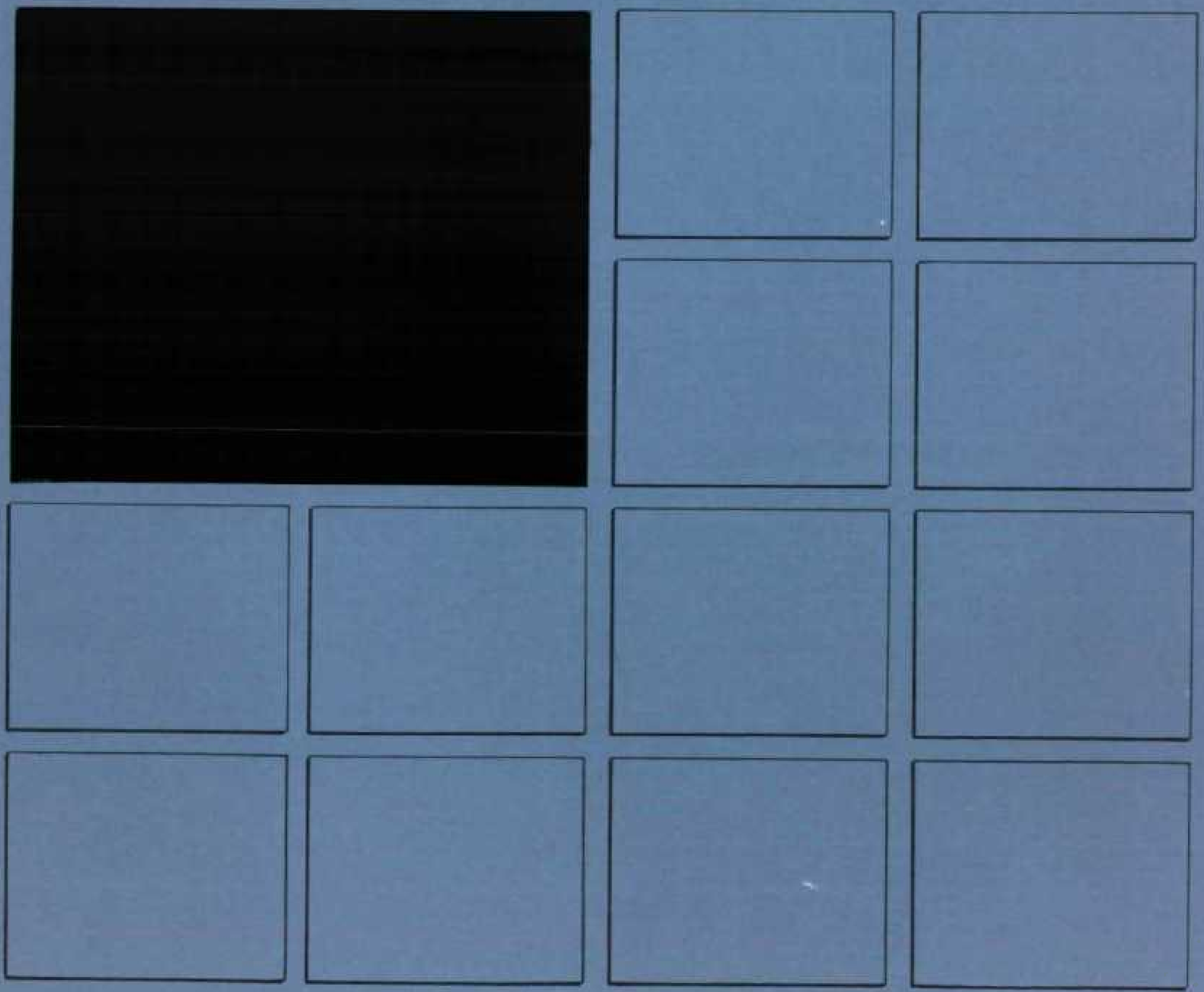
INSTITUTE of
HYDROLOGY



Joint dependence of supply reliability on aquifer recharge and peak demand

by J.R.M. Hosking

A report to the Southern Water Authority

July 1982

THE NATURAL ENVIRONMENT RESEARCH COUNCIL

Institute of Hydrology
Maclean Building
Crowmarsh Gifford
Wallingford Oxon
OX10 8BB

Telephone Wallingford 38800
STD Code 0491
Cables Hycycle Wallingford
Telex 849365 Hydrol G

Mr P W Herbertson
Resource Planning
Southern Water Authority
Guildbourne House
Chatsworth Road
Worthing
BN11 1LD

Your ref P 23035

Our ref S 33-335/DWR

Date 8 July 1982

Dear Peter

WATER SUPPLY RELIABILITY STUDIES

I enclose two copies of a report entitled "Joint dependence of supply reliability on aquifer recharge and peak demand". This represents the outcome of our study under job P 23035; the other study, of climate influences on peak demand, is continuing and I enclose a brief note on findings to date.

As you will see, the joint dependence study has proceeded along the lines envisaged in our proposal of 19 May 1982 with the agreed exception that we should not attempt to incorporate the supply manager's perception of the relative severity of particular years. We are confident that the report demonstrates the statistical technique for representing the joint dependence of supply reliability on resource availability and peak demand, as required. However, our reservations remain about the use of residual rainfall as a measure of resource availability. Although these are brought home in the latter part of the report I am anxious that the casual reader is not misled by Figure 6.

The figure shows contours of various degrees of supply unreliability as a result of high demand and low recharge in the East Sussex water supply area. Also shown are the basic data of aquifer recharge and peak week demand factors for the years 1964 to 1981. The figure indicates that 1979 was an extremely difficult year for meeting peak demands, with 1981 the next most severe. The eye is, of course, drawn to 1976 which ranks only moderately difficult by this analysis. We doubt that this will be in agreement with the supply manager's perception of the relative severity of supply problems in 1976!

Closer scrutiny of Figure 6 shows that the 1976 value of aquifer recharge (defined here as the combined residual rainfall over the two previous winters) was slightly above average for the period 1964-1981. This feature is difficult to reconcile with the 1975/76 drought. Possibly the calculations of residual rainfall are unreliable for this period ? However, the anomaly serves to support our view that it is the state of the aquifer, rather than a rate of recharge, that is crucial to supply availability.

