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WIDE-ANGLE AND WAVELENGTH-INDEPENDENT PERFECT ABSORPTION AT METAMATERIAL SURFACES

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Abstract. In this paper, the reflection and absorption properties of a metamaterial layer on a metallic substrate is investigated theoretically and numerically. Perfect absorption is achieved for any frequency and for any angle of incidence when specified conditions are satisfied. These conditions are as follows: (i) the real permeability Re μ of the metamaterial is very small as compared with the imaginary part Im μ and (ii) the metamaterial thickness is very thin as compared with the wavelength of the incident radiation. Expressions for reflection and absorption coefficients are derived in detail. In the numerical results, the mentioned coefficients are computed and illustrated as a function of angle of incidence when Re μ, Im μ, and metamaterial thickness change.

Key words: absorption, metallic substrate, metamaterial, permeability, reflection.

1. INTRODUCTION

An electromagnetic absorber is a device that can efficiently absorb incident radiation with negligible reflection and transmission [1]. The performance of an absorber depends on its thickness, morphology and the high possibility of the materials fabrication used [2]. Traditional absorbers can be categorized into two types: *resonant absorber* and *broadband absorbers* [3]. Resonant absorbers rely on the material interacting with the incident radiation in a resonant way at a specific frequency. Broadband absorbers generally rely on materials whose properties are frequency independent and therefore can absorb radiation over a large bandwidth [4]. These absorbers are usually relatively thick and heavy, especially for longer wavelengths *e.g*. the Salisbary absorber [5] and the Dällenbach absorber [6] have a thickness of a quarter wavelength.

Electromagnetic metamaterials [7, 8] are arrays of structured subwavelength elements, which may be described as effective materials *via* ε (ω) and μ (ω), the electric permittivity and magnetic permeability, respectively [4]. Although the initial interest in metamaterials was due to their ability to exhibit exotic electromagnetic effects impossible to achieve with natural materials, they are excellent candidates for electromagnetic wave absorbers. Metamaterial-based absorbers [9] offer benefits over conventional absorbers such as further miniaturization, wider adaptability, and increased effectiveness. Intended applications for the metamaterial absorber include emitters, sensors, spatial light modulators, infrared camouflage, wireless communication, and use in solar photovoltaics and thermophotovoltaics. Due to these benefits, the metamaterial-based absorbers have achieved remarkable and growing interest. For example, Tao *et al*. [10] have shown experimentally highly flexible wide angle of incidence terahertz metamaterial absorber. Landy *et al*. [11] have performed measurements of a polarization-insensitive absorber for terahertz imaging. Meng *et al*. [12] have studied polarization-sensitive perfect absorbers at near-infrared wavelengths. Chen [13] has provided a general formulation for metamaterial perfect absorbers and Alu *et al*. [14] have studied epsilon-near-zero metamaterials and electromagnetic sources. Liu *et al*. [15] have demonstrated a spatially dependent metamaterial perfect absorber operating in the infrared regime and Huang *et al*. [16] have investigated numerically the impact of resonator geometry and its coupling with ground plane on the performance of metamaterial perfect absorbers. Avitzour *et al*. [17] have described metamaterial-based approach in making a wide-angle absorber of infrared radiation.

In this study, we perform theoretical and analytical investigations of a perfect absorption based on a metamaterial layer placed on a metallic substrate. The electromagnetic radiation that is obliquely incident from vacuum on the structure is considered to be reflected by the structure and propagate within the metamaterial layer. We have proved that maximum absorption and minimum reflection are demonstrated if the permeability of the metamaterial has vanishing real part but a large imaginary part as well as the metamaterial thickness is very thin as compared with the long wavelength of the incident radiation. Maxwell's equations are used to determine the electric and magnetic fields in each region. Then, Snell's law is applied and boundary conditions are imposed at each interface to obtain the reflection and absorption coefficients in a closed form. The behavior of the mentioned coefficients against the incidence angle and the metamaterial layer thickness is computed and presented in numerical results showing the effect of the real and imaginary parts of the permeability of the metamaterial. The analysis corresponding to these numerical computations is in agreement with the experimental results [1].

