IUG Journal of Natural Studies
Peer-reviewed Journal of Islamic University-Gaza

ISSN 2409-4587

IUGNES
Vol. 25, No 2, 2017, pp 233-238

Accepted on (14-03-2017)

New Types of Alexandroff Lattice Spaces

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Abstract
In this paper, we introduce and investigate new types of Alexandroff spaces using well known types of posets; under the conditions that the corresponding poset is complete lattice, distributive lattice and Boolean Algebra. We present some results about these types. Some characterizations of $A_{cl}-$spaces, $A_{dL}-$spaces, $A_{mL}-$space and $A_{BL}-$spaces are obtained and it mainly shown that a T_0 $A-$space is $A_{BL}-$space if and only if it is homeomorphic to a product of n copies of Sierpinski space if and only if every join-irreducible element is an atom.

Keywords:
Alexandroff space,
Lattice,
Complete lattice,
Distributive lattice,
Boolean algebra.

1. Introduction and Preliminaries:

An *Alexandroff space* (Alexandroff, 1937) (briefly $A-$space) X is a topological space in which the arbitrary intersection of open sets is open. In this space, each element x possesses a smallest neighborhood $V(x)$ which is the intersection of all open sets containing x. For every T_0 $A-$space (X,τ), there is a corresponding poset $(X,\leq_\tau)$ in one to one and onto way, where each one of them is completely determined by the other. If (X,τ) is a T_0 A-space, we define the corresponding partial order $\leq_\tau$, called *(Alexandroff) specialization order*, by: $a\leq_\tau b$ iff $a\in\overline{\{b\}}$ iff $b\in V(a)$. On the other hand, if $(X,\leq)$ is a poset, then the collection $\mathcal{B}$ = $\{\uparrow x: x\leq X\}$ forms a base for a T_0 A-space on X, denoted by $\tau_\leq$. In this case, $V(x)$ = $\uparrow x=\{y: y\geq x\}$ and $\overline{x}=\downarrow x=\{z: z\leq x\}$.If X is a T_0 $A-$space, the the collection of closed sets forms a T_0 $A-$space, denoted by X^∂, and the induced order on X^∂ is the reverse order on X. We consider $(X,\tau_{(\leq)})$ to be a T_0 $A-$space (X,τ) together with its corresponding poset $(X,\leq)$.

A poset $(X,\leq)$ satisfies the *ascending chain condition* (briefly ACC), if for any increasing sequence $x_1\leq x_2\leq x_3\leq\cdots$ in X, there exists $k\in\mathbb{N}$ such that x_k = x_{k+1} = $\cdots$. X satisfies the *descending chain condition* (briefly DCC), if for any decreasing sequence $x_1\geq x_2\geq x_3\geq\cdots$ in X, there exists $k\in N$ such that x_k = x_{k+1} = $\cdots$ A T_0 $A-$space whose corresponding poset satisfies the ACC (resp. DCC is called *Artinian* (resp. *Notherian*) T_0 A-space (Mahdi & Elatrash, 2005). Given a poset $(X,\leq)$, the set of all maximal elements is denoted by $M(X)$ (or simply by M) and the set of all minimal elements is denoted by $m(X)$ (or simply by m). If X is a Artinian (resp. Noetherian) T_0 A-space, then M (resp. m) is non-empty.

For posets $(X_i,\leq_i)$, i = $1,2,\cdots,n$, we can formulate many types of partial orders on the cartesian product $\prod_i^n X_i$ = $X_1 \times X_2 \cdots \times X_n$. The most famous one is the *coordinatewise order* $\leq_c$. For two elements a = $(a_1,a_2,\cdots,a_n)$ and b = $(b_1,b_2,\cdots,b_n)$ in $\prod_i^n X_i$, we have that $a \leq_c b$ iff $a_i \leq_i b_i$ $\forall$ $i=1,2,\cdots,n$.

If X is a poset and $A \subseteq X$, we define $A^l = y : y \leq x, \forall x \in A$ and $A^u = z : z \geq x, \forall x \in A$. A poset X is said to be a *lattice* if $\forall x,y \in X$, bothe $x \wedge y$ = $\sup\{x,y\}^l$ and $x \vee y$ = $\inf\{x,y\}^u$ exist in X $\forall x,y \in X$. A poset is *bounded* if it has a maximum (a top) $\top$ and a minimum (a bottom) $\perp$ elements. Any finite lattice is bounded. A subset of a lattice is called a *sublattice* if it is closed under the meet and the join operations. A lattice is *distributive* if the meet operator distributes over the join operator. That is, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$.

