(7,4) HAMMING LIKE CODE FOR QPSK MODULATION

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Abstract

Quadrature Phase Shift Keying (QPSK) is one of the most popular digital modulation techniques. It is widely used in existing technologies because of its spectral efficiency. In this paper, we present a (7, 4) code which can be directly applied on the phase of the QPSK constellation points. The code is based on the fundamental idea of the binary Hamming code. The decoding performance is enhanced by considering the minimum Euclidean distance between the received codeword and all codewords that that can be corrected using the same syndrome. The code can correct all the single symbol errors and 96.3% of double symbol errors at E_b/N_0 = 8 dB. By simulation, it is shown that this approach can guarantee a coding gain of 1.5 dB with respect to uncoded QPSK.

Index Terms—Coded-Modulation, Hamming, Block coding, QPSK.

I. INTRODUCTION

Increasing need for higher capacity and reliability of communication over fading channels has drawn attention of researchers to develop efficient coding schemes. The classical approach in communication systems is to separate the coding and modulation processes. The modulator/demodulator converts analog channel into a discrete channel and the encoder/decoder corrects errors that occur on the discrete channel [1].

Combining coding and modulation techniques for digital transmission has been evolved by introducing the Trellis Coded Modulation (TCM) in 1976 [2]. The principles of TCM were published in 1982 [3] and further description followed in 1985 [4]. The basic idea behind TCM is to increase the Euclidean distance between modulated symbol sequences whilst preserving the bandwidth [3]-[4]. This is done by allowing for extra constellation points for MPSK modulation. The TCM combines the convolutional code with a chosen signal constellation [5].

The 2[nd] approach for coded-modulation is to combine binary block components with a signal constellation as in [5]-[7]. This scheme is called Block Coded Modulation (BCM) which suffers from high decoding complexity for a long blocklength [8].

However, in both schemes, the receiver should perform a maximum likelihood (ML) sequence detection based on the minimum squared Euclidean distance. This decoding scheme suffers from high complexity.

In this paper, a simple block coding technique is directly applied on QPSK constellation phases. The first step in the decoding process is done by calculating the syndrome. In this approach, we allow for more than one error pattern to be assigned to a unique syndrome. The correct codeword is chosen by considering the minimum Euclidean distance between the received codeword and every codeword that assigned to the same syndrome. This minimizes the number of searched sequences and hence reduces the ML decoding complexity. Moreover, it allows efficient use of soft-decoding approach on block codes whereas it is usually done in case of convolutional codes.

II. (7, 4) HAMMING-LIKE CODE FOR QPSK

In a binary (7, 4) Hamming code, three parity bits are added for each four data bits to assign every single error pattern to a unique syndrome. The main principle of that code is applied in this paper.

Instead of encoding the binary bits directly, the bits are first modulated into their corresponding constellation points using gray encoding (see Fig. 1.). The gray encoding is to ensure that each two adjacent symbols only differ by one bit. The coding procedure is applied on the phases of the modulated symbols. This is done by considering the fact that the group of phases of the QPSK constellation points forms an abelian group under addition modulo 2π. This can be easily verified as by adding or subtracting (modulo 2π) any two elements of the group $\{0, \pi/2, \pi, 3\pi/2\}$, the result is also an element of the Z4 group. There are several researches about coding over Z4 [9]-[11]. Most of which don't take advantage from well established binary codes.

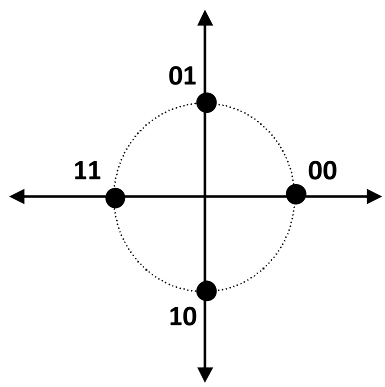

Figure 1: Constellation diagram for QPSK with gray encoding.

