Inverse analysis of the nonlinear response of laterally loaded pile

Αντίστροφη ανάλυση της μη γραμμικής απόκρισης πασσάλου σϵ ϵγκάρσια φόρτιση

by

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in the
Geotechnical Department
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της

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Abstract

School of Civil Engineering
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Aim of thesis is the calculation of soil parameters, required for designing pile foundation under lateral load, with speed and acceptable geotechnical accuracy, utilizing the respective incremental static in-situ load test.

Towards this achievement, algorithm was designed, which, given the pile's response, calibrates the soil parameters, converging the reproduced response into the given, by inverse analysis. This algorithm is based on the phenomenological Winkler-type constitutive model BWGG, that considers the inelastic behavior of both soil and pile. The model was reformulated with Implicit Finite Difference equations, for the purpose of compatibility with the optimization techniques.

The technical adequacy of the algorithm was verified using as input source, the feedback results. The physical accuracy was validated by the satisfactory convergence of calculated response into the results of finite element analysis, monotonic static load test, with the soil described by the constitutive model Hardening Soil.

Keywords: pile, lateral load, inverse analysis, calibration of soil parameters, BWGG, soil and pile inelasticity, algorithm design

ΕΘΝΙΚΟ ΜΕΤΣΟΒΙΟ ΠΟΛΥΤΕΧΝΕΙΟ

Περίληψη

Σχολή Πολιτικών Μηχανικών
Τομέας Γεωτεχνικής

Αντίστροφη ανάλυση της μη-γραμμικής απόκρισης πασσάλου σε εγκάρσια φόρτιση

της Κωνσταντίνας Σ. Ζολώτα

Στόχο της εργασίας αποτελεί ο υπολογισμός των εδαφικών παράμετρων, απαραίτητων για τον σχεδιασμό θεμελίωσης εγκαρσίως φορτιζομένου πασσάλου, με ταχύτητα και ικανοποιητική γεωτεχνική ακρίβεια, αξιοποιώντας την αντίστοιχη δοκιμαστική επιτόπια στατική επαυξητική φόρτιση.

Για την επίτευξή του, σχεδιάστηκε αλγόριθμος, ο οποίος, δεδομένης της απόκρισης του πασσάλου, βαθμονομεί τις εδαφικές παραμέτρους, συγκλίνοντας την αναπαραγμένη απόκριση στην δεδομένη, με αντίστροφη ανάλυση. Αυτός ο αλγόριθμος βασίζεται στο φαινομενολογικό ελατηριωτό καταστατικό προσομοίωμα BWGG, που θεωρεί ανελαστική τη συμπεριφορά εδάφους και πασσάλου. Το προσομοίωμα επαναδιατυπώθηκε με εξισώσεις Πεπερασμένων Διαφορών Έμμεσης μορφής, για λόγους συμβατότητας με τις τεχνικές βελτιστοποίησης.

Η τεχνική αρτιότητα του αλγορίθμου επαληθεύτηκε χρησιμοποιώντας, ως πηγή δεδομένων, τα αποτελέσματα ανατροφοδότησης. Η φυσική ορθότητά του επικυρώθηκε από την ικανοποιητική σύγκλιση των υπολογισμένων καμπυλών απόκρισης στα αποτελέσματα ανάλυσης πεπερασμένων στοιχείων, μονοτονικής στατικής φόρτισης, με έδαφος που περιγράφεται από το καταστατικό προσομοίωμα Hardening Soil.

Λέξεις Κλειδιά: πάσσαλος, εγκάρσια φόρτιση, αντίστροφη ανάλυση, βαθμονόμηση εδαφικών παράμετρων, BWGG, ανελαστικότητα εδάφους και πασσάλου, σχεδιασμός αλγορίθμου

Acknowledgements

First and foremost, I would like to express my sincere gratitude to my supervisor, *Dr Nikos Gerolymos*, for his continuous support, patience and motivation whilst creative freedom was granted. His knowlegde and multiple-perspective thinking were enlightening, making this cooperation an honor for me.

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Furthermore, I would like to express my appreciation to the National Technical University of Athens's community overall. Without the access in services of Virtual Private Networks(VPN) and Cloud Computing, powered by the Network Management and Computer Center, respectively, this thesis would not have been possible.

Appreciation is ,also, absolute to the open-source community. Their software were extensively used all through this project.

Last but not least, I am indebted to my beloved people: family and friends, for their support throughout writing this thesis and in life generally. I thank, additionally, the youngest members of this group, nieces and nephews, who learn me to take pleasure in simplicity.

Contents

List of Figures

List of Tables

To my goddaughter,
little yet brave
Στρατονίκη

Chapter 1

Introduction

"πάντα ρεῖ, πάντα χωρεῖ καὶ οὐδὲν μένει"
(i.e. there is nothing permanent except change, free translation)
Heraclitus, (4th century)

"ce que nous connaissons est peu de chose; ce que nous ignorons est immense"
(i.e. what we know is little; what we do ignore is immense)
Pierre-Simon Laplace, (1827)

Fluidity and uncertainty in soil behaviour, in the theories that describe it and in the choice of their parameters, are granted in geotechnical engineering. Thus, inverse analysis of in-situ tests, which calibrates soil constitutive models' parameters and verifies their results, is critical.

In this thesis, *aim* is the optimization of the soil's parameters in pile design, so that their use predict the real pile response, through inverse analysis of single-pile, statically icremental, *lateral* load test, considering the nonlinearity of both soil and pile. Towards this achievement, algorithm was designed, based on the phenomenological Winkler-type constitutive model BWGG, reformulated with Implicit Finite Difference equations, and embodied with a, derivative-free and MATLAB codified, local minimizer algorithm.

Thesis *layout* follows, summarizing the content of the chapters and apprendices:

Algorithm Design Algorithm design process and content, along with the asumptions, delimitatins and limitations made, of which the awarenessis is crutial.

Algorithm Evaluation The two phases of evaluation. First, its technical adequacy's verification and, afterwards, its physical accuracy's validation.

Conclusions Summary of work and proposals of furure relatives themes.

Matlab Code The codified algorithm in MATLAB language, along with a brief guide.

Finite Elements 3D analysis The analysis used in validation process. Results' report and figures.

Chapter 2

Algorithm Design

The algorithm design consists of two parts: designing the forward analysis algorithm and designing the analysis inverse algorithm.

The first is the algorithm of solving the classical laterally loaded pile problem: given the lateral load, and estimating the soil reactions, pile's response is computed. The second allows the inverse process: given pile's response, and the lateral load, parameters that govern the soil reactions are computed, based on the forward analysis and optimization technics.

The codes utilize the phenomenological constitutive model BWGG [1–4] , since it allows the nonlinearity of the soil and the inelasticity of the pile to be expressed in a synoptic, clear way (see p. 7). This constitutive model was implemented through implicit finite-difference equations, as described in the first section of the chapter. Crutial is the awareness of the asumptions, delimitatins and limitations made (section 2.3).

2.1 Forward Analysis Algorithm

A laterally loaded single pile is a soil-structure interaction problem, since the soil reaction is dependent on the pile movement, and the pile movement is dependent on the soil reaction. The solution, i.e. the computation of pile's response, must satisfy a nonlinear differential equation as well as equilibrium and compatibility conditions.

In the next paragraphs, a brief background theory on the governal differential equation is presented. Then, n pile's nodes and i iterations are intoduced, dicretizing the pile and the loading in smaller elements and steps, respectively. Afterwards, implicit finite-difference equations [5] describe the governal differential equation and the boundary

