British Geological Survey
NATURAL ENVIRONMENT RESEARCH COUNCIL

Separation of oceanic and continental crustal field signatures using Slepian functions

Ciaran Beggan[1] (ciar@bgs.ac.uk), Jarno Saarimäki[2] and Frederik Simons[3]
[1]British Geological Survey, West Mains Road, Edinburgh EH9 3LA, United Kingdom
[2]School of GeoSciences, University of Edinburgh, UK
[3]Dept. of Geosciences, Princeton University, USA

Models of the crustal magnetic field are typically represented using spherical harmonic coefficients. Rather than spherical harmonics, spherical Slepian functions can be employed to produce a locally and also globally orthogonal basis in which to optimally represent the data in a region at a given degree. The region can have *any* arbitrary shape and size. We use Slepian functions to optimally separate a crustal field model into its oceanic and continental regions in order to investigate the spectral content of each. Spherical harmonic coefficients are transformed into Slepian coefficients, separated into the appropriate regions and transformed back to spherical harmonic coefficients representing the space-limited extent of the oceans and continents. The spectral power of each region is examined over degrees L=16-72.

1. Introduction

Models of the crustal magnetic field are typically represented using spherical harmonic coefficients. Rather than spherical harmonics, spherical Slepian functions can be employed to produce a locally and also globally orthogonal basis in which to optimally represent the data in a region at a given degree. Slepian functions can be tailored to be either band- or space-limited, allowing a trade-off between spectral and spatial concentration in the region and leakage beyond. Only *N* Slepian coefficients are required to be solved for to optimally concentrate the energy of the Slepian functions into the region of interest $N = (L+1)^2R$; where N is the Shannon Number (ShN) and R is the fraction of the full sphere.

For positions (θ,φ) on the unit sphere, we have data *d(r)* with structure to maximum degree L. Using spherical harmonics basis functions (Y_{lm}) to represent real functions on a sphere requires $(L+1)^2$ coefficients (f_{lm}). However, using Slepian basis functions (G_α), which are locally and globally orthogonal, the data in a region can be *optimally* represented with coefficients (s_{ShN}), where ShN is the Shannon Number.

Spherical harmonic solutions and Slepian solutions are related by:

$$d(r) = \sum_{lm}^{(L+1)^2} f_{lm} Y_{lm}(r) = \sum_{\alpha=1}^{(L+1)^2} s_\alpha G_\alpha(r)$$

where

$$G_\alpha(r) = \sum_{lm}^{(L+1)^2} g_{lm}^{\alpha} Y_{lm}(r)$$

$$s_\alpha = \sum_{lm} g_{lm}^{\alpha} f_{lm}$$

→ Sort Slepian coefficients. Find ShN and use it to separate the Spherical Harmonic Coefficents into the two regions. →

$$s = \sum_{\alpha=1}^{ShN} g_{lm}^{\alpha} f_{lm}$$

$$\bar{s} = \sum_{\alpha=ShN+1}^{(L+1)^2} g_{lm}^{\alpha} f_{lm}$$

We seek to optimally separate the spherical harmonic coefficients from a magnetic crustal field model into its oceanic and continental regions. We generate Slepian basis functions for the continental regions with the oceanic regions being complementary.

In order to investigate the spectral content of each, the input spherical harmonic coefficients are rotated (in order to move Antarctica off the South pole for algorithm reasons), then transformed into Slepian coefficients, separated into the appropriate regions using the Slepian basis functions and the appropriate Shannon Number and finally transformed back to spherical harmonic coefficients representing the space-limited extent of the continents (s) and the oceans ($\bar{s}$). The spectral power of each region is examined over degrees L = 16-72.

Input Data: MF7 coefficients

We use the spherical harmonic coefficients from the MF7 crustal model available at www.geomag.org.

The MF7 model is based on the methodology of Maus *et al.* (2007) and is derived from CHAMP satellite data up to April 2010. Figure 1 shows the radial component of the crustal magnetic field from degree L = 16 -72 at the Earth's surface.