Groundwater level at 1 April would, in our opinion, provide a more direct and meaningful measure of resource availability in a given summer period. We would welcome the opportunity to confirm this by a further study, taking advantage of the very long well records that exist in Southern England. We are confident that the end product would be more in accord with supply managers' perception than Figure 6.

Jon Hosking has, I think, produced a precise and readable report. We recognize the importance that you attached to presentation (when accepting our proposals) and are prepared to revise the report should it be too indigestible for the circulation that you had in mind. However, given the choice, I would prefer that any change in reporting style accompanied our re-working the analysis in terms of groundwater level.

Yours sincerely

Dr D W Reed

Enc.

DWR/RS

CONTENTS

Introduction

Unreliability of a water-supply system is manifested in the inability of the system to meet the demand for water made by the consumers in the area supplied by the system. Such failure of the system may be caused by an exceptionally high demand for water by the consumers; by the system having insufficient yield, due to an unusually low level of storage, even to meet an unexceptional demand; or by some combination of the two.

For the purposes of water-supply management and planning it would be convenient if an "index of unreliability" could be defined which, given values of demand and storage, would yield a single figure giving an indication of the severity of water-supply management problems likely to arise from that combination of demand and storage. The availability of such an index would enable the supply manager to establish whether a certain combination of demand and storage would cause supply problems as great as those encountered in some previous year; and would enable the planner to determine those demand/storage combinations which should be allowed for over any given planning horizon.

This study defines an index of unreliability and establishes its probability distribution. In order to produce a yearly index, the analysis is carried out on yearly data. The measure of demand used is the average daily demand during the peak summer week. Winter peaks are excluded: since their cause is qualitatively different from that of summer peaks they are likely to require separate analysis (as it happened, no winter peak occurred in the data used for the study). The measure of storage, or potential yield, of the supply system used is the recharge to the aquifer supplying the system during the winter or winters preceding the summer peak. Since, for the area considered in the study the water supply is drawn almost entirely from groundwater sources, no measure of surface water yield was deemed necessary. The resulting index of unreliability may be thought of as a measure of the stress placed on the water supply system during the week in which the heaviest demands are made on it.

The study was carried out for the East Sussex Water and Drainage Division, for which 18 years of demand data and 40 years of recharge data were available. The results obtained may be expected to hold in qualitatively similar form for other water supply units of comparable size.

2 Definition of peak demand

Since problems in water-supply management tend to arise when a high level of demand is maintained for a period of a week or more, the main quantity of interest in the pattern of a year's water demands is the average daily demand during the peak week of demand. For the East Sussex Water and Drainage Division, peak week demands were obtained from the Division's daily records, which cover the period 1964 to 1981.

The available data are for consumption (the amount of water actually used) rather than demand (the amount that consumers would have liked to use). Consumption may be less than demand if there are restrictions on water use (such as hosepipe bans or pressure reductions). Apart from a hosepipe ban in the period July-September 1976, there is no evidence of significant restrictions affecting East Sussex during 1964-1981 (SWA Section 24 report, 1982, Table 7.14). In 1976 the peak week for demand, 26 June - 2 July, fell outside the hosepipe ban period, and inspection of the data suggested that this week would still have been the peak week had there been no hosepipe ban. Accordingly the consumption data were accepted as giving a true indication of demand for water.

Demand for water varies year by year throughout the period considered, so to facilitate comparison between years it is better to work with the "peak week factor" D, defined by

$$D = \frac{\text{Average daily consumption in peak week}}{\text{Average daily consumption over the year}},$$

the yearly average being calculated over the calendar year in which the peak week falls. The yearly average rises fairly smoothly from year to year, so a high value of the peak week factor consistently indicates a high peak demand rather than, say, a low base demand during the preceding winter.

For the East Sussex Division, the eighteen available values of D range from 1.16 in 1965 to 1.38 in 1979. The higher values tend to occur towards the end of the sequence, and may reasonably be ascribed to the fine summers of recent years; there is no evidence of any significant underlying upward trend in the data.

3 Definition of aquifer recharge

The source of replenishment of an aquifer is the rain which falls on it, and in particular that residual part of the rainfall which is not lost by evaporation or surface runoff. This residual rainfall, in millimetres has been estimated for each of the Southern Water Authority's resource areas for each month for the period 1941 to 1981 by P Midgley of the Southern Water Authority, and these values are used as the basis for the data for aquifer recharge.

The East Sussex Water and Drainage Division draws its water from resource areas 16 (Lower Rother) and 17 (Upper Rother). The average of the values for these two areas was used to give a single value for each month for the residual rainfall over the whole Division.

Winter recharge was defined as the sum of the residual rainfalls for the months August to April inclusive (residual rainfall during the remaining months of May, June and July is negligible), giving a single value for each year. The state of an aquifer (the amount of water stored in it) depends on recharge over a longer period than just the preceding winter, so for each year, recharge values were found for the previous winter, the previous two winters, and the previous three winters, and the demand-recharge relationship was investigated for all three. The most statistically convenient of these in terms of its lack of correlation with the peak week demand factor, and the most satisfactory in the ease with which a credible index of water-supply systems unreliability could be derived from it, was recharge over the two previous winters, so the following sections take this to be the definition of aquifer recharge:

R = sum of residual rainfalls in the two previous winters.

4 Distribution of peak demand

The peak week factor D may be defined as the maximum over all summer weeks of the ratio of weekly demand to average daily demand for the year. This suggests that the distribution of D may be deduced from the theory of extreme values to be the extreme-value, or Gumbel, distribution.

For extreme-value theory to apply exactly to D, four conditions must be satisfied:

(1) demands for water in successive weeks should be independent;

(2) demands in different summer weeks should have identical distributions;

(3) peak week factors in successive years should be independent;

(4) peak week factors in different years should have identical distributions.

Conditions (3) and (4) may reasonably be expected to hold, and the data for East Sussex provide no evidence that they do not. Conditions (1) and (2), however, are unlikely to hold: (1) because the weeks for which demand factors are found overlap and are therefore not independent; (2) because different patterns of water use apply at different times during the summer.