2. THEORY

We consider a metamaterial slab with thickness d located between a semi-infinite half free space and a metallic substrate. A parallel polarized plane wave in region 1 is incident on the plane $x = 0$ at some angle θ relative to the normal to the boundary (Fig. 1). The permittivity and permeability of free space in region 1 are ε_o and μ_o, respectively. For the metamaterial in region 2, the relative permittivity and permeability tensors $\overline{\overline{\varepsilon}}_2$ and $\overline{\overline{\mu}}_2$ are described by [1]:

$$\overline{\overline{\varepsilon}}_2 = \varepsilon_{xx}\hat{x} + \varepsilon_{yy}\hat{y} + \varepsilon_{zz}\hat{z}, \tag{1}$$

$$\overline{\overline{\mu}}_2 = \mu_{xx}\hat{x} + \mu_{yy}\hat{y} + \mu_{zz}\hat{z}. \tag{2}$$

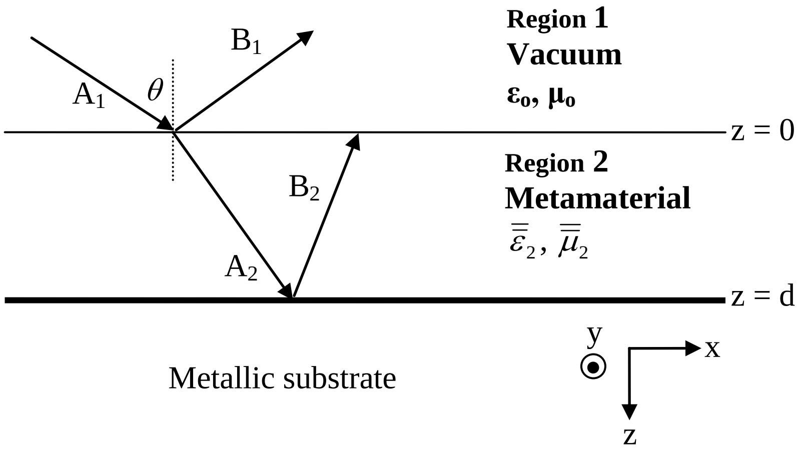


Fig. 1 – Oblique incidence of electromagnetic wave on a metamaterial embedded between semi-infinite free space and metallic substrate.

The magnetic field of the incident and reflected waves in regions 1, 2 can be written as [18, 19]:

$$\vec{H}_\ell = \left(A_\ell e^{ik_{\ell z}z} + B_\ell e^{-ik_{\ell z}z}\right) e^{i(k_{\ell x}x - \omega t)}\hat{y}. \tag{3}$$

To find the corresponding electric field $\vec{E}_\ell$, we start with Maxwell's equation $\vec{\nabla}\times\vec{H}_\ell = \partial\vec{D}/\partial t$, substituting $\vec{D} = \varepsilon_\ell \vec{E}_\ell$ and solving for $\vec{E}_\ell$ yield:

$$\vec{E}_{\ell x} = \frac{1}{\varepsilon_{\ell x}\omega}\left(A_\ell k_{\ell z} e^{ik_{\ell z}z} - B_\ell k_{\ell z} e^{-ik_{\ell z}z}\right) e^{i(k_{\ell x}x - \omega t)}, \tag{4}$$

$$\vec{E}_{\ell z} = \frac{-1}{\varepsilon_{\ell z}\omega}\left(A_\ell k_{\ell x} e^{ik_{\ell z}z} + B_\ell k_{\ell x} e^{-ik_{\ell z}z}\right) \; e^{i(k_{\ell x}x - \omega t)}, \tag{5}$$

where A_ℓ and B_ℓ are the amplitude of forward and backward traveling waves ($\ell = 1, 2$), $k_\ell = n_\ell \omega/c$ is the wave vector inside the material, n_ℓ is the refractive index of the corresponding material, ω is the angular frequency, c is the speed of light in vacuum, and $\varepsilon_{1x} = \varepsilon_{1z} = \varepsilon_o$, $\varepsilon_{2x} = \varepsilon_{xx}$, $\varepsilon_{2z} = \varepsilon_{zz}$.