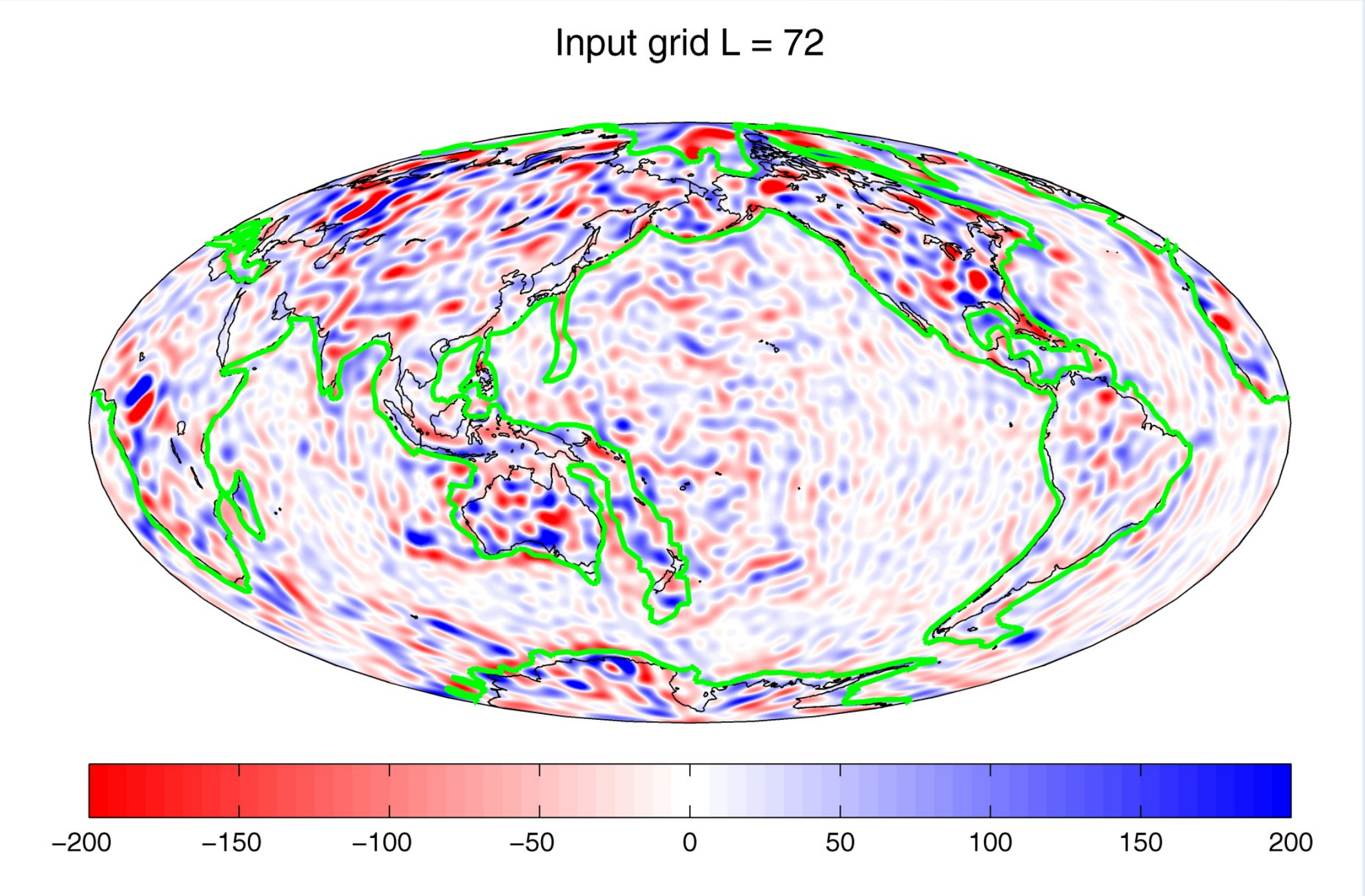


Figure 1: MF7 model of the crustal field (radial component [nT]) from degree 16-72 at the Earth's surface. The areas of the crust considered to be continental (including submarine continental shelves) are outlined in green. Continents shown for reference.

