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ORIGINAL PAPER

Numerical study of a structure containing left-handed material waveguide

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Abstract: In this paper a waveguide structure consisting of a pair of left-handed material (LHM) and dielectric slabs inserted in vacuum is investigated theoretically. Maxwell's equations are used to determine the electric and magnetic fields of the incident waves at each layer. Snell's law is applied and the boundary conditions are imposed at each layer interface to calculate the reflected and transmitted powers of the structure. Numerical results are illustrated to show the effects of frequency, angle of incidence and LHM thickness on the transmitted power when the refractive index of the dielectric layer changes. The same procedure is repeated to show the variation of the transmitted power with the change in the mentioned parameters under different values of dissipation factor of the lossy LHM. Consequently, two cases of the LHM are considered, loss-less case and loss case. The results obtained, are in agreement with the law of conservation of energy.

Keywords: Electromagnetic waves; Left-handed material; Frequency; Transmitted power

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1. Introduction

Metamaterials (sometimes termed as left-handed materials (LHMs)) are materials whose permittivity ε and permeability μ are both negative and consequently they have negative index of refraction. These materials are artificial and theoretically discussed first by Veselago [1] more than 40 years ago. The first realization of such materials, consisting of split-ring resenators (SRRs) and continuous wires, was first introduced by Pendry et al. [2, 3]. Regular materials are materials whose ε and μ are both positive and so, they are termed as right handed materials (RHMs). Shelby et al. [4] have studied negative refraction in LHMs. Shadrivov [5] has investigated nonlinear guided waves in LHMs. Garcia and Nieto-Vesperinoas [6] have shown that LHMs do not make a perfect lens. Kong [7] has provided a general formulation for the interaction of electromagnetic wave with stratified metamaterial structures. Mousa and Shabat [8] have discussed nonlinear TE surface waves in a LHM and magnetic super lattice waveguide structure. Kourakis and Shukla [9] have investigated a nonlinear propagation of electromagnetic waves in negative-refractive index LHM. Raghuwanshi [10] has performed a comparative study of asymmetric versus symmetric planar slab dielectric optical waveguide. Cory and Zach [11] and Sabah and Uckun [12] have estimated high reflection coatings of multilayered structure. Oraizi and Abdolali [13] have obtained zero reflection from multilayered metamaterial structures. Characteristics of other waveguides have been also reported [14, 15].

In this paper a structure consisting of LHM and dielectric slabs inserted in vacuum is considered. A plane polarized wave is obliquely incident on it. Two cases of the LHM are studied, namely, loss-less case and loss case. Maxwell's equations are used to determine the electric and magnetic fields of the incident waves at each layer. The boundary conditions for the fields are matched at each layer interface. Then, Snell's law is applied to obtain a number of equations with unknown parameters. The equations are solved for the unknown parameters to calculate the reflection and transmission coefficients of the structure. These coefficients are then used to determine the reflected

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and transmitted powers. The effects of many parameters like frequency, angle of incidence etc. on the transmitted power are studied in detail by changing the refractive index of the dielectric layer and subsequently by changing the dissipation factor of the LHM. Throughout the computations, the frequency dependence of permittivity and permeability of the LHM is taken into account. It is found that, the numerical results are in agreement with the law of conservation of energy given by [11, 16, 17]. It is also noticed that the numerical results of Fig. 4 are similar to that reported in Ref. [16]. This is another evidence of the validity of the performed computations. The propagation of electromagnetic waves through a structure containing LHM and dielectric were considered by many previous studies. The current work concentrates on the powers of the structure and the role of the properties of the dielectric and LHM layers. Thus, the variation of the transmitted power with the incident angle, the frequency and LHM thickness is computed and presented in numerical results, with the emphasis on the refractive index of the dielectric layer and the dissipation factor of the LHM.

