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Soft βc-Generalized Closed Sets

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Abstract
Previously, we introduced and studied the concept of soft $\beta c-$open sets in soft topological spaces. This paper is a continuation study of this concept. We introduce and investigate the notion of soft $\beta c-$generalized closed sets. We discuss some soft properties of this type of soft sets. Also, we introduce the concepts of soft $\beta c-T_{\frac{1}{2}}$ space and soft $\beta c-T_{\circ}$ space. In addition, we introduce the notion of soft $\beta c-$Kernel of a soft set and use it to get a characterization of soft sets to be soft $\beta c-$generalized closed sets.

Keywords:
Soft topology, Soft $\beta-$open sets, Soft $\beta c-$open sets, Soft $\beta c-$generalized closed sets, Soft $\beta c-T_{\frac{1}{2}}$ space.

1. Introduction:

Molodtsove (Molodtsov, 1999) initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainly. He successfully applied the soft set theory into several directions such as smoothness of functions, game theory, Riemann Integration, theory of measurement, and so on. Soft set theory and its applications have shown great development in recent years. This is because of the general nature of parametrization expressed by a soft set. Shabir and Naz (Shabir and Maz, 2011) introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. The authors in (Aygnolu and Aygn, 2012) continued the study of properties of soft topological space. Weaker forms of soft open sets were first studied by Chen in (Chen, 2013). He investigate soft semi-open sets in soft topological spaces and studied some properties of them. Arockiarani and Arokialancy are defined soft $\beta-$open sets and continued to study other weaker forms of soft open sets in soft topological space. Later, Akdag and Ozkan (Akdag and Ozkan, 2014b) defined soft $\alpha-$open sets (Akdag and Ozkan, 2014a).

2. Preliminary Notes About Soft Sets:

Definition 2.1 *(Molodtsov, 1999)* Let X be an initial universe set and E a set of parameters. Let $\mathrm{P}(X)$ be the power set of X and A a nonempty subset of E. A pair (F,A), denoted by F_A, is called a *soft set* over X if F is a mapping given by $F:A\to \mathrm{P}(X)$. Shortly, a soft set over X is a parameterized family of subsets of the universe X. The family of all these soft sets over X is denoted by $SS(X)_A$. For a particular $e\in A$, the collection $\{F(e): F \text{ is a soft set}\}$ is considered to be *the set of e-approximate elements* of the soft sets. If $e\notin A$, then $F(e)=\varnothing$.

Let I be an arbitrary indexed set and $L=\{(F_i,A): i\in I\}$ be a subfamily of $SS(X)_A$.

The union of L is the soft set (H,A) (Maji et al., 2003), where $H(e)=\bigcup_{i\in I}F_i(e)$ for each $e\in A$. We write $\tilde{\bigcup}_{i\in I}(F_i,A)=(H,A)$. *The intersection* of L is the soft set (M,A) (Maji et al., 2003), where $M(e)=\bigcap_{i\in I}F_i(e)$ for each $e\in A$. We write $\tilde{\bigcap}_{i\in I}(F_i,A)=(M,A)$. If (G,A), (F,A) are two soft sets over X, we say (F,A) is *a soft subset* of (G,B) (or (G,B) is said to be *a soft superset* of (F,A)) (Maji et al., 2003), denoted by $(F,A)\tilde{\subseteq}(G,B)$, if $A\subseteq B$ and $F(e)\subseteq G(e)$, $\forall e\in A$. Also (F,A) and (G,B) are called *soft equal*, if $A=B$ and $F(e)=G(e)$, $\forall e\in A$ (Maji et al., 2003). For a soft set (F,A) over X, we define $(F,A)^c=(F^c,A)$ to be the soft complement of (F,A) such that $F^c(e)=X-F(e)$ (Ali et al., 2009). $(F,A)\tilde{\subseteq}(G,A)$ iff $(G,A)^c\tilde{\subseteq}(F,A)^c$. A soft set (F,A) over X is called *a null soft set* (Maji et al., 2003) (resp. *absolute soft set* (Maji et al., 2003)), denoted by Φ_A (resp. denoted by X_A), if for all $e\in A$, $F(e)=\varnothing$ (resp. if for all $e\in A$, $F(e)=X$). Clearly, $X_A^c=\Phi_A$. *A soft point* (Das and Shmanta, 2013), denoted by x_e, is a soft set where $x\in X$ and $e\in A$ defined by $x_e(e)=\{x\}$ and $x_e(e')=\varnothing$ $\forall$ $e'\neq e$ in A. $x_e\tilde{\in}(G,A)$ if for the element $e\in A$, $\{x\}\subseteq G(e)$. Note that any soft point $x_e\tilde{\in}X_A$. The *difference of two soft sets* (Pei and Miao, 2005) (F,A) and (G,A) over a common universe X, denoted by $(F,A)-(G,A)$ is the soft set (H,A) where for all $e\in A$, $H(e)=F(e)-G(e)$.