Association: IAGA Div. V
Symposium:A14.1
Poster: #2666 [06 Jul 2011]

IUGG Melbourne, Australia
28th June - 7th July 2011

2. Results

The continental crust is approximately 40% of the global area and its optimal Shannon number is 2170 (out of 5329). The Slepian coefficients are transformed back to Spherical Harmonic coefficients and plotted on the sphere. Figure 2 shows the spatial separation of (a) continental and (b) ocean crust. Note that some leakage from one region into another always occurs, though it is relatively low amplitude. The spectral power of each region is computed using the Lowes-Mauersberger formula. Figure 3 shows the input spectra from MF7, and the separated continental and oceanic spectra.

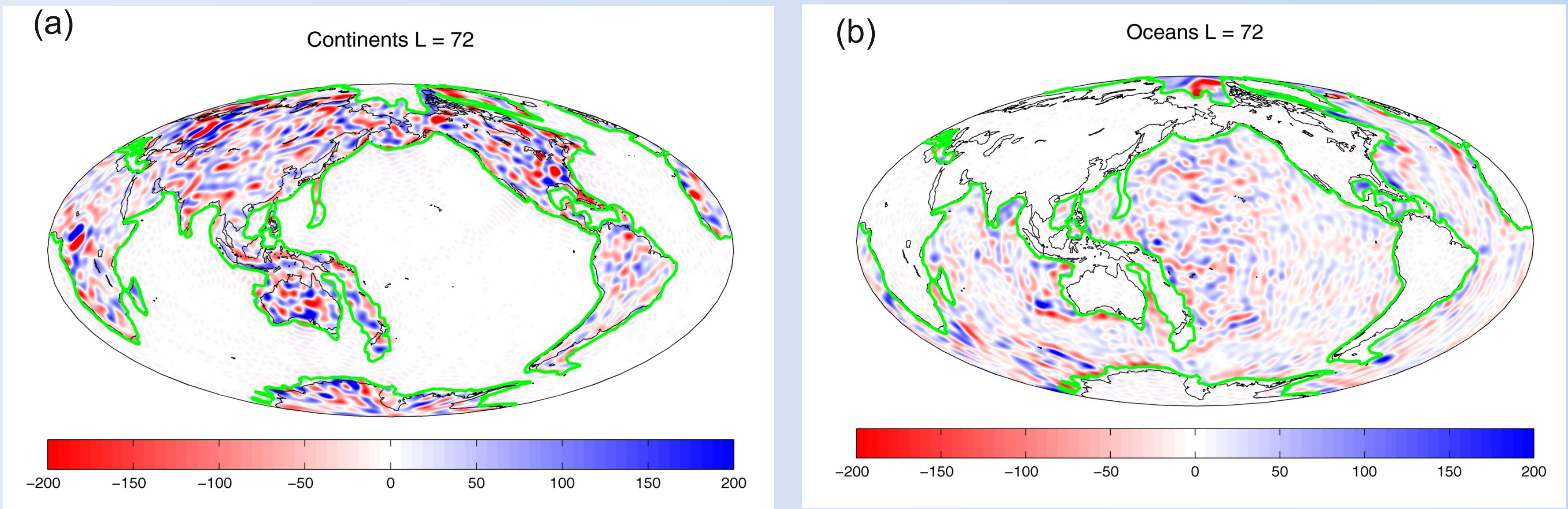


Figure 2: MF7 model of the crustal field (radial component [nT]) from degree 16-72 at the Earth's surface, optimally divided into (a) the continental and (b) oceanic crustal regions.