2. Theory

We consider four regions each with permittivity ε_l and permeability μ_l, where l represents the order of the region. Region 1 and 4 are vacuums (ε_o, μ_o), Region 2 is a regular dielectric (ε_2, μ_2), Region 3 is a metamaterial ($\varepsilon_3(\omega)$, $\mu_3(\omega)$). A plane polarized wave in Region 1 incident on the plane $z = 0$ at some angle θ with the normal to the boundary (see Fig. 1).

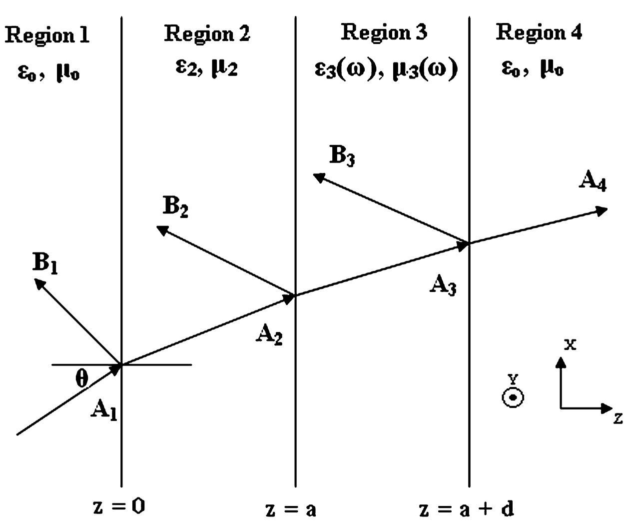

Fig. 1 Wave propagation through a structure consisting of a pair of dielectric and metamaterial embedded in vacuum

The electric field in each region is [7, 11, 18]:

$$E_l = \left(A_l e^{ik_{\ell z}z} + B_\ell e^{-ik_{\ell z}z}\right) e^{i(k_{\ell x}x-\omega t)}\,\hat{y} \tag{1}$$

We use Maxwell's equation as is done by [19] to find corresponding magnetic field H_ℓ :

$$\begin{aligned} H_\ell = &\frac{1}{\mu_\ell\omega}\left[(A_\ell k_{\ell x} e^{ik_{\ell z}z} + B_\ell k_{\ell x} e^{-ik_{\ell z}z})\hat{z}\right. \\ &\left.+(-A_\ell k_{\ell z} e^{ik_{\ell z}z} + B_\ell k_{\ell z} e^{-ik_{\ell z}z})\hat{x}\right] e^{i(k_{\ell x}x-\omega t)} \end{aligned} \tag{2}$$

where A_ℓ and B_ℓ are the amplitudes of the forward and backward travelling waves respectively. $k_\ell = n_\ell\omega/c$ is the wave vector inside the material and n_ℓ is the refractive index of it. Matching the boundary conditions for E and H fields at each layer interface, that is at $z = 0$, $E_1 = E_2$ and $H_1 = H_2$ and so on, yields six equations with six unknown parameters [11, 13, 19]:

$$A_1 + B_1 = A_2 + B_2 \tag{3}$$

$$\frac{k_{1z}}{\mu_1}(A_1 - B_1) = \frac{k_{2z}}{\mu_2}(A_2 - B_2) \tag{4}$$

$$A_2 e^{ik_{2z}d_2} + B_2 e^{-ik_{2z}d_2} = A_3 e^{ik_{3z}d_2} + B_3 e^{-ik_{3z}d_2} \tag{5}$$

$$\frac{k_{2z}}{\mu_2}\left(A_2 e^{ik_{2z}d_2} - B_2 e^{-ik_{2z}d_2}\right) = \frac{k_{3z}}{\mu_3}\left(A_3 e^{ik_{3z}d_2} - B_3 e^{-ik_{3z}d_2}\right) \tag{6}$$

$$A_3 e^{ik_{3z}(d_2+d_3)} + B_3 e^{-ik_{3z}(d_2+d_3)} = A_4 e^{ik_{4z}(d_2+d_3)} \tag{7}$$

$$\frac{k_{3z}}{\mu_3}\left(A_3 e^{ik_{3z}(d_2+d_3)} - B_3 e^{-ik_{3z}(d_2+d_3)}\right) = \frac{k_{4z}}{\mu_4} A_4 e^{ik_{4z}(d_2+d_3)} \tag{8}$$

where $k_{1x} = k_{2x} = k_{3x} = k_{4x} \equiv$ Snell's law.