Nonetheless the extreme-value distribution is a reasonable starting point when analysing the demand data. The observed sample exhibits the moderate skewness to be expected of an extreme-value distribution. An extreme-value distribution was therefore fitted to the East Sussex demand data: the distribution has the form

$$P(D \leqslant d) = \exp\left[-\exp\left\{-(d-\xi)/\theta\right\}\right], \; -\infty < d > \infty,$$

where $P(D \leqslant d)$ is the probability that the peak week factor D will exceed a given value d. The parameters of the distribution were estimated by the method of maximum likelihood:

location parameter $\xi = 1.205$, standard error 0.012;
scale parameter $\theta = 0.048$, standard error 0.009.

The fitted distribution gives an adequate fit to the data (see Figure 1), and with only 18 data points available no other distribution is likely to give a substantially better fit than this. (Normal and gamma distributions were also fitted to the data, but did not give a noticeably better fit than the extreme-value distribution).

The fitted distribution for D can take values less than 1, whereas D itself clearly cannot. The probability of the fitted distribution taking a value less than 1 is, however, vanishingly small (10^{-32}) and can be ignored for all practical purposes.

5 Distribution of aquifer recharge

Although the measure of aquifer recharge used in this study is the sum of residual rainfalls over two winters, the fitting of a distribution to the recharge values is best done using each winter's residual rainfall separately. This is because residual rainfalls for separate winters are almost independent whereas there is a strong correlation between successive yearly values of two-year residual rainfalls; and correlation in a sample can cause bias and inefficiency in the estimation of statistical distributions. Accordingly a distribution was fitted to the one-year residual rainfall values and the distribution of two-year residual rainfall totals deduced from this.

There are 40 years of residual rainfall data available for the East Sussex Water and Drainage Division, from winters 1941-42 to 1980-81 inclusive. Although the principal interest resides in the 18 years for which both demand and recharge data are available, all 40 years of data were used when fitting the distribution of residual rainfall in order to make maximum use of the available information. To check that the data were homogeneous over the entire period, the hypothesis that the first 22 years' and the last 18 years' values followed the same distribution was tested statistically, using the Kolmogorov-Smirnov two-sample test and the Wilcoxon rank-sum test; no evidence of inhomogeneity was discovered.

There is no theoretical reason to favour any particular distribution for residual rainfall. A gamma distribution was chosen on the pragmatic grounds that it takes only postive values and can model the moderate skewness exhibited by the observed sample. The probability of the gamma distribution is given by

$$f(r) = \frac{r^{\alpha-1} e^{-r/\beta}}{\beta^{\alpha}\Gamma(\alpha)}, \quad 0 < r < \infty,$$

Γ being the gamma function. The parameters of the distribution were estimated by maximum likelihood:

shape parameter $\alpha = 3.16$, standard error 0.67;
scale parameter $\beta = 85$, standard error 20.

The observed and fitted distributions are superimposed in Figure 2, and it can be seen that a reasonably good fit is obtained.

From the fitted distribution of residual rainfall, the distribution of aquifer recharge R may be deduced as the sum of two independent gamma distributions, both having the same parameters α and β. The resulting distribution is another gamma distribution, with shape parameter $2\alpha = 6.31$ and scale parameter $\beta = 85$. The observed and fitted distributions of recharge are compared in Figure 3.

6 Dependence between demand and recharge

The joint distribution of demand and recharge is a function of the separate marginal distributions of demand and recharge, and in addition of the dependence between demand and recharge. Any dependence, such as a tendency for a winter with low recharge to be followed by a summer with a high peak week demand factor, will complicate the modelling of the joint distribution of demand and recharge.

Figure 4 is a graph of the observed values of demand and recharge for East Sussex, and shows little evidence of dependence between the two. To verify this, a linear regression of demand on recharge was performed; a test for zero slope of the regression line gave a t statistic of 0.92 (16 degrees of freedom), implying that there is no statistically significant dependence of demand on recharge.

It is therefore safe to conclude that demand and recharge may be taken as independent, and that their joint distribution is determined solely by their marginal distributions.

7 Definition of an index of unreliability

The raison d'être of an index of water-supply unreliability has been explained in section 1; the present section attempts to derive a meaningful index from the available measures of demand and recharge.

An index of unreliability U may be written as U(D,R) to emphasize its dependence on demand D and recharge R. Since unreliability is caused by high demand and low recharge, U(D,R) should be an increasing function of D, and a decreasing function of R. The exact definition of U should not be critical provided it gives an indication of relative severity of supply problems which is consistent with the supply managers' experience.

Each definition of U(D,R), when applied to the 18 years of available data, gives an ordering of those years in terms of the severity of water-supply problems encountered in them. All that will be attempted here is to find a simple definition of U(D,R) which gives an ordering of the years which "looks reasonable".

Perhaps the simplest definition is $U = D/R$. This definition is not satisfactory; the variability of R is so much greater than that of D that the ordering of years in terms of U is essentially the same as the ordering of R values (see Table 1).

Instead one might consider the definition $U = (D-1)/R$. This is reasonable: all D values must exceed 1, so negative values of U cannot occur, while replacing D by D-1 serves to increase the variability of the numerator of the ratio defining U. This definition of U is some improvement, but there is still too much dependence on R rather than D: 1976, the year with the second highest D values, is ranked only 7th in order of severity (see Table 1).

A further reduction in the influence of R may be achieved by replacing R by $R + R_o$ for some constant R_o, leading to the definition $U = (D - 1)/(R + R_o)$. This corresponds to aquifer recharge having a base level over and above which the effect of residual rainfall applies. In terms of ordering of the years and their severity of water-supply problems, values of R_o in the range 300 to 1000 give a balanced influence of R and D on the index of unreliability, U. A value $R_o = 543$ was chosen since 543 is the mean value of R for the observed sample; this choice has no particular significance in itself except insofar as it lies within the preferred range, but may be useful in suggesting an initial choice of R_o should the definition of U be extended to other water supply units. The ordering of the years 1964 to 1981 according to this definition of U is given in Table 1, and indicates that the three years of most severe water-supply problems were 1979, 1981 and 1976.

In practice a definition of $U = 10^5(D - 1)/(R + 543)$ is recommended, so that the observed U values fall into a convenient range ($U = 10$ to $U = 50$).