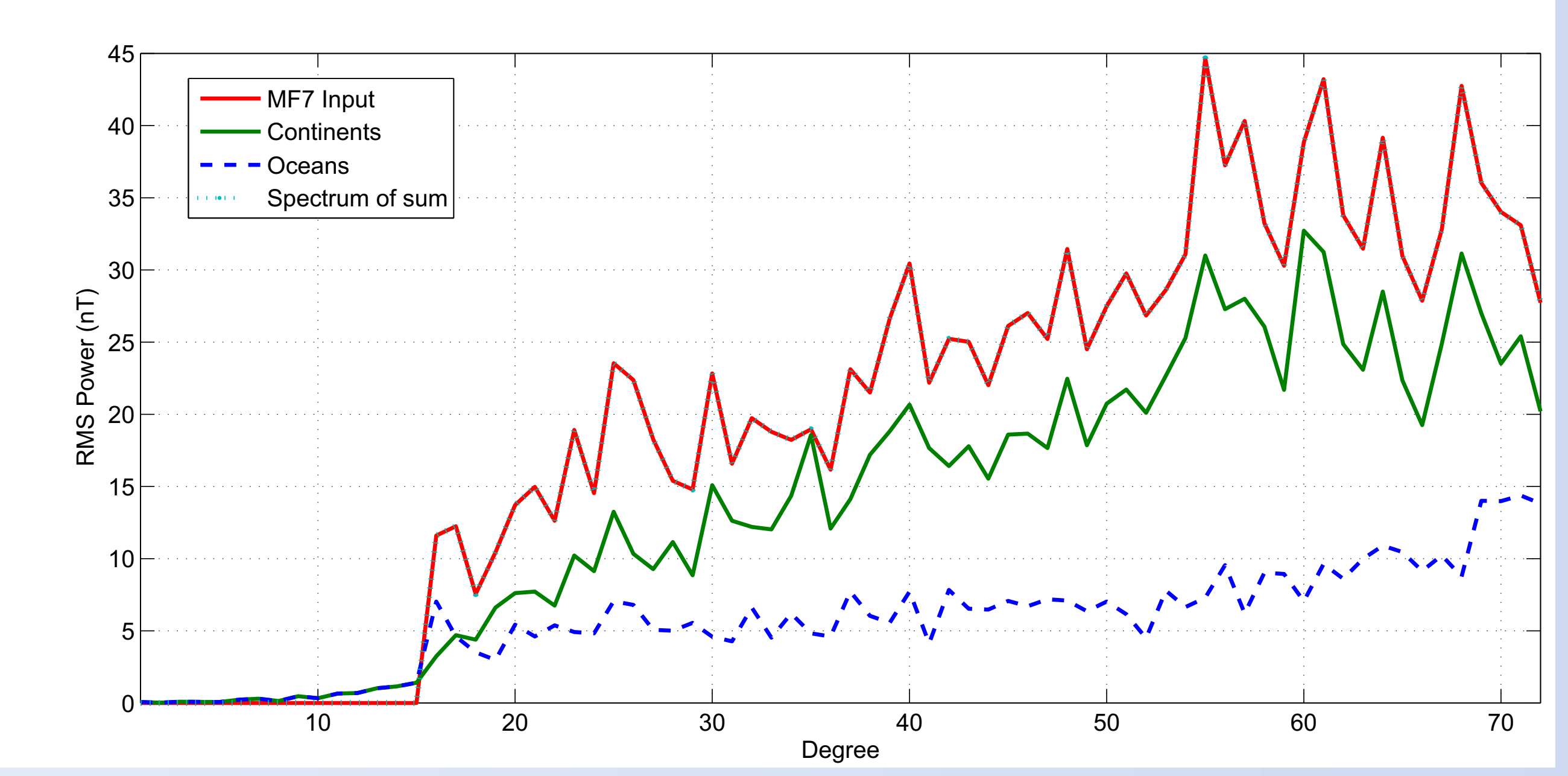