Putting $A_1 = 1$ and solving the resulting equations for the unknown parameters, enables us to calculate the reflection and transmission coefficients B_1 and A_4 [11, 19]. The reflected power R and the transmitted power T are given by [11, 19]:

$$R = B_1 B_1^*, \quad T = A_4 A_4^* \tag{9}$$

where B_1^* and A_4^* are the complex conjugates of B_1 and A_4 respectively.

The law of conservation of energy is given by [11, 16, 17]:

$$R + (k_{4z}/k_{1z})T = 1 \tag{10}$$

where

$$k_{\ell z} = \frac{\omega}{c}\sqrt{n_\ell^2 - n_1^2\sin^2\theta_1} \tag{11}$$

3. Numerical results

For the LHM in region 3, a dispersive one is employed with ε_3 and μ_3 appeared in [2, 3, 19]:

$$\varepsilon_3(\omega) = 1 - \frac{F_e \omega_{ep}^2}{\omega^2 - \omega_{eo}^2 + i\gamma_e \omega} \quad (12)$$

$$\mu_3(\omega) = 1 - \frac{F_m \omega_{mp}^2}{\omega^2 - \omega_{mo}^2 + i\gamma_m \omega} \quad (13)$$

where ω_{ep} and ω_{mp} are the electric and magnetic plasma frequencies, ω_{eo} and ω_{mo} are the electric and magnetic resonance frequencies. F_e and F_m are the scaling filling parameters. γ_e and γ_m are the electric and magnetic dissipation factors. The following parameters appearing in [19] are used: $\omega_{mp} = 2\pi \cdot 10.95$ GHz, $\omega_{mo} = 2\pi \cdot 10.1$ GHz, $F_m = .26$, $\omega_{ep} = 2\pi \cdot 13.3$ GHz, $\omega_{eo} = 2\pi \cdot 10.3$ GHz, $F_e = .37$. Two cases of the LHM are considered, loss-less case ($\gamma_e = \gamma_m = \gamma = 0$) and loss case ($\gamma_e = \gamma_m = \gamma \neq 0$). The central frequency is selected to be 11 GHz. At This frequency both ε_3 and μ_3 of the LHM are simultaneously negative for loss-less case and loss case. Regions 1, 2 and 4 given in Fig. 1 are assumed to be loss-less. The thicknesses of each of dielectric and LHM layers are equal to 3λ where λ is the wavelength of the incident waves at the central frequency.

In Fig. 2a a calculation of the transmitted power of the considered structure as a function of frequency is shown for two different values of refractive indices of the dielectric ($n_2 = 1.883$ and $n_2 = 2.55$), loss-less case of the LHM and for an angle of incidence of 30°. The frequency is varied between 9 and 12 GHz, because the simultaneity of the negative values of the permittivity and permeability of the LHM can be realized in this range, according to Eqs. (12) and (13). In this example the frequency range extends from 10.3 to 11.5 GHz. As shown in the graph the transmission is nullified above 11.5 GHz frequency, where all incident radiation is reflected. In this band $\varepsilon_3(\omega)$ and $\mu_3(\omega)$ are different in signs, and hence n_3 is imaginary, as because $n_3 = \sqrt{\varepsilon_3 \mu_3 / \varepsilon_0 \mu_0}$. In 10.3–11.5 GHz band the transmission is very good. In this band $\varepsilon_3(\omega)$ and $\mu_3(\omega)$ are both negative, and hence n_3 is real. At 10.1 and 10.2 GHz frequencies the transmitted power is zero because $\mu_3(\omega)$ is positive and $\varepsilon_3(\omega)$ is negative and then n_3 is imaginary. In 9–10 GHz band the transmitted power is not zero because both $\varepsilon_3(\omega)$ and $\mu_3(\omega)$ are positive and so n_3 is real. This means that the electromagnetic waves will only propagate in a medium that has a real index of refraction [19].