Theorem 1.1 *(Grätzer, 2011) A lattice is distributive if and only if neither the diamond nor the pentagon is embedded to be a sublattice.*

A lattice is *modular* if and only if the pentagon can not be embedded as a sublattice, (see Figure 1). So every distributive lattice is modular lattice.

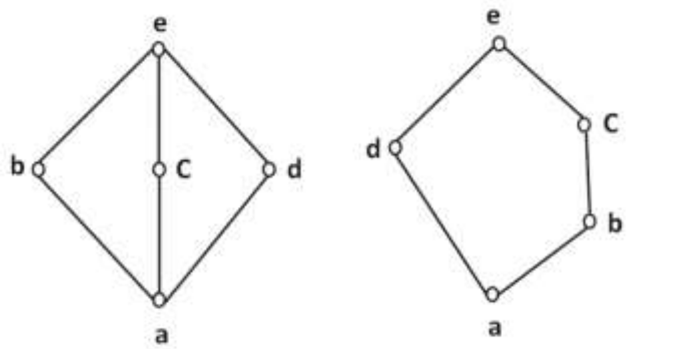

Figure 1 *(a) Diamond and (b) Pentagon*

Let $(X,\leq)$ be a poset and $S \subseteq X$. Then we say $c = \bigvee S$ if $x \leq c \; \forall x \in X$ and if $r \in X$ such that $r \geq x \; \forall x \in S$, then $r \geq c$. Similarly, d = $\bigwedge S$ $d \leq x \; \forall x \in X$ and if $t \in X$ such that $t \leq x \; \forall x \in S$, then $t \leq d$. A lattice X is *complete* if for any $S \subseteq X$ both $\bigvee S$ and $\bigwedge S$ exists in X. Let $(L,\leq)$ be a bounded lattice and $a \in L$. If there exists $b \in L$ such that $a \vee b = \top$ and $a \wedge b = \perp$, we say that a has a *complement* element b. In general an element may have more than one complement. However, in a bounded distributive lattice every element has at most one complement.

Definition 1.2 *(Grätzer, 2011) A Boolean lattice (or Boolean algebra) is a bounded distributive lattice such that each element has a complement.*

The most famous example of a Boolean lattice is a power set $P(S)$ of a non-empty set S.

Theorem 1.3 *(Grätzer, 2011) A finite bounded lattice is Boolean iff it is lattice isomorphic to a Boolean lattice of all subsets of some finite set.*

Theorem 1.4 *(Grätzer, 2011) Let S be a finite set with n elements and $(P(S),\subseteq)$ the Boolean lattice of power set of S. If $A = \{0,1\}$ is the poset such that $0 \leq 1$, then $P(S)$ is lattice isomorphic to the product of n copies of A with coordinatewise order.*

Theorem 1.5 *(Mahdi, 2010) If $(X,\tau_x(\leq_x))$ and $(Y,\tau_y(\leq_y))$ are two T_0 $A-$spaces with corresponding posets $(X,\leq_x)$, $(Y,\leq_y)$ respectively, then $X \times Y$ is a T_0 $A-$space induces a specialization order $\leq_p$ coincides with the coordinatewise order of the product of the corresponding posets.*

2. Alexandroff Complete Lattice Spaces:

Definition 2.1 *(Othman & Mahdi, 2017) Let $(X,\tau_{(\leq)})$ be a T_0 A-space with corresponding poset $(X,\leq)$. We say that X is an Alexandroff lattice space (briefly, A_L-space) if the corresponding poset $(X,\leq)$ is lattice.*

Theorem 2.2 *Let $(X,\tau_{(\leq)})$ be a T_0 A-space with corresponding poset $(X,\leq)$ and $S \subseteq X$. Then $\bigvee S$ exists if and only if there exists $u \in X$ such that $V(u) = \bigcap_{x \in S} V(x)$.*

Proof. $(\Rightarrow)$ Let $S \subseteq X$ and $\bigvee S$ = u. Then $x \leq u$ $\forall x \in S$. Equivalently, $u \in \bigcap_{x \in S} V(x)$, so $V(u) \subseteq \bigcap_{x \in S} V(x)$.