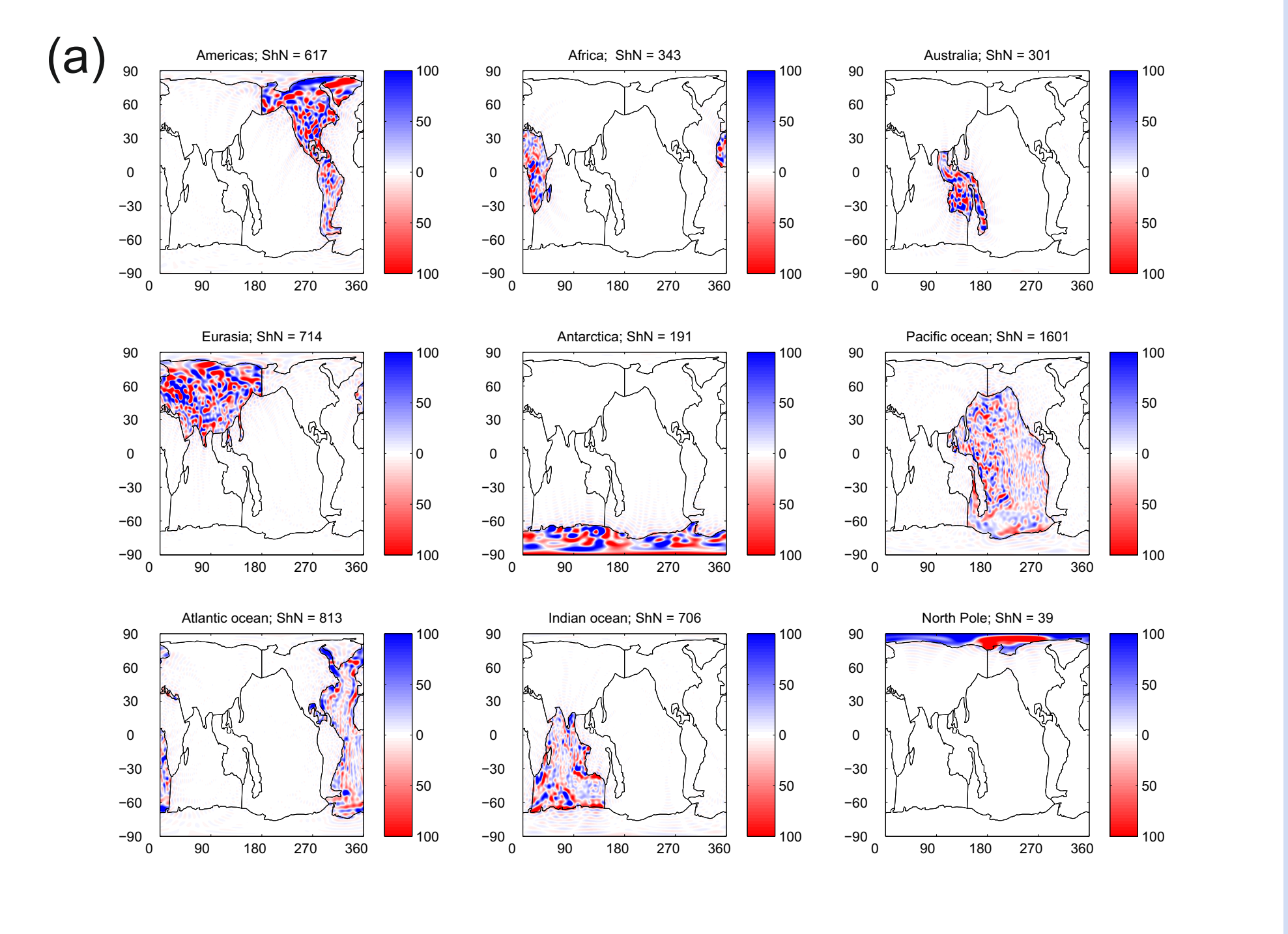