Theorem 2.2 *(Ali et al., 2009) If (F,A) and (G,A) are two soft sets in $SS(X)_A$, then*

1. $\left((F,A)\tilde{\cup}(G,A)\right)^c=(F,A)^c\tilde{\cap}(G,A)^c$.
2. $\left((F,A)\tilde{\cap}(G,A)\right)^c=(F,A)^c\tilde{\cup}(G,A)^c$.

Theorem 2.3 *(Ali et al., 2009) Let (F,A) and (G,A) be soft sets in $SS(X)_A$. Then the following are true:*

1. $(F,A)\tilde{\cap}\phi_A=\phi_A$.
2. $(F,A)\tilde{\cap}X_A=(F,A)$.
3. $(F,A)\tilde{\cup}\phi_A=(F,A)$.
4. $(F,A)\tilde{\cup}X_A=X_A$.

Theorem 2.4 *Let (F,A), (G,A), (H,A) and (S,A) $\in SS(X)_A$. Then the following are true:*

1. If $(F,A)\tilde{\cap}(G,A)=\phi_A$, then $(F,A)\tilde{\subseteq}(G,A)^c$ (Zorlutuna etal., 2012).
2. $(F,A)\tilde{\cup}(F,A)^c=X_A$ (Ali et al., 2011).
3. If $(F,A)\tilde{\subseteq}(G,A)$ and $(G,A)\tilde{\subseteq}(H,A)$, then $(F,A)\tilde{\subseteq}(H,A)$ (Zorlutuna etal., 2012).
4. $(F,A)\tilde{\subseteq}(G,A)$ iff $(G,A)^c\tilde{\subseteq}(F,A)^c$.

Theorem 2.5 *(Zorlutuna etal., 2012) Let (F,A) and (G,A) be soft sets in $SS(X)_A$. Then the following are true:*

1. $(F,A)\tilde{\subseteq}(G,A)$ iff $(F,A)\tilde{\cap}(G,A)=(F,A)$.
2. $(F,A)\tilde{\subseteq}(G,A)$ iff $(F,A)\tilde{\cup}(G,A)=(G,A)$.

Theorem 2.6 *Let $x_e\tilde{\in}X_A$ and $(G,A)\tilde{\subseteq}X_A$. Then the following are true:*

1. If $x_e\tilde{\in}(G,A)$, then $x_e\tilde{\notin}(G,A)^c$ (Zorlutuna etal., 2012).
2. If $x_e\tilde{\notin}(G,A)^c$, then $x_e\tilde{\in}(G,A)$.

3. Soft Topology:

Definition 3.1 *(Shabir and Maz, 2011)* Let τ be a collection of soft sets over a universe X with a fixed set of parameters A. Then τ is said to be a *soft topology* on X, if

1. Φ_A, X_A belong to τ.
2. The union of any number of soft sets in τ belongs to τ.
3. The intersection of any two soft sets in τ belongs to τ.

