a finite number of steps. (In the event that $M(s) = 1$, then $u(KT)$ settles to a constant.)

It may be necessary to impose constraints on the transient response that arise from practical implementation issues.

To find the control signal for the 2DOF system we will discretize the reference signal $R(s)$, the plant $P(s)$ and the filter $M(s)$ then rearrange the block diagram for the system as shown in Figure 3.2. The controller may be realized using the configuration shown in Figure 3.2.

We note that $N_1(q)$ *and* $N_2(q)$ may be implemented as FIR filters. A polynomial in $q$ corresponds to a rational function in z with all the poles at the origin, where ($q = z^{-1}$).

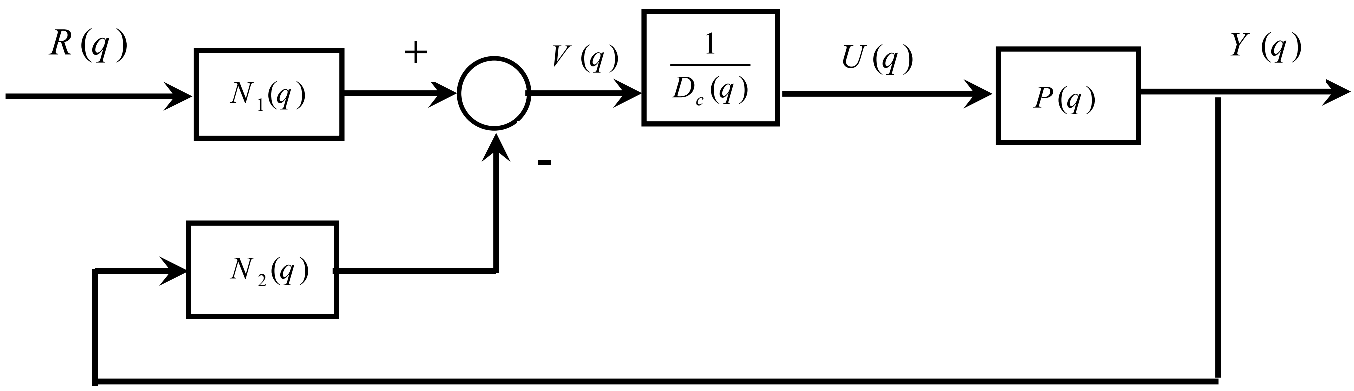


Figure 3.2: Implementation of the Deadbeat controller.

Now we can find the 2DOF control law as the following form

$$U(q) = \frac{1}{D_c(q)} V(q) \tag{3.18}$$

$$V(q) = N_1(q) R(q) - N_2(q) Y(q) \tag{3.19}$$

From (3.18) and (3.19), we get the transfer function of the control signal $U(q)$ as follow:

$$U(q) = \frac{N_1(q)R(q) - N_2(q)Y(q)}{D_c(q)} \qquad (3.20)$$

### 3.4.3 Solution for ripple-free deadbeat control problem (RFDCP)

The controller polynomials are obtained by solutions of the Diophantine equations [11]

$$N_p(q)N_1(q) + D_r(q)Q_1(q) = 1 \qquad (3.21)$$

$$N_p(q)N_2(q) + D_p(q)D_c(q) = 1 \qquad (3.22)$$

where $Q_1$ is a polynomial.

The solution of the RFCP requires the solution of two Diophantine equations. Any $N_1, N_2$ and $D_c$ that satisfy the above Diophantine equations provide a solution to the RFCP. Since the Diophantine equation has an infinite number of solutions, we will seek specific solutions that provide desired transient behavior and robustness.

In general, Diophantine equations such as (3.21) and (3.22) have unique, minimum-order solutions. Thus, there exist unique $N_1 \min(q), Q_1 \min(q), N_2 \min(q)$ *and* $D_c \min(q)$ that solve (3.21) and (3.22) with

$$\deg(N_{1\min}) < \deg(D_r),\ \deg(Q_{1\min}) < m \qquad (3.23)$$

$$\deg(N_{2\min}) < n,\quad \deg(D_{C\min}) < m \qquad (3.24)$$

Our two Diophantine equations have solutions that may thus be parameterized by

$$N_1(q) = N_{1\min}(q) - D_r(q)\nu_1(q) \qquad (3.25)$$

$$Q_1(q) = Q_{1\min}(q) + N_p(q)\nu_1(q) \qquad (3.26)$$

$$N_2(q) = N_{2\min}(q) - D_p(q)\nu_2(q) \qquad (3.27)$$

$$D_c(q) = D_{c\min}(q) + N_p(q)\nu_2(q) \qquad (3.28)$$

While $v_1$ *and* $v_2$ are arbitrary polynomials of degrees that define the degree of freedom in the design or free parameters.

The number $N_a = \deg(v_1)+1$ defines the number of steps above the minimum order solution for the first Diophantine equation and so we have $N_a$ free parameters introduced in this equation. Here, we define $\deg(0)=-1$. Likewise with the second Diophantine equation, we have $N_b = \deg(v_2)+1$ defining the number of free parameters. If we applied the controller in (3.26), we obtain the transfer function

$$\frac{Y(q)}{R(q)} = \frac{\left(N_1/D_c\right)\left(N_P/D_P\right)}{1+\left(N_2/D_C\right)\left(N_P/D_P\right)} = \frac{N_1 N_P}{N_2 N_P + D_P D_C} = N_1 N_P \tag{3.29}$$

by (3.20) we also have the error signal

$$\begin{aligned} E(q) &= R(q) - Y(q) = (1 - N_1 N_P) R(q) \\ &= (D_r Q_1)\frac{N_r}{D_r} = N_r Q_1 \end{aligned} \tag{3.30}$$

Which is a polynomial implying that $e(KT)=0 \; for \; K \geq N_s = \deg(N_r Q_1)$. Now we define the polynomial

$$D_{P0}(q) = \frac{D_P}{D_r} \tag{3.31}$$

which looks like a rational function, but is actually a polynomial. Using 3.22 when $N_p(q)N_2(q)+D_p(q)D_c(q)=1$ , we have the control signal transfer function

$$\frac{U(q)}{R(q)} = \frac{\left(N_1/D_C\right)}{1+\left(N_2/D_C\right)\left(N_P/D_P\right)} = \frac{D_P N_1}{N_P N_2 + D_P D_C} = D_P N_1 \tag{3.32}$$

again by (3.22), We thus have the control signal

$$U(q) = D_P N_1 \frac{N_r}{D_r} = D_{P0} N_r N_1 \tag{3.33}$$

which is a polynomial. This implies that the control signal $U(kT) = 0$ *for all* $k \geq N_u = \deg(D_{p0} N_r N_1)$.

## 3.5 Multivariable ripple-free deadbeat control

Salgado and Oyarzun [12] presented a new method to demonstrate a multivariable ripple-free deadbeat control. Their study was applied to discrete-time, stable, linear and time invariant plant model. A simple parameterization of all stabilizing ripple-free deadbeat controllers of a given order was considered. The free parameter was then optimized in the sense that a quadratic index is kept minimal. The optimality criterion had the advantage of accounting for both tracking performance and magnitude of the control effort.

A control strategy leads to settle the tracking error sequence to zero in a minimum number of time steps, which is sometimes termed as minimum prototype control.

Since this formulation is based on pole zero cancellations between controller and plant model, it has problems when dealing with non minimum phase and unstable plants but this problem is solved later by many conditions and authors. All previous approach didn't consider the inter sample behaviors and undesirable ripple which may appear in the output. This issue is dealt within [15,33], where fair general parameterizations of ripple-free deadbeat controllers are given. The basic idea behind this approach is in order to avoid any intersample ripple after the settling time; the control sequence must also reach its steady state in, at most, the same number of samples.

Salgado and Oyarzun [12] dealt with optimal ripple-free deadbeat control for MIMO, stable, linear and time invariant plants. First, a simple characterization of ripple-free deadbeat controllers was derived for stable plants and constant reference signals. Despite our characterization is less general than that of [33], it proves to be useful in solving an optimal control problem that minimizes a two objective performance measure. This

measure accounts for the energy of the tracking error, which has been widely used as a performance measure [34], and also for the energy of the control signal.
This implies that the derived control law exhibits ripple-free deadbeat behaviors while attaining an optimal performance in terms of both tracking behaviors and control effort. We stress that the latter property is important from an application point of view and that the excessive control magnitudes, typical from deadbeat control, have been one of the main criticisms to this control technique. In this research, we introduce the novelty of dealing with a combined optimality criterion and ensuring a MIMO ripple-free deadbeat response.

### 3.5.1 Assumptions and definitions

We consider the plant model $G(z)$ to be a stable $p \times p$ discrete-time transfer matrix. We will assume that $G(z)$ is represented in right coprime polynomial matrix fraction description as

$$G(z) = B(z)A(z)^{-1} \tag{3.34}$$

where $A(z)$ *and* $B(z)$ are right coprime polynomial matrices of dimension $p \times p$. We can find methods which can be used to build coprime polynomial factorizations of transfer matrices in [35, 36, and 37].

We assume that $G(z)$ has no zeros on $z = 1$, $i.e.$ $B(1)$ is nonsingular and, without loss of generality, we further assume that $B(1) = I$. The non singularity of $B(1)$ is a standard condition necessary for being able to track constant reference signals.

Given a proper transfer matrix $M(z)$, we define the *right degree interactor* (**RDI**) of $M(z)$ as a polynomial matrix $E(z)$ such that the product $M(z)E(z)$ is biproper, i.e.

$$\lim_{z \to \infty} M(z)E(z) = D \tag{3.35}$$

where $0 < \det\{D\} < \infty$.