Table 1. Ordering of years according to various indices of unreliability

Definition of U:	1/R Order	D/R Order	(D - 1)/R Order	(D - 1)/(R + 543) Order	(D - 1)/(R + 543) Values
	1973	1973	1979	1979	41.8
	1981	1981	1981	1981	32.0
	1972	1972	1973	1976	27.0
	1980	1979	1972	1964	26.3
	1979	1980	1964	1973	23.6
	1974	1974	1980	1975	22.9
	1966	1964	1976	1969	22.4
	1965	1966	1974	1977	21.8
	1964	1965	1969	1972	21.6
	1969	1969	1975	1974	19.6
	1976	1976	1966	1980	19.2
	1971	1971	1977	1971	18.7
	1978	1975	1965	1978	18.7
	1975	1978	1971	1967	18.2
	1977	1977	1978	1966	17.0
	1968	1968	1967	1965	15.7
	1967	1967	1970	1970	14.6
	1970	1970	1068	1968	12.7

Years are listed in decreasing order of U. Final column gives observed values of U for the definition $U = 10^5 (D - 1)/(R + 543)$.

Distribution of an index of unreliability

In order to establish how extreme an observed value of the index of unreliability, U, may be, it is necessary to determine the long-run distribution of U as a basis for comparison. Given the definition of U as a function of D and R, its distribution can be determined from the fitted joint distribution of D and R. A mathematical expression can be derived in this way for the distribution of U, but since it is algebraically rather intractable the distribution *of U was determined by computer simulation instead.*

A sample of 10 000 values of the peak week demand factor D was generated from the extreme-value distribution described in section 4. A sequence of 10 001 residual rainfall values was generated from the gamma distribution described in section 5, and the 10 000 sums of two successive residual rainfalls formed a sample of 10 000 recharge values R. For each pair of (D,R) values the index of unreliability was calculated as $U = 10^5(D - 1)/(R + 543)$ and the distribution of these 10 000 values was taken to be the long-run distribution of U. The distribution is illustrated in Figure 5, and its basic properties are given in Table 2.

In Figure 6, contours of equal unreliability (ie lines joining equal values of U) are plotted on a graph of the observed values of demand and recharge, to show how the observed data compare with the index of unreliability fitted to them. The contours are straight lines radiating from the point D = 1, R = -543, and are drawn for values of U corresponding to return periods of 5, 10, 20, 50 and 100 years. From the diagram one may deduce that, for example, a supply problem equal to or worse than that of 1979, the most extreme year in the East Sussex data set, may be expected to occur about once in 50 years.

Table 2. Properties of the simulated distribution of U.

Mean	22.4
Standard deviation	7.3

Values of U for various return periods

(Return Period) (years)	(U)
2	21.2
5	27.8
10	32.2
20	36.0
50	41.3
100	45.4

Note: standard error of return period T is $0.01\ T^2\{T^{-1}(1 - T^{-1})\}^{\frac{1}{2}}$. eg for T = 50 years, standard error is 3.5, so U value of 41.3 has a return period of 50 ± 3.5 years.

A single value of U can arise from many different combinations of D and R. For planning purposes it is useful to define, for a given value U, a pair of values (D,R) which are typical of the demand/recharge combinations giving rise to that value of U. From the joint distribution fitted to demand and recharge, one can determine the distribution of those (D,R) pairs giving rise to any given value of U. The mode of this distribution identifies the most probable (D,R) pair conditional on the given value of U, and gives a suitable choice of a representative pair of (D,R) values. Representative values of D and R for increasingly rare values of U are given in Table 3.

Table 3 Representative values of demand and recharge leading to particular levels of unreliability, U

U		D		R	
return period	value	value	return period	value	return period
5	27.8	1.248	3	349	5
10	32.2	1.276	5	313	7
20	36.0	1.300	8	291	9
50	41.3	1.335	16	268	13
100	45.4	1.362	27	254	16

Notes (i) Given U, and the parameters α, β, ξ, θ and R_o defined in Sections 4-7, the representative R is obtained by solving the equation:

$$\frac{2\alpha - 1}{R} + \frac{U}{10^5\theta} \exp \left\{\frac{\xi - 1}{\theta} - \frac{U(R + R_o)}{10^5\theta}\right\} = \frac{1}{\beta} + \frac{U}{10^5\theta},$$

and the representative D then obtained as $D = 1 + 10^{-5} U(R + R_o)$.

(ii) Return periods quoted for D and R are by reference to the marginal distributions of demand and recharge.

9 Conclusions

The principal conclusions of this study may be summarized as follows.

1) Peak week demand factors may be adequately modelled by an extreme-value distribution.

2) Residual rainfalls during a winter may be adequately modelled by a gamma distribution.

3) There is no significant dependence between the peak week demand factor D and aquifer recharge R, where recharge is defined as residual rainfall over the two preceding writers.

4) An index of unreliability U defined by $U = (D - 1)/(R + R_o)$ gives an indication of the severity of problems in water-supply management resulting from a combination of values of demand D and recharge R. A suitable initial choice for the constant R_o is the mean value of R.

The small size of the data set available for analysis, and the heuristic nature of the definition of the index of unreliability, require that caution be exercised in any wider application of the above conclusions. Some comments on, and caveats about, the conclusions should be mentioned.

a) A sample size of only 18 for the demand data is insufficient to give confidence that the data truly follow an extreme-value distribution. A wider investigation to validate the conclusion is called for, using data from other locations.

b) The estimated values of residual rainfall give cause for concern. Although the winter of 1975-76 is generally thought to be the driest of recent years, the figures provided by S.W.A. for the East Sussex Division indicate that of the eleven winters between 1970-71 and 1980-81, six had a lower residual rainfall than 1975-76. This is largely responsible for the unexpectedly low ranking of 1976 in the orders of severity given in Table 1.

c) The adequacy of aquifer recharge as an indicator of potential problems in water-supply is called into question by the difficulties encountered in defining an acceptable index of unreliability (section 7). It is possible that a more useful quantity would be some measure of the amount of water stored in the aquifer at the start of the summer, eg groundwater level at April 1st. (Long-term well records are available for the Southern W.A. area).

d) The final definition of the index of unreliability was determined by its leading to a reasonable ordering of the years 1964 to 1981 in terms of their water-supply problems. It would be worthwhile to compare the final order with a corresponding ordering as perceived by water-supply managers, and to revise the definition of U in the light of the managers' comments. It is hoped that the Southern Water Authority can provide such an ordering.

e) The definition of the index of unreliability depends on the constant R_o, which must be suitably chosen. The choice of R_o as the mean recharge is reasonable under the stated criteria, for the East Sussex Water and Drainage Division, and can be recommended as an initial choice should such an index be required for other water-supply units.

f) It should be noted that the analysis was carried out with peak week demands, and is not applicable to the assessment of water-supply reliability in weeks other than the peak week for demand in any year.

g) At the start of the summer, the recharge value R for that year (the previous two winters' residual rainfall) will be known and it may be desired to establish the distribution of the possible values of U as the demand D varies, conditional on the observed value of R. Such a procedure is not recommended: the conditional distribution of U for fixed R is much more sensitive than the unconditional distribution of U to small changes in the definition of U(D,R), and a data set of the size used in this study is insufficient to determine the conditional distribution of U to a reasonable degree of accuracy.