To investigate the effect of the dissipation factor γ, Fig. 2b shows a calculation of the transmitted power as a function frequency, for $\theta = 30°$, $n_2 = 1.883$ and for three different values of γ ($\gamma = 0$, .1, .2). These values of γ are chosen from [19]. As confirmed from the figure the transmitted power decreases when γ increases.

Figure 3a illustrates the variation of transmitted power with the angle of incidence for 11 GHz frequency when the

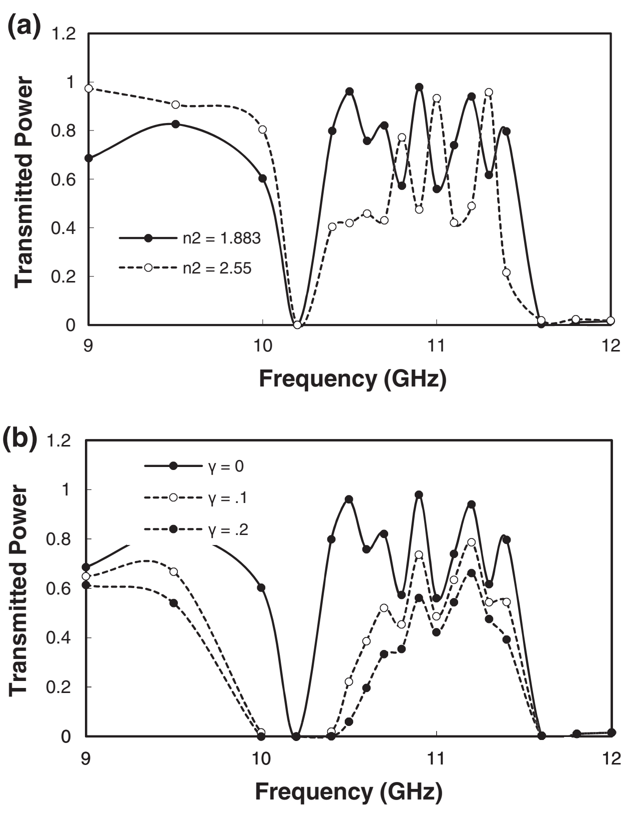

Fig. 2 Transmitted power variation with frequency (**a**) for two values of n_2 and (**b**) for three values of γ

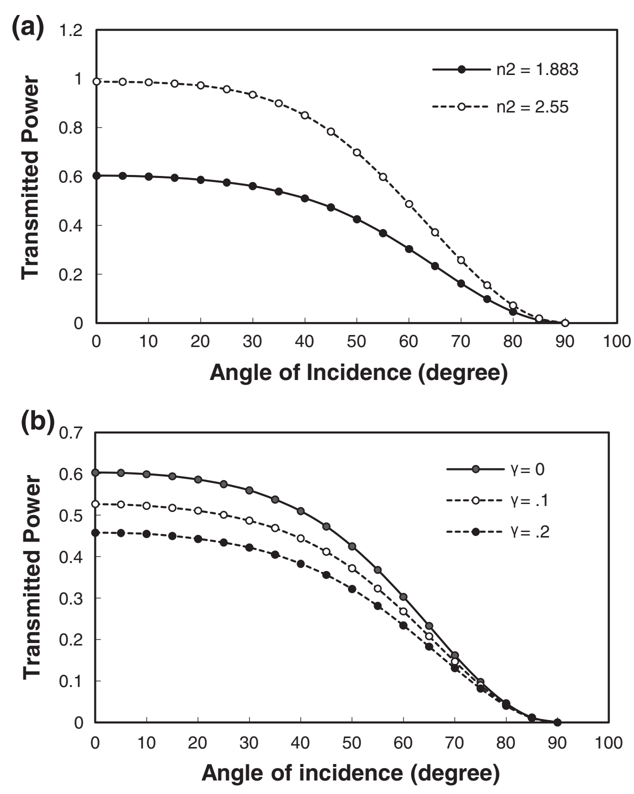

Fig. 3 Transmitted power variation with angle of incidence (**a**) under two values of n_2 and (**b**) under three values of γ