Algorithms to build different types of **RDI** matrices can be found in [38].

## 3.5.2 Deadbeat control essentials

Consider a continuous-time plant with transfer function $G_c(s)$ which is digitally controlled through a zero order sample and hold device with transfer function $G_{ho}(s)$, by a linear discrete-time feedback controller with transfer function $C(z)$. We can present sampled data control loop system in the block diagram as shown in Figure 3.3.

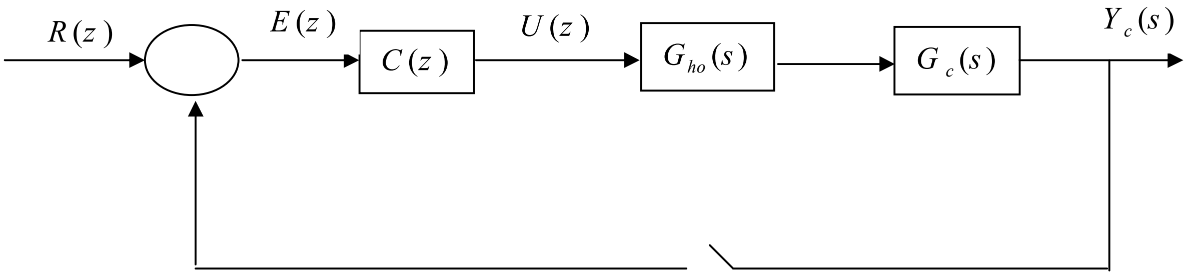


Figure 3.3: Multivariable Sampled data control loop

Now, we will find the transfer function of the sampled data open loop system as follow:

$$G(z) = \mathrm{Z}\{G_{ho}(s)G_c(s)\} = B(z)A(z)^{-1} \tag{3.36}$$

where $B(z)$ *and* $A(z)$ are right coprime polynomial matrices. Given that the reference vector signal is assumed to be step function, i.e. $r(k) = \upsilon\mu(k), \upsilon \in \mathbb{R}^p$, then having a ripple-free deadbeat control loop means that $y_c(t)$ satisfies

$$y_c(t) = \upsilon \quad , \forall t > N\Delta \tag{3.37}$$

where $N \in \mathbb{N}$ is called the deadbeat horizon of the control system. A controller will be designed to achieve a ripple-free deadbeat class which provides perfect steady state tracking at D.C. and makes the output of the plant to settle in a finite number of samples, while avoiding any intersample ripple beyond the deadbeat horizon. Ripple in a deadbeat response arises when the controller cancels the minimum phase zeros of $G(z)$. Those cancelled zeros appear as closed loop poles and generate the intersample response of the continuous-time output. The response of the system can be shown in Figure 3.4, where it is noticeable that, although the sampled output settles in one sample, the continuous-time output exhibits considerable ripple. The discrete-time model $G(z)$ usually contains sampling zeros located in the negative real axis; therefore its cancellation leads to oscillatory modes in the deadbeat response.

To avoid the intersample ripple a sufficient condition must be applied to the control sequence which settles in N samples. This condition is equivalent to

$$u(k)=u_{ss}, \quad \forall k > N \tag{3.38}$$

where the constant vector $u_{ss}$ is the steady state value of the control sequence.

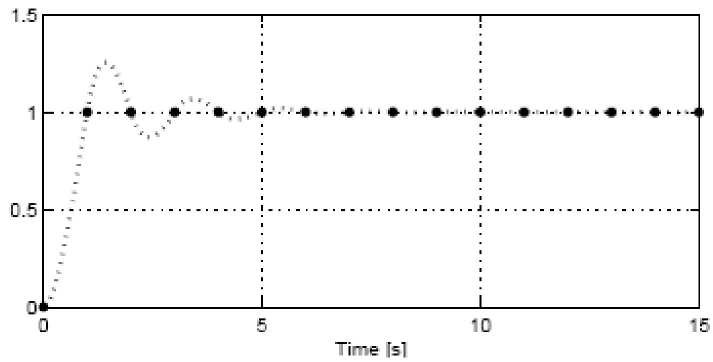


Figure 3.4: Continuous-time deadbeat response with ripple.

### 3.5.3 Characterization of MIMO ripple-free deadbeat controllers

The ripple-free condition (3.38) implies that the control sensitivity must have the form

$$S_u(z) = \frac{K(z)}{z^N} \tag{3.39}$$

where the deadbeat horizon $N \in \mathbb{N}$ *and* $K(z)$ is a polynomial matrix with degree such that $S_u(z)$ is proper. On the other hand, the tracking error signal must also settle in a finite horizon, so that the complementary sensitivity function

$$T(z)=G(z)S_u(z)=B(z)A(z)^{-1}\frac{K(z)}{z^N} \qquad (3.40)$$

must have all its poles at $z=0$. This means that $K(z)$ must be factored as $K(z)=A(z)V(z)$ with $V(z)$ being a polynomial matrix. Thus

$$S_u(z)=\frac{A(z)V(z)}{z^N} \qquad (3.41)$$

For convenience, we set $N=n+\ell$ *where* $n$ is the degree of $A(z)$ *and* $\ell\in\mathbb{N}_0$. Let also $V(z)$ be written as $V(z)=E(z)W(z)$, *with* $E(z)$ being a **RDI** of $A(z)/z^n$, hence making $A(z)E(z)/z^n$ biproper. These definitions lead to

$$S_u(z)=\frac{A(z)}{z^n}E(z)\frac{W(z)}{z^\ell} \qquad (3.42)$$

from where we have that the properness of $S_u(z)$ depends on the properness of $W(z)/z^\ell$. It is worth noting that the degree of $A(z)$ is always equal to that of $A(z)E(z)$. The form of $T(z), S_u(z)$ and the multivariable deadbeat controller $C(z)$ can then be obtained simply as

$$T(z)=\frac{B(z)E(z)W(z)}{z^N} \qquad (3.43)$$

$$S_u(z)=\frac{A(z)E(z)W(z)}{z^N} \qquad (3.44)$$

$$C(z)=S_u(z)\left(I-T(z)\right)^{-1} \qquad (3.45)$$

$$C(z)=A(z)E(z)W(z)\left(z^N I-B(z)E(z)W(z)\right)^{-1} \qquad (3.46)$$

Since we also need perfect steady state tracking of constant references, we must force $T(1)=I$, which, using (3.43) and the fact that $B(1)=I$, implies that $W(1)=E(1)^{-1}$. The controller given in (3.46) is then a general form of a MIMO deadbeat controller for stable plants and constant reference signals. We need to build the **RDI** $E(z)$ which will be used in (3.46). Using this formulation we have the advantage to provide a unitary $E(z)$ that also

Satisfies $E(1)=I$ , which certainly simplifies the condition imposed on $W(z)$ to $W(1)=I$ . In the sequel, we will always assume that $E(z)$ is a unitary **RDI** [38].

Moreover, from (3.44) it is clear that since $N=n+\ell$ , then the minimum deadbeat horizon is $N_{\min}=n$ , that is, the degree of $A(z)$ . This is the multivariate version of the fact that for SISO systems, the minimum deadbeat horizon is given by the plant order. Next lemma gives a characterization of all polynomial matrices $W(z)$ that yield the minimum horizon ripple-free deadbeat controller.

**Lemma 3.3:** Consider a stable transfer matrix $G(z)$ and the MIMO deadbeat controller of (3.44). Then the minimum horizon deadbeat controller is given by:

$$C_{\min}(z)=A(z)E(z)\left(z^{n}I-B(z)E(z)\right)^{-1} \tag{3.47}$$

and it is achieved by choosing $W(z)=z^{\ell}I, \forall\ell\in\mathbb{N}_0$ in (3.46).

### Proof:

Suppose $W(z)=z^{\ell}I \ with \ \ell\in\mathbb{N}_0,$ then $W(1)=I$ and substituting in (3.44) gives

$$S_u(z)=\frac{A(z)E(z)}{z^{n}} \tag{3.48}$$

Which is equivalent to choose $N=N_{\min}=n$ . Substituting $W(z)$ in (3.46) gives $C_{\min}(z)$ as in (3.47).

From Lemma 3.3 it holds that we can achieve a minimum horizon deadbeat response if we choose $W(z)=z^{\ell}I$. Nevertheless, larger deadbeat horizons can be attained with different choices for $W(z)$. Hence, if we write

$$W(z)=\sum_{i=0}^{\ell}w_i z^{i} \tag{3.49}$$

and consider the constraint $W(1) = E(1)^{-1}$, we conclude that there is a set of $\ell$ free design parameters yielding a ripple-free response of $N = n + \ell$ samples. The polynomial matrix $W(z)$ can be used for many purposes, such as to avoid the cancellation of certain plant poles or to choose the zeros of the complementary sensitivity function (recall from (3.43) that every zero of $W(z)$ is also a zero of $T(z)$). So, by using $W(z)$ we can build a ripple-free deadbeat controller that minimizes a two objective quadratic cost function.

# CHAPTER 4

# Simulation and Result

In this chapter, results and MATLAB simulations for multirate ripple free deadbeat control design will be discussed. Using the solution of the Diophantine equations, we can achieve a multirate ripple free deadbeat control. We propose a hybrid, two-degree off freedom(2DOF) controller for the fixed-order constrained optimization problem addressing performance and robustness specifications.

We will compare between single rate and multirate then we will see the advantages of multirate control over single rate.