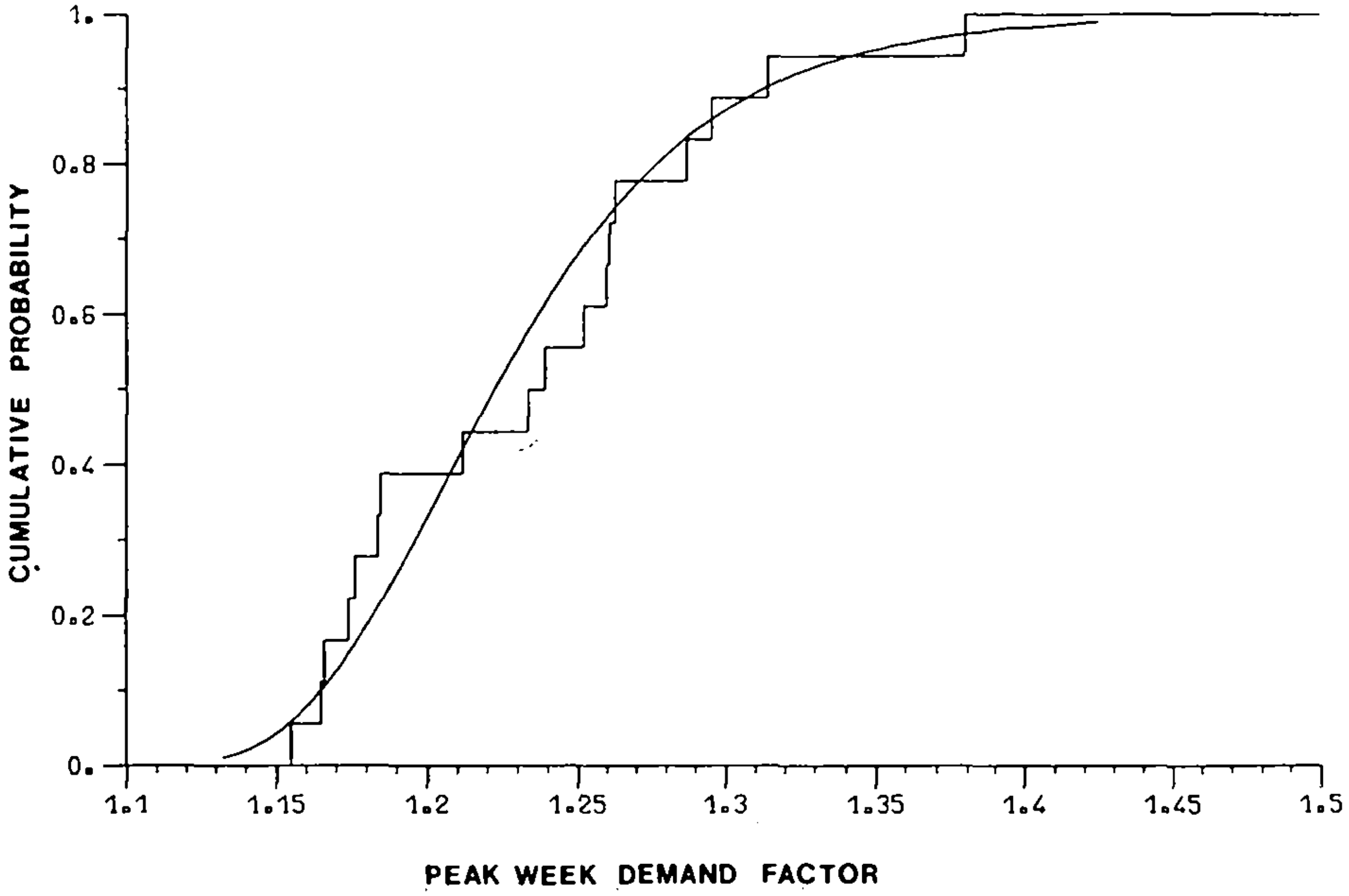


Figure 1 Empirical distribution function of peak week demand factor (stepped line) and its fitted extreme-value distribution (smooth curve).