The triple (X,τ,A) (briefly, $\tilde{X}$) is called *a soft topological space over* X. The members of τ are called *soft open sets*. A soft complement of a soft open set (F,A) is called *a soft closed set* in $\tilde{X}$. If (F,A) belongs to τ, we write $(F,A)\tilde{\in}\tau$. A soft set (F,A) which is both soft open and soft closed is called *soft clopen set*.

Definition 3.2 *(Shabir and Maz, 2011)* Let $\tilde{X}$ be a soft topological space over X and (F,A) a soft set over X. Then the *soft closure* of (F,A), denoted by $\overline{(F,A)}$, is the intersection of all soft closed supersets of (F,A). Clearly $\overline{(F,A)}$ is the smallest soft closed set in (X,τ,A) which contains (F,A).

Definition 3.3 *(Shabir and Maz, 2011)* Let $\tilde{X}$ be a soft topological space over X and (F,A) a soft set over X. Then *soft interior* of a soft set (F,A) is denoted by $(F,A)^{\circ}$ and is defined as the union of all soft open sets contained in (F,A). Clearly $(F,A)^{\circ}$ is the largest soft open set contained in (F,A).

Definition 3.4 *(Yumak and Kaymakc, 2013)* A soft set (F,A) in a soft topological space (X,τ,A) is called *soft $\beta-$open set* if $(F,A)\tilde{\subseteq}\overline{\left(\overline{(F,A)}\right)^{\circ}}$.

Definition 3.5 *(Fayad and Mahdi, 2017)* Let $\tilde{X}$ be a soft topological space and $(F,A)\in SS(X)_A$. Then (F,A) is called *a soft $\beta c-open$* if (F,A) is a soft $\beta-$open set and for each $x_e\tilde{\in}(F,A)$ there is a soft closed set (H,A) in $SS(X)_A$ such that $x_e\tilde{\in}(H,A)\tilde{\subseteq}(F,A)$. The complement of a soft $\beta c-$open set is called a *soft $\beta c-$closed set.*

Theorem 3.6 *(Fayad and Mahdi, 2017) An arbitrary union of soft $\beta c-open$ sets is a soft $\beta c-open$ set.*

Definition 3.7 *(Fayad and Mahdi, 2017)* Let $\tilde{X}$ be a soft topological space over X and (B,A) a soft set over X. The *soft $\beta c-closure$* of (B,A), denoted by $Cl_{\beta c}(B,A)$, is the intersection of all soft $\beta c-$closed supersets of (B,A). Clearly $Cl_{\beta c}(B,A)$ is the smallest soft $\beta c-$closed set in $\tilde{X}$ which contains (B,A).

Theorem 3.8 *(Fayad and Mahdi, 2017) Let $\tilde{X}$ be a soft topological space, (F,A) a soft set over X and $x_e\tilde{\in}X_A$. Then, the following two statement are equivalent:*

1. $x_e\tilde{\in}Cl_{\beta c}(F,A)$.
2. For any soft $\beta c-$open set (G,A) over X containing x_e we have, $(F,A)\tilde{\cap}(G,A)\neq\phi_A$.

Theorem 3.9 *(Fayad and Mahdi, 2017) Let $\tilde{X}$ be a soft topological space and let (F,A) and (G,A) be soft sets over X. Then*

1. (F,A) is a soft $\beta c-$closed set if and only if $(F,A)=Cl_{\beta c}(F,A)$.
2. $Cl_{\beta c}(\phi_A)=\phi_A$ and $Cl_{\beta c}(X_A)=X_A$.
3. $Cl_{\beta c}(Cl_{\beta c}(F,A))=Cl_{\beta c}(F,A)$.
4. $(F,A)\tilde{\subseteq}(G,A)$ implies $Cl_{\beta c}(F,A)\tilde{\subseteq}Cl_{\beta c}(G,A)$.
5. If $Cl_{\beta c}(F,A)\tilde{\cap}Cl_{\beta c}(G,A)=\phi_A$, then $(F,A)\tilde{\cap}(G,A)=\phi_A$.
6. $Cl_{\beta c}(F,A)\tilde{\cup}Cl_{\beta c}(G,A)\tilde{\subseteq}Cl_{\beta c}((F,A)\tilde{\cup}(G,A))$.
7. $Cl_{\beta c}((F,A)\tilde{\cap}(G,A))\tilde{\subseteq}Cl_{\beta c}(F,A)\tilde{\cap}Cl_{\beta c}(G,A)$.