## 4.1 Multivariable ripple-free deadbeat control

There are many advantages of multirate control over conventional single-rate control. First of them is achieving specified minimum settling time. Multirate input mechanism can yield shorter settling time than single-rate control using the same frequency of sampling. However, multirate control often exhibits intersample ripple. The undesirable effect of multirate input on the steady-state response can be removed completely to accomplish ripple-free deadbeat, keeping the settling time short using multirate mechanism at the same time. Furthermore, a multirate ripple-free deadbeat control guarantees robustness against continuous-time model uncertainty and disturbance [10].

Salgado and Oyarzun [12] presented a new method to design an optimal multivariable deadbeat control. Given a discrete-time, stable, linear and time invariant plant model, we give a simple parameterisation of all stabilising ripple-free deadbeat controllers of a given order. The free parameter is then optimised in the sense that a quadratic index is kept minimal. The optimality criterion has the advantage of accounting for both tracking performance and magnitude of the control effort. The proposed design procedure is simple to use and allows the tuning of the controller with a scalar weighting factor.

All formulation, parameterization and assumption were presented and discussed in details in the previous chapter.
Now we will show the example which enables us to apply the mathematical demonstration and the theorem which discussed in Chapter 3.

## **Illustrative example 1:** [12]

To illustrate the controller design procedure proposed in this paper, consider the continuous time plant model

$$G_c(s)=\begin{bmatrix}\frac{24}{s^2+6s+5} & \frac{108}{s^2+11s+30}\\ \frac{-162}{s^2+11s+30} & \frac{30}{s^2+7s+6}\end{bmatrix} \tag{4.1}$$

We choose a sampling time of 0.1 second as it's used in this example by Salgado and Oyarzun. The zero-order hold discrete-time version of $G_c(s)$ using a sampling time of 0.1second is given by

$$G(z)=\begin{bmatrix}\frac{0.098812(z+0.8189)}{(z-0.9048)(z-0.6065)} & \frac{0.37755(z+0.6928)}{(z-0.6065)(z-0.5488)}\\ \frac{-0.56632(z+0.6928)}{(z-0.6065)(z-0.5488)} & \frac{0.11979(z+0.7922)}{(z-0.9048)(z-0.5488)}\end{bmatrix} \tag{4.2}$$

In this case, the right coprime polynomial factors of $G(z)$ are given by

$$A(z)=\begin{bmatrix}-2.05+9.39z-14.04z^2+6.81z^3 & 1.48-6.76z+10.11z^2-4.91z^3\\ -2.22+10.14z-15.16z^2+7.36z^3 & -1.97+9.01z-13.47z^2+6.5z^3\end{bmatrix} \tag{4.3}$$

$$B(z)=\begin{bmatrix}-2.04-0.41z+3.45z^2 & -.133-0.65z+1.98z^2\\ 1.99+0.98z-2.98z^2 & -2.12-0.44z+3.56z^2\end{bmatrix} \tag{4.4}$$

After we find the right coprime polynomial factors of $G(z)$, we will find the multivariable deadbeat controller $C(z)$ which is demonstrated in section 3.5.3. Lemma 3.3 gives the minimum horizon deadbeat controller which is given by:

$$C_{\min}(z) = A(z)E(z)\left(z^{n}I - B(z)E(z)\right)^{-1} \tag{4.5}$$

and it is achieved by choosing $W(z) = z^{\ell}I, \forall \ell \in \mathbb{N}_0$ in (3.46).

A according to $E(z)$, we get it from the following formula which is derived from the block diagram in Figure (3.3).

$$E(z) = R(z) - Y(z) \tag{4.6}$$

So, by substitute (4.3), (4.4) and (4.6) in (4.5), we get the minimum horizon deadbeat controller $C_{\min}(z)$.

Note that $B(z)$ satisfies $B(1) = I$ and in this case $n = 3$. Hence, the minimum achievable deadbeat horizon is $N_{\min} = 3$. Figures 4.1 and 4.2 show the simulation results with $r(k) = \left[\mu(k) \quad \mu(k-20)\right]^{T}$ for the minimum horizon deadbeat controller (3.45) in Lemma 3.3. It can be seen that the ripple-free response is achieved in the minimum number of samples.

Figure 4.1 shows the continuous time output of optimal deadbeat control loop with $N = 3$. We note that the output signal tracks the input step signal in short settling time but with high overshoot. The second drawback is the large sampling time ($N = 3$) that requires to settle the signal. It's clear that the response is a periodic and the time domain specification for the output signal is illustrated below:

$$\text{Overshoot} = \frac{4.5-1}{1} \times 100\% = 350\%$$

Settling time = 2.5 s

Rise time = 0.1 - 0.08 = 0.02 s

Steady state error = 0

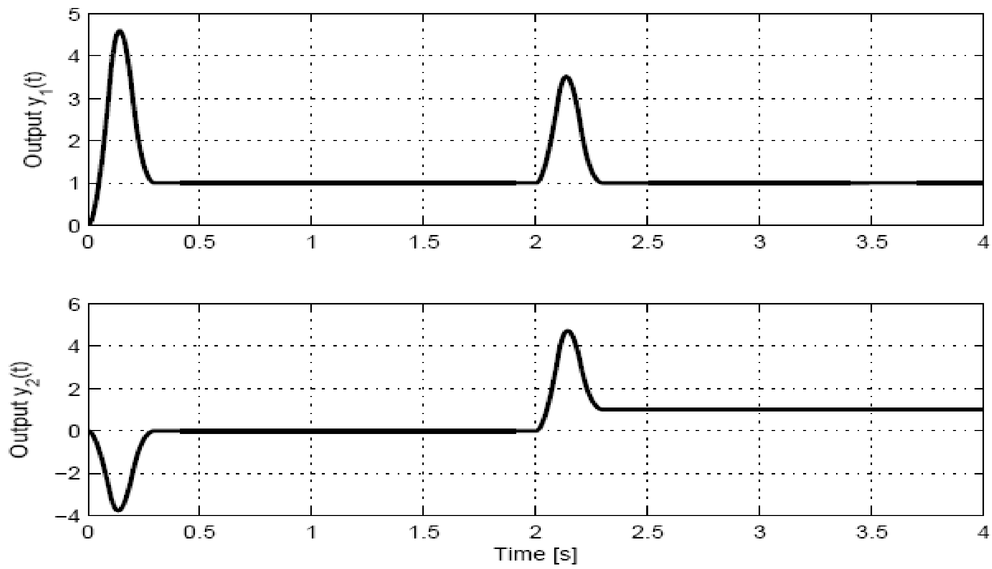


Figure 4.1: Continuous time output of optimal deadbeat control loop with $N = 3$

Figure 4.2 shows the control sequences of optimal deadbeat control loop with $N = 3$. We show that the signal settles in short time which is 2.5s while the steady state error is zero.

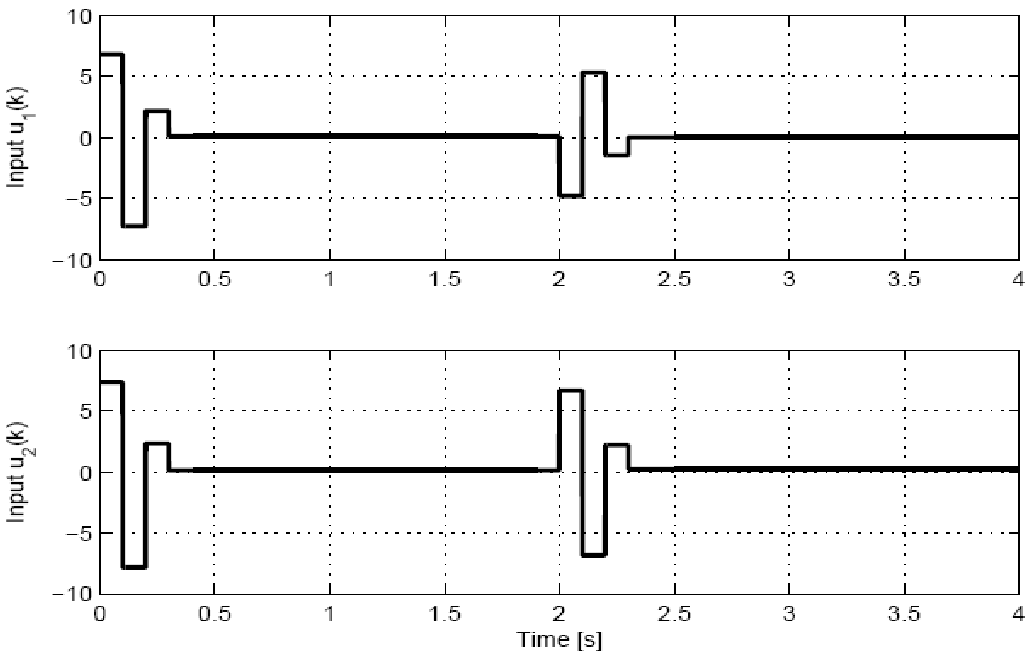


Figure 4.2: Control sequences of optimal deadbeat control loop with $N = 3$

## 4.2 Ripple free Deadbeat tracking control

It is convenient to refer to trajectory following problems by one of the three technical terms, the particular term used depending on the nature of the desired trajectory. If the plant output are to follow a class of desired trajectories, for example, all polynomials up to a certain order, the problem is referred to as a servo (servomechanism) problem; if the desired trajectory is a particular prescribed function of time, the problem is called a tracking problem. When the outputs of the plant are to follow the response of another plant (or model), the problem is referred to as the model-following problem [39].

Paz [11] presented a new method using Diophantine equation to solve single rate ripple-free deadbeat tracking control. A hybrid, two-degree-off freedom (2DOF) controller was proposed for the fixed-order constrained optimization problem addressing performance and robustness specifications.