We match the boundary conditions for $\vec{H}$ and $\vec{E}$ fields at each layer interface, that is, at $z = 0$, $H_{1y} = H_{2y}$ and $E_{1x} = E_{2x}$ and at $z = d$, $E_{2x} = 0$ (since the presence of the metallic substrate make the transmission coefficient T to be zero [1]) . This yields the following equations [18, 20, 21]:

$$A_1 + B_1 = A_2 + B_2 , \tag{6}$$

$$\frac{k_{1z}}{\varepsilon_{1x}}\left(A_1 - B_1\right) = \frac{k_{2z}}{\varepsilon_{2x}}\left(A_2 - B_2\right), \tag{7}$$

$$\frac{k_{2z}}{\varepsilon_{2x}}\left(A_2 e^{ik_{2z}d} - B_2 e^{-ik_{2z}d}\right) = 0, \tag{8}$$

where $k_{1x} = k_{2x} = k_x \equiv$ Snell's law and

$$\frac{{k_x}^2}{\varepsilon_{zz}} + \frac{{k_{2z}}^2}{\varepsilon_{xx}} = {k_1}^2 \mu_{yy}. \tag{9}$$

The reflection coefficient r of the structure is given by [22]:

$$r = \frac{B_1}{A_1} = \frac{\varepsilon_{xx}k_{1z} + ik_{2z}\tan(k_{2z}d)}{\varepsilon_{xx}k_{1z} - ik_{2z}\tan(k_{2z}d)}. \tag{10}$$

From the law of conservation of energy [23, 24], the relation between the reflection $R = |r|^2$ and the absorption A is given by (remember that, the transmission T is zero)

$$A + R = 1. \tag{11}$$

Using eq. (9) together with $k_{1z} = k_1\cos\theta$, $k_x = k_1\sin\theta$, and performing the approximation $\tan(k_{2z}d) \approx k_{2z}d$, when $|k_{2z}d| < 0.5$, then eq. (10) becomes:

$$r = \frac{\varepsilon_{zz}\cos\theta + i\varepsilon_{zz}k_1\mu_{yy}d - ik_1 d\sin^2\theta}{\varepsilon_{zz}\cos\theta - i\varepsilon_{zz}k_1\mu_{yy}d + ik_1 d\sin^2\theta}. \tag{12}$$

For normal incidence $\theta = 0$, the above equation reduces to

$$r = \frac{1 + ik_1\mu_{yy}d}{1 - ik_1\mu_{yy}d}. \tag{13}$$

The solution of this equation for perfect absorption condition leads to $\mu_{yy} = i\lambda/(2\pi d)$, where λ is the wavelength in free space. To get the second perfect absorption condition, we substitute $\mu_{yy} = i\lambda/(2\pi d)$ into eq. (9) and use the approximation condition $|k_{2z}d| < 0.5$ to yield $|d\varepsilon_{xx}|<\lambda/(8\pi)$. From the above discussion, two main conditions are required to get perfect absorption. First, the thickness d and the permittivity ε_{xx} must be small enough to meet the approximation condition $|d\varepsilon_{xx}|<\lambda/(8\pi)$, and secondly, μ_{yy} must be equal to $i\lambda/(2\pi d)$, which is purely imaginary and inversely proportional to thickness d.

A careful examination of eq. (10) shows that, the reflection coefficient r is independent of the wavelength λ. This can be explained as follows, if the metamaterial thickness $d = \lambda/m$, $m = 1, 2, 3, \ldots$, then, $\mu_{yy} = im/(2\pi)$. Moreover, from eq. (9):

$$k_{2z} = k_1\sqrt{\varepsilon_{xx}(\mu_{yy} - \sin^2\theta/\varepsilon_{zz})} \text{ and } k_{2z}d = 2\pi/m\sqrt{\varepsilon_{xx}(\mu_{yy} - \sin^2\theta/\varepsilon_{zz})}\ ,$$

where $k_1 = 2\pi/\lambda$. Clearly, the reflection is determined by permeability μ_{yy}, thickness d, and their permittivities ε_{xx} and ε_{zz}.

