Hybrid Takagi-Sugeno Fuzzy FED PID Control of Nonlinear Systems

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Abstract: The new method of proportional–integral-derivative (PID) controller is proposed in this paper for a hybrid fuzzy PID controller for nonlinear system. The important feature of the proposed approach is that it combines the fuzzy gain scheduling method and a fuzzy fed PID controller to solve the nonlinear control problem. The resultant fuzzy rule base of the proposed controller contains one part. This single part of the rules uses the Takagi-Sugeno method for solving the nonlinear problem. The simulation results of a nonlinear system show that the performance of a fed PID Hybrid Takagi-Sugeno fuzzy controller is better than that of the conventional fuzzy PID controller or Hybrid Mamdani fuzzy FED PID controller.

Keywords: Fuzzy gain scheduling, fuzzy fed proportional–integral-derivative (PID),Takagi-Sugeno, nonlinear system.

I. INTRODUCTION

PID control is widely used in industrial applications although it is a simple control method. Stability of PID controller can be guaranteed theoretically, and zero steady-state tracking error can be achieved for linear plant in steady-state phase. Computer simulations of PID control algorithm have revealed that the tracking error is quite often oscillatory, however, with large amplitudes during the transient phase.

To improve the performance of the PID controllers, several strategies have been proposed, such as adaptive and supervising techniques.

Fuzzy control methodology is considered as an effective method to deal with disturbances and uncertainties in terms of ignorance and ambiguity. Fuzzy PID controller combining fuzzy technology with traditional PID control algorithm has become the most effective domain in artificial intelligence control [1],[15],

The most common problem resulted early depending on the complexity of FLC is the tuning problem. It is hard to design and tune FLCs manually for the most machine problems especially used in industries like nonlinear systems. For alleviation of difficulties in constructing the fuzzy rule base, there is the conventional nonlinear design method [2] which was inherited in the fuzzy control area, such as fuzzy sliding control, fuzzy scheduling [3],[8], and adaptive fuzzy control [4],[14]. The error signal for most control systems is available to the controller if the reference input is continuous. The analytical calculations present two-inputs FLC employing proportional error signal and velocity error signal. PID controller is the most common controller used in industries, most of development of fuzzy controllers revolve around fuzzy PID controllers to insure the existence of conventional controllers in the overall control structure, simply called Hybrid Fuzzy Controllers [5],[6].

The key idea of the proposed method is as follows: First, the fuzzy gain scheduling method is applied to linearize the nonlinear system at frozen times. A fuzzy fed PID controller is designed for each frozen system by replacing the conventional PID controller by an incremental FLC, the integral part of the PID controller is fed by a deferential feedback gain, this fed PID controller is the new method used in this paper and it gives the best results any way. Second, fuzzification of the reference input is performed for the system, while the control space of error signals is linearly partitioned after normalization. Third, the fuzzy rule base is constructed in a recursive way to obtain better nonlinear control as well as to guarantee closed-loop stability of the frozen system.

It should be emphasized that because the proposed approach utilizes some modern control theorems, tuning the hybrid fuzzy controller is much easier than tuning a conventional fuzzy logic controller.

In Section II, the gain scheduling method is introduced as an effective nonlinear control method for nonlinear systems. In Section III, a novel fuzzy fed PID controller is proposed. We show that recursive design of the fuzzy rule base can guarantee stability of local closed-loop systems. In Section IV, control of a pole-balancing robot illustrates how the proposed design method can be easily applied to a nonlinear robotics system. Concluding remarks are given in the last section.

II. NONLINEAR CONTROL PROBLEM

Generally, most of robotics systems are nonlinear systems. One common task in robotics system control is to demand the robot or parts of the body to follow a given reference trajectory [12]. Tracking control of system dynamics may change significantly. Hence, instead of trying to model the system, a more feasible solution is to schedule the gains at each operating point. Since human expert can describes the system in a natural language better than mathematical equations, fuzzy control is also commonly used in nonlinear control of robotics systems [9],[13].

A. Gain Scheduling Method

Nonlinear systems can be generally expressed by the following nonlinear autonomous system equation:

$$\dot{x} = f(x) + g(x)u \qquad (1)$$

Where $x=[x_1,x_2,..x_m]^T$ is the state vector, $u=[u_1,u_2,..u_m]^T$ is the control input vector, f(x) and g(x) are vector functions of states.