On the other hand, let $a \in \bigcap_{x\in S} V(x)$. Then $x \leq a \ \forall x \in S$. This implies that $u \leq a$ and hence $a \in V(u)$. Therefore, $\bigcap_{x\in S} V(x) \subseteq V(u)$. Conversely, if $V(u) = \bigcap_{x\in S} V(x)$, then $V(u) \subseteq V(x) \ \forall x \in S$. So $u \geq x \ \forall x \in S$. Suppose that $y \geq x \ \forall x \in S$. Then $V(y) \subseteq V(x) \ \forall x \in S$, and then $V(y) \subseteq \bigcap_{x\in S} V(x) = V(u)$. Therefore, $y \geq u$. $\supset$

Corollary 2.3 *Let $(X, \tau_{(\leq)})$ be a T_0 A-space with corresponding poset $(X, \leq)$ and $S \subseteq X$. Then $\wedge S$ exists if and only if there exists $v \in X$ such that $\overline{\{v\}} = \bigcap \{\overline{\{x\}} : x \in S\}$.*

Definition 2.4 *Let $(X, \tau_{(\leq)})$ be a T_0 A-space with corresponding poset $(X, \leq)$. We say that X is an Alexandroff complete lattice space (briefly, A_{cl}-space) if the corresponding poset $(X, \leq)$ is complete lattice.*

Corollary 2.5 *A T_0 $A-$space (X, τ) is A_{cl}-space if and only if for all $S \subseteq X$, there exist two elements c, d such that $V(c) = \bigcap \{V(x) : x \in S\}$ and $\overline{\{d\}} = \bigcap \{\overline{\{x\}} : x \in S\}$.*

Corollary 2.6 *A T_0 $A-$space is A_{cl}-space iff the intersection of any collection of minimal open sets $\{V(x) : x \in A \text{ and } A \subseteq X\}$ equals a minimal open $V(y)$ for some $y \in X$ and the intersection of any collection of minimal closed sets $\{\overline{x} : x \in B \text{ and } B \subseteq X\}$ equals a minimal closed $\overline{z}$ for some $z \in X$.*

Theorem 2.7 *If X is A_{cl}-space, then X is bounded.*

Corollary 2.8 *If X is $A_{cl}-$space, then X is BB T_0 $A-$space.*

In general, a poset can be bounded even when it is neither lattice nor complete lattice as illustrated in Figure 2:

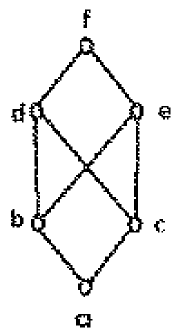

Figure 2 *Bounded poset*

Example 2.9 *A bonded A_L $-$space need not be A_{cl} space. Let $\mathbb{Q}$ be the set of rational numbers, and $X = [0,2] \cap \mathbb{Q}$ with it usual order. Then X is a bounded A_L-space. Take $S = [0, \frac{1}{\sqrt{2}}) \cap \mathbb{Q}$. Then $\vee S$ dose not exist in X. So X is not A_{cl}-space.*

Example 2.10 *An A_{cl}-space is not necessarily Artinian or Noetherian. Let $X = \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$ with the usual order $\leq$. Then both X and X^{∂} are A_{cl}-space. But X is not Noetherian and X^{∂} is not Artinian.*

If X is a finite lattice, then X is complete. Hence we get the following theorem.

Theorem 2.11 *A finite A_L-space is A_{cl}-space.*

Theorem 2.12 *Let $(X, \tau_{(\leq)})$ be an A_{cl}-space and S any subset of X. Then $B = S \cup \{\vee S, \wedge S\}$ is A_L-subspace.*

Proof. Let $x, y \in B$. If $x \vee y \in S$ we are done. If not, then $x \vee y = \vee S$. You should note that $x \vee y$ in X need not be equal $x \vee y$ in B. Similarly with meet. $\supset$

A subspace of A_{cl}-space need not be A_{cl}-space as shown in the following example:

Example 2.13 *Let $X = [0,2]$ with usual order. By completeness property of $\mathbb{R}$, X is complete lattice and hence the induced T_0 A-space is A_{cl}-space. Let $A = [0,2] - \{1\}$ and let $S = [01) \subseteq A$. Then $\vee S$ does not exist in A. So A as a subspace is not A_{cl}-space.*