If the metallic substrate is removed, then the structure becomes vacuum / metamaterial / vacuum (regions 1, 3 are vacuum and region 2 is metamaterial). In this case the transmission cannot be neglected. The reflection and transmission coefficients are given by [22]:

$$r = \frac{r_{12} + r_{23}\exp(i2k_{2x}d)}{1 + r_{12}r_{23}\exp(i2k_{2z}d)}, \tag{14}$$

$$t = \frac{t_{12}t_{23}\exp(ik_{2x}d)}{1 + r_{12}r_{23}\exp(i2k_{2z}d)}, \tag{15}$$

where r and t are the interface reflection and transmission coefficients, respectively. For the parallel polarization r and t are given by [22]:

$$r_{ij} = \frac{\varepsilon_j k_{iz} - \varepsilon_i k_{jz}}{\varepsilon_j k_{iz} + \varepsilon_i k_{jz}}, \tag{16}$$

$$t_{ij} = \frac{2\varepsilon_j k_{iz}}{\varepsilon_j k_{iz} + \varepsilon_i k_{jz}}, \tag{17}$$

where i, j correspond to any two adjacent media and $\frac{k_x^{\,2}}{\varepsilon_{zz}} + \frac{k_{2z}^{\,2}}{\varepsilon_{xx}} = k_1^{\,2}\mu_{yy}$.

3. NUMERICAL RESULTS AND APPLICATIONS

In this section, the computations of the above theory have been carried out for the parallel polarization (TM wave). Specifically, the reflection and absorption coefficients for the structure shown in Fig. 1 are computed as a function of angle of incidence, Re (μ), Im (μ), and metamaterial thickness. The calculations are performed using the exact formula (10). Here, the computed reflection and transmission coefficients are represented by the relations R = 10 log$|r|^2$ and T = = 10 log$|t|^2$, respectively. In the calculations, the angle of incidence is changed between 0° and 90° to realize all possible angles of incidence. The relative permittivity and permeability of region 1 are equal to 1 (free space). For simplicity, the permittivity of the metamaterial in region 2 is assumed to have a moderate normal value, for example, $\varepsilon_{xx} = \varepsilon_{zz} = 1$ [1].

Figure 2 shows the reflection as a function of the angle of incidence for m = 90, Re μ = 0, and for various imaginary parts of the permeability of the metamaterial (Im μ = 1, 9, 14.3, 20, 50). It can be seen that, the reflection at normal incidence (0° angle of incidence) is minimized (–38, *i.e.*, 99.98 absorption) when μ_{yy} = 14.3i, which is in agreement with our calculations (m = 90, $\mu_{yy} = im/(2\pi)$ = 14.3i). For μ_{yy} values larger than 14.3i, the reflection increases (absorption decreases). On the other hand for μ_{yy} values smaller than 14.3i, the reflection increases (the absorption decreases) as μ_{yy} decreases.

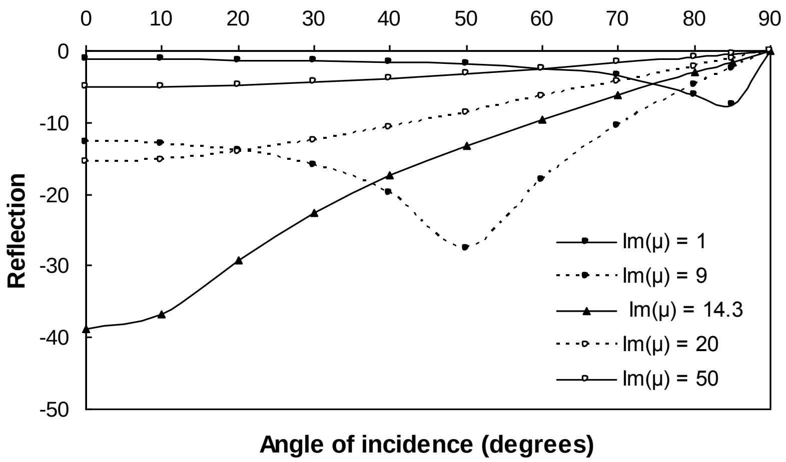


Fig. 2 – The reflection as a function of the angle of incidence for m = 90, Re μ = 0 and for various values of Im μ = 1, 9, 14.3, 20, 50.