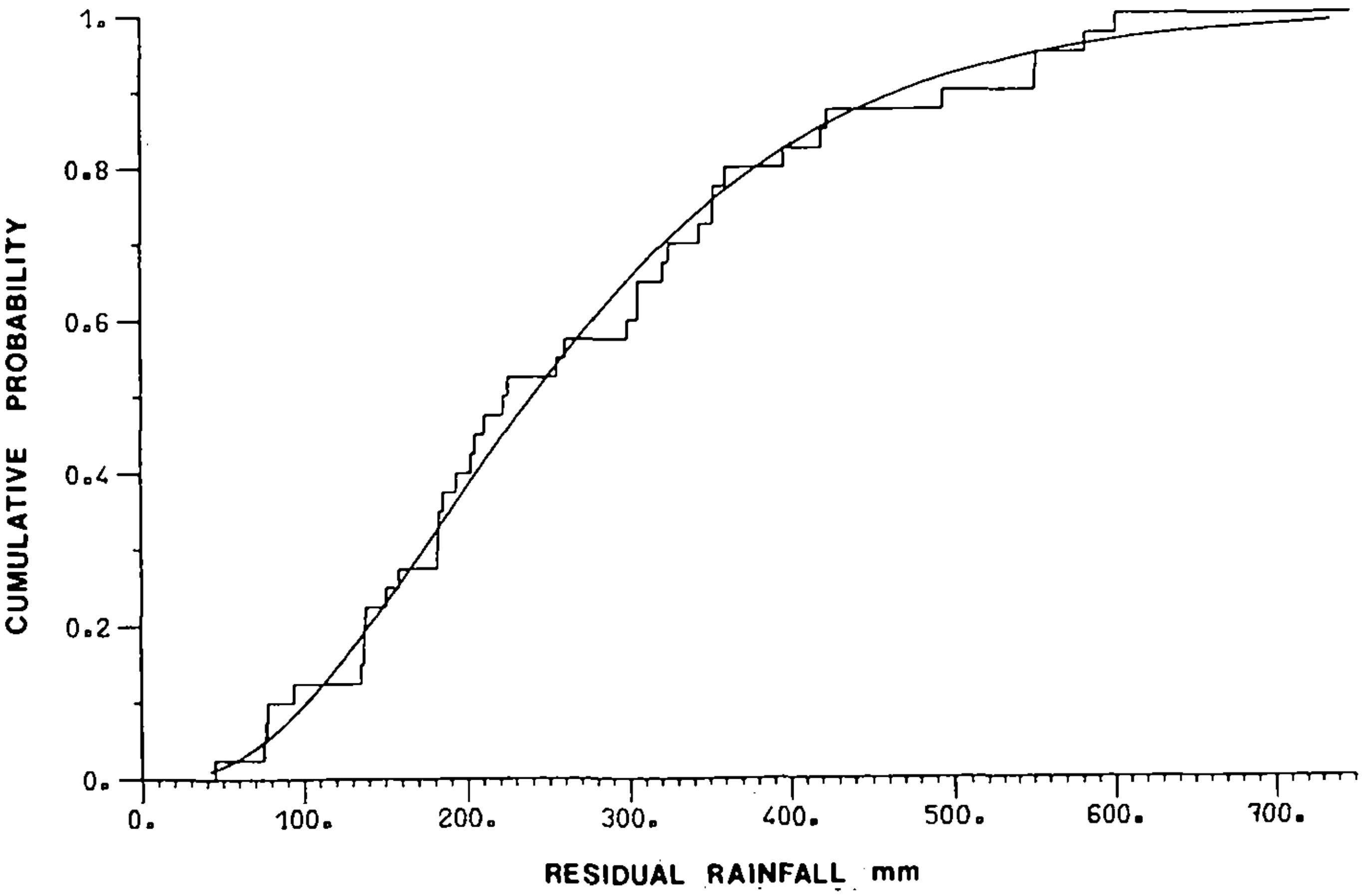


Figure 2 Empirical distribution function of residual rainfall (stepped line) and its fitted gamma distribution (smooth curve)

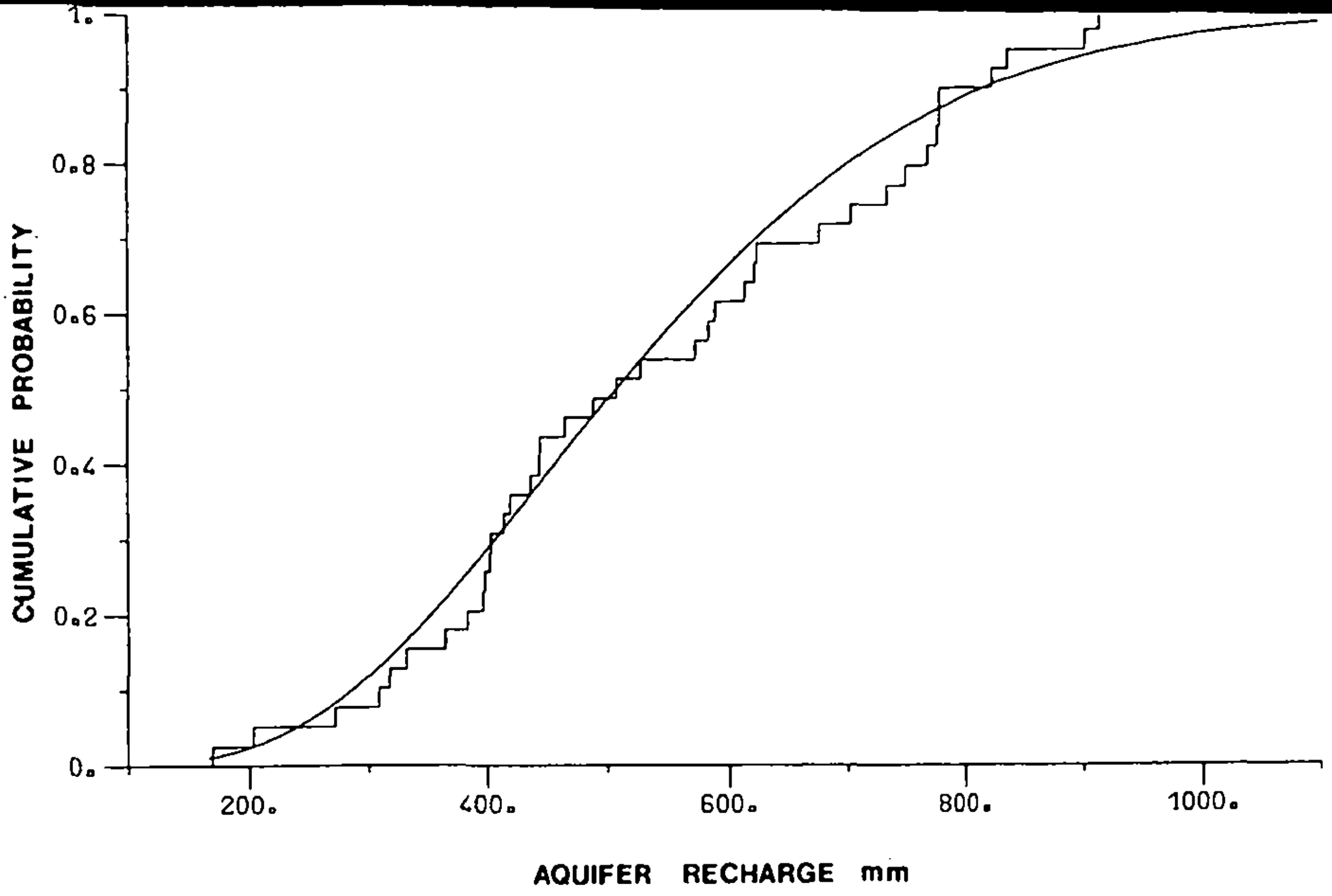


Figure 3 Empirical distribution function of aquifer recharge (stepped line) and its fitted gamma distribution (smooth curve).