All formulation, parameterization and assumption were presented and discussed in details in the previous chapter.

Now we will show the example which enables us to apply the mathematical demonstration and the theorem which discussed in Chapter 3.

## Illustrative example 2: [11]

We consider the unstable, time-delayed system

$$G(s) = e^{-0.13s}\frac{200}{s^2 - 2s + 2} \tag{4.7}$$

and we wish to track the sinusoid $r(t) = \sin(2t)$ with the control magnitude constraint

$$\|U\|_\infty \le 1.4 \tag{4.8}$$

and settles in $t_s = 2/3$, and minimizes the control energy $\|U\|_2^2$. we use the tracking model

$$M(s) = \frac{100}{s^2 + 4} \tag{4.9}$$

The system is simulated using Matlab while the block diagram is drwan using Simulink . We can see all over the system in Figure 4.3.

If we choose the sampling period (From example 1 by Paz) $T = {t_s}/{8} = 0.0833$, then we have the discrete-time model

$$P(q) = \frac{N_p(q)}{D_p(q)} = \frac{0.041508q(1+10.22q)(1+1.034q)(1+0.1046q)}{(1-2.166q+1.181q^2)(1-1.972q+q^2)} \tag{4.10}$$

and discrete-time reference signal

$$R(q) = \frac{1-0.9861q}{1-1.972q+q^2} \tag{4.11}$$

Computing the minimum order solutions to the Diophantine equations, we obtain

$$N_{1\min}(q) = 3.2041 - 2.3583q \tag{4.12}$$

$$Q_{1\min}(q) = 1 + 1.8393q + 1.2155q^2 + 0.1082q^3 \tag{4.13}$$

$$N_{2\min}(q) = 14.0092 - 30.6059q + 24.4865q^2 - 6.9368q^3 \tag{4.14}$$

$$D_{c\min}(q) = 1 + 3.5571q + 2.9354q^2 + 0.2694q^3 \tag{4.15}$$

After computing the discrete-time reference signal and the Diophantine equation parameters, we will subsitute them in the control signal which yeilded in 3.20. The figure 3.2 showed that we need three parameter from the Diophantine equations which are $N_1(q), N_2(q)$ *and* $D_c(q)$ and the minimum order solutions will be used as in (4.12, 4.14 and 4.15). A formula of the controlller will be written again as follow:

$$U(q) = \frac{N_1(q)R(q) - N_2(q)Y(q)}{D_c(q)} \tag{4.16}$$

$$= \frac{2.75q^9+13q^8-105.2q^7+201.7q^6-91.4q^5-145q^4+182.6q^3-43q^2-27.4q+12}{q^5+8.924q^4-7.283q^3-11.43q^2+5.884q+3.712} \tag{4.17}$$

Then, we can get the ripple- free deadbeat control for the hybrid (2DOF) system.

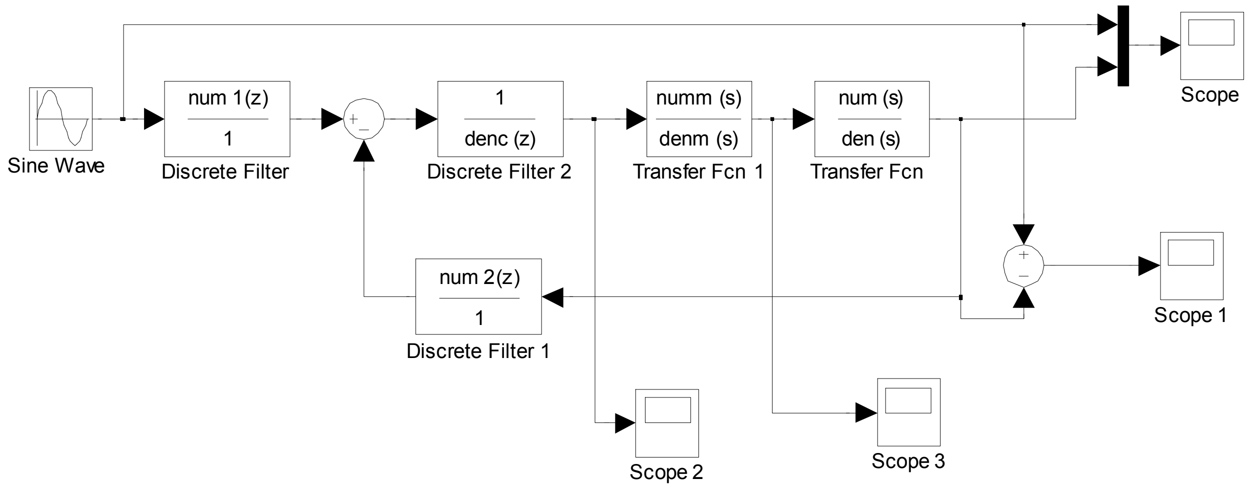


Figure 4.3: Over all system using Simulink

Figure 4.4 shows the time response for the plant. We note that the output signal tracks the input sinusoidal signal in short settling time. It's clear that the response is a periodic and the time domain specification for the output signal is illustrated below:

$$\text{Overshoot} = \frac{1.6-1}{1}\times 100\% \quad = 60\ \%$$

Settling time = 0.70 s

Rise time = 0.27 - 0.14 = 0.13 s

Steady state error = 0

Also, the time response for the error signal $e(t)$ between input and output signal is shown where the steady state error is zero while the settling time is 0.7s. The control signal $u(t)$ which guarantees the minimum settling time with ripple free is drawn where the steady state error is zero while the settling time is 0.667s. Moreover, the output of the filter $f(t)$ is shown where the settling time is 0.64s while the steady state error is zero.

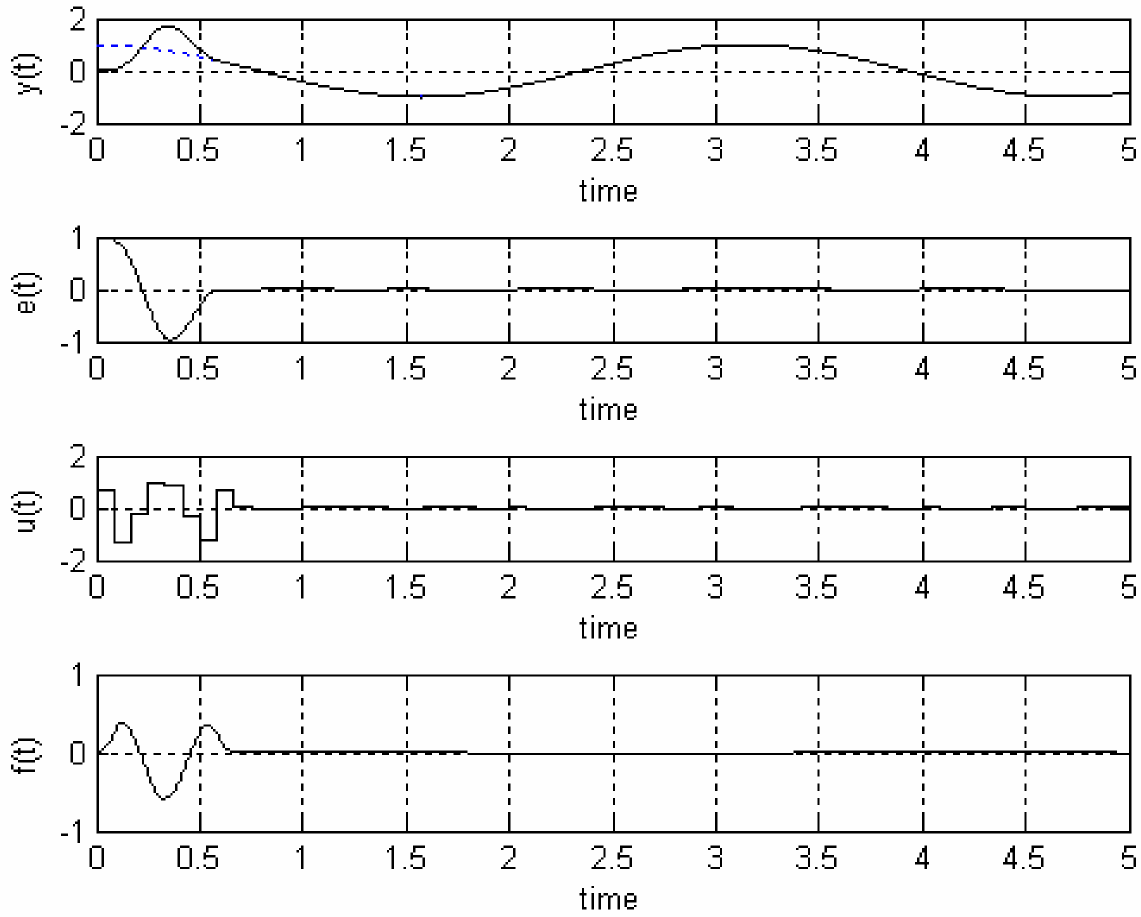


Figure 4.4: Time response for Illustrative example 2

## 4.3 Multirate Ripple-Free Deadbeat Control

In this section, a multirate ripple free deadbeat control is introduced. A combination between Paz approach with Salgado and Oyarzun approach is proposed for simulation of single rate ripple – free deadbeat control which was developed by Paz as illustrated in example in section 4.2. Aftere that, I have developed a Matlab code and a block diagram in Simulink to simulate the multirate ripple – free deadbeat control. This code for multirate consists of several functions, one of them is the computing of the Diophantine equation parameters.