4. Soft $\beta c-$Generalized Closed Sets:

Definition 4.1 Let $\tilde{X}$ be a soft topological space and $(F,A)\tilde{\in}SS(X)_A$. Then (F,A) is called a *soft $\beta c-$ generalized closed* (briefly, a *soft $\beta c-g.closed$*) set if $Cl_{\beta c}(F,A)\tilde{\subseteq}(G,A)$ whenever $(F,A)\tilde{\subseteq}(G,A)$ and (G,A) is a soft $\beta c-$-open set over X. The complement of a soft $\beta c-$g.closed set is called *soft $\beta c-generalized$ open set.*

Theorem 4.2 *In a soft topological space $\tilde{X}$, every soft $\beta c-closed$ set is soft $\beta c-g.closed$ set.*

Proof. Let (F,A) be a soft $\beta c-$closed set and (G,A) a soft $\beta c-$open set such that $(F,A)\tilde{\subseteq}(G,A)$. Then, $Cl_{\beta c}(F,A)=(F,A)\tilde{\subseteq}(G,A)$. Therefore, (F,A) is a soft $\beta c-$g.closed set.

Remark 4.3 *The converse of the above theorem need not be true in general as shown in the following example:*

Example 4.4 *Let $X=\{a,b,c\}$ and $A=\{e\}$ with a soft topology $\tau=\{\phi_A,X_A,(e,\{a\}),(e,\{b\}),(e,\{a,b\}),(e,\{a,c\})\}$. Then, the soft $\beta c-open$ sets over X are ϕ_A, X_A, $(e,\{b\})$ and $(e,\{a,c\})$. If $(F,A)=(e,\{a\})$, then $(e,\{a,c\})$ and X_A are the soft $\beta c-open$ sets contain (F,A). Moreover $Cl_{\beta c}(F,A)=(e,\{a,c\})$. So, $Cl_{\beta c}(F,A)\tilde{\subseteq}(e,\{a,c\})$ and $Cl_{\beta c}(F,A)\tilde{\subseteq}X_A$.*

Therefore, (F,A) *is a soft* $\beta c-$*g.closed set, which is not soft* $\beta c-$*closed set.*

Theorem 4.5 *Let* $\tilde{X}$ *be a soft topological space and* (F,A) *a soft set over* X*. If* (F,A) *is a soft* $\beta c-$*open and soft* $\beta c-$*g.closed set, then* (F,A) *is a soft* $\beta c-$*closed set.*

Proof. Since $(F,A)\tilde{\subseteq}(F,A)$ and $Cl_{\beta c}(F,A)\tilde{\subseteq}(F,A)$, $Cl_{\beta c}(F,A)=(F,A)$. Therefore, (F,A) is a soft $\beta c-$closed set.

Theorem 4.6 *Let* $\tilde{X}$ *be a soft topological space and let* (F,A) *and* (G,A) *be soft sets over* X*. If* (F,A) *is a soft* $\beta c-$*g.closed set and* (G,A) *is a soft* $\beta c-$*closed set, then* $(F,A)\tilde{\cap}(G,A)$ *is a soft* $\beta c-$*g.closed set.*