Figure 3 illustrates the variation of reflection with the angle of incidence when Re μ changes (Re μ = 0, 1, 2, 3, 10). The values m = 90 and Im μ = 14.3 are kept fixed. As confirmed from the figure, a smaller real μ gives a smaller reflection (a higher absorption). Thus a large absorption is achieved at a zero real part of μ.

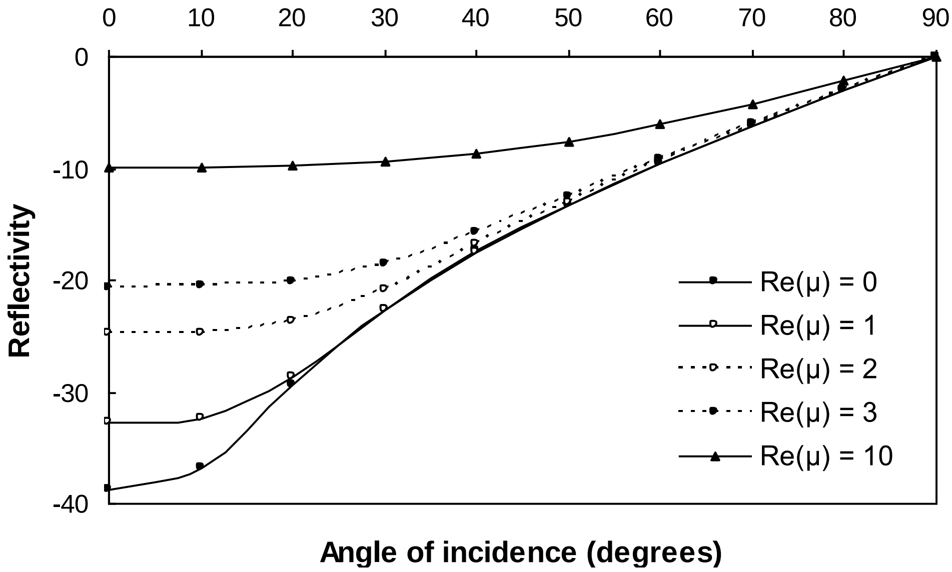


Fig. 3 – The variation of reflection with the angle of incidence when Re μ changes (Re μ = 0, 1, 2, 3, 10). The values m = 90 and Im μ = 14.3 are kept fixed.

Figure 4 presents the reflection versus the angle of incidence for Re μ = 0, Im μ = 14.3 and for different values of thickness (m = 4, 30, 90, 200, 1000). It can be observed that, at the value of metamaterial thickness that meets our calculations (m = 90, $\mu_{yy} = im/(2\pi)$ = 14.3i) and for normal incidence, the reflection is minimum (the absorption is maximum). When the thickness increases beyond m = 90 (m = 4, 30), the reflection rises up, implying lower absorption. For thickness less than m = 90 (m = 200, 1000), the lowest reflection occurs at large oblique incidence.

Figure 5 shows the reflection against the angle of incidence for different values of slab thickness (m = 4, 30, 90, 200, 1000) and their corresponding values of imaginary part of μ (Im μ = .64, 4.8, 14.3, 31.8, 159.2) with Re μ = 0. As can be seen from Fig. 5, the highest absorption is obtained for the thinnest case and its corresponding largest value of Im μ, which is in agreement with the two mentioned above conditions for perfect absorption.