Definition 2.14 *Let X be a T_0 $A-$space and let c be a not minimum element (resp. a not maximum element) in X. Then c is called join-irreducible (resp. meet-*

irreducible) if whenever $V(c)=V(x)\cap V(y)$ *(resp. if whenever* $\overline{c}=\overline{x}\cap\overline{y}$*), then either* $c=x$ *or* $c=y$*.*

Definition 2.15 *Let* X *be a* T_0 $A-$*space with a bottom element* $\perp$ *and* $x\in X$*. Then* x *is called atom if* $\overline{\{x\}}-\{x\}=\perp$*.*

Theorem 2.16 *Let* X *be a* T_0 $A-$*space with a bottom element* $\perp$ *and* $x\in X$*. If* x *is atom, then* x *is join-irreducible.*

Proof. Suppose that $V(x)=V(a)\cap V(b)$. Then $V(x)\subseteq V(a)$ and $V(x)\subseteq V(b)$. If $x\neq a$ and $x\neq b$, then $a,b\in\overline{\{x\}}-\{x\}$. That is $a=b=\perp$ and $V(x)=V(\perp)=X$. This implies that $x=\perp$, which is a contradiction. ⊃

The converse need not be true. Consider a linear order poset with more than three elements. Each element is join-irreducible, while at most there is one atom.

3. Distribution in Alexandroff Lattice Spaces:

Definition 3.1 *Let* $(X,\tau_{(\leq)})$ *be an* A_L*-space. A subset* E *of* X *is called non-distributive set if the following conditions hold:*

1. E contains exactly five elements.
2. There exists $a\in E$ such that $E\subseteq V(a)$.
3. There exists $e\in E$ such that $V(e)\cap E=\{e\}$.
4. There exist two elements c,d different from a,e such that $\overline{\{c\}}\cap\overline{\{d\}}=\overline{\{a\}}$ and $V(c)\cap V(d)=V(e)$.

If E is not non-distributive set, then it is called *distributive set.*

Theorem 3.2 *The set* E *in* A_L-*space* X *is non-distributive iff* E *describes a pentagon or a diamond in the corresponding lattice.*

Proof. Consider the corresponding lattice $(X,\vee,\wedge)$, and let $E=\{a,b,c,e,d\}$ be a non-distributive subset of X. By condition 2, we have $a\leq b$, $a\leq c$, $a\leq d$, and $a\leq e$. By condition 3, we have $e\geq c$, $e\geq b$, and $e\geq d$. Condition 3 implies that c and d are incomparable. Therefore, we have the following three cases:

Case 1: If b is comparable with c or with d (but not both), then E -as a poset- is a pentagon.

Case 2: If b is incomparable with c and d, then E is a diamond.

Case 3: If b is comparable with both c and d, then we will see that this case is impossible. Under this case, we have the following three subcases:

Subcase 3.1: $c\leq b$ and $d\leq b$. Then $b\in V(c)\cap V(d)=V(e)$. Hence $b\in V(e)\cap E=\{e\}$, which is a contradiction.

Subcase 3.2: $b\leq c$ and $b\leq d$. Then $b\in\overline{\{c\}}\cap\overline{\{d\}}=\overline{\{a\}}$. Hence $b\leq a$, but $b\neq a$, then $b\notin V(a)$, which contradicts condition 1.

Subcase 3.3: $c<b<d$ or $d<b<c$. Then $V(c)\cap V(d)$ is either $V(c)$ or $V(d)$. Hence $\neq V(e)$, which is a contradiction. ⊃

Definition 3.3 *An* A_L*-space* X *is called Alexandroff distributive lattice space (briefly,* A_{dL}*-space) if every subset of* X *is distributive.*

Corollary 3.4 *An* A_L*-space X is* A_{dL}*-space if and only if its corresponding poset* $(X,\leq)$ *is distributive lattice.*

Proof. The proof comes directly from Theorem 1.1 and Theorem 3.2. ⊃

Definition 3.5 *Let* $(X,\tau_{(\leq)})$ *be an* A_L*-space. A subset* F *of* X *is called non-modular set if the following conditions hold:*

1. F contains exactly five elements.
2. There exists $a\in F$ such that $F\subseteq V(a)$.
3. There exists $e\in F$ such that $V(e)\cap F=\{e\}$.
4. There exist two elements c,d different from a,e such that $\overline{\{c\}}\cap\overline{\{d\}}=\overline{\{a\}}$ and $V(c)\cap V(d)=V(e)$.
5. $V(c)\subseteq V(b)$.