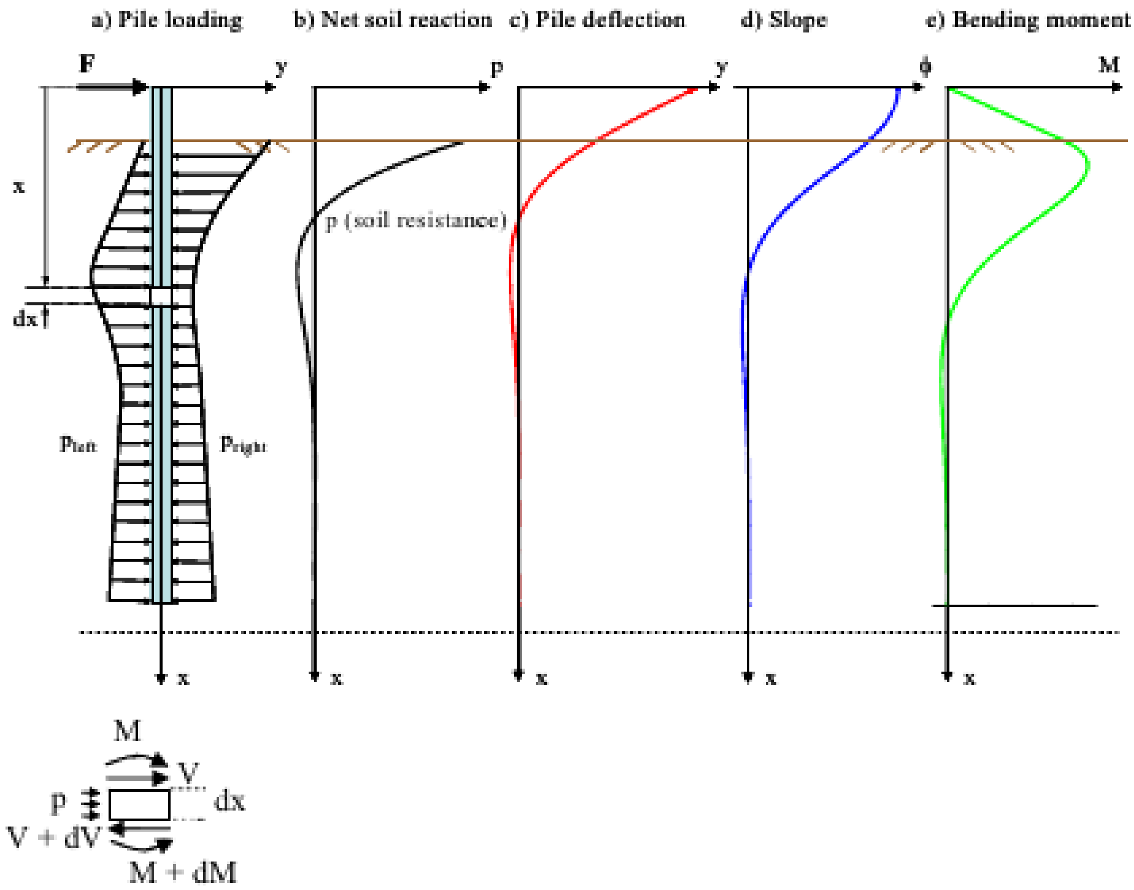


FIGURE 2.1.1: Laterally loaded pile problem

conditions. Nonlinearity of pile and soil is, then, added, using the BWGG model. Finally, all the equations are summerized in a system. The overall process of the algorithm, though, is better understood in the flowchart fig.2.1.5.

The governal equation The differential equation is created from the pile's consideration as an elastic, initially, beam and the soil reaction as the distributed load along the beam, derived by Hetenyi (1946) [6]).

This equation can be obtained by considering moment equilibrium of the infinitesimal element of pile length (dz)(as shown in figure .2.1.1), and by calculating the bending moment M, as integral of the normal stresses, σ_z, acting within the cross section of area A ($M = \int_A \sigma_z y dA$). Its final expression is the eq.2.1.1.

$$\frac{\partial^2}{\partial z^2}\left(EI\frac{\partial^2 y}{\partial z^2}\right) + p = 0 \tag{2.1.1}$$

$$
\begin{cases}
M = EI\frac{\partial^2 y}{\partial z^2} \\
\\
Q = \frac{\partial M}{\partial z} \\
\\
q = -\frac{\partial Q}{\partial z} = \frac{\partial^2 M}{\partial z^2} \\
\\
p = -q
\end{cases}
$$

Dicretizion of pile and loading: n nodes, i iterations

$$p_{n,i} = k_{n,i} y_{n,i} \tag{2.1.2}$$

$$
\begin{aligned}
0 &= \frac{\partial^2}{\partial z^2}\left(EI\frac{\partial^2 y}{\partial z^2}\right) + ky \\
&= \frac{\partial^2 EI}{\partial z^2}\frac{\partial^2 y}{\partial z^2} + EI\left(\frac{\partial^4 y}{\partial z^4}\right) + ky \\
&\approx \left(\frac{EI_{n-1,i} - 2EI_{n,i} + EI_{n+1,i}}{\Delta z^2}\right)\left(\frac{y_{n-1,i} - 2y_{n,i} + y_{n+1,i}}{\Delta z^2}\right) \\
&\quad + EI_{n,i}\left(\frac{y_{n-2,i} - 4y_{n-1,i} + 6y_{n,i} - 4y_{n+1,i} + y_{n+2,i}}{\Delta z^4}\right) + k_{n,i}y_{n,i} \\
\Leftrightarrow 0 &= y_{n-2,i} + y_{n-1,i}(-4 + t_1) + y_{n,i}(6 + \frac{k_{n,i}\Delta z^4}{EI_{n,i}} - 2t_1) \\
&\quad + y_{n+1,i}(-4 + t_1) + y_{n+2,i}
\end{aligned} \tag{2.1.3}
$$

whereas $t_1 = \dfrac{EI_{n-1,i} - 2EI_{n,i} + EI_{n+1,i}}{EI_{n,i}}$

Boundary conditions

Head of Pile $z \equiv z_{Head} = 0 \Leftrightarrow node = 3$[1]

- $Q = P_{lateral\,load}$

$$\begin{aligned} P &= \frac{\partial M}{\partial z} \\ &= \left[\frac{\partial}{\partial z}\left(EI\frac{\partial^2 y}{\partial z^2}\right)\right]_{z=0} \\ &= \left[\frac{\partial EI}{\partial z}\frac{\partial^2 y}{\partial z^2} + EI\left(\frac{\partial^3 y}{\partial z^3}\right)\right]_{z=0} \\ &\approx \left(\frac{EI_{4,i} - EI_{2,i}}{2\Delta z}\right)\left(\frac{y_{2,i} - 2y_{3,i} + y_{4,i}}{\Delta z^2}\right) \\ &\quad + EI_{3,i}\left(\frac{-y_{1,i} + y_{2,i} - y_{4,i} + y_{5,i}}{2\Delta z^3}\right) \\ \Leftrightarrow \frac{P2\Delta z^3}{EI_{3,i}} &= -y_{1,i} + y_{2,i}(2 + t_2) + y_{3,i}(-2t_2) \\ &\quad + y_{4,i}(-2 + t_2) + y_{5,i} \qquad (2.1.4) \\ \text{whereas} \quad t_2 &= \frac{EI_{4,i} - EI_{2,i}}{EI_{3,i}} \end{aligned}$$

- $M = 0$

$$\begin{aligned} 0 &= \left[\left(EI\frac{\partial^2 y}{\partial z^2}\right)\right]_{z=3} \\ \Leftrightarrow 0 &= y_{2,i} - 2y_{3,i} + y_{4,i} \qquad (2.1.5) \end{aligned}$$

Pinpoint of Pile $z \equiv z_{Head} = L \Leftrightarrow node = NumLayers = nn - 2$

- $Q = 0$

$$\begin{aligned} 0 &= \left[\left(EI\frac{\partial^3 y}{\partial z^3}\right)\right]_{z=L} \\ \Leftrightarrow 0 &= -y_{nn-4,i} + 2y_{nn-3,i} - 2y_{nn-1,i} + y_{nn,i} \qquad (2.1.6) \end{aligned}$$

- $M = 0$

$$\begin{aligned} 0 &= \left[\left(EI\frac{\partial^2 y}{\partial z^2}\right)\right]_{z=L} \\ \Leftrightarrow 0 &= y_{nn-3,i} - 2y_{nn-2,i} + y_{nn-1,i} \qquad (2.1.7) \end{aligned}$$

[1] Physical nodes' number = pile's layers' number ($NumLayers$). Whilst, total number of nodes $nn \neq NumLayers$, but $nn = NumLayers + 4$, since 4 plasmatic nodes are created (see last assumption on p.13)

Non-Linearity Using the BWGG model, whose parameters n, a, b, g control the p-y curve (figures 2.1.3, 2.1.2, 2.1.4, [3]). The ζ parameters define the nonlinear response, i.e pile's bending moment $M = \alpha_p EI \frac{\partial^2 y}{\partial z^2} + (1-\alpha_p) M_y \zeta_p$ and soil reaction (per length) $p = \alpha_s k_x y + (1-\alpha_s) p_y \zeta_s$.

Pile κ:curvature

$$\begin{aligned} \kappa &= \frac{\partial^2 y}{\partial z^2} \\ &\approx \frac{y_{n+1,i} - 2y_{n,i} + y_{n-1,i}}{\Delta z^2} \\ d\kappa &= \kappa_{n,i+1} - \kappa_{n,i} \end{aligned}$$

$$\begin{aligned} \frac{d\zeta_p}{d\kappa} &= \frac{1 - |\zeta_p|^{n_p}(b_p + g_p sign(d\kappa\, d\zeta_p)}{\kappa_o} \\ \zeta_{p\,(n,i+1)} &= \zeta_{p\,(n,i)} + d\zeta_p \\ EI_{n,i} &= \alpha_p EI_{o\,(n,i)} + (1-\alpha_p) EI_{o\,(n,i)} \{1 - |\zeta_p|^{n_p}(b_p + g_p sign(d\kappa\, d\zeta_p) \end{aligned} \tag{2.1.8}$$

Soil

$$dy = y_{n,i+1} - y_{n,i}$$

$$\begin{aligned} \frac{d\zeta_s}{dy} &= \frac{1 - |\zeta_s|^{n_s}(b_s + g_s sign(dy\, d\zeta_s)}{y_o} \\ \zeta_{s\,(n,i+1)} &= \zeta_{s\,(n,i)} + d\zeta_s \\ k_{n,i} &= \alpha_s k_{o\,(n,i)} + (1-\alpha_s) k_{o\,(n,i)} \{1 - |\zeta_s|^{n_s}(b_s + g_s sign(dy\, d\zeta_s) \end{aligned} \tag{2.1.9}$$