Proof. If (M,A) is a soft $\beta c-$open set over X such that $(F,A)\tilde{\cap}(G,A)\tilde{\subseteq}(M,A)$, then $(F,A)\tilde{\subseteq}(M,A)\tilde{\cup}(G,A)^c$ where $(M,A)\tilde{\cup}(G,A)^c$ is a soft $\beta c-$open set. Since (F,A) is a soft $\beta c-$g.closed set, $Cl_{\beta c}(F,A)\tilde{\subseteq}(M,A)\tilde{\cup}(G,A)^c$. So, $Cl_{\beta c}((F,A)\tilde{\cap}(G,A))$ $\tilde{\subseteq}Cl_{\beta c}(F,A)\tilde{\cap}Cl_{\beta c}(G,A)=Cl_{\beta c}(F,A)\tilde{\cap}(G,A)\tilde{\subseteq}((M,A)\tilde{\cup}(G,A)^c)$ $\tilde{\cap}\ (G,A)=((M,A)\tilde{\cap}(G,A))\tilde{\cup}((G,A)^c\ \tilde{\cap}(G,A))\tilde{\subseteq}((M,A)$. Therefore, $(F,A)\tilde{\cap}(G,A)$ is a soft $\beta c-$g.closed set.

Theorem 4.7 *Let* $\tilde{X}$ *be a soft topological space and let* (F,A) *and* (G,A) *be soft sets over* X*. If* (F,A) *is a soft* $\beta c-$*g.closed set such that* $(F,A)\tilde{\subseteq}(G,A)\tilde{\subseteq}Cl_{\beta c}(F,A)$*, then* (G,A) *is a soft* $\beta c-$*g.closed set.*

Proof. If (M,A) is a soft $\beta c-$open set over X such that $(G,A)\tilde{\subseteq}(M,A)$, then $(F,A)\tilde{\subseteq}(M,A)$. Also, $Cl_{\beta c}(F,A)\tilde{\subseteq}Cl_{\beta c}(G,A)\tilde{\subseteq}Cl_{\beta c}(F,A)\tilde{\subseteq}(M,A)$. Therefore, (G,A) is a soft $\beta c-$g.closed set.

Theorem 4.8 *Let* $\tilde{X}$ *be a soft topological space. For each* $x_e\tilde{\in}X_A$*, either* $\{x_e\}$ *is a soft* $\beta c-$*closed set or* $\{x_e\}^c$ *is a soft* $\beta c-$*g.closed set.*

Proof. If $\{x_e\}$ is not soft $\beta c-$closed set, then $\{x_e\}^c$ is not soft $\beta c-$open set and so the only soft $\beta c-$open set containing $\{x_e\}^c$ is X_A. So, $Cl_{\beta c}(\{x_e\}^c)\tilde{\subseteq}X_A$. Therefore, $\{x_e\}^c$ is a soft $\beta c-$g.closed set.

Theorem 4.9 *Let* $\tilde{X}$ *be a soft topological space and* (F,A) *a soft set over* X*. Then,* (F,A) *is a soft* $\beta c-$*g.closed set if and only if for each* $x_e\tilde{\in}Cl_{\beta c}(F,A)$ *we have* $Cl_{\beta c}(\{x_e\})\tilde{\cap}(F,A)\neq\phi_A$*.*

Proof. Let (F,A) be a soft $\beta c-$g.closed set and $x_e\tilde{\in}Cl_{\beta c}(F,A)$ such that $Cl_{\beta c}(\{x_e\})\tilde{\cap}(F,A)=\phi_A$. Since $Cl_{\beta c}(\{x_e\})$ is a soft $\beta c-$closed set, $\left(Cl_{\beta c}(\{x_e\})\right)^c$ is a soft $\beta c-$open set and $(F,A)\tilde{\subseteq}\left(Cl_{\beta c}(\{x_e\})\right)^c$. But (F,A) is a soft $\beta c-$g.closed set, so that $Cl_{\beta c}(F,A)\tilde{\subseteq}\left(Cl_{\beta c}(\{x_e\})\right)^c$. Hence, $x_e\tilde{\notin}Cl_{\beta c}(\{x_e\})$ which is a contradiction. Conversely, let (M,A) be a soft βc-open set such that $(F,A)\tilde{\subseteq}(M,A)$. Since $Cl_{\beta c}(\{x_e\})\tilde{\cap}(F,A)\neq\phi_A$, there exists $y_{e'}\tilde{\in}(F,A)\tilde{\subseteq}(M,A)$ and $y_{e'}\tilde{\in}Cl_{\beta c}(\{x_e\})$. Then, by Theorem 3.8, $(M,A)\tilde{\cap}\{x_e\}\neq\phi_A$ and so, $x_e\tilde{\in}(M,A)$ which implies that, $Cl_{\beta c}(F,A)\tilde{\subseteq}(M,A)$. Therefore, (F,A) is a soft $\beta c-$g.closed set.