A nother important contribution is linking Simulink with m – file using (sim) command. An easy and flexiable methode to export the Diophantine equation parameters for all plants in the multirate system in Simulink block diagram is introduced.

The procedures for doing this process is to open m – file of each Simulink block, so the properties of the block diagram components can be changed such as: discrete filter, transfer function and especially scope. By changing these properties, you can draw the response results and add comments like title, axes name and supplot.

The plant of second order in (4.1) will be used in this simulation. A multirate system has two input so we can represent the system as an LTI system with two states. The state equation of an LTI system in state-variable form is [40]

$$x'(t) = A\,x(t) + B\,u(t) \tag{4.18}$$

$$y(t) = C\,x(t) + D\,u(t) \tag{4.19}$$

An over all system is presented using Matlab Simulink as shown in Figure 4.5.

The goal of the controller is to track the two input sinsodial signals. In Figure 4.5, we can see how we represent the two inputs by applying the concept in (4.18 and 4.19). Also, we divided the four transfer function of the plant by applying the concept in (4.18 and 4.19). Assume the thransfer function of the plant is in the form of:

$$\begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix} \tag{4.20}$$

where

$$T_1 = \frac{24}{s^2 + 6s + 5} \tag{4.21}$$

$$T_2 = \frac{108}{s^2 + 11s + 30} \tag{4.22}$$

$$T_3 = \frac{-162}{s^2 + 11s + 30} \tag{4.23}$$

$$T_4 = \frac{30}{s^2 + 7s + 6} \tag{4.24}$$

There are many steps which be followed to get the controller. Illustrative example 2 shows these steps in details. However, we will introduce it a gain. First step is finding the discrete-time reference signal. The second step is to compute the discrete-time model for every plant. For multirate plant in (4.19) The discrete-time model for all plants will be computed as follow:

$$G(z)=\begin{bmatrix}\dfrac{0.098812(z+0.8189)}{(z-0.9048)(z-0.6065)} & \dfrac{0.37755(z+0.6928)}{(z-0.6065)(z-0.5488)} \\ \dfrac{-0.56632(z+0.6928)}{(z-0.6065)(z-0.5488)} & \dfrac{0.11979(z+0.7922)}{(z-0.9048)(z-0.5488)}\end{bmatrix} \tag{4.25}$$

After that a Matlab function will compute the Diophantine equation parameters. We use $N_1(q), N_2(q)$ *and* $D_c(q)$ for finding the controller which is introduced in (4.17). Then, we can get the ripple- free deadbeat control for the first plant in the system. These steps must be applied for all other plants.

From Figure (4.7) to (4.10) we can see that the output signals track the input sinsodial signals in minimum settling time ( about 0.7 second ). Also, we can see the error sinal $e(t)$ and the control signals $u(t)$ for every plant. Moreover, the output of the filter $f(t)$ is shown. Accodring to the output for all system (multirate output ), we apllied the concept in (4.18 and 4.19) and this is illustrated in Figure 4.6. We add summation between the output from input 1 and between the output from input 2. The time response for the output 1 from input 1 is shown in Figure 4.11. Also, the time response for output 2 from input 2 is shown in Figure 4.12. We note that the output signals in Figure 4.11 and 4.12 is track the two inputs sinsodial signal in minimum settling time ( about 0.7 second ) and this time is achieved the requirement. For more details in the block diagram of the all system, I see the block diagram of the first output which presnted by (4.18) in Figure 4.5.