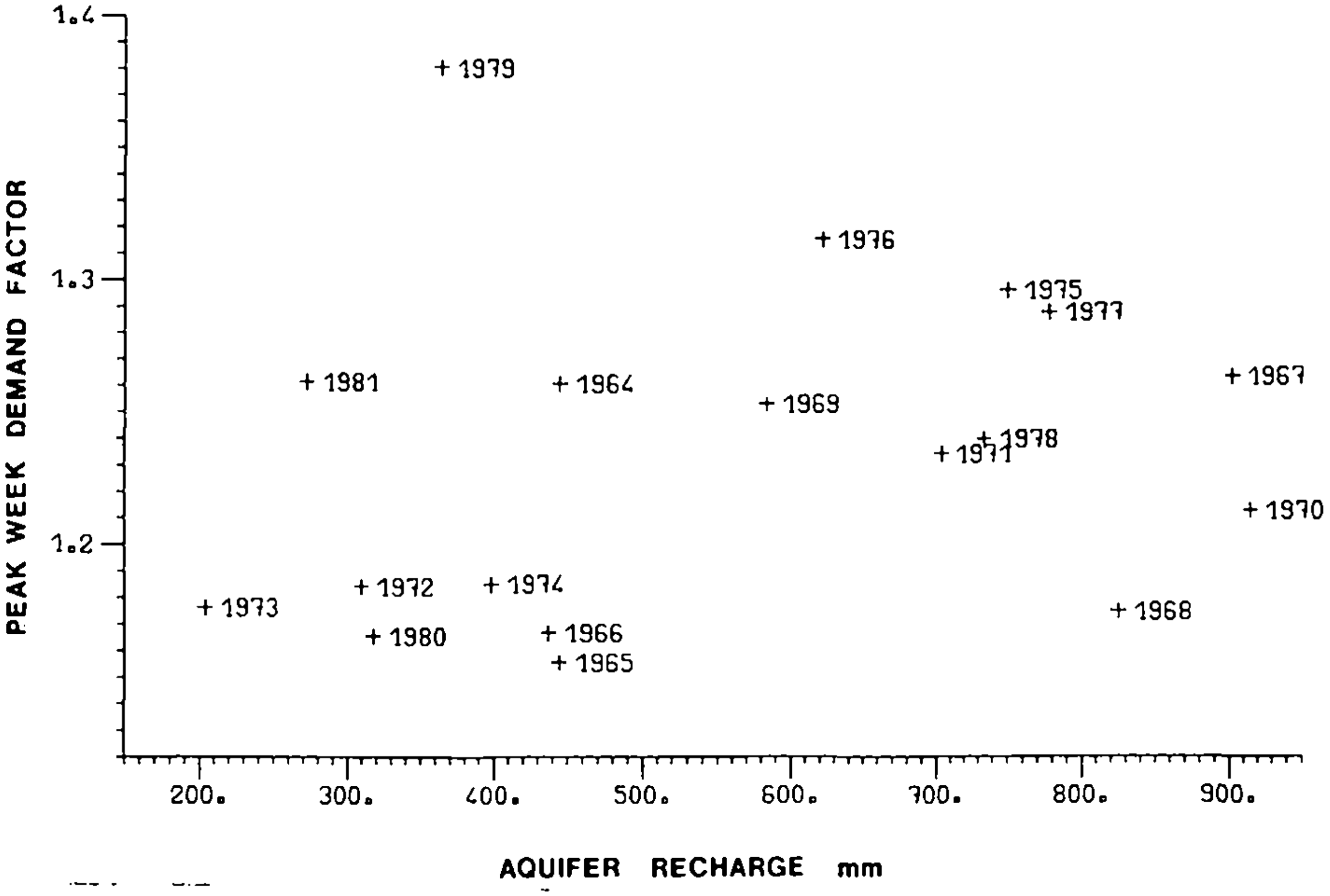


Figure 4 Observed values of demand and recharge for the years 1964-81 in the East Sussex Water and Drainage Division.