Let $\theta_1, \theta_2, \theta_3$, and θ_4 denote the phases for the symbols, X_1, X_2, X_3, and X_4, respectively. Extra three parity symbols can be added such as

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$$\emptyset_1 = -\theta_1 - \theta_2 - \theta_3, \quad (1)$$
$$\emptyset_2 = -\theta_2 - \theta_3 - \theta_4, \quad (2)$$
$$\emptyset_3 = -\theta_1 - \theta_3 - \theta_4, \quad (3)$$

where $\emptyset_i$ is the phase for the parity symbol "P_i". The syndromes (S_1, S_2, and S_3) at the receiver can be calculated by

$$S_1 = \emptyset_1 + \theta_1 + \theta_2 + \theta_3, \quad (4)$$
$$S_2 = \emptyset_2 + \theta_2 + \theta_3 + \theta_4, \quad (5)$$
$$S_3 = \emptyset_3 + \theta_1 + \theta_3 + \theta_4. \quad (6)$$

Let us denote the group elements {0, π/2, π, 3π/2} as {0, 1, 2, 3}and define the multiplication "*" and addition "+"operations in Z4 as given in table I.
The generator matrix and the parity check matrix in Z4 mathematics can be constructed from equations (1)-(6) and table I as:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 3 & 0 & 3 \\ 0 & 1 & 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 1 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 1 & 0 & 3 & 3 \end{bmatrix}, \quad [7]$$

and

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}. \quad [8]$$

The syndrome can be calculated as:

$$S = vH^T, \quad [9]$$

where v is the received vector and S is the syndrome.
It is obvious that the syndrome is zero when there is no errors. Moreover, there is a unique nonzero syndrome for all the single error patterns and extra 35 syndromes which can be assigned for double error patterns.
Fig.2 shows Bit Error Rate (BER) performance of the Hamming like coded Before Correction (BC) and the uncoded. It is clear that the code is not efficient compared to the binary Hamming code. Up to E_b/N_0 = 8.33 dB, the uncoded modulation has a better performance. The coding gain is only about 0.4 dB at 10^{-5} which is even lower than the gain provided by (7, 4) binary Hamming code which is around 0.5 dB at the same BER. In the next section, it is demonstrated that by taking a simple soft approach of considering the Euclidean Distance (ED) between codewords into account, the code performance can be significantly improved.

III. Euclidean Distance (ED) Correction

The main reason behind the poor performance of (7, 4) Hamming-like code is that when the decoder chooses the wrong codeword, it actually creates more errors. Let us demonstrate this by an example.
If we assume [3 3 0 3] is to be transmitted. Using equation (1)-(3), the added parity symbols for this message is found to be [0 0 0]. When a random simulation is run in MATLAB R2009b at E_b/N_0 = 2 dB, the received signal vector is given in Table I. The demodulator decides on [1 1 0 3 0 0 0] which has two symbol errors compared to the original (2 bit errors). Using equations (4)-(6), the syndrome is found to be [2 3 3] which is assigned to the error pattern [0 0 2 1 0 0 0]. Hence, the corrected codeword after subtracting the phase error

Table I:Addition and multiplication in Z4 group

+	0	1	2	3	*	0	1	2	3
0	0	1	2	3	**0**	0	0	0	2
1	1	2	3	0	**1**	0	1	2	3
2	2	3	0	1	**2**	0	2	1	0
3	3	0	1	2	**3**	0	3	0	1

pattern is [**1 1 3 1** 0 0 0] which has 4 symbol errors or 5 bit errors.

Compared to the demodulated signal, it is obvious that the decoder has added extra 3 bit errors. However, the new symbol errors increase the Euclidean distance between the received code vector (before demodulation) and the corrected code vector. Therefore, all the double error patterns are assigned to their corresponding syndrome. As a result, more than two error patterns may be assigned to the same syndrome. In case of collision (caused by multiple wrong symbols), a