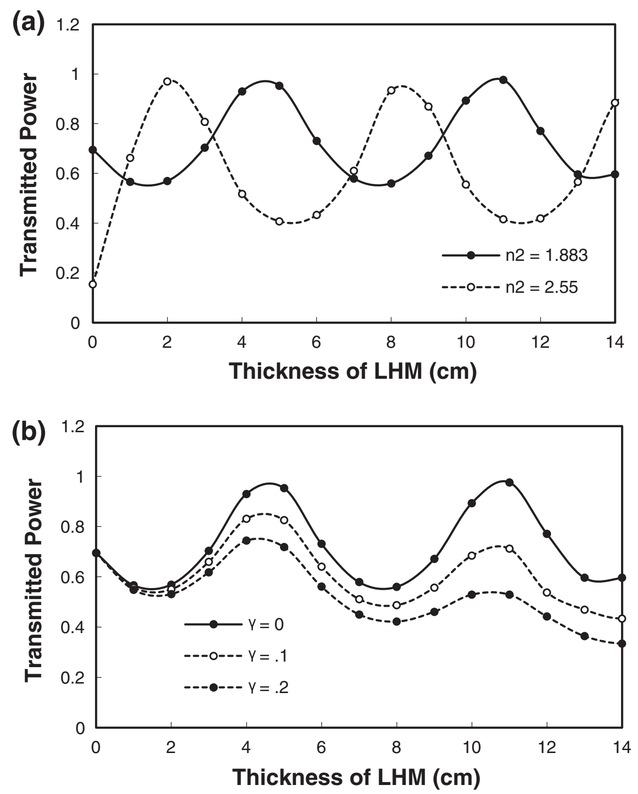

Fig. 4 Transmitted power variation with thickness of LHM (**a**) when n_2 changes and (**b**) when γ changes

refractive index of the dielectric changes. The angle of incidence is varied between 0 and 90° to realize all possible angles of incidences. Clearly the transmitted power decreases with the increase in the angle of incidence and becomes zero at 90° for any value of dielectric refractive index.

Figure 3b illustrates the transmitted power versus the angle of incidence, when the dissipation factor γ changes, with $\omega = 11$ GHz and $n_2 = 1.883$. As mentioned before the transmitted power decreases with the increasing of the dissipation factor. It is seen that the variation in the dissipation factor affects the initial values of the transmitted power.

Figure 4a demonstrates the effect of thickness of LHM on the transmitted power for 30° angle of incidence and 11 GHz frequency for two different values of refractive index of the dielectric. The thickness of the LHM is varied from zero to approximately 5λ (13.6 cm). It is evident from the figure that the transmitted power varies periodically with the thickness of LHM. For $n_2 = 1.883$ it varies between .57 and .95 while for $n_2 = 2.55$ it varies between 0.407 and 0.93.

Figure 4b shows the transmitted power as a function of LHM thickness, for $\theta = 30°$, $\omega = 11$ GHz, $n_2 = 1.883$ and for three different values of γ.

4. Conclusions

The transmission and reflection of electromagnetic waves by a multilayered structure consisting of a pair of LHM and dielectric slabs situated in free space have been studied. The present study is based on Maxwell's equations and matching the boundary conditions for the electric and magnetic fields of the incident waves at each layer interface. The frequency dependence of ε and μ of the LHM is taken into account. The dependence of the transmitted power of the considered structure on various parameters have been investigated. The law of conservation of energy given in [11, 16, 17] is obeyed by our results. The present problem is useful in applications which require controlling of reflected and transmitted powers like antenna radome, microwave, millimeter wave and optical devices.

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