Assume $x^d(t) \in R^{nx1}$ is the given reference trajectory whose corresponding reference input is $u^d(t)$

$$\overset{*}{x}^d = f(x^d) + g(x^d)u^d \tag{2}$$

Taking Lyapunov linearization around the operating points (x^d, u^d), we have

$$\dot{x} = \dot{x}^d + A(x^d)(x - x^d) + B(x^d)(u - u^d) \tag{3}$$

Where

$$A(x^d) = \left.\frac{df}{dx}\right|_{x=x^d}$$

$$B(x^d) = g(x^d) \tag{4}$$

Let $e = x - x^d, \dot{e} = \dot{x} - \dot{x}^d$ and (5)

System (3) is equivalent to

$$\dot{e} = A^d e + B^d u^e \tag{6}$$

where A^d and B^d are assumed to can be transformed into diagonal CCF, which is valid for many robotics systems. Because the reference trajectory x^d is a function of time, the nonlinear system (1) can be linearized at frozen time τ so that the tracking problem of the nonlinear system is transformed into a stabilization problem of the linear system (6) in the error state space. The equilibrium points are shifted from the reference trajectory points $x^d(\tau)$ to the origin. However, the aforementioned conventional gain-scheduling method employs linearization between two consecutive operating points. If the system states vary significantly along the time axis, global stability will be a problem. An alternative solution is to utilize fuzzy rules containing expert knowledge to perform smoother interpolation of all the operating points in the control envelope [2],[11].

B. Fuzzy Gain Scheduling

At some frozen times τ_i the corresponding control input can be approximated by (2), which is $x^d(\tau_i)$ or x_i shortly. In partitioning the state space, this xi will be the centers of membership functions (MFs), LX_i [16]. The nonlinear system given by (1) can, therefore, be transformed into several local linearized systems

$$R^i : IF\ x^d\ is\ LX^i\ ,\ THEN \quad \dot{e} = Ae + Bu^e \tag{7}$$

where A_i and B_i are system state matrices corresponding to x_i.

The control law to be designed is

$$R^i : IF\ x^d\ is\ LX^i\ ,\ THEN \quad u = u^d + u^e \tag{8}$$

where u^d is the control input corresponding to the reference input x^d and u^e is the control input derived from error inputs.

The conventional approach of using the gain scheduling method is to design a linear controller for each local system in (7). The main advantage of this approach is that the powerful linear control theory may be applied. However, some simple nonlinear controllers like fuzzy PID controllers could be a better choice for handling the system nonlinearities. Then, the fuzzy PID controllers for local systems may be embedded in the global fuzzy gain scheduling rules to improve the integrity of the design, Moreover; the fuzzy fed PID controller will give the optimal solution more than any previous controller.

III. HYBRID FUZZY CONTROLLER DESIGN

In this section, a fuzzy fed PID controller is proposed for enhanced control of the local linearized systems. By employing recursive feedback and appropriate tuning of conventional derivative gain, the fuzzy fed PID controller guarantees sector conditions of the output [10],[13]. Local stability analysis also explores the relationship between the conventional derivative gain and the fuzzy gain. Although the proposed controller is developed as a hybrid fuzzy fed PID controller, the overall structure shows its potential to be a new form of stand alone fed FLC as depicted in Fig.1.

A fuzzy PID controller with fuzzy is proposed by constructing from simple linear fuzzy rules in an incremental way. But in this section, a new type of fuzzy PID controller is proposed based on fuzzy fed PID control structure using the Takagi-Sugeno (T-S).

The fuzzy fed PID controller is constructed in an incremental way by employing both error signals and recursive feedback signals as inputs to fed PID the main idea is found in the integral side where the integral side when fed by a deferential feedback gives us a null overshoot and steady state error, the enhancement is very significant using Fuzzy fed PID controller. The most widely adopted conventional PID controller structure used in industrial applications is the following structure [7]:

$$u_{PID}(t) = K_P^C e_v(t) + K_I^C e_p(t) + K_D^C e_a(t) \tag{9}$$

where K_P, K_I, and K_D are the conventional proportional, integral, and derivative gains, respectively, and $u_{PID}(t)$ is the controller output and $e_v(t)$ is the velocity error signal, $e_p(t)=\int e_v(t)$ is the proportional error signal and $e_a(t)=de_v(t)/dt$ is the acceleration error signal.