Lateral pile displacements in all depths The system of equatations is completed, in the following page, whereas:

$$\begin{aligned} kk_{n,i} &= 6 + \frac{k_{n,i}\, 2\Delta z^4}{EI_{n,i}} \\ t_1 &= \frac{EI_{n-1,i} - 2EI_{n,i} + EI_{n+1,i}}{EI_{n,i}} \\ t_2 &= \frac{EI_{4,i} - EI_{2,i}}{EI_{3,i}} \end{aligned}$$

.

$$
\begin{array}{r}
\\
2.1.4 \\ 2.1.5 \\ \\ 2.1.3 \\ \\ 2.1.7 \\ 2.1.6
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccccc}
\scriptstyle 1 & \scriptstyle 2 & \scriptstyle 3 & \scriptstyle 4 & \scriptstyle 5 & \dots & \scriptstyle n-2 & \scriptstyle n-1 & \scriptstyle n & \scriptstyle n+1 & \scriptstyle n+2 & \dots & \scriptstyle nn-4 & \scriptstyle nn-3 & \scriptstyle nn-2 & \scriptstyle nn-1 & \scriptstyle nn
\end{array} \\
\begin{pmatrix}
-1 & 2+t_2 & -2t_2 & -2+t_2 & 1 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -2 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & 1 & -4+t_1 & kk_{n,i} & -4+t_1 & 1 & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & -2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & -1 & 2 & 0 & -2 & 1
\end{pmatrix}
\end{array}
$$

$$
* \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \\ \vdots \\ y_{nn-1} \\ y_{nn} \end{Bmatrix} = \begin{Bmatrix} \frac{P\,2\Delta z^3}{EI_{3,i}} \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \end{Bmatrix}
$$

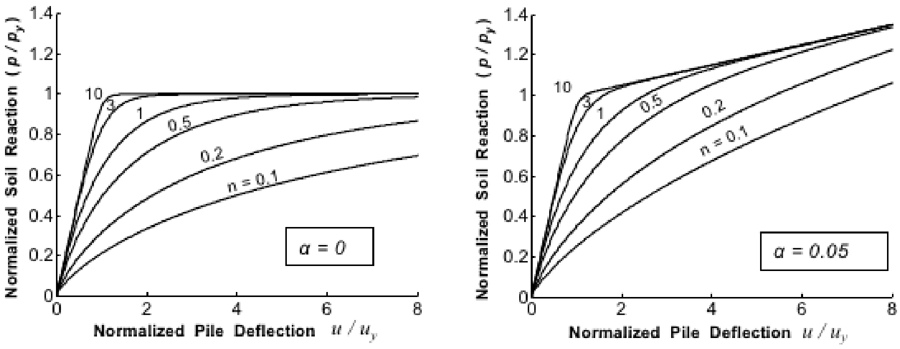


FIGURE 2.1.2: Effect of "n" parameter on the sharpness of the transition from the linear to nolinear range during virgin loading

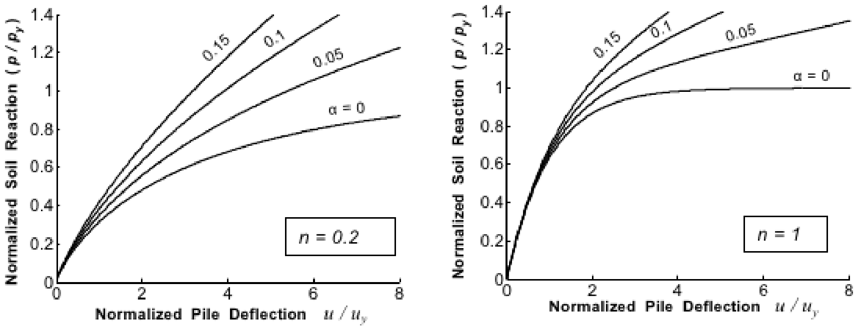


FIGURE 2.1.3: Effect of the "a" parameter, control of the post yielding shear stiffness

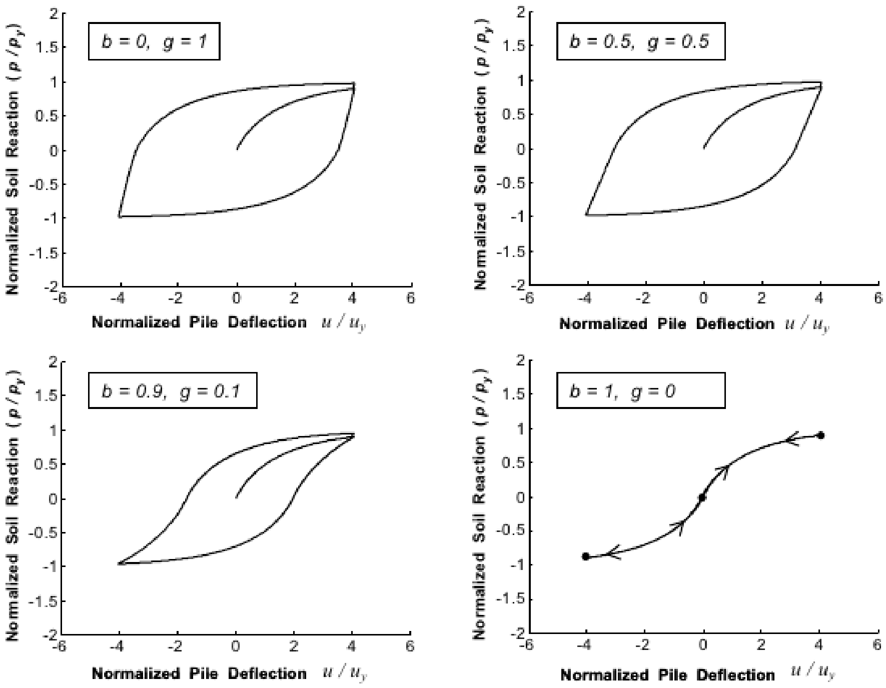


FIGURE 2.1.4: Effect of " b","g" parameters on the unloading-reloading curve shape

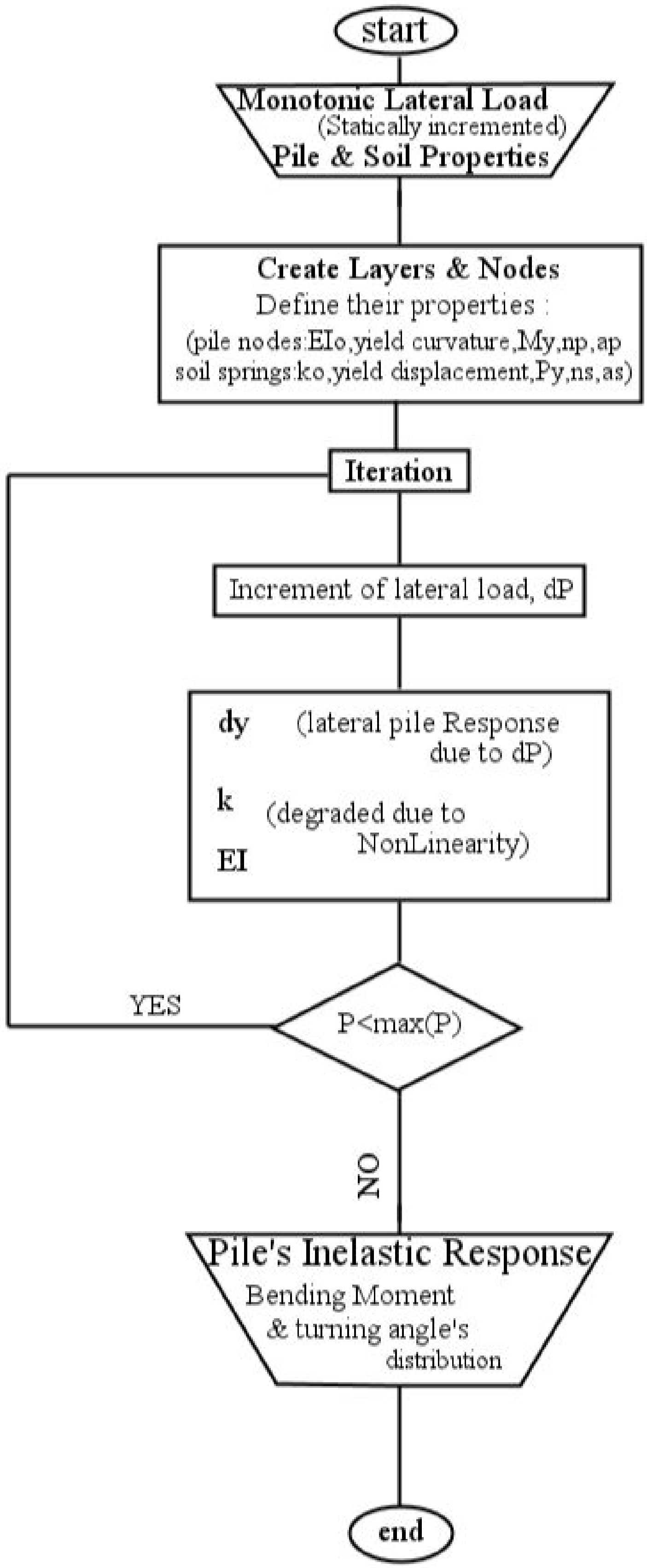


FIGURE 2.1.5: Forward analysis flowchart

2.2 Inverse Analysis Algorithm

In inverse analysis, constitutive parameter values are optimized in the way that the error/ deviation between the recorded response, by insitu measurements, and the computed data is minimized. The process is better understood in the flowchart fig.2.2.2.