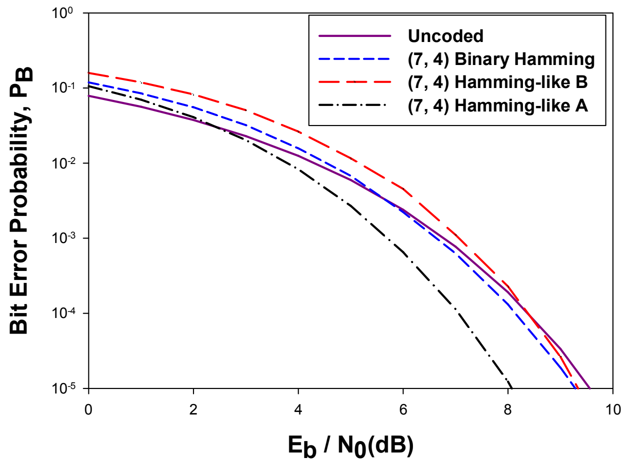

Figure 2: Bit error rate versus E_b/N_0 performance for a) uncoded QPSK, b) binary Hamming code with hard decision c) and d) Hamming-Like code, B: Before considering ED and **A** after considering ED.

minimum Euclidean distance decision is taken from all the patterns generating that syndrome.

For our example, the four two-error patterns that can produce a syndrome, S = [2 3 3] are illustrated in Table III. Each with its corresponding corrected code vector. By intuition, the vector with smallest ED to the received vector is the correct one.

IV. Code Performance and Limitations

The BER versus E_b/N_0 for Hamming like is depicted on Fig. 2. The code after the ED correction starts to perform better than the uncoded at E_b/N_0 = 2.359 dB. The coding gain is 1.526 dB at BER = 10^{-5}. The percentage of corrected single double error patterns is depicted in Fig. 3. Even at E_b/N_0= 0 dB, the code after ED correction can correct up to 94% of single error patterns and 68% of double error patterns. At

E_b/N_0= 8 dB, it can correct 100 % of single error patterns and 96.3% of double error patterns.

V. CONCLUSION

A novel Hamming-like code was presented in this study. A simple soft decoding scheme considering both the minimum Hamming distance and the Euclidean distance is applied. The code has lower spectral efficiency than TCM and BCM. However, the decoding scheme has lower complexity than the Trellis decoder used in BCM and TCM and it provides a significant coding gain.

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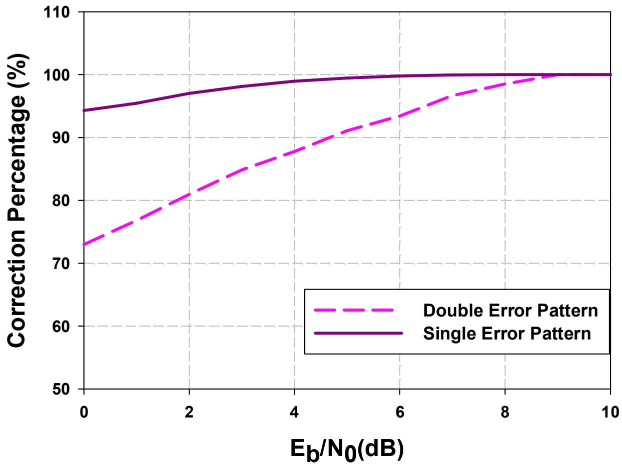

Figure 3: Percentage of single and double error patterns that can be corrected after ED correction.

Table II: Transmitted, received and demodulated sequences

Transmitted	3	3	0	3	0	0	0
Received	-0.26 + 0.54i	-0.25 + 0.38i	1.35 - 0.16i	-1.63 + 0.15i	1.38 - 0.41i	1.85 + 0.47i	1.25 - 0.60i
Demodulated	1	1	0	3	0	0	0

Table III: Error Pattern for *S* = [2 3 3]

Error Pattern							**Corrected Codeword**							**ED**
0	0	2	1	0	0	0	<u>1</u>	<u>1</u>	<u>3</u>	<u>1</u>	0	0	0	7.5710
0	0	0	3	2	0	0	<u>1</u>	<u>1</u>	0	<u>2</u>	<u>3</u>	0	0	7.6129
0	0	3	0	3	0	0	<u>1</u>	<u>1</u>	<u>1</u>	<u>3</u>	<u>1</u>	0	0	7.2187
3	3	0	0	0	0	0	3	3	0	3	0	0	0	4.9696

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