Theorem 4.10 *Let* $\tilde{X}$ *be a soft topological space and* (F,A) *a soft set over* X*. Then,* (F,A) *is a soft* $\beta c-$*g.closed set if and only if* $Cl_{\beta c}(F,A)-(F,A)$ *does not not containing any non-null soft* $\beta c-$*closed set.*

Proof. Let (F,A) be a soft $\beta c-$g.closed set and there exists a non-null soft $\beta c-$closed set (G,A) such that $(G,A)\tilde{\subseteq}Cl_{\beta c}(F,A)-(F,A)$. Then $(G,A)\tilde{\subseteq}(F,A)^c$ and so, $(F,A)\tilde{\subseteq}(G,A)^c$. Therefore, $Cl_{\beta c}(F,A)\tilde{\subseteq}(G,A)^c$ which implies, $(G,A)\tilde{\subseteq}\left(Cl_{\beta c}(F,A)\right)^c$ Hence, $(G,A)\tilde{\subseteq}Cl_{\beta c}(F,A)\tilde{\cap}\left(Cl_{\beta c}(F,A)\right)^c=\phi_A$ and so $(G,A)=\phi_A$ which is a contradiction. Conversely, if (M,A) is a soft $\beta c-$open set such that $(F,A)\tilde{\subseteq}(M,A)$ and $Cl_{\beta c}(F,A)\tilde{\cup}(M,A)$. Then $Cl_{\beta c}(F,A)\tilde{\cap}(M,A)^c\neq\phi_A$ and $Cl_{\beta c}(F,A)\tilde{\cap}(M,A)^c$ $\tilde{\subseteq}Cl_{\beta c}(F,A)\tilde{\cap}(F,A)^c=Cl_{\beta c}(F,A)-(F,A)$. Therefore,

$Cl_{\beta c}(F,A)\tilde{\cap}(M,A)^c \tilde{\subseteq} Cl_{\beta c}(F,A)-(F,A)$ and $Cl_{\beta c}(F,A)\tilde{\cap}(M,A)^c$ is a non-null soft $\beta c-$closed set which is a contradiction.

Theorem 4.11 *Let $\tilde{X}$ be a soft topological space and (F,A) a soft set over X. Then, the following are equivalent:*

1. Every soft set over X is a soft $\beta c-$g.closed set.
2. (F,A) is a soft $\beta c-$open set if and only if (F,A) is a soft $\beta c-$closed set.

Proof. Direct using Definition 4.1.

5. Soft $\beta c-T_{\frac{1}{2}}$ Space:

Definition 5.1 A soft topological space $\tilde{X}$ is called *soft $\beta c-T_{\frac{1}{2}}$ space* if every soft $\beta c-$g.closed set over X is soft $\beta c-$closed set over X.

