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Sensitivity enhancement in optical waveguide sensors using metamaterials

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Abstract We consider a four-layer waveguide structure as an optical waveguide sensor. One of the layers is a metamaterial with negative permittivity and permeability. We show that the sensitivity of the proposed optical waveguide sensor can be dramatically enhanced by using a metamaterial layer between the guiding and the cladding layers. The variation of the sensitivity of the proposed waveguide sensor with different parameters of the waveguide is studied.

1 Introduction

The non-communication applications of slab waveguides as optical sensors have drawn considerable attention in the past few years [1, 2]. The use of slab waveguides as optical sensors offers numerous advantageous features such as small size, ruggedness, potential for realizing various optical functions on a single chip (integration with other optical components), multi-channel sensing, etc. In one class of commonly used optical waveguide chemical sensors, an analyte (the material to be detected) is placed in the evanescent field of the waveguide. As a result, changes take place in the absorption or in the phase of the electromagnetic wave propagating in the structure. Measuring this change between the analyte and a reference material is used to determine the change in the effective refractive index N of the waveguide structure. The measured change in the effective refractive index allows one to determine the refractive index of the analyte through the characteristic equation of the optical waveguide structure. Optical waveguide sensors have been used in a wide range of applications such as detection of harmful gases (methane, SO_2), monitoring pollutants and other compounds in water, pH detection, and detection of certain chemicals in blood.

Recently, the concept of double-negative (negative ε and negative μ) materials has achieved remarkable importance due to the exhibition of unusual electromagnetic properties different from the known materials. These phenomena are observed in microwave, millimeter-wave, and optical frequency bands. The materials of double negativity are called metamaterials or left-handed materials (LHMs). The history of these materials begins with the work of Veselago [3], who proposed a medium with simultaneously negative ε and μ and studied the propagation of electromagnetic waves in such a medium. Pendry et al. [4] presented an artificial metallic construction of periodic rods which shows negative permittivity and they also presented a structure of split rings which exhibits negative permittivity [5]. Smith et al. [6] constructed a LHM using the combination of periodic rods and split rings and they performed many experiments in the microwave range to point out that the nature of this material is unlike any existing material. The first experimental investigation of negative index of refraction was achieved by Shelby et al. in 2001 [7]. The interaction of electromagnetic waves with stratified isotropic LHMs was studied by Kong [8]. The theory of LHMs and their electromagnetic properties, possible future applications, physical remarks, and intuitive justifications were provided by Engheta in 2003 [9]. Chew [10] analyzed the energy-conservation property of a LHM and the realistic Sommerfeld problem of a point source over a LHM half space and a LHM slab. In 2006, Sabah et al. [11] studied the effects of the structure parameters, incidence angle, and the frequency on the reflected and transmitted powers for a lossless LHM. The electromagnetic wave propagation through a frequency-dispersive and

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lossy double-negative slab embedded between two different semi-infinite media was presented in 2007 [12].

In this work, we show that the sensitivity of the slab waveguide sensors can be dramatically enhanced by inserting a layer of left-handed material between the cladding and the guiding layers.

2 Theory

We consider a guiding layer with permittivity ε_f, permeability μ_f, and thickness d_1 sandwiched between a semi-infinite substrate with permittivity ε_s and permeability μ_s and a semi-infinite cladding with permittivity ε_c and permeability μ_c. An additional layer of metamaterial with negative permittivity ε_m, negative permeability μ_m, and thickness d_2 is inserted between the cladding and the guiding layers. All the materials are assumed to be lossless. We also consider s-polarized (TE) waves in which the electric field $\boldsymbol{E}$ is polarized along the y-axis. It is straightforward to show that the dispersion relation of the structure is given by

$$\begin{aligned}\gamma_f d_1 = {}&\tan^{-1}\left(\frac{\gamma_s\mu_f}{\gamma_f\mu_s}\right)\\&+\tan^{-1}\left(\frac{\gamma_m\mu_f}{\gamma_f\mu_m}\right.\\&\left.\times\frac{(\gamma_m\mu_c+\gamma_c\mu_m)-(\gamma_m\mu_c-\gamma_c\mu_m)e^{-2\gamma_m d_2}}{(\gamma_m\mu_c+\gamma_c\mu_m)+(\gamma_m\mu_c-\gamma_c\mu_m)e^{-2\gamma_m d_2}}\right)\\&+m\pi,\end{aligned}\tag{1}$$

where $\gamma_c=\sqrt{\beta^2-\varepsilon_o\varepsilon_c\mu_c\omega^2}$, $\gamma_m=\sqrt{\beta^2-\varepsilon_o\varepsilon_m\mu_m\omega^2}$, $\gamma_f=\sqrt{\varepsilon_o\varepsilon_f\mu_f\omega^2-\beta^2}$, and $\gamma_s=\sqrt{\beta^2-\varepsilon_o\varepsilon_s\mu_s\omega^2}$. Here $m=0,12,\ldots$ is the mode order and β is the propagation constant in the x-direction.