Output are the optimized parameters: k_o, the initial, refference, soil spring stiffness, m, the power that determines the way soil stiffness differs in depth and is critical [7] to the pile's response, n, the BWGG parameter (p. 9) and Py, the ultimate soil reaction.

Optimization technics *Unconstrained nonlinear* optimization was used, i.e. finding the minimum of a scalar function (the error) of several variables, starting at an initial estimate, without the user to define upper and lower limits of the variables.

The MATLAB® function "fminsearch" was selected, which is a local minimize and uses the, derivative-free, Nelder-Mead simplex algorithm. This function uses the simplex search method of Lagarias et al. (1998, [8]). This is a direct search method that does not use numerical or analytic gradients.

The way it operates, in summary, is: If n is the length of vector x (n are the parameteres to be optimized), a simplex in n-dimensional space is characterized by the $n+1$ distinct vectors that are its vertices." In two-space, a simplex is a triangle; in three-space, it is a pyramid. At each step of the search, a new point in or near the current simplex is generated. The function value at the new point is compared with the function's values at the vertices of the simplex and, usually, one of the vertices is replaced by the new point, giving a new simplex. This step is repeated until the diameter of the simplex is less than the specified tolerance", fig.2.2.1 [9].

Scaling the variables and the subject function was also nessecary for numerical "equality" [10].

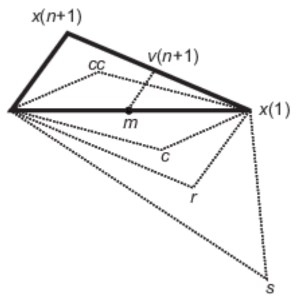


FIGURE 2.2.1: A graphical simplified explanation of the calculations that fminsearch does in the procedure, along with each possible new simplex. The original simplex has a bold outline. The iterations proceed until they meet the stopping criterion.

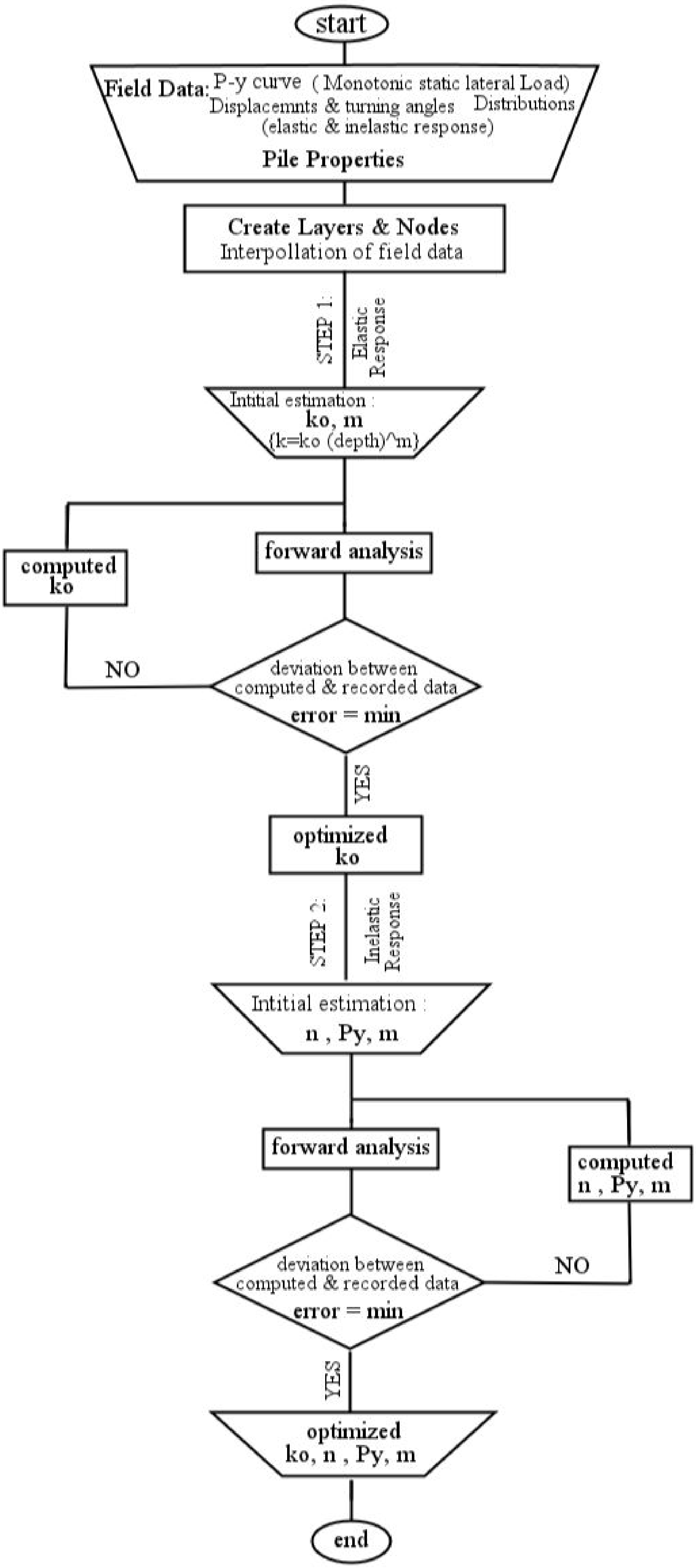


FIGURE 2.2.2: Inverse analysis flowchart

2.3 Assumptions, Delimitations and Limitations

Assumptions

- Lateral, static and monotonic load of single pile.
- Soil drained, single-layered, homogenous but not uniform, modelised by Winkler-type nonlinear springs ($p = ky$, fig. 2.3.1, [6]). The soil spring stiffness (k) is increased in depth (z) in an optimum way ($k(z) = k^{ref} * z^m$, whereas m is part of the output) and is decreased during the loading stages due to nonlinearity (eq. 2.1.9).
- Pile's nonlinearity of the flexural stiffness (eq. 2.1.8).
- Free pile head.
- Four plasmatic nodes are created in analysis because of the finite difference method's approach of the 4th grade governal equation (eq.2.1.1), two above the head of pile and two below its pinpoint, with properties of the nearest physical node.

Delimitations

- Cycling loading is not suported. Thus, load-induced anisotropy, seperation (gapping) of the pile from the soil (figure 2.3.2, [11]) and cyclic strength degration (figure 2.3.3, [1]) are not expressed.
- Dynamic loading is also not supported. Thus, damping (hysteretic and radiation) is not expressed. The reason of excluding cyclic and dynamic analysis is the reduction of the parameters that will be optimized. However, the static analysis excludes the time-domain analysis and, therefore, the explicit expression of finite difference method, leading to the, more complicate, implicit expression in the design.
- The effect of the pile cross-section shape (Reese and Van Impe, 2001, figure 2.3.4, [6]) and the surface roughness are not considered.
- Vertical load was from self-weight and was considered negligible.

Limitations

- Quality of the output depends on the quality of the input: the recorded pile response and the initial estimation of the wanted parameters. The first is affected by the type and methology of meusurement instrumentation and the pile's installation conditions, for field-harvested data, or by the simulation's assumptions, for simulation-harvested data. Due to the local minimizer used in the optimization, for numerical balance and speed, neccessary is a rational intitial estimation, i.e. positive parameters within the range of bibliographical records.

- Because of the spatial sensitivity of the finite difference method, the pile's plastic hinge leads to numerical imbalance, since the part of pile above the hinge deflects, then, independently and unconnected to the part below the hinge. Thus, the lateral load could be big enough to provoke inelastic pile response, but not reach ultimate level.

- Due to the macroscopic approach, stresses, strains and the related microscopic quantities are not computed.

- The continuous nature of the soil is not explicitly modeled, since soil is simulated by a series of nonlinear springs.