Theorem 5.2 *A soft topological space $\tilde{X}$ is soft $\beta c-T_{\frac{1}{2}}$ space if and only if for each $x_e \tilde{\in} X_A$, x_e is either soft $\beta c-$closed set or x_e is soft $\beta c-$open set.*

Proof. Suppose that x_e is not soft $\beta c-$closed set. Then, by Theorem 4.8, x_e^c is a soft $\beta c-$g.closed set. Since $\tilde{X}$ is a soft $\beta c-T_{\frac{1}{2}}$ space, x_e^c is a soft $\beta c-$closed set. Therefore, x_e is a soft $\beta c-$open set. Conversely, Let (F,A) be a soft $\beta c-$g.closed set over X and let $x_e \tilde{\in} Cl_{\beta c}(F,A)$. If x_e is a soft $\beta c-$closed set, then $x_e \tilde{\notin}(F,A)$ which implies that $x_e \tilde{\in} Cl_{\beta c}(F,A)-(F,A)$ which contradicts Theorem 5.2. Therefore, $x_e \tilde{\in}(F,A)$ and $Cl_{\beta c}(F,A)=(F,A)$. On the second case, if x_e is a soft $\beta c-$open where $x_e \tilde{\in} Cl_{\beta c}(F,A)$, then $x_e \tilde{\cap}(F,A)\neq \phi_A$. Therefore, $x_e \tilde{\in}(F,A)$ and (F,A) is a soft $\beta c-$closed set.

Corollary 5.3 *Let $\tilde{X}$ be a soft topological space and (F,A) a soft set over X. Then, the following are equivalent:*

1. $\tilde{X}$ is a soft $\beta c-T_{\frac{1}{2}}$ space.
2. (F,A) is a soft $\beta c-$closed (resp. a soft $\beta c-$open) set if and only is (F,A) is a soft $\beta c-$g.closed (resp. a soft $\beta c-$g.open) set.

Definition 5.4 Let $\tilde{X}$ be a soft topological space and $x_e, y_{e'} \tilde{\in} X_A$ such that $x_e \neq y_{e'}$. Then $\tilde{X}$ is called a *soft $\beta c-T_\circ$ space* if there exist soft βc-open sets (F,A) and (G,A) such that either $x_e \tilde{\in}(F,A)$ and $y_{e'} \tilde{\notin}(F,A)$ or $y_{e'} \tilde{\in}(G,A)$ and $x_e \tilde{\notin}(G,A)$.

Theorem 5.5 *Every soft $\beta c-T_{\frac{1}{2}}$ space is soft $\beta c-T_\circ$ space.*

Proof. Let $\tilde{X}$ be a soft $\beta c-T_{\frac{1}{2}}$ space and let $x_e \neq y_{e'}$ over X. Then, by Theorem 5.2, x_e is either soft $\beta c-$closed or soft $\beta c-$open set. If x_e is a soft $\beta c-$closed set, then x_e^c is a soft $\beta c-$open set contains $y_{e'}$ and not x_e. Hence, $\tilde{X}$ is a soft $\beta c-T_\circ$ space. In the second case, if x_e is a soft $\beta c-$open set, then $x_e \tilde{\in} x_e$ and $y_{e'} \tilde{\notin} x_e$. Therefore, $\tilde{X}$ is a soft $\beta c-T_\circ$ space.

Remark 5.6 *The converse of the above theorem need not be true in general as illustrated in the following example:*

Example 5.7 *Let $X=\{a,b,c\}$ and $A=\{e\}$ with a soft topology $\tau=\{\phi_A, X_A,(e,\{a\}),(e,\{b\}),(e,\{a,b\})\}$. Then, the soft $\beta c-$open sets are ϕ_A, X_A, $(e,\{a,c\})$ and $(e,\{b,c\})$. Then $\tilde{X}$ is a soft $\beta c-T_\circ$ space but not soft $\beta c-T_{\frac{1}{2}}$ space because $(e,\{c\})$ is soft $\beta c-$g.closed set, which is not soft $\beta c-$closed set.*

Theorem 5.8 *A soft topological space* $\tilde{X}$ *is a soft* $\beta c - T_\circ$ *space if and only if for each pair of distinct soft points* x_e *and* $y_{e'}$ *over* X, $Cl_{\beta c}(x_e) \neq Cl_{\beta c}(y_{e'})$.