If F is not non-modular set, then it is called *modular set.*

Depending on Theorem 3.2, it is easy prove that a set A is non-modular iff it describes a pentagon in the corresponding lattice.

Definition 3.6 *An* A_L*-space* X *is called Alexandroff modular lattice space (briefly,* A_{mL}*-space) if all subsets of* X *are modular. In this case, the corresponding poset is modular lattice.*

Since every distributive lattice is modular lattice, we have the following remark:

Remark 3.7 *Every* A_{dL}*-space is* A_{mL}*-space.*

The converse need not be true in general. To see this, consider the diamond lattice as A_L-space. Clearly it is A_{mL}-space, which is not A_{dL}-space. In fact an $A_{mL}-$space is $A_{dL}-$space iff it contains no diamond subspace.

In (Othman & Mahdi, 2017), we prove that a subspace of A_L-space need not be A_L-space.

Theorem 3.8 *Let X be an $A_{dL}-$space (resp. $A_{mL}-$space) and $B \subseteq X$. If B as a subspace of X is A_L-space, then B is $A_{dL}-$space (resp. $A_{mL}-$space).*

Proof. Suppose that (B, τ_B) is non-distributive A_L-space. So there exists a non-distributive subset A of B. Hence A is also non-distributive subset of X. Therefore $(X, \tau_{(\leq)})$ is non-distributive. ⊃

4. Boolean Alexandroff Spaces:

Definition 4.1 *Let $(X, \tau_{(\leq)})$ be a bounded T_0 A-space and $a \in X$. If there exists $b \in X$ such that $\overline{\{a\}} \cap \overline{\{b\}} = \{\perp\}$ and $V(a) \cap V(b) = \{\top\}$ then b is called a complement of a.* The complement need not be unique as the following example shows:

Example 4.2 *In a diamond space, b has two complements, d and c. And in a pentagon space, d has two complements, b and c.*

Lemma 4.3 *Let $(X, \tau_{(\leq)})$ be a bounded A_{dL}-space and $a \in X$. If a has a complement, then it is unique.*

Proof. If c, b are two complement elements of a, then the subset $A = \{\top, \perp, a, b, c\}$ with its induced order forms a non-distributive subset in X, which is a contradiction.
⊃

Definition 4.4 *A bounded A_{dL}-space X is called Boolean Alexandroff space (briefly, $A_{BL}-$space) if each element has a complement.*

If X is $A_{BL}-$space, then the corresponding poset is Boolean lattice. Moreover, from Theorem 1.3 and the above definition one can easly see that if X is a finite $A_{BL}-$space, then the cardinality of X is $|X| = 2^n$ for some natural number n.

A *Sierpinski space* is a space $X = \{a, b\}$ with a topology $\tau = \{\varnothing, X, \{b\}\}$. It is T_0 $A-$space with induced order $a \leq b$.

Theorem 4.5 *A finite T_0 $A-$space X is A_{BL}-space iff it is homeomorphic to a product of n copies of a Sierpinski space.*

Proof. The corresponding poset $(X, \leq)$ is Boolean lattice. So, by Theorem 1.3, there exists a set S with n elements such that $(X, \leq)$ is a lattice isomorphic to $(P(S), \subseteq)$, where $P(S)$ is the collection of all subsets of S. By Theorem 1.4, the Boolean lattice $P(S)$ is a lattice isomorphic of the cartesian product of n copies of the poset $\{0,1\}$ where $0 \leq 1$. Using Theorem 1.5, X is homeomorphic to a product of n copies of the induced Sierpinski topology on $\{0,1\}$. Conversely, the corresponding poset $(X \leq)$ has an isomorphism with the product of n copies of the induced poset $\{0,1\}$ where $0 \leq 1$ of the Sierpinski space with coordinatewise order. By Theorem 1.3, X is A_{BL}-space.
⊃

Theorem 4.6 *Let $(X, \tau_{(\leq)})$ be a bounded finite A_{dL}-space. Then the following statements are equivalent:*

1. $(X, \tau_{(\leq)})$ is an A_{BL}-space.
2. Every element in $X - \{\perp\}$ is a join of atoms.
3. Every join-irreducible element is an atom.
4. For each $x \in X$, there exists $y \in X$ such that $x \vee y = \top$ and $x \wedge y = \perp$.