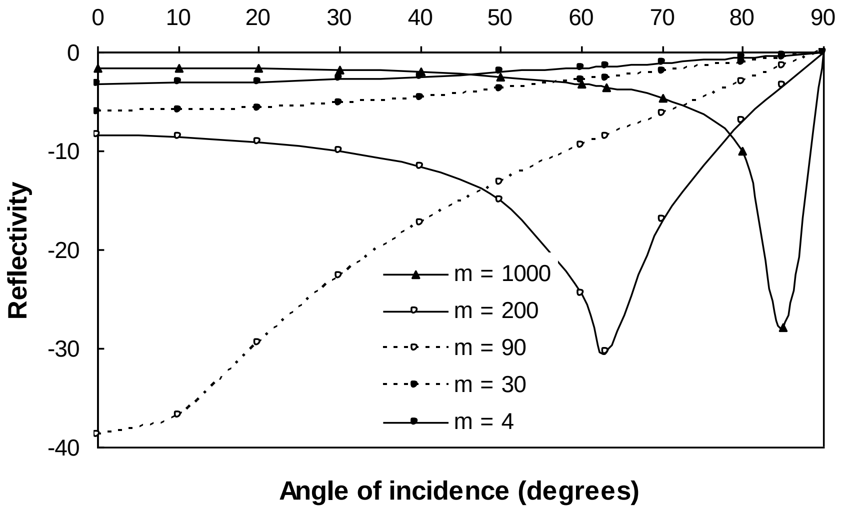


Fig. 4 – The reflection versus the angle of incidence for different values of thickness (m = 4, 30, 90, 200, 1000). Here Re μ = 0 and Im μ = 14.3.

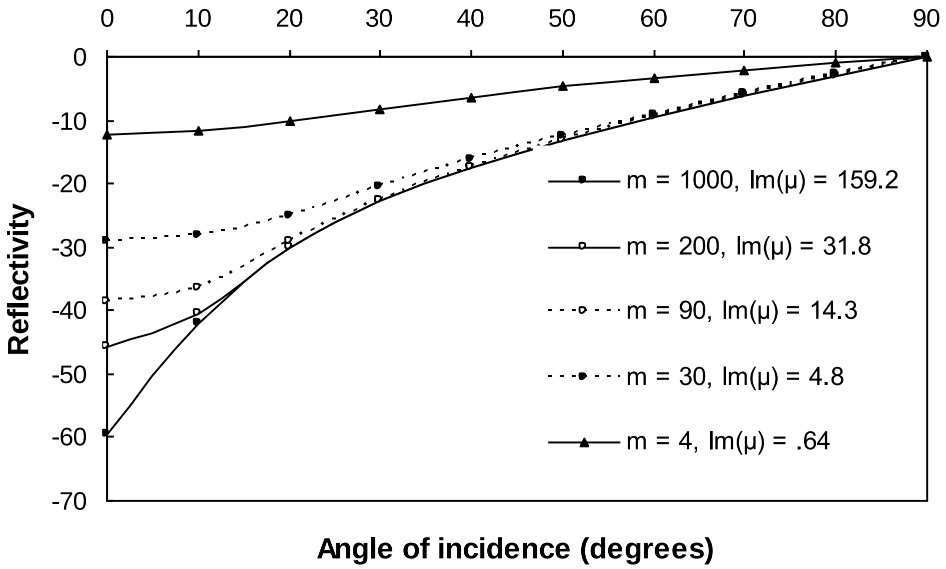
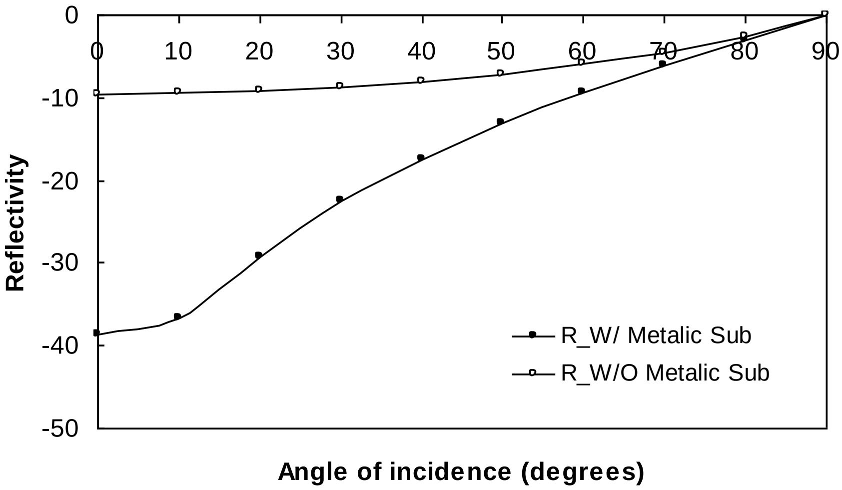
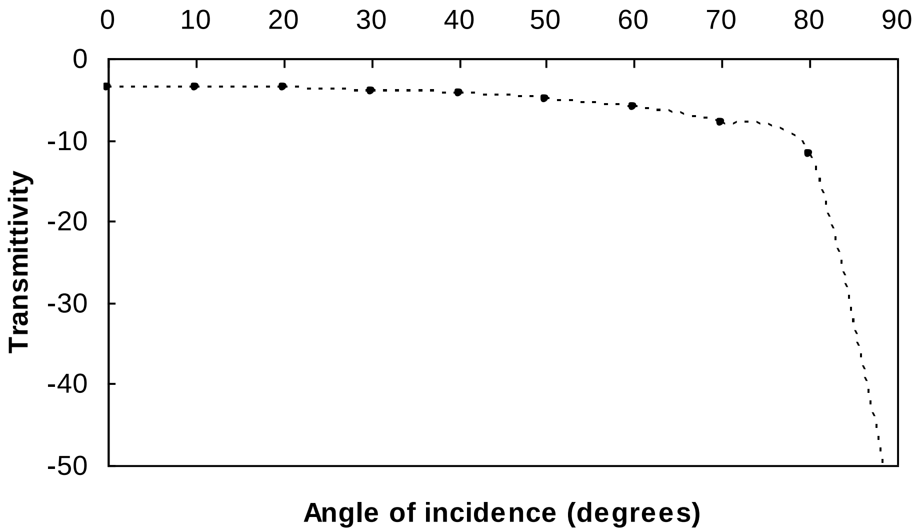