Proof. There is a soft $\beta c -$open set (U, A) over X containing x_e but not $y_{e'}$ which implies, $y_{e'} \tilde{\in} (U, A)^c$ where $(U, A)^c$ is a soft $\beta c -$closed set. Hence, $Cl_{\beta c}(y_{e'}) \tilde{\subseteq} (U, A)^c$ and $x_e \tilde{\notin} Cl_{\beta c}(y_{e'})$. Therefore, $Cl_{\beta c}(x_e) \neq Cl_{\beta c}(y_{e'})$. Conversely, if $Cl_{\beta c}(x_e) \tilde{\cup} Cl_{\beta c}(y_{e'})$, then $\left(Cl_{\beta c}(y_{e'})\right)^c$ is a soft $\beta c -$open set such that $y_{e'} \tilde{\notin} \left(Cl_{\beta c}(y_{e'})\right)^c$ and $x_e \tilde{\in} \left(Cl_{\beta c}(y_{e'})\right)^c$ (if $x_e \tilde{\notin} \left(Cl_{\beta c}(y_{e'})\right)^c$, then $x_e \tilde{\in} Cl_{\beta c}(y_{e'})$). So, $Cl_{\beta c}(x_e) \tilde{\subseteq} Cl_{\beta c}(y_{e'})$ which contradicts our assumption. Therefore, $\tilde{X}$ is a soft $\beta c - T_\circ$ space.

Definition 5.9 Let $\tilde{X}$ be a soft topological space and (F, A) a soft set over X. Then, the intersection of all soft $\beta c -$open sets containing (F, A) is called the *soft* $\beta c - Kernal$ of (F, A) and its denoted by $Ker_{\beta c}(F, A)$.

Theorem 5.10 *A soft set* (F, A) *is a soft* $\beta c -$*g.closed set if and only if* $Cl_{\beta c}(F, A) \tilde{\subseteq} Ker_{\beta c}(F, A)$.

Proof. Let $x_e \tilde{\in} Cl_{\beta c}(F, A)$ such that $x_e \tilde{\notin} Ker_{\beta c}(F, A)$. Then, there exists a soft $\beta c -$open set (V, A) such that $(F, A) \tilde{\subseteq} (V, A)$ and $x_e \tilde{\notin} (V, A)$. But (F, A) is a soft $\beta c -$g.closed set which implies, $Cl_{\beta c}(F, A) \tilde{\subseteq} (V, A)$ and so, $x_e \tilde{\notin} Cl_{\beta c}(F, A)$ which is a contradiction. Conversely, let (M, A) be a soft $\beta c -$open set over X such that $(F, A) \tilde{\subseteq} (M, A)$. Then, $Cl_{\beta c}(F, A) \tilde{\subseteq} Ker_{\beta c}(F, A) \tilde{\subseteq} (M, A)$. Therefore, (F, A) is a soft $\beta c -$g.closed set.

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المجموعات المغلقة المعممة اللينة من النوع βc

سابقاً، قمنا بتقديم ودراسة مفهوم المجموعات المفتوحة اللينة من النوع βc في فضاءات التبولوجيا اللينة. هذا البحث هو تكملة لدراسة هذا المفهوم. قمنا فيه بتقديم ودراسة مفهوم المجموعات المغلقة المعممة من نوع βc. ناقشنا خلاله بعض الخصائص اللينة لهذا النوع من المجموعات اللينة. أيضاً قمنا بتعريف ودراسة مفاهيم الانفصال اللين $-\beta c$ T_0 و $-\beta c$ $T_{\frac{1}{2}}$. إضافة لذلك، قمنا بتقديم مفهوم النواة اللينة من النوع βc واستخدامه للحصول على وصف للمجموعات اللينة لتكون مجموعات لينة مغلقة معممة من نوع βc.

كلمات مفتاحية:
التبولوجيا اللينة،
المجموعات المفتوحة اللينة من النوع β،
المجموعات المفتوحة اللينة من النوع βc،
المجموعات المغلقة المعممة اللينة من نوع βc،
الفضاء اللين $-\beta c$ $T_{\frac{1}{2}}$.