Proof. $(1 \Rightarrow 2)$ Using Theorem 1.3, we can take $X = P(S)$ for some finite set S where the join operation is the union operation. So the atoms are the singleton sets. Let $B \in P(S)$, then B is the union of the singleton sets of the elements in B.

$(2 \Rightarrow 3)$ Suppose to contrary that a is a join-irreducible which is not atom. Then by (2), there exist distinct atoms $a_1, a_2, \cdots, a_k, k > 1$ such that $a = a_1 \vee a_2 \vee \cdots \vee a_k$. Set $c_2 = a_2 \vee a_3 \vee \cdots \vee a_k$. Then $a = a_1 \vee c_2$. This implies that $V(a) = V(a_1) \cap V(c_2)$, [To see this, if $x \in V(a)$, then $x \geq a$. So $x \geq a_1$ and $x \geq c_2$. On the other hand, if $x \in V(a_1) \cap V(c_2)$, then $x \geq a_1$ and $x \geq c_2$. Hence, $x \geq a_1 \vee c_2 = a$]. Since a is join-irreducible, either $a = a_1$ or $a = c_2$. Since a is not atom, $a = c_2 = a_2 \vee a_3 \vee \cdots \vee a_k$. Again, set $c_3 = a_3 \vee a_4 \vee \cdots \vee a_k$. We get $a = a_2 \vee c_3$, to get that $a = c_3$. Continue this process, we must have $a = a_k$ and a is atom, which is a contradiction.

$(3 \Rightarrow 2)$ Let $x \neq \perp$. If x is atom, we done. Otherwise x is not join-irreducible. So, there exist $a, b \in X$ such that $V(x) = V(a) \cap V(b)$ and $a \neq x, b \neq x$. In this case, $x = a \vee b$. For the elements a and b and similar to x, if a is atom,take $a_1 = a$. Otherwise, we get $a = a_1 \vee a_2$ where $a_1 \neq a$ and $a_2 \neq a$. If b is atom, take $b_1 = b$. Otherwise, $b = b_1 \vee b_2$. Repeat this process for a_i and b_j, we must terminate and we get $x = a_1 \vee a_2 \vee \cdots \vee a_r \vee b_1 \vee \cdots \vee b_s$ for some atoms $a_i, i = 1,2,\cdots,r$ and b_j, $j = 1,2,\cdots,s$.

$(2 \Rightarrow 4)$ If $x = \mathrm{T}$, take $y = \perp$. So, suppose that $x \neq \mathrm{T}$ and $x \neq \perp$. Then $x = a_1 \vee a_2 \vee \cdots \vee a_k$ for some atoms a_i. If AT is the set of atoms of X, then $AT - \{a_1, a_2, \cdots a_k\} \neq \varnothing$ (otherwise, $x = \mathrm{T}$). Suppose that $AT - \{a_1, a_2, \cdots a_k\} = \{b_1, b_2, \cdots, b_r\}$. Set $y = b_1 \vee b_2 \vee \cdots \vee b_r$. Then $x \vee y = \mathrm{T}$ and $x \wedge y = \perp$.

$(4 \Rightarrow 1)$ Just the definition.

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أنواع جديدة لفضاءات التس الكساندرووف

في هذا البحث، قمنا بتعريف ودراسة أنواع جديدة من فضاءات الكساندرووف من خلال استخدام أنواع معروفة من المجموعات المرتبة مثل اللاتس المتكامل واللاتس القابل للتوزيع والجبر البوليني. قمنا بإثبات بعض النتائج على هذه الأنواع. حصلنا على وصف للفضاءات A_{cl}، A_{dL}، A_{mL}، وكذلك A_{BL}. بشكل أساسي، تم إثبات أن الفضاء T_0 الكساندرووف يكون فضاء A_{BL} إذا وفقط إذا كان متماثل توبولوجياً مع فضاء الضرب لعدد n من نسخ فضاء سيربنسكي إذا وفقط إذا كان كل عنصر غير قابل للاختزال الإنضمامي يكون ذرة.

كلمات مفتاحية:
فضاء الكساندرووف،
اللاتس،
اللاتس المتكامل،
اللاتس القابل للتوزيع،
الجبر البوليني.