Fig. 5 – The reflection against the angle of incidence for different values of slab thickness (m = 4, 30, 90, 200, 1000) and their corresponding values of imaginary part of μ (Im μ = .64, 4.8, 14.3, 31.8, 159.2) with Re μ = 0.

To check the effect of the metallic substrate, Fig. 6 shows a calculation of the reflection (with and without the metallic substrate, see the panel (a) of Fig. 6) and of transmission (without the metallic substrate, see the panel (b) of Fig. 6) as a function of the angle of incidence for Re $\mu = 0$, $m = 90$, and Im $\mu = 14.3$. As seen in Fig. 6, both the reflection and transmission increase when the metallic substrate is removed.



a)



b)

Fig. 6 – a) The reflection with and without the metallic substrate; b) the transmission without the metallic substrate.

4. CONCLUSIONS

In this paper, a perfect absorption at metamaterial surface is proposed for a wide range of incidence angles and for any wavelength of the electromagnetic radiation, where a metamaterial slab is arranged between vacuum and metallic substrate. The structure parameters are defined and the required equations for the electromagnetic wave propagation are derived using Maxwell's equations. The boundary conditions are imposed and Snell's law is applied to obtain the reflection and transmission coefficients in the closed form. We have shown that the metamaterial permeability of a vanishing real part but a large imaginary part as well as a very thin metamaterial slab are required for perfect absorption. Finally, the reflection and absorption coefficients as a function of the incidence angle, metamaterial slab thickness, and metamaterial permeability with its real and imaginary parts are studied numerically to demonstrate the effects of the perfect absorption conditions. As it can be seen from the theoretical and numerical results, if the mentioned conditions are satisfied, the minimum reflection and maximum absorption will be clearly observed.