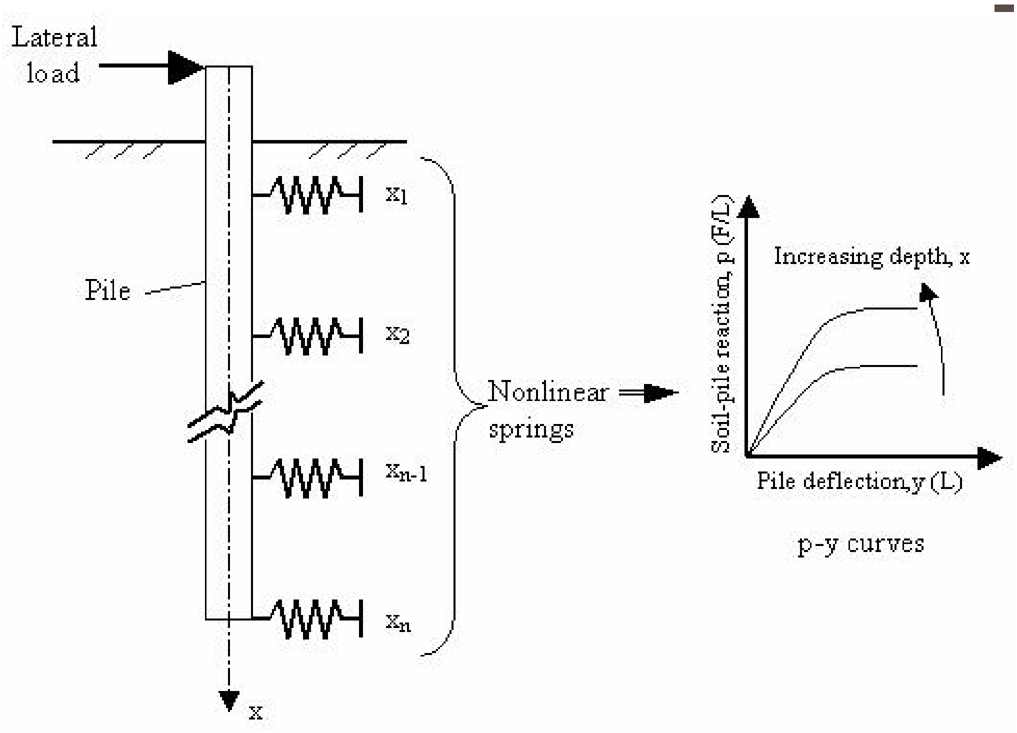


FIGURE 2.3.1: Winkler-type nonlinear soil model

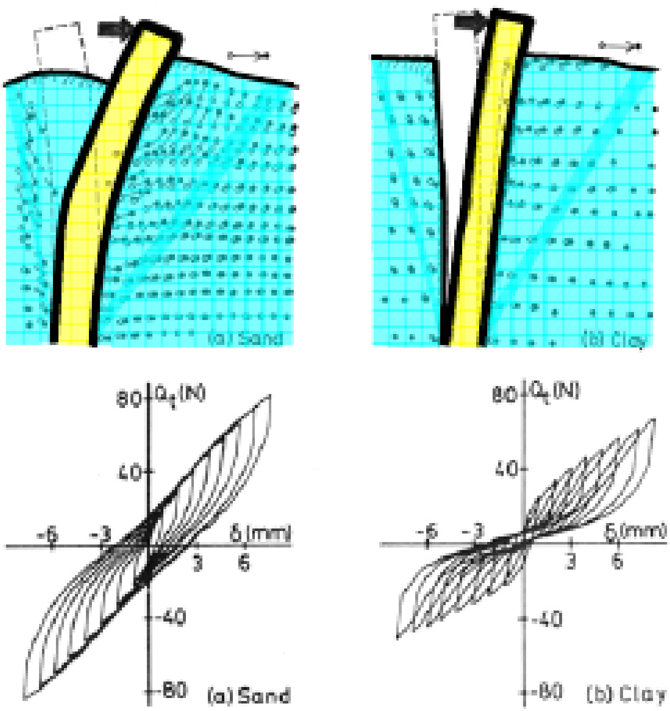


FIGURE 2.3.2: Pile's response to cyclic lateral load in cohesion-less (left) and cohesive (right) soil.

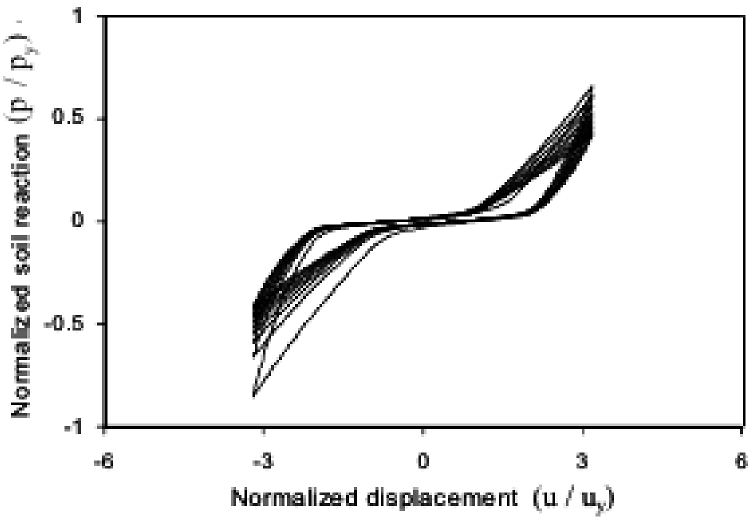


FIGURE 2.3.3: Hysteretic component of a typical soil reaction on a pile in stiff clay with gapping effect and stregth deterioration, computed with the BWGG model

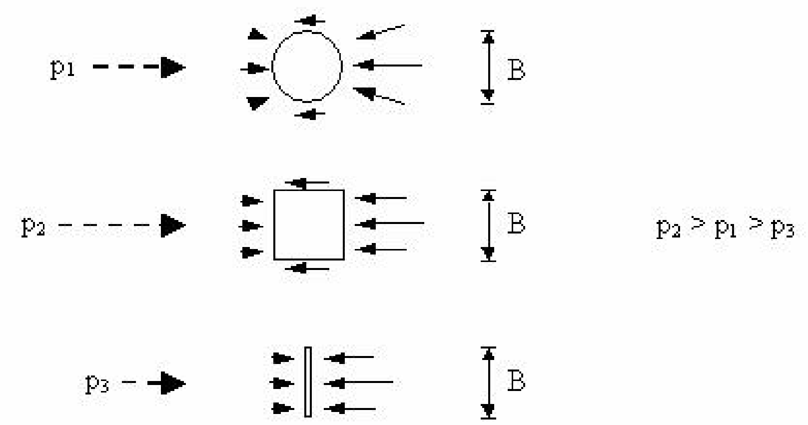


FIGURE 2.3.4: The influence of pile's cross-sectional shape on the soil reaction p

Chapter 3

Algorithm Evaluation

The designed algorithm was evaluated in two phases. First, its technical adequacy was verified and, afterwards, the physical accuracy was validated.

3.1 Verification

Through the verification process, technical adequacy, in terms of performance, stability and velocity, was examined. Time-consuming functions and code's "hot-spots", regions where high proportion of executed instructions occur, were minimized. Simplicity and articulacy in the input demand and the output expression were, also, examined. In the required multiple analyses, pile responses, created with the forward analysis algorithm, were used as data source. Thus, the ability of adjustment in the unknown soil behaviour has not evaluated yet,in this phase.

Such an inverse analysis is presented in the following subsections.

3.1.1 Input data creation

Major input of the inverse analysis is the pile's response to lateral load and soil reaction.

This response, expressed by the curves of load-head displacement $(p - y)$ and depth-displacement $(z - y)$, was created by the forward analysis algorithm implementation, using as input the values of table 3.1.1.

Table 3.1.1: Forward Analysis Input

pile	geometry	$L = 24\ m$ $d = 1\ m$
	strength	$E = 21\ GPa$ $My = 4000\ kNm$ $\alpha_p = 0.001$
soil	$k_o = k_o^{ref} * depth^m$	$k_o^{ref} = 30\ MN/m^2$ $m = 1$ $Py = 400\ kN$ $n = 0.6$ $\alpha_s = 0.001$
load	maximum iterations	$P = 1500\ kN$ 1000

Table 3.1.2: Verification Case Input

pile	geometry	$L = 24\ m$ $d = 1\ m$
	strength	$E = 21\ GPa$ $My = 4000\ kNm$ $\alpha_p = 0.001$
soil	(initial estimation)	$k_o^{ref} = 100\ MN/m^2$ $m = 0.5$ $Py = 500\ kN$ $n = 0.5$
		$\alpha_s = 0.001$
analysis's iterations		1000
pile's response		$p - y$ curve $z - y$ curve

Table 3.1.3: Verifiation Case Output

```
OPTIMIZATION results:
The no.1 Soil Material has n=0.6 .
The no.1 Soil Material has Pyo=395.583 kN.
The no.1 Soil Material has ko=30000 kN/m2.
The no.1 Soil Material has power (kx=ko*depth^power)=1.
Elapsed time is 145.744490 seconds.
```

3.1.2 Inverse analysis

When inverse analysis code was executed, using as input the values of table 3.1.2, the output graphical figures 3.1.2 and 3.1.3 were created, in which blue represent the calculated and red the recorded, given in input, data.

Synopsis of the optimized soil parameteres, was displayed, then, on the Command Window of the MATLAB® environment, as showed in table 3.1.3.

3.1.3 Output assessment

Assessing the results, absolute curve fitting and 98% $\sim$ 100% accuracy in defining the wanted soil parameteres were achieved, as expected by a refeed analysis. Results were expressed in both, user-friendly, text and graphical way. Analysis lasted less than three minutes.

Reffering to time, counter is activated, when OK bottom of the pile's input dialog box (figure 3.1.1) is pressed, and gets deactivated, when optimization and graphs creation are completed. Moreover, the code was runned through the CloudFront service of the NTUA's Computer Center and, thus, its hardware was used, boosting velocity.