Figure 3: Lowes-Mauersberger power spectra of the MF7 input model of the crustal field and optimally separated continental and oceanic crustal regions.

The oceanic spectrum is approximately flat to degree 60, with an increase between 60–72. The continental spectrum has greater power at all degrees above L > 18.

The spectra in Figure 3 suggest that the oceanic crust has approximately equal power at all degrees up to L = 60, while the continental crustal spectrum shows approximately increasing power up to L = 55 with a flatter curve between L = 56–72. Note there is some leakage into degrees below L < 16.

Figure 4 shows further analysis with the separation of the crustal model into nine individual continental and ocean regions. Many of the regions are approximately similar in size.



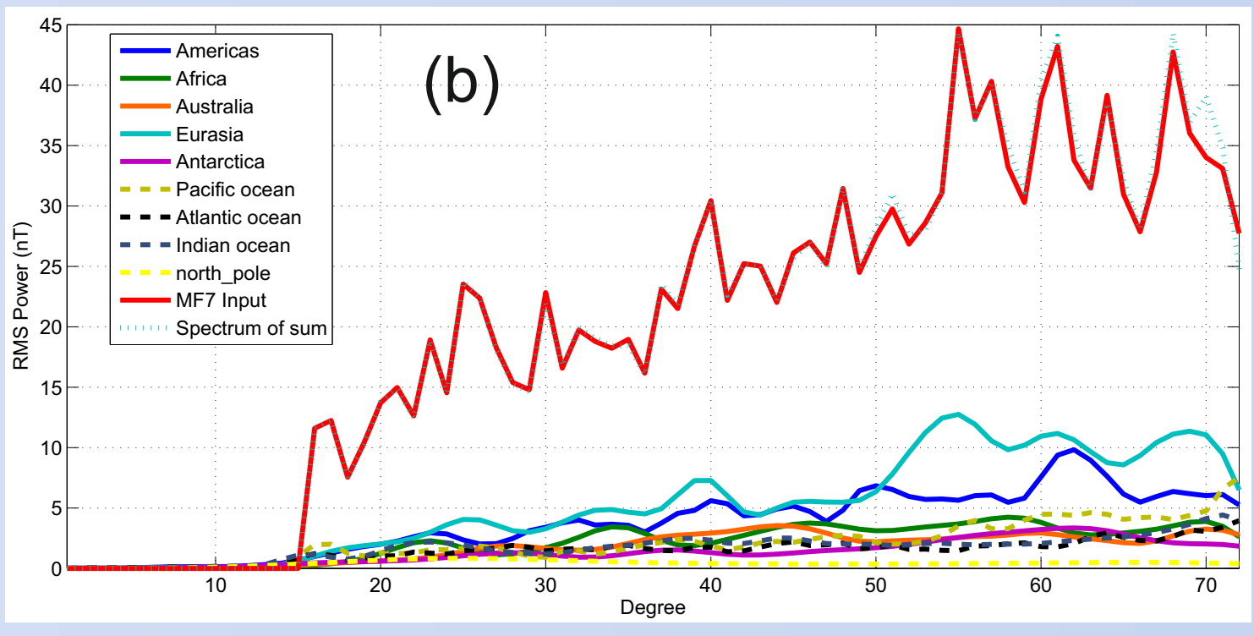


Figure 4: (a) Separation of the crust into nine discrete continental regions and oceanic basins.
(b) Lowes-Mausberger power spectra of the MF7 input model of the nine optimally separated continental and oceanic crustal regions.

3. Discussion

The reason for approximately equal power at all degrees in the ocean basins is not understood but may be a combination of the changing rate of field reversals, non-uniform plate motions, smoothing from both measurement at satellite altitudes and from downward continuation to the Earth's surface. The results from the oceans, [c.f. Figures 3 and 4 (b)] imply that studies which assume the oceanic crust has lower power are correct, and hence that imaging smaller features of the core field beneath the oceans is possible. The Pacific ocean region shows an increase in power at L = 70–72, perhaps associated with larger magnetic anomalies of the Cretaceous.
The continental crust spectra show most of the power is in the American and Eurasian regions, with distinctive 'peaks' at L = 62 and 55, respectively. The summation of the spectra shows that very little leakage occurs overall, though inevitably the optimal Shannon number is a trade-off between spectral and spatial concentration.
Future work will place constraints and error bars on the curves to evaluate the expected leakage from each degree into the others.

References

Dahlen, F. A. & F. J. Simons, Spectral estimation on a sphere in geophysics and cosmology, *Geophys. J. Int.*, **2008**, 174, 774–807
Maus, S., H. Lühr, M. Rother, K. Hemant, G. Balasis, P. Ritter, and C. Stolle, Fifth-generation lithospheric magnetic field model from CHAMP satellite measurements, **2007**, *Geochem. Geophys. Geosyst.*, 8, Q05013
Simons, F, J., Dahlen, F. A. & M. A. Wieczorek, Spatiospectral concentration on a sphere, *SIAM Review*, **2006**, 48, 504–536