REFERENCES

1. S. Zhong and S. He, *Ultrathin and lightweight microwave absorbers made of mu-near-zero metamaterial*, Scientific Reports **3**, 1–5 (2013).
2. K. B. Alici, F. Bilotti, L. Vegni, and E. Ozbay, *Experimental verification of metamaterial based subwavelength microwave absorbers*, J Appl. Phys. **108**, 083113 (2010).
3. G. T. Ruck, D. E. Barrick, and W. D. Stuart, *Radar Cross Section*, Handbook, Plenum Press, New York, 1970.
4. G. M. Watts, X. Liu, and W. J. Padilla, *Metamaterial electromagnetic wave absorbers*, Advanced Optical Materials **24**, OP98–OP120 (2012).
5. B. A. Munk, *Frequency Selective Surfaces: Theory and Design*, Wiley-Interscience Publication, New York, 2000.
6. E. Knott, J. F. Shaeffer, and M. T. Tuley, *Radar Cross Section*, 2nd ed., SciTech Publishing, Raleigh, 2004.
7. J. B. Pendry, A. J. Holden, W. J. Stewart, and I. Youngs, *Extremely low frequency plasmons in metallic mesostrctures*, Phys. Rev. Lett. **76**, 4773–4776 (1996).
8. J. B. Pendry, A. J. Holden D. J. Robbins, and W. J. Stewart, *Magnetism form conduction and enhanced nonlinear phenomena*, IEEE Transactions on Microwave Theory and Techniques **47**, 2075–2084 (1999).
9. N. I. Landy, S. Sajuyigbe, J. J. Mock, D. R. Smith, and W. J. Padilla, *Perfect metamaterial absorbers*, Phys. Rev. Lett. **100**, 207402 (2008).
10. H. Tao *et al.*, *Highly flexible wide angle of incidence terahertz metamaterial absorber: design, fabrication, and characterization*, Phys. Rev. B **78**, 241103 (2008).
11. N. I. Landy *et al.*, *Design, theory, and measurement of a polarization-insensitive absorber for terahertz imaging*, Phys. Rev. B **79**, 125104 (2009).
12. L. Meng, D. Zhao, Q. Li, and M. Qiu, *Polarization-sensitive perfect absorbers at near-infrared wavelengths*, Opt. Express **21**, A111–A122 (2013).
13. H. T. Chen, *Interference theory of metamaterial perfect absorbers*, Opt. Express **20**, 7165–7172 (2012).

14. A. Alu *et al*., *Epsilon-near-zero metamaterials and electromagnetic sources: Tailoring the radiation phase pattern*, Phys. Rev. B **75**, 155410 (2007).
15. X. Liu, T. Starr, A. F. Starr, and W. J. Padilla, *Infrared spatial and frequency selective metamaterial with near-unity absorbance*, Phys. Rev. Lett. **104**, 20403 (2010).
16. L. Huang, D. R. Chowdhury, S. Ramani, M. T. Reiten, S-N. Luo, A. K. Azad, A. J. Taylor, and H.-T. Chen, *Impact of resonator geometry and its coupling with ground plane on ultrathin metamaterial perfect absorbers*, Appl. Phys. Lett. **101**, 101102 (2012).
17. Y. Avitzour, Y. A. Urzhumov, and G. Shvets, *Wide-angle infrared absorber based on a negative-index plasmonic metamaterial*, Phys. Rev. B **79**, 045131 (2009).
18. R. A. Shelby, PhD Thesis, University of California, San Diego, *Microwave Experiments with Left-Handed Materials*, Bell and Howell Information and Learning Company, 2001.
19. J. A. Kong, *Theory of Electromagnetic Waves*, EMW Publishing, Massachusetts, 2005.
20. M. F. Ubeid, M. M. Shabat and M. O. Sid-Ahmed, *Effect of negative permittivity and permeability on the transmission of electromagnetic waves through a structure containing left-handed material*, Natural Science **3**, *4*, 328–333 (2011).
21. M. F. Ubeid, M. M. Shabat, and M. O. Sid-Ahmed, *Numerical study of negative-refractive index ferrite waveguide*, Journal of Nano- and Electronic Physics **4**, *1*, 01009 (2012).
22. C. Caloz and T. Itoh, *Electromagnetic Metamaterials*, John Wiley and Sons, New Jersey, 2006.
23. C. Sabah and S. Uckun, *Electromagnetic wave propagation through frequency-dispersive and loss double-negative slab*, Opto-Elec. Rev. **15**, *3*, 133–143 (2007).
24. D. D. Stancil and A Prabhakar, *Spin Waves*, Springer, New York, 2009.