On the other hand, despite of the above good technical results, refeed analyses can not test the physical accuracy. Thus, additional analysis, using , even simulation's, experimental results was imperative and is presented in the following section.

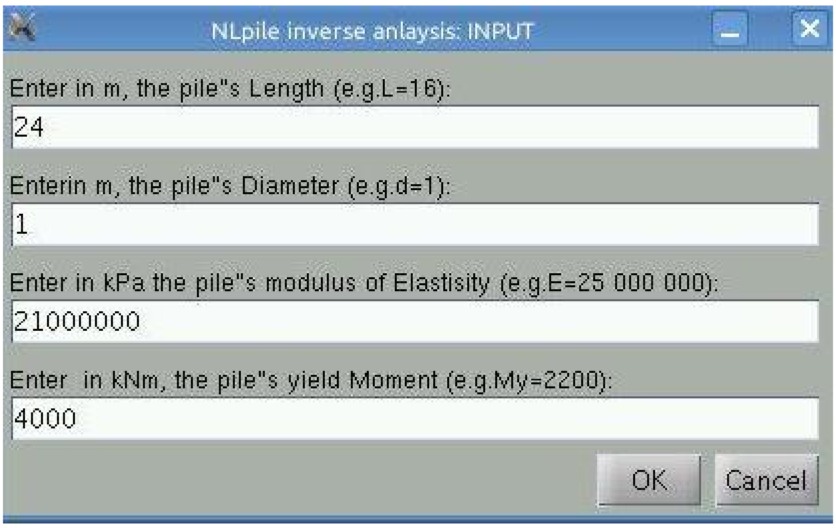


Figure 3.1.1: Verification case: Dialog box gathering input

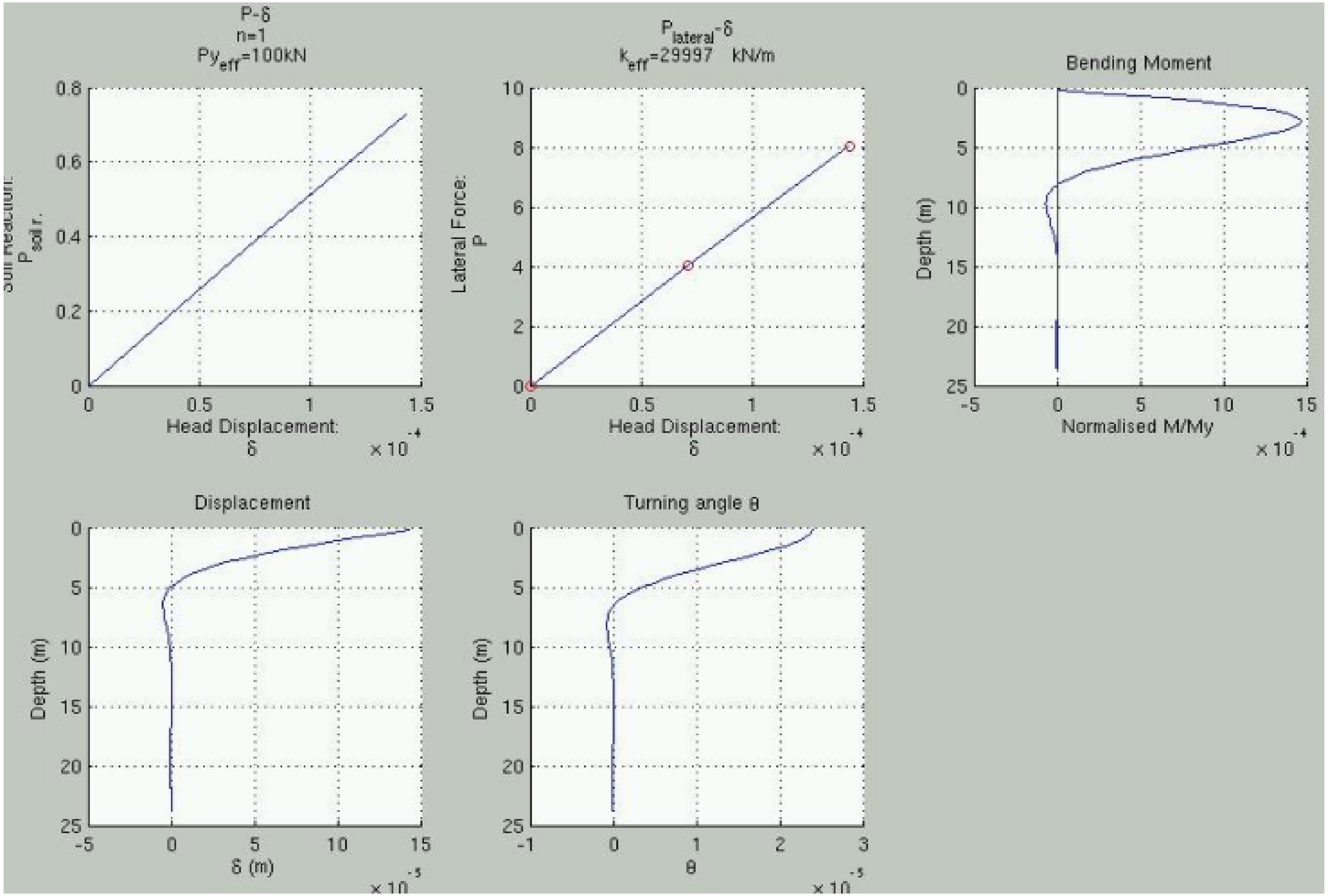


FIGURE 3.1.2: Verification case: optimum $k_{elastic}$

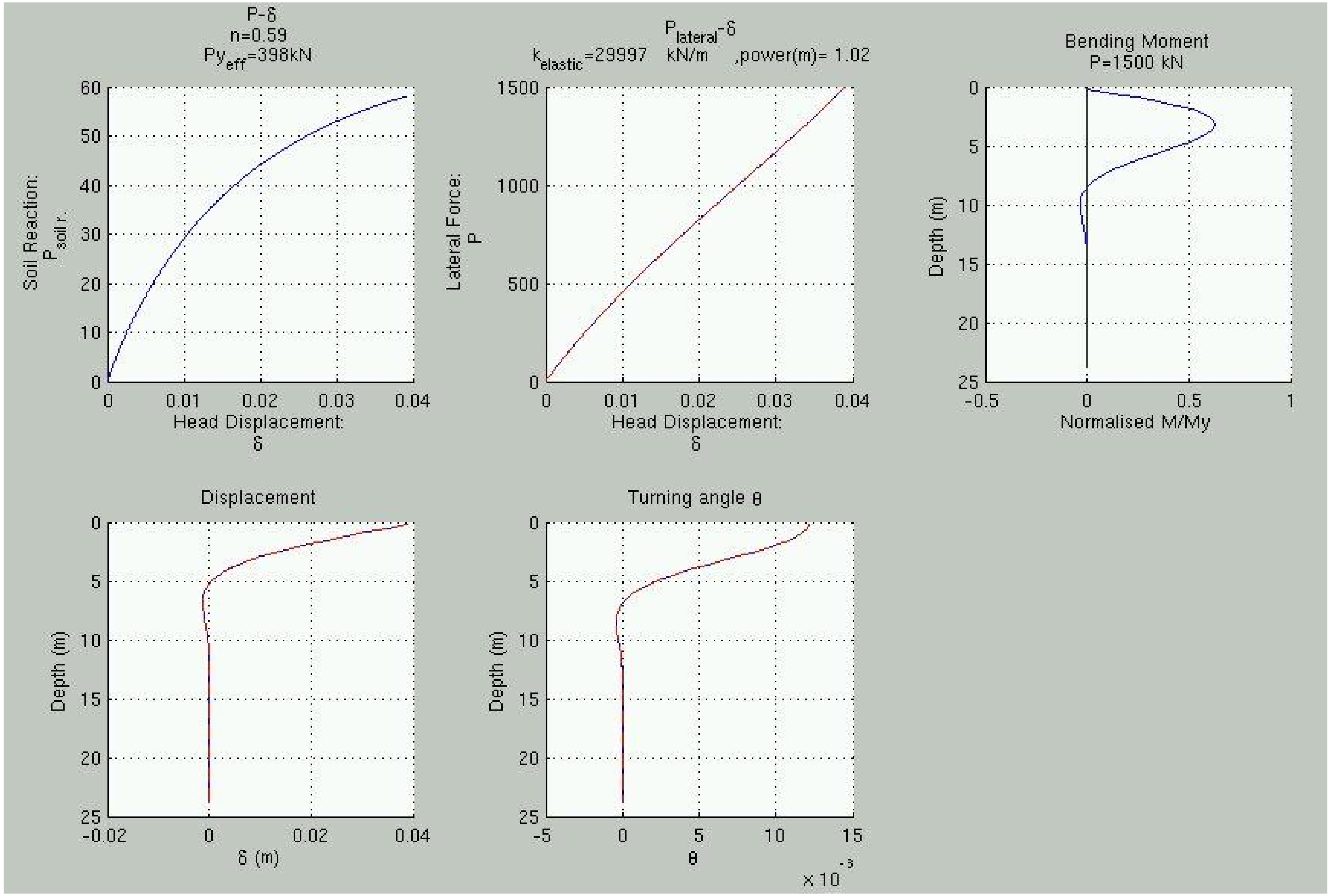


FIGURE 3.1.3: Verification case: optima n, P_y, m

3.2 Validation

Through the validation process, physical accuracy, in macroscopic terms, was examined. This accuracy was measured by the fitting rate of the calculated response into the recorded pile behaviour and by the relationship of the optimized soil parameters to the parameters proposed in empirical methods.