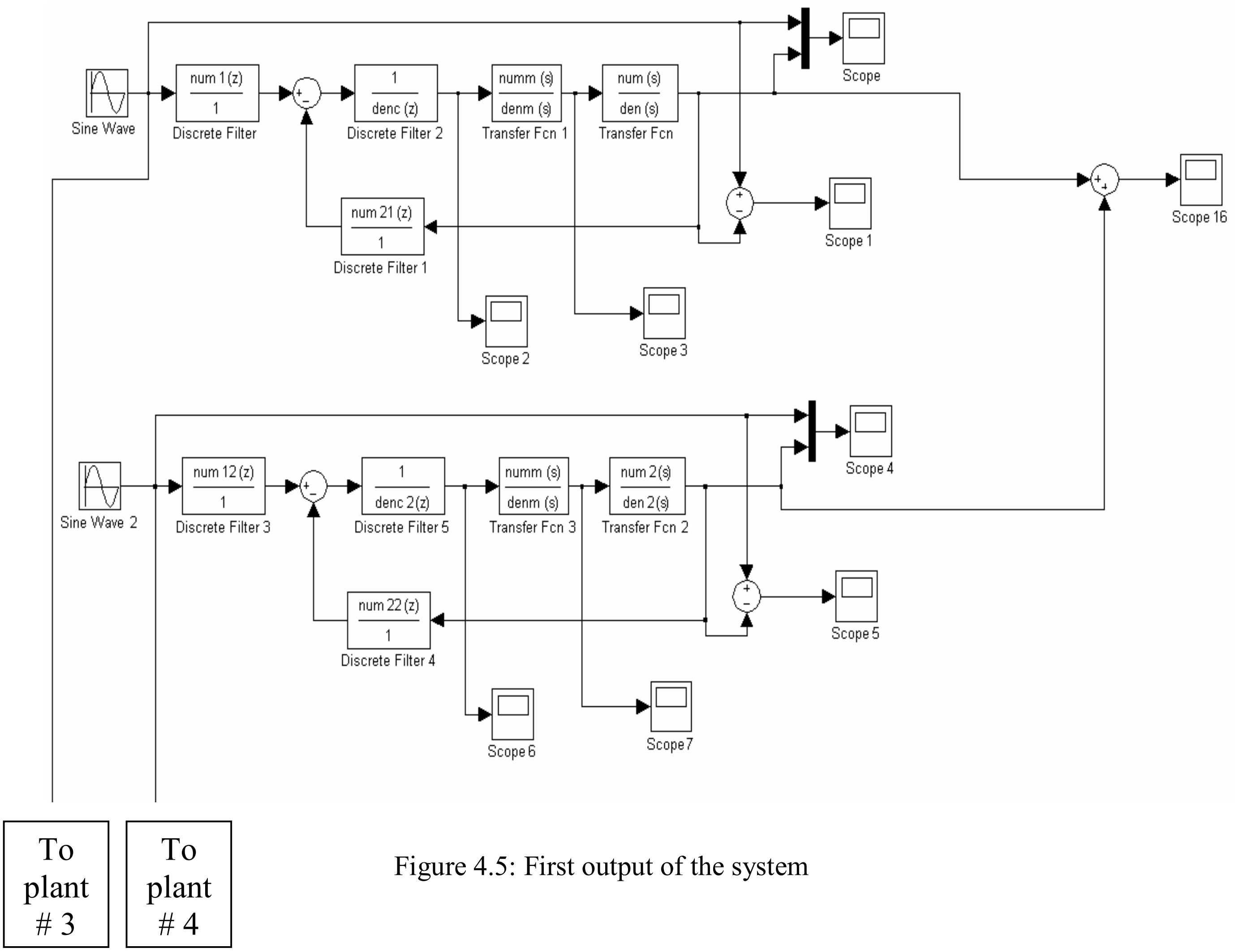


Figure 4.5: First output of the system

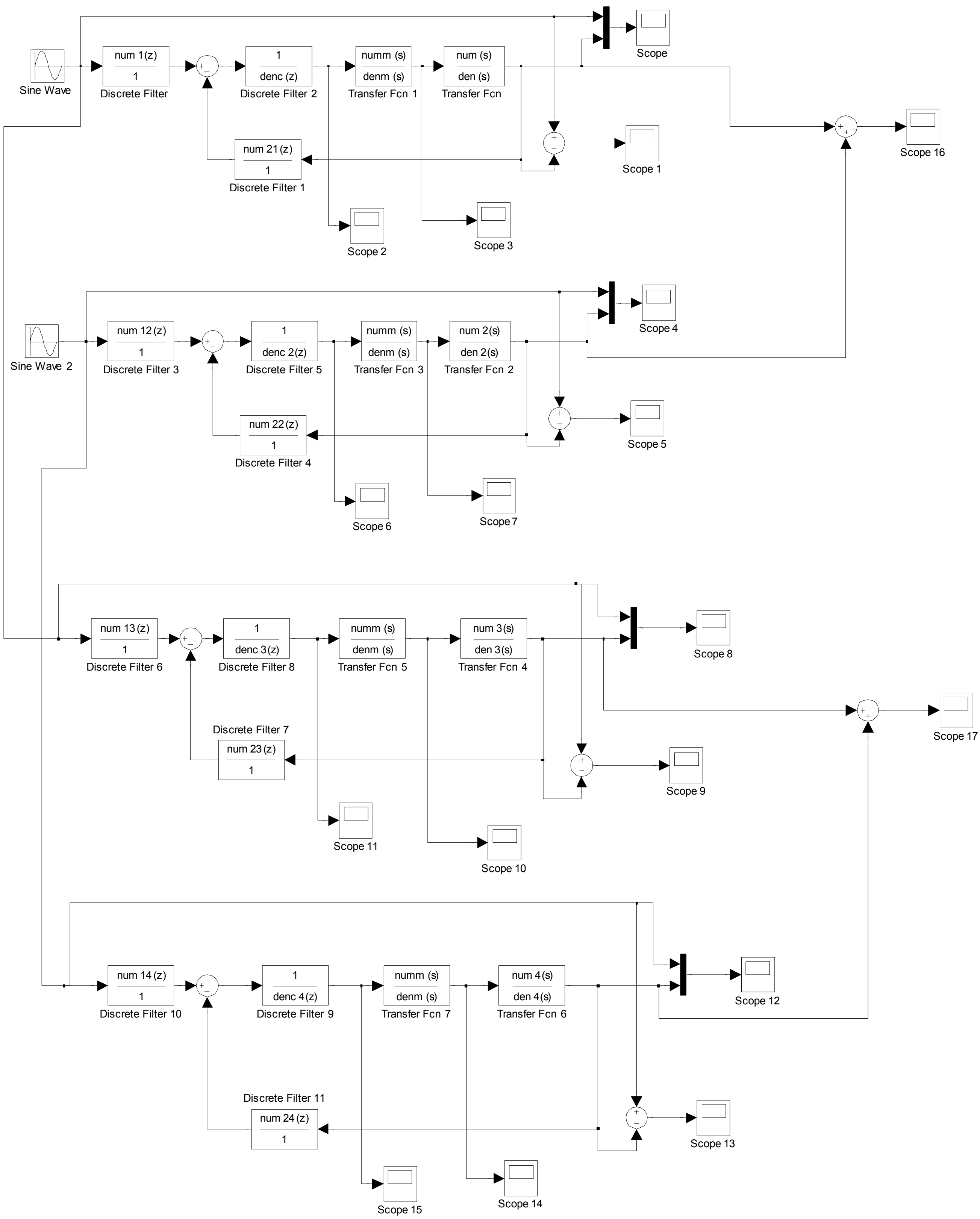


Figure 4.6: Over all system using Simulink

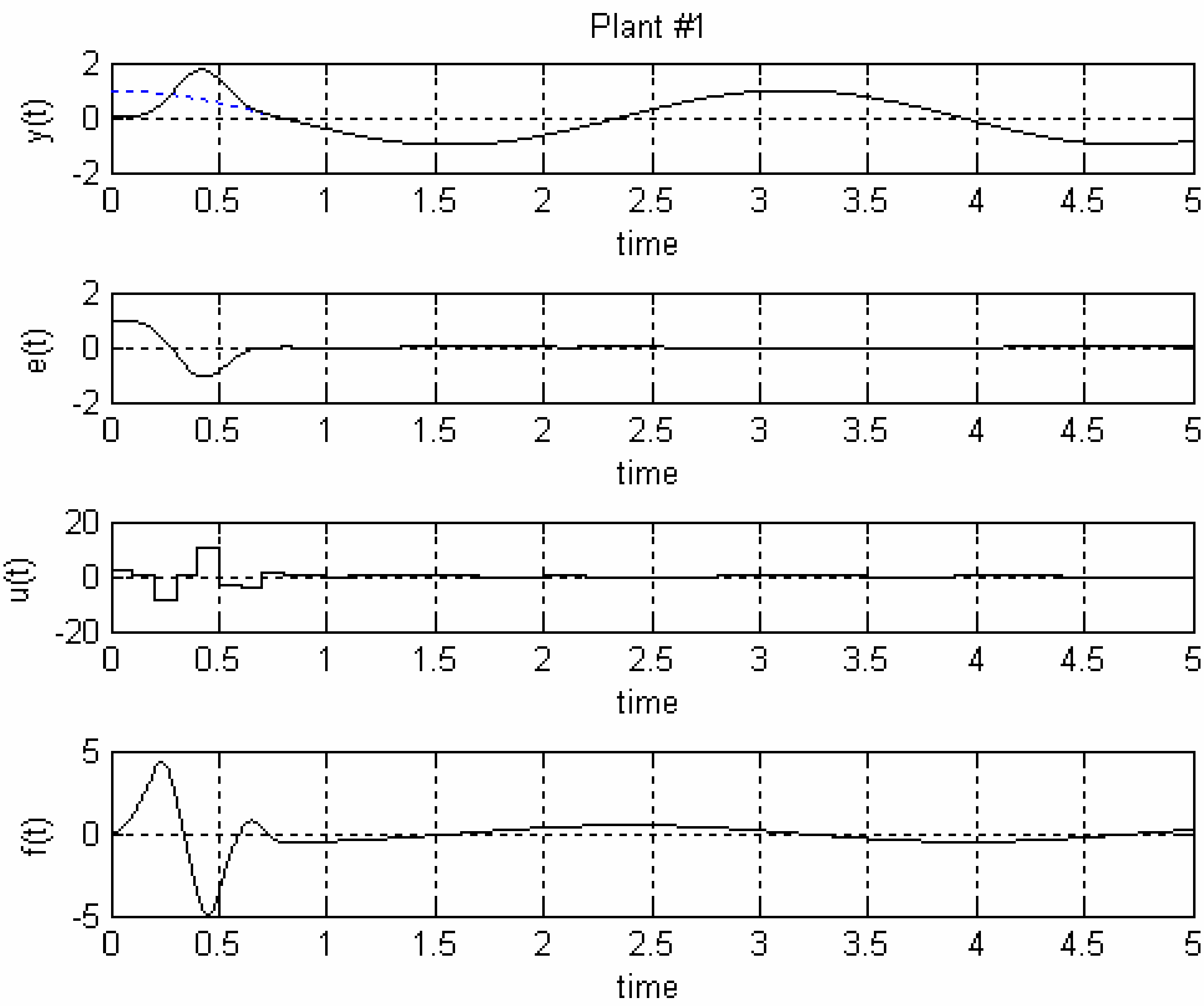


Figure 4.7: Time response for plant #1

Figure 4.7 shows the time response for the first plant, $T_1$. We note that the output signal tracks the input sinusoidal signal in short settling time. It's clear that the response is a periodic and the time domain specification for the output signal is illustrated below:

$$\text{Overshoot} = \frac{1.74-1}{1}\times 100\% \quad = 74\ \%$$

Settling time = 0.709 s

Rise time = 0.285- 0.153 = 0.132 s

Steady state error = 0

Also, the time response for the error signal $e(t)$ between input and output signal is shown where the steady state error is zero while the settling time is 0.7s. The control signal $u(t)$ which guarantees the minimum settling time with ripple free is drawn where the steady state error is zero while the settling time is 0.8s. Moreover, the output of the filter $f(t)$ is shown where the settling time is 0.78s while the steady state error is 0.5.

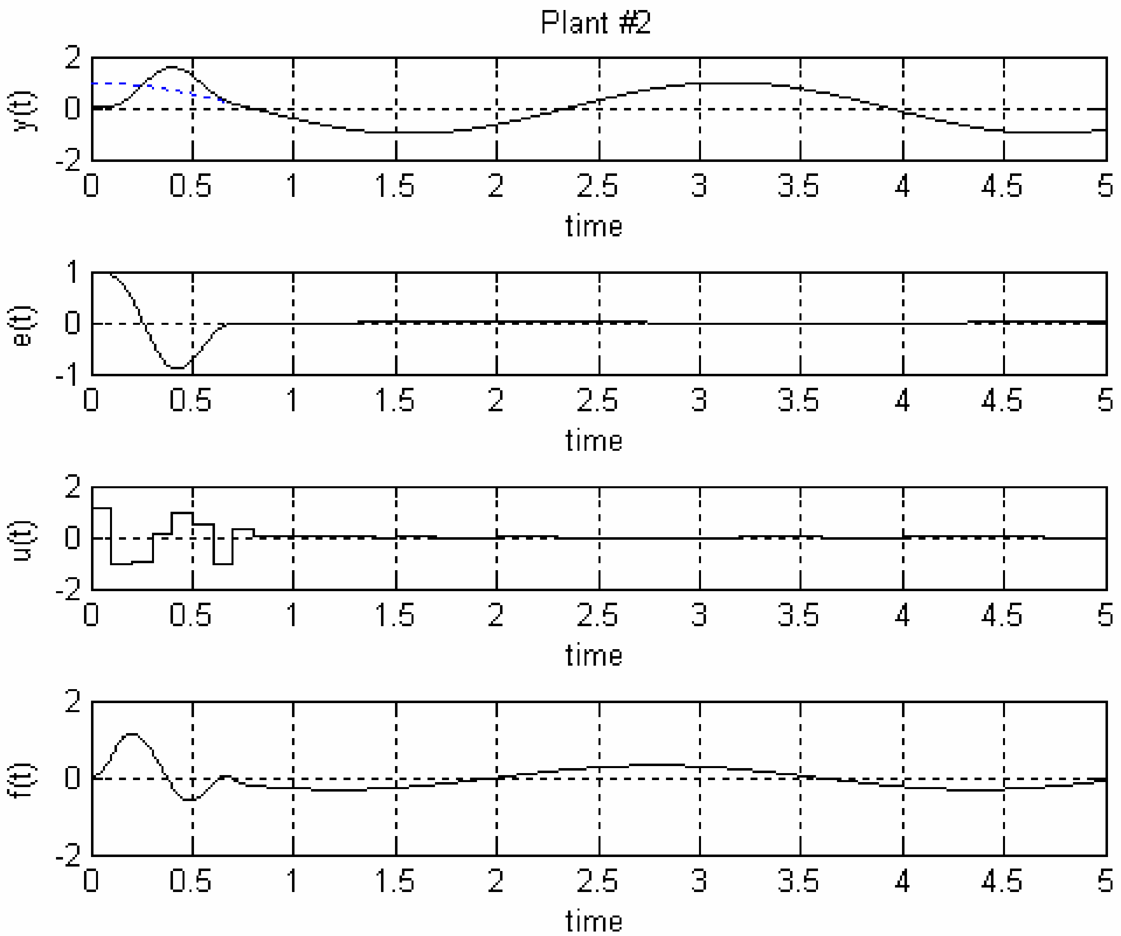


Figure 4.8: Time response for plant #2

Figure 4.8 shows the time response for the second plant, $T_2$. We note that the output signal tracks the input sinusoidal signal in short settling time. It's clear that the response is a periodic and the time domain specification for the output signal is illustrated below:

$$\text{Overshoot} = \frac{1.58-1}{1}\times 100\% \quad = 58\ \%$$

Settling time = 0.702 s

Rise time = 0.263 - 0.129 = 0.134 s

Steady state error = 0

Also, the time response for the error signal $e(t)$ between input and output signal is shown where the steady state error is zero while the settling time is 0.71s. The control signal $u(t)$ which guarantees the minimum settling time with ripple free is drawn where the steady state error is zero while the settling time is 0.8s. Moreover, the output of the filter $f(t)$ is shown where the settling time is 0.75s while the steady state error is 0.32.