The items in (9) form the PID control and can be replaced by the following linear fuzzy rules:

$$R^j : IF\ e_p\ is\ LE_p^j\ AND e_v ..,THEN u_{PID}\ is\ DU_{PID}^j \tag{10}$$

Where $LE_p^j\ and\ LE_v^j$ are the linguistic values of error signals of the j^{th} fuzzy rule and DU_{PID}^j is the desired function value of the output $u_{PID}(t)$

The first look of fed PID gives the following equation:

$$u_{PID}(t) = K_P^C e_v(t) + (0.5)K_I^C e_p(t) + K_D^C e_a(t) \tag{11}$$

But the real output is differ when the fed PID controller is used where the fed PID controller has overshoot and steady state error less than the conventional PID controller.

Note that the output feedback from the integrator is taken from the output of the defuzzification process which gives the best results showing in the illustrative example.

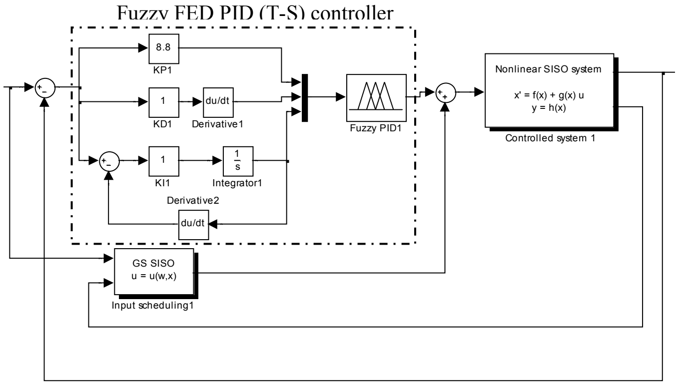

Fig. 1: Overall Control Structure

IV. ILLUSTRATIVE EXAMPLE

In the example, the proposed controller is used to Takagi-Sugeno fuzzy control with an inverted pendulum robot, that robot is used in the most of our applications because of nonlinearity problem and marginally stability. The dynamic equation of the inverted pendulum robot is given by

$$\ddot{\theta} = \frac{(m_p + m_c) g \sin\theta - m_p \dot{\theta}^2 \, l \sin\theta - F\cos\theta}{(m_p + m_c) l (4/3 - m_p \cos^2\theta)}$$

Where θ is the angle between the pendulum and the vertical, the angular velocity is expressed by $\dot{\theta}$, the force which acts on the cart is F, the gravity acceleration g is 9.8m/sec2 , m_c and m_p are the mass of cart and the mass of pole respectively, and l is the half length of the pendulum. The system equation is written as follow:

$$\dot{x} = f(x) + g(x)u$$

Where

$$f(x) = \begin{bmatrix} \dot{\theta} \\ \dfrac{(m_p + m_c) g \sin\theta - m_p \dot{\theta}^2 \, l \sin\theta\cos\theta}{(m_p + m_c) l (4/3 - m_p \cos^2\theta)} \end{bmatrix}$$

$$g(x) = \begin{bmatrix} 0 \\ \dfrac{-F\cos\theta}{(m_p + m_c) l (4/3 - m_p \cos^2\theta)} \end{bmatrix}$$

The last two equations are used for simulation without a previous technique of linearization because of two methods are used, the first one is the gain scheduling method which divides the system into small areas to relent using of iterations, the second method is the fuzzy PID controller which use the linguistic formulas and it by default make a linearization of the nonlinear system. The addition of the two methods is called hybrid fuzzy PID controller. Beside the point that the amount of masses and measurements of the pendulum, the most point to be focused is the fed Fuzzy PID controller is make lower overshoot and minimum steady state error., this technique always make the best results shown in Fig 3, the fuzzy rules of the fed PID controller using Takagi-Sugeno shown bellow is better than the results of Hybrid fuzzy FED PID controller [17]:

For the fuzzy proportional integrator differentiator:

1. If (I1 is -ve) and (I2 is -ve) and (I3 is -ve) then (O/P1 is F1)
2. If (I1 is -ve) and (I2 is -ve) and (I3 is zero) then (O/P1 is F1)
3. If (I1 is -ve) and (I2 is -ve) and (I3 is +ve) then (O/P1 is F1)
4. If (I1 is -ve) and (I2 is zero) and (I3 is -ve) then (O/P1 is F1)
5. If (I1 is -ve) and (I2 is zero) and (I3 is zero) then (O/P1 is F2)
6. If (I1 is -ve) and (I2 is zero) and (I3 is +ve) then (O/P1 is F2)
7. If (I1 is -ve) and (I2 is +ve) and (I3 is -ve) then (O/P1 is F2)
8. If (I1 is -ve) and (I2 is +ve) and (I3 is zero) then (O/P1 is F3)
9. If (I1 is -ve) and (I2 is +ve) and (I3 is +ve) then (O/P1 is F3)
10. If (I1 is zero) and (I2 is -ve) and (I3 is -ve) then (O/P1 is F1)
11. If (I1 is zero) and (I2 is -ve) and (I3 is zero) then (O/P1 is F2)
12. If (I1 is zero) and (I2 is -ve) and (I3 is +ve) then (O/P1 is F2)
13. If (I1 is zero) and (I2 is zero) and (I3 is -ve) then (O/P1 is F2)
14. If (I1 is zero) and (I2 is zero) and (I3 is zero) then (O/P1 is F2)
15. If (I1 is zero) and (I2 is zero) and (I3 is +ve) then (O/P1 is F2)
16. If (I1 is zero) and (I2 is +ve) and (I3 is -ve) then (O/P1 is F2)
17. If (I1 is zero) and (I2 is +ve) and (I3 is zero) then (O/P1 is F2)
18. If (I1 is zero) and (I2 is +ve) and (I3 is +ve) then (O/P1 is F3)
19. If (I1 is +ve) and (I2 is -ve) and (I3 is -ve) then (O/P1 is F1)
20. If (I1 is +ve) and (I2 is -ve) and (I3 is zero) then (O/P1 is F2)
21. If (I1 is +ve) and (I2 is -ve) and (I3 is +ve) then (O/P1 is F3)
22. If (I1 is +ve) and (I2 is zero) and (I3 is -ve) then (O/P1 is F2)
23. If (I1 is +ve) and (I2 is zero) and (I3 is zero) then (O/P1 is F2)
24. If (I1 is +ve) and (I2 is zero) and (I3 is +ve) then (O/P1 is F3)
25. If (I1 is +ve) and (I2 is +ve) and (I3 is -ve) then (O/P1 is F3)
26. If (I1 is +ve) and (I2 is +ve) and (I3 is zero) then (O/P1 is F3)
27. If (I1 is +ve) and (I2 is +ve) and (I3 is +ve) then (O/P1 is F3)

The output (O/P) constants are -0.5, 0, and 0.5 for the three functions respectively.

Fig 2 illustrates the membership functions of the inputs (I's) to the controller desired:

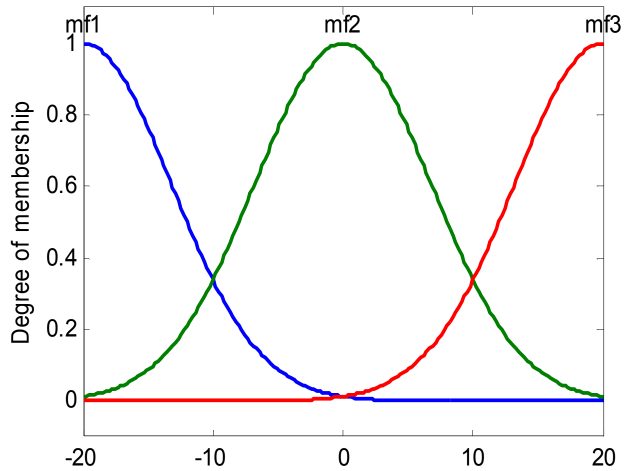

Fig.2: membership functions of the inputs to the controller

Fig 3 illustrates the step response of hybrid fuzzy FED PID controller versus conventional PID controller using mamdani technique:

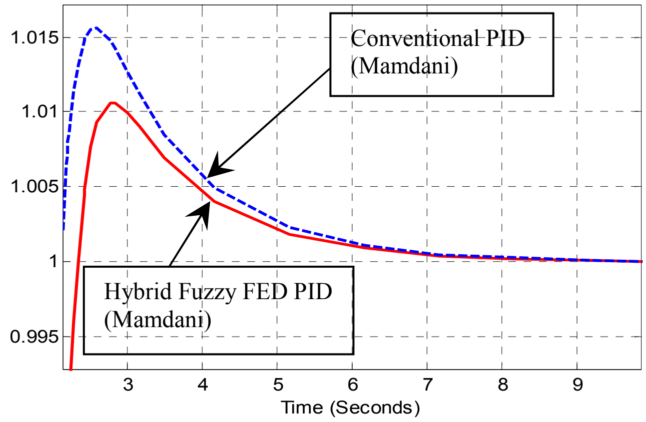

Fig.3: Stabilization control of the PID versus FED PID (Mamdani)

Fig 4 illustrates the step response of hybrid fuzzy FED PID controller (Takagi-Sugeno) versus conventional PID controller:

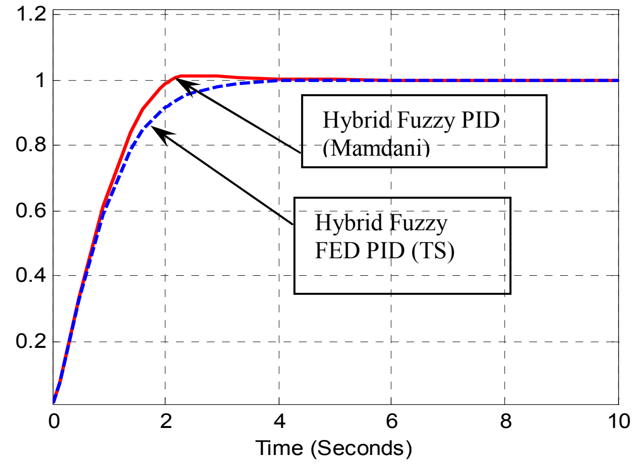

Fig.4: Stabilization control of the PID versus FED PID (T-S)

Fig 5 illustrates the step response of hybrid fuzzy FED PID controller (Takagi-Sugeno) versus hybrid fuzzy FED PID controller (Mamdani):

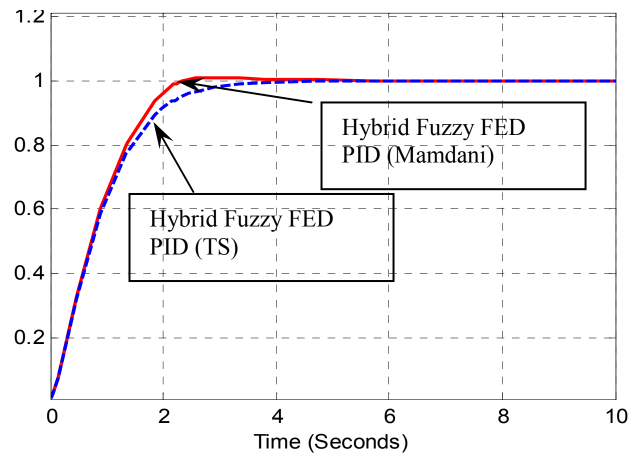

Fig.5: Stabilization control of the FED PID (M) versus FED PID (T-S)

V. CONCLUSION

In this paper, the new approach of control design of a hybrid fuzzy FED PID controller is proposed using the T-S method. The proposed design method focuses on constructing the fuzzy rule base. The proposed controller demonstrates excellent control performance for nonlinear robot which depends on the hybridizing of the gain scheduling method and fed PID T-S controller which gives the best control specifications towards the conventional PID, fuzzy PID and hybrid fuzzy PID . The proposed problem is considered one of the hottest and useful topics in the area of fuzzy control field related with robotics systems.

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