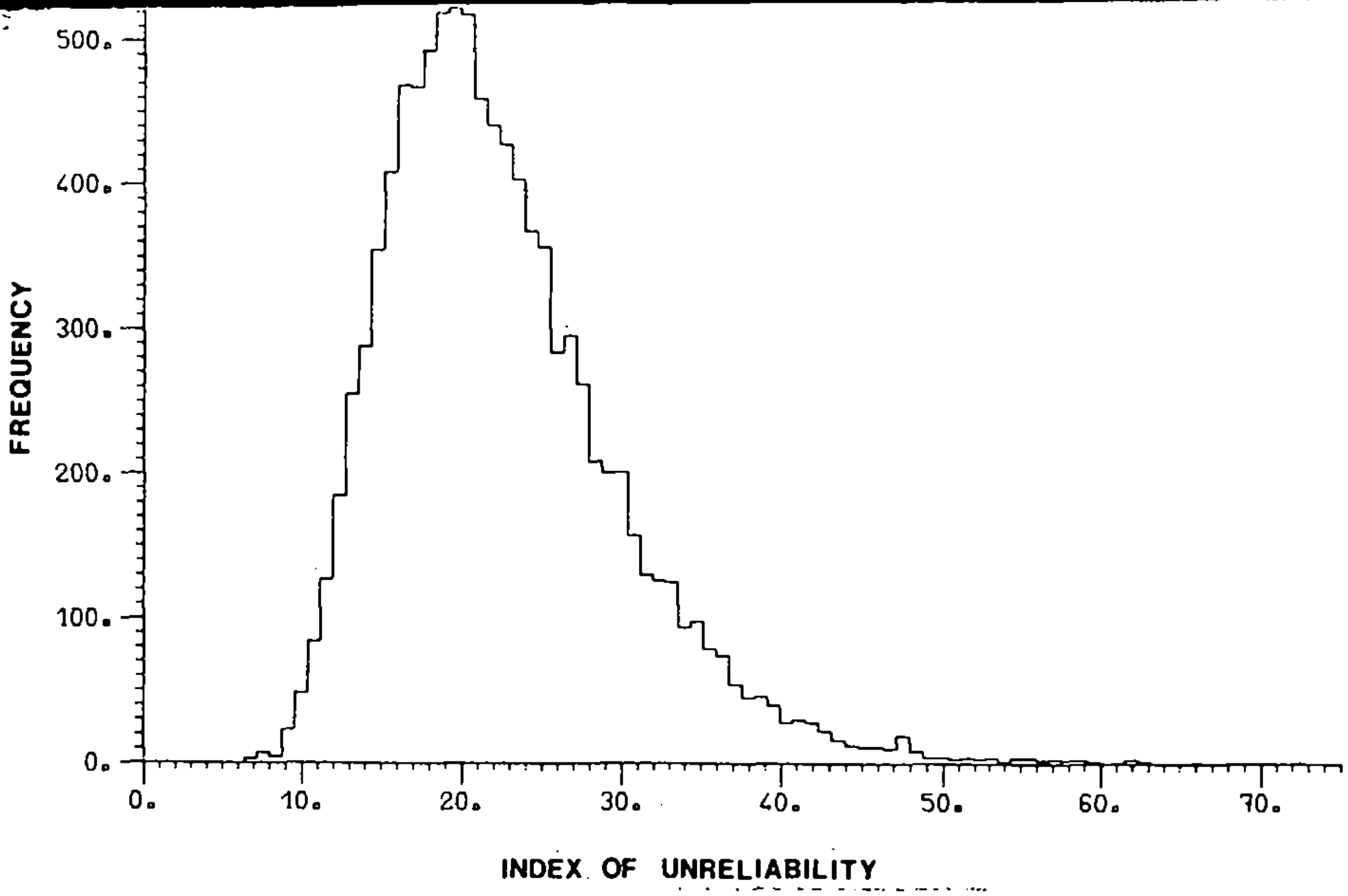


Figure 5 Histogram of 10 000 simulated values of index of unreliability $U = 10^5 \ (D - 1)/(R + 543)$

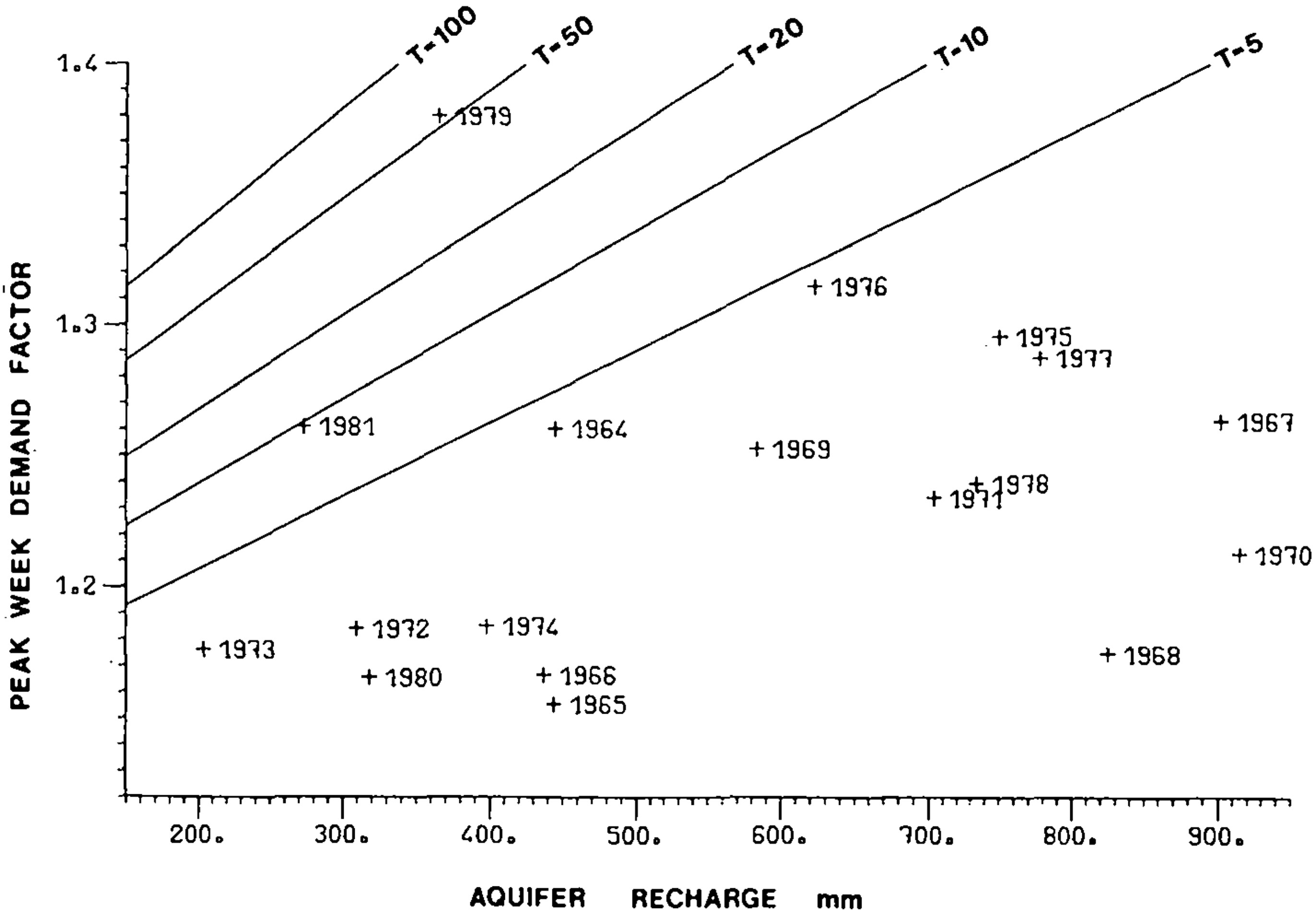


Figure 6 Observed values of demand and recharge, with contours of equal unreliability for various return periods T (in years).

Institute of Hydrology Wallingford Oxfordshire OX10 8BB UK
Telephone Wallingford (STD 0491) 38800 Telegrams Hycycle Wallingford Telex 849365 Hydrol G

The Institute of Hydrology is a component establishment of the Natural Environment Research Council