For the sake of simplicity in the evaluation of the sensitivity, we assume that the cladding, the film, and the substrate are non-magnetic materials and the permeability of the metamaterial is given by $\mu_m=n\mu_o$, where n is a negative number. We also assume that $\gamma_c=k_oq_c$, $\gamma_m=k_oq_m$, $\gamma_f=k_oq_f$, and $\gamma_s=k_oq_s$, where k_o is the free-space wave number, $q_c=\sqrt{N^2-\varepsilon_c}$, $q_m=\sqrt{N^2-n\varepsilon_m}$, $q_f=\sqrt{\varepsilon_f-N^2}$, and $q_s=\sqrt{N^2-\varepsilon_s}$. To obtain the sensitivity of the proposed sensor in a condensed form, we define three normalized effective indices X_s, X_c, and X_m and three asymmetry parameters a_s, a_c, and a_m as

$$\begin{aligned}&X_s=\frac{q_s}{q_f}, &&X_c=\frac{q_c}{q_f}, &&X_m=\frac{q_m}{q_f},\\&a_s=\frac{\varepsilon_s}{\varepsilon_f}, &&a_c=\frac{\varepsilon_c}{\varepsilon_f}, \quad\text{and} &&a_m=\frac{\varepsilon_m}{\varepsilon_f}.\end{aligned}\tag{2}$$

In the light of these assumptions, (1) can be written as

$$k_oq_fd_1=\arctan(X_s)+\arctan\left(\frac{X_m}{n}\frac{b_1}{b_2}\right)+m\pi,\tag{3}$$

where $b_1=(X_m+nX_c)-(X_m-nX_c)e^{-2k_oX_mq_fd_2}$, and $b_2=(X_m+nX_c)+(X_m-nX_c)e^{-2k_oX_mq_fd_2}$.

In the case of homogeneous sensing, the sensitivity S is defined as the rate of change of the modal effective index N under an index change of the cover n_c. The sensitivity $S_2=\partial N/\partial n_c$ of the proposed sensor is calculated by differentiating (3) with respect to N. After some algebraic manipulations, the sensitivity can be written as

$$S_2=\frac{\sqrt{a_c}\sqrt{1+X_c^2}X_m[b_2-b_1+e^{-f}(b_1+b_2)]}{X_c\sqrt{a_c+X_c^2}[(A_{mTE}+\frac{1}{X_s})(b_2^2+\frac{X_m^2b_1^2}{n^2})+G_1+G_2+\frac{b_1b_2C_1}{nX_m}]},\tag{4}$$

where $G_1=(b_2-b_1)(\frac{C_1}{n}+\frac{C_2X_m}{X_c})$, $G_2=\frac{e^{-f}(b_1+b_2)}{n}[\frac{(X_m-nX_c)f}{X_m}-C_1+\frac{C_2X_mn}{X_c}]$, $f=2k_oX_mq_fd_2$, $A_{mTE}=\arctan(X_s)+\arctan(\frac{X_m}{n}\frac{b_1}{b_2})+m\pi$, $C_1=1+X_m^2$, and $C_2=1+X_c^2$.

As d_2 approaches zero, i.e. no metamaterial is available, (4) reduces to

$$S_1=\frac{\sqrt{a_c}}{X_c\sqrt{a_c+X_c^2}\sqrt{1+X_c^2}(\arctan(X_s)+\arctan(X_c)+m\pi+\frac{1}{X_c}+\frac{1}{X_s})}.\tag{5}$$

Equation (5) gives the sensitivity of the conventional three-layer waveguide sensor without the left-handed material. To evaluate the enhancement effect due to the left-handed material, we define the sensitivity enhancement factor F_{en} as $F_{\text{en}}=S_2/S_1$.

3 Results and discussion

In the analysis below, we will assume the guiding layer to be Si_3N_4 ($\varepsilon_f=4$), the free space wavelength to have the value 1550 nm, and $m=0$, which corresponds to the fundamental