In situ static lateral load test was numerically simulated in Finite Elements Method (FEM) software, PLAXIS 3D©. The simulation, its result's implementation into the inverse algorithm and the output assessemnt are presented in the following subsections.

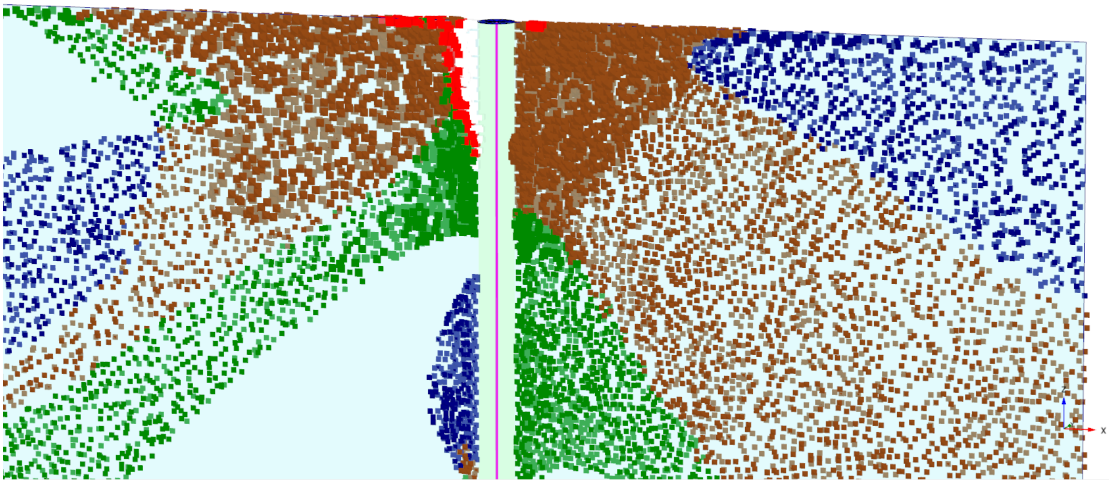
3.2.1 Input data creation

"Recorded" pile behaviour was created by numerical simulation of the, lateral load, pile test.

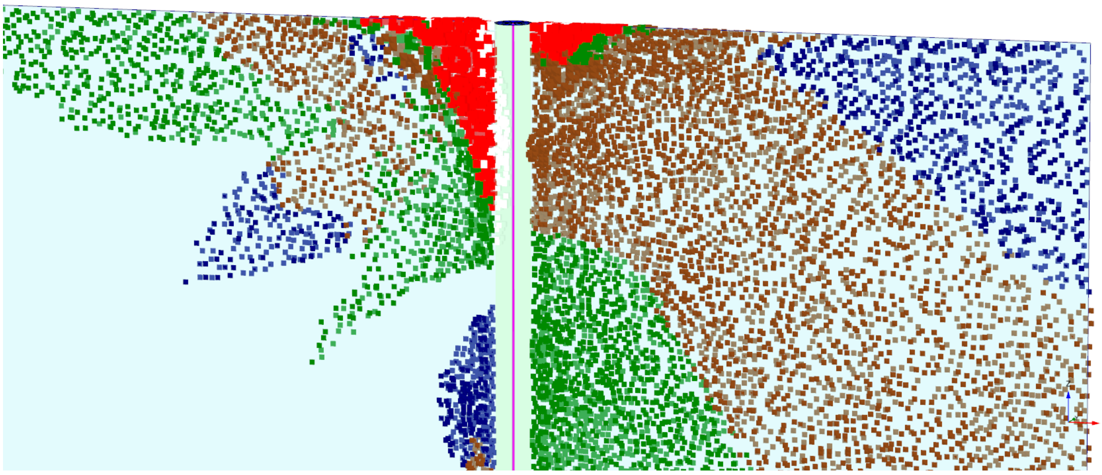
Four materials were defined, fig.3.2.2, detailly reported in Apprendix **??**. The *Soil* material was following the Hardening Soil constitutive model [12]. The reinforced concrete *Pile* was simulated by soil-interface Mohr-Coulomb elements. The *Head* of the pile was simulated by plate material and the *Inclinometer* in pile axis by beam elements, which modulus of elasticity is the $\frac{1}{10000}$ of the pile's modulus, so as not to affect the pile's deflection.

In particular for the reinforced concrete pile, concrete was assumed to have characteristic strength (f_{ck}) 30 MPa, the steel strength 500 MPa ($S500s$ or $StIV$) and the steel bar section area percentage (ρ) to be 1.5%. Proposed values, which determine failure envelope in the M-N space, are the cohesion (c) 15262 kPa and the tensible strength (σ_t) 7534 kPa [13]. From this envelope, fig.3.2.4, without axial force ($N = 0\,MN$), the yield moment (M_y) of the pile is 2200 kNm.

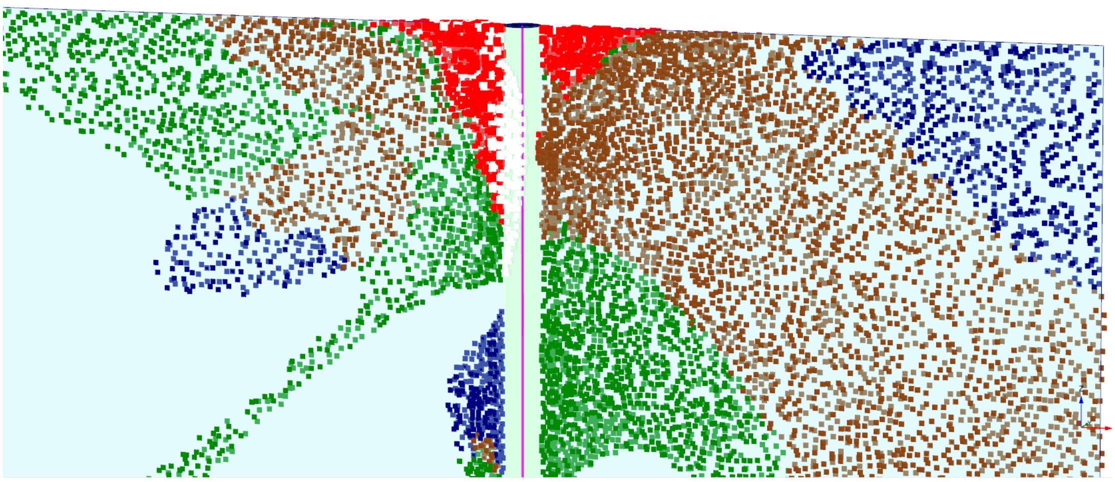
Brom's method confirmation An interesting notice over the figure 3.2.1, which showes the soil's plastic points created during the loading stages, is that the prolongation of line that delimits the plastified soil, in the side of the passive pressure, leads to the plastic hinge of pile. This is a confirmation of Broms' method (1964) over the single, long, free-head pile in cohesionless soil. According to which, the ultimate soil resistance develops from the ground surface to the point of plastic hinge, till the depth f [14]. Moreover, at this point the shear is zero and the bending moment is maximum . Thus, depth f is computed by solving the system of the moments balance and the horizontal forces balance at the hinge depth, i.e. $Hf = M_y + \frac{f}{3}\frac{3d\sigma'_\nu k_p\, f}{2}$, $H = \frac{3d\sigma'_\nu k_p\, f}{2}$ (fig. 3.2.3b, [15]). In this case, $f = 3,2\,m$ (Broms' method), close to the $f = 3,6\,m$ (3D FEM).