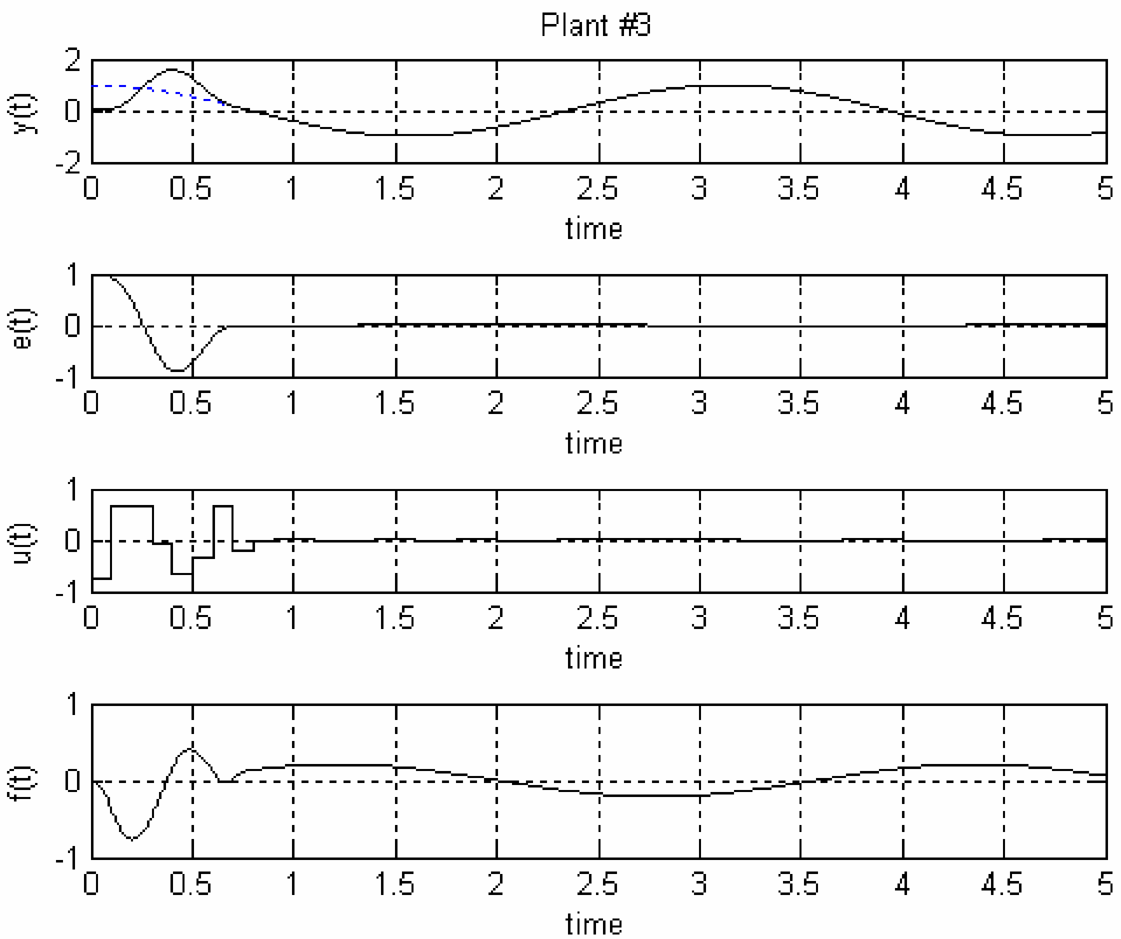


Figure 4.9: Time response for plant #3

Figure 4.9 shows the time response for the third plant, $T_3$. We note that the output signal tracks the input sinusoidal signal in short settling time. It's clear that the response is a periodic and the time domain specification for the output signal is illustrated below:

$$\text{Overshoot} = \frac{1.57-1}{1}\times 100\% \quad = 57\ \%$$

Settling time = 0.69 s

Rise time = 0.262 - 0.127 = 0.135 s

Steady state error = 0

Also, the time response for the error signal $e(t)$ between input and output signal is shown where the steady state error is zero while the settling time is 0.68 s. The control signal $u(t)$ which guarantees the minimum settling time with ripple free is drawn where the steady state error is zero while the settling time is 0.8s. Moreover, the output of the filter $f(t)$ is shown where the settling time is 0.7s while the steady state error is 0.21.

Figure 4.10 shows the time response for the fourth plant, $T_4$. We note that the output signal tracks the input sinusoidal signal in short settling time. It's clear that the response is a periodic and the time domain specification for the output signal is illustrated below:

$$\text{Overshoot} = \frac{1.71-1}{1}\times 100\% \quad = 71\ \%$$

Settling time = 0.71 s

Rise time = 0.282 - 0.146 = 0.136 s

Steady state error = 0

Also, the time response for the error signal $e(t)$ between input and output signal is shown where the steady state error is zero while the settling time is 0.68 s. The control signal $u(t)$ which guarantees the minimum settling time with ripple free is drawn where the steady state error is zero while the settling time is 0.8s. Moreover, the output of the filter $f(t)$ is shown where the settling time is 0.75 s while the steady state error is 0.4.

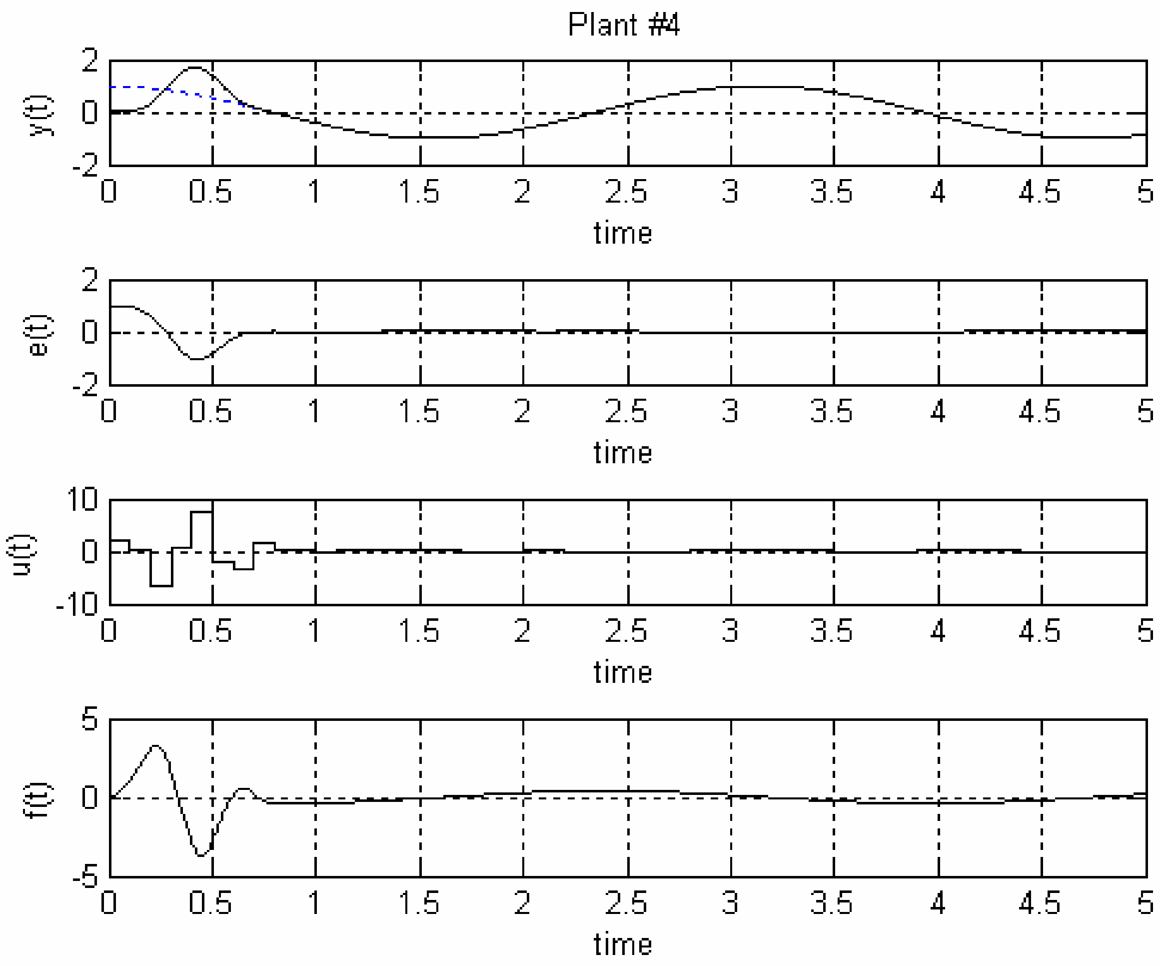


Figure 4.10: Time response for plant #4

Figure 4.11 shows the time response for the first output from first sinusoidal input. We note that the output signal tracks the input sinusoidal signal in short settling time. It's clear that the response is a periodic and the time domain specification for the output signal is illustrated below:

$$\text{Overshoot} = \frac{3.3-2}{2}\times 100\% \quad = 65\ \%$$

Settling time = 0.72 s

Rise time = 0.21 - 0.11 = 0.1 s

Steady state error = 0

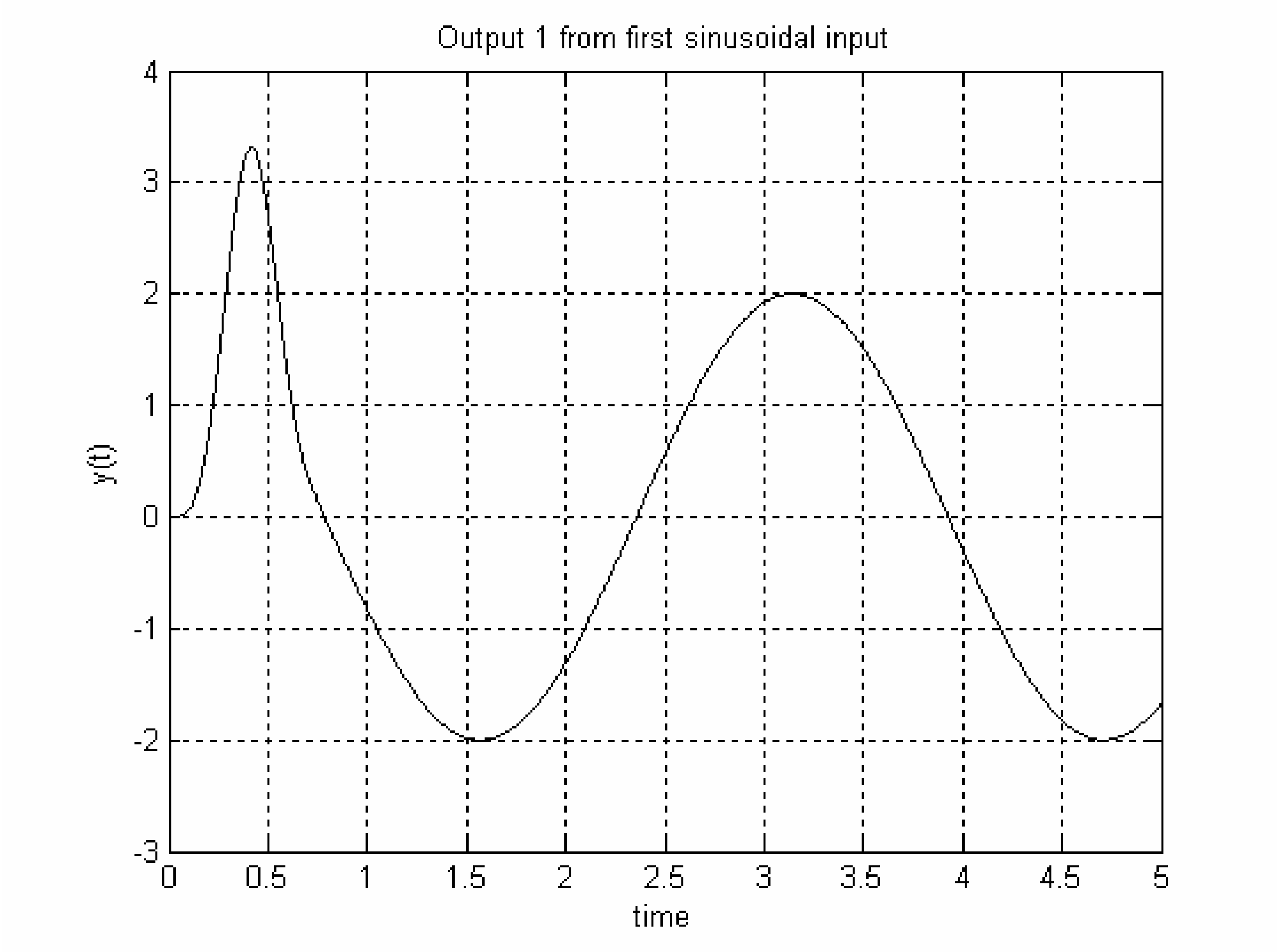


Figure 4.11: Time response for the first output from first sinusoidal input

Figure 4.12 shows the time response for the second output from second sinusoidal input. We note that the output signal tracks the input sinusoidal signal in short settling time. It's clear that the response is a periodic and the time domain specification for the output signal is illustrated below:

$$\text{Overshoot} = \frac{3.29-2}{2} \times 100\% \ = 64.5\ \%$$

Settling time = 0.73 s

Rise time = 0.212 - 0.11 = 0.102 s

Steady state error = 0

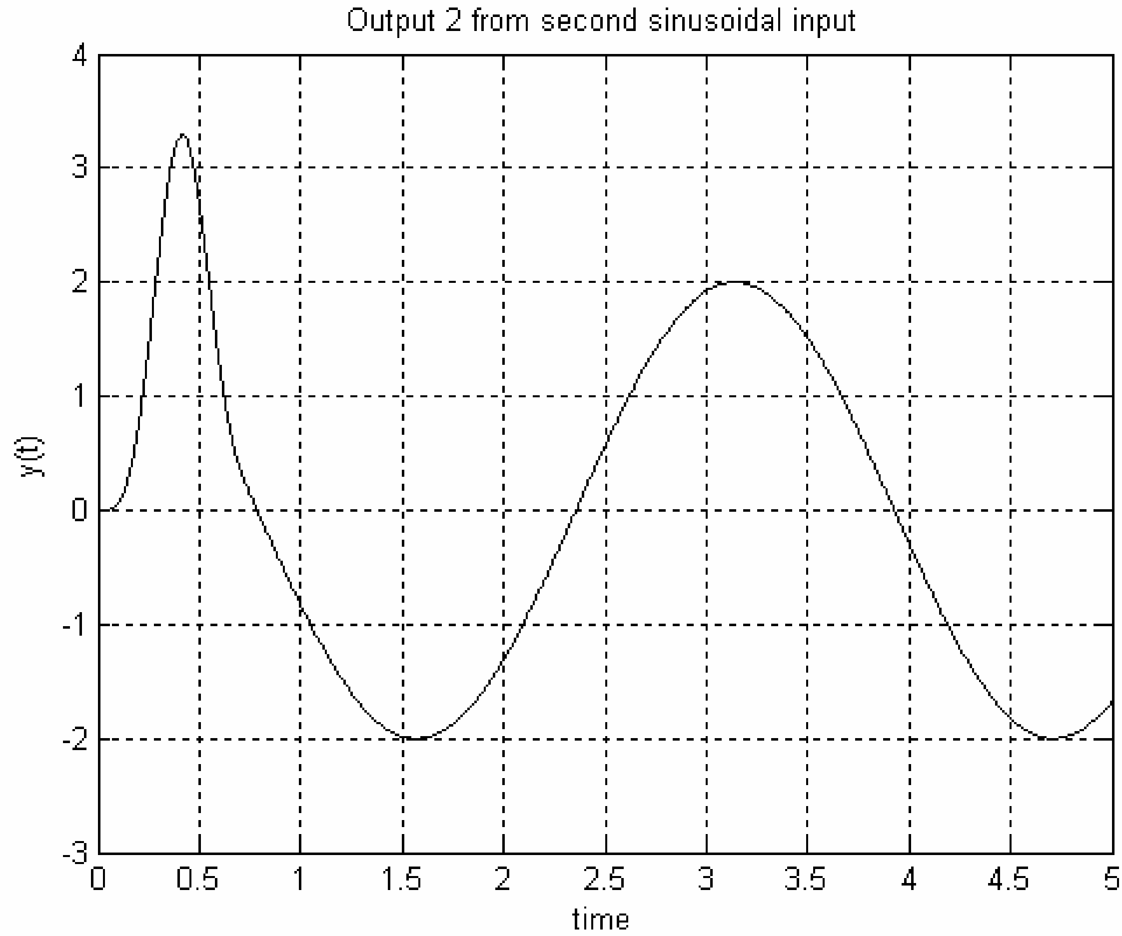


Figure 4.12: Time response for the second output from second sinusoidal input

# CHAPTER 5

# Conclusion

In this thesis, a new approach shows promise for solving robust multirate ripple-free deadbeat control (MRFDC) problems using Diophantine equation parameters. This thesis proposed a hybrid two degree of freedom controller for the fixed-order constrained optimization problem addressing performance and robustness specifications utilizing the parameters of Diophantine equation to build a robust multirate ripple-free deadbeat control.
A combination between the concept of multirate which was demonstrated by Salgado and Oyarzun and robust single rate which was demonstrated by Paz was proposed.
Simulation results showed that the output signal tracked the input sinusoidal signal in short settling time either in single rate or multirate. Also the ripple problem which caused by intersample was solved.The time domain specification for the output signal, control signal, error signal and the output of the filter signal were computed and satisfied that it was guaranteed the requirement and constraint.
A time delay was also presented with simulation and was solved by using deadbeat controller based on solving Diophantine equation parameters.

Future research can be done in the third order system with time delay. Moreover, the effect of the noise in the system can be studied since the noise affected in the stability and efficiency of the system especially in the high frequency applications. comparison between two main approach for solving multirate ripple – free deadbeat control which is time domain approach and the polynomial approach can be introduced.

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