(A) $\Sigma M_{stage} = 0.518$



(B) $\Sigma M_{stage} = 0.726$



(C) $\Sigma M_{stage} = 1$, (full load gets active)

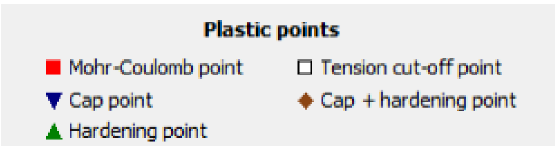
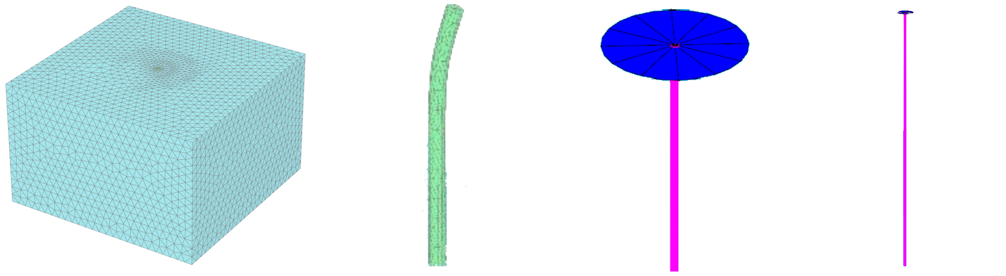
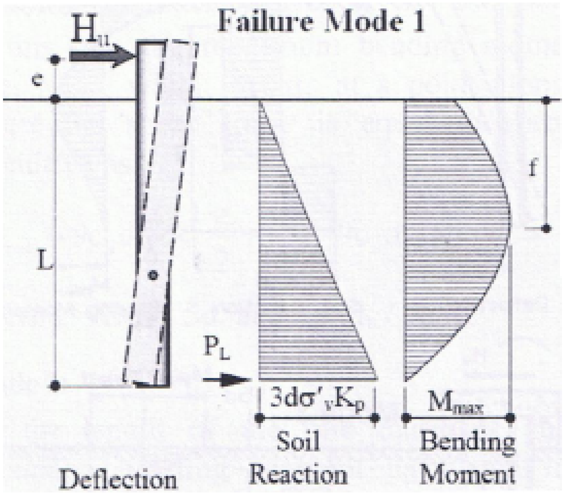


FIGURE 3.2.1: 3D Finite Element Analysis: Plastic points

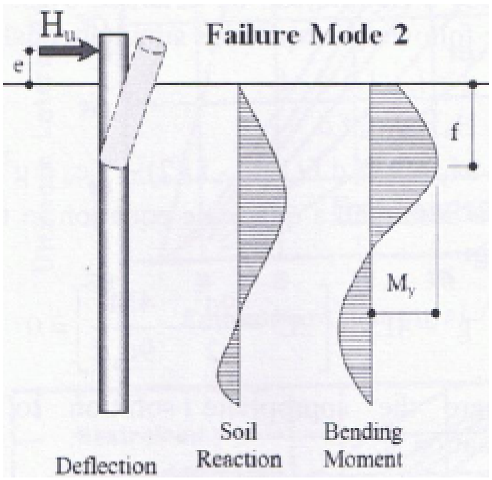


(A) Soil (B) Pile (C) Head plate (D) Inclinometer

FIGURE 3.2.2: 3D Finite Element Analysis: Material elements



(A)



(B)

FIGURE 3.2.3: Broms method for analysis of single, unrestrained against rotation, piles in cohesionless soil, whereas $\sigma'_\nu = \gamma' z$ is the active vertical stress and $K_p = tan^2(45 + \frac{\phi}{2})$ is the coefficient of passive earth pressure, (Rankine 1857)

Table 3.2.1: 3D Finite Elements Analysis Input

pile	geometry	$L = 16\ m$ $d = 1\ m$
	strength	$E = 25\ GPa$ $c = 15262\ kPa$ $\sigma_t = 7534\ kPa$
soil		drained $\gamma = 20\ kN/m^3$ $\phi = 32°$
	HS model parameters	$E_{50}^{ref} = 50\,000\ kN/m^2$ $m = 0.5$
head	plate	$E = 25\ GPa$
inclinometer	beam	$E = 2,5\ MPa$

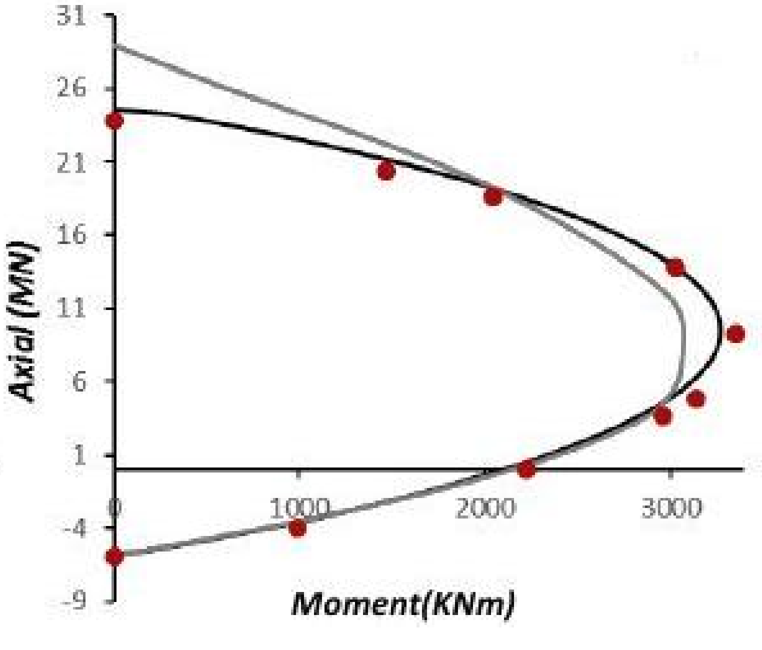


Figure 3.2.4: M-N failure envelope of the reinforced concrete pile

3.2.2 Inverse analysis

When inverse analysis code was executed, using as input the values of table 3.2.3, the output graphical figures, fig.3.2.6 and fig.3.2.7, were created, in which blue and continues lines (i.e. —) represent the calculated data, whilst the circular red points (i.e. ∘) and the broken (i.e. −−) lines represent the recorded, given as input, data.

Synopsis of the optimized soil parameteres, was displayed, then, on the Command Window of the MATLAB® environment, as showed in table 3.2.2.

3.2.3 Output assessment

The physical rightness, in terms of the fitting rate of the calculated response into the recorded pile behaviour, is adequately confirmed in fig.3.2.7.

A numerical assessement could be only indirect, in contrast to the verification case, since the "correct" soil parameters are not distinctly predictable. To begin with, the soil spring's initial stiffness k_o is within the, bibliographically proposed, range for cohessionless materials, and specifically for, relatively dense, sand ([16], combined with the Makris & Gazetas, 1992, relationship $k_s = 1.2\,E_s$). The exponential power $m = 0.5$, which defines the distribution of k with depth, seems logical for cohesionless material [7]. The $n = 0.75$ for the parameter, that governs the sharpness of transition from the linear to nonlinear range during initial loading, intuitively only, seems logical. Last but not least, the ultimate, reference (when depth $z = 1\,m$), soil reaction $P_{yo} = 127\,kN$, is also within the bibliographical range [11], since $127 = 1.95\ tan^2(45 + \frac{32}{2})20$, using Broms (1964) expressions.

TABLE 3.2.2: Validation Case Output

```
OPTIMIZATION results:
The no.1 Soil Material has n=0.75 .
The no.1 Soil Material has Pyo=127.188 kN.
The no.1 Soil Material has ko=53938 kN/m2.
The no.1 Soil Material has power(kx=ko*depth^power)=0.5 .
Elapsed time is 467.561652 seconds.
```

TABLE 3.2.3: Validation Case Input

pile	geometry	$L = 16\ m$ $d = 1\ m$
	strength	$E = 25\ GPa$ $My = 2200\ kNm$ $\alpha_p = 0.001$
soil	(initial estimation)	$k_o^{ref} = 100\ MN/m^2$ $m = 0.5$ $Py = 500\ kN$ $n = 0.5$
		$\alpha_s = 0.001$
pile's response	(Apprendix B.1.1) (Apprendix B.1.2)	$p - y$ curve $z - y$ curves

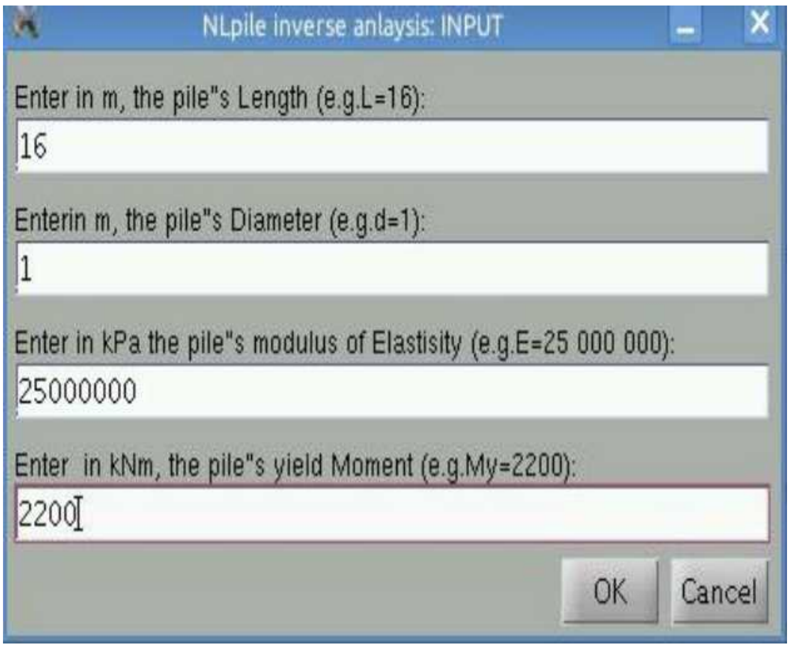


FIGURE 3.2.5: Validation case: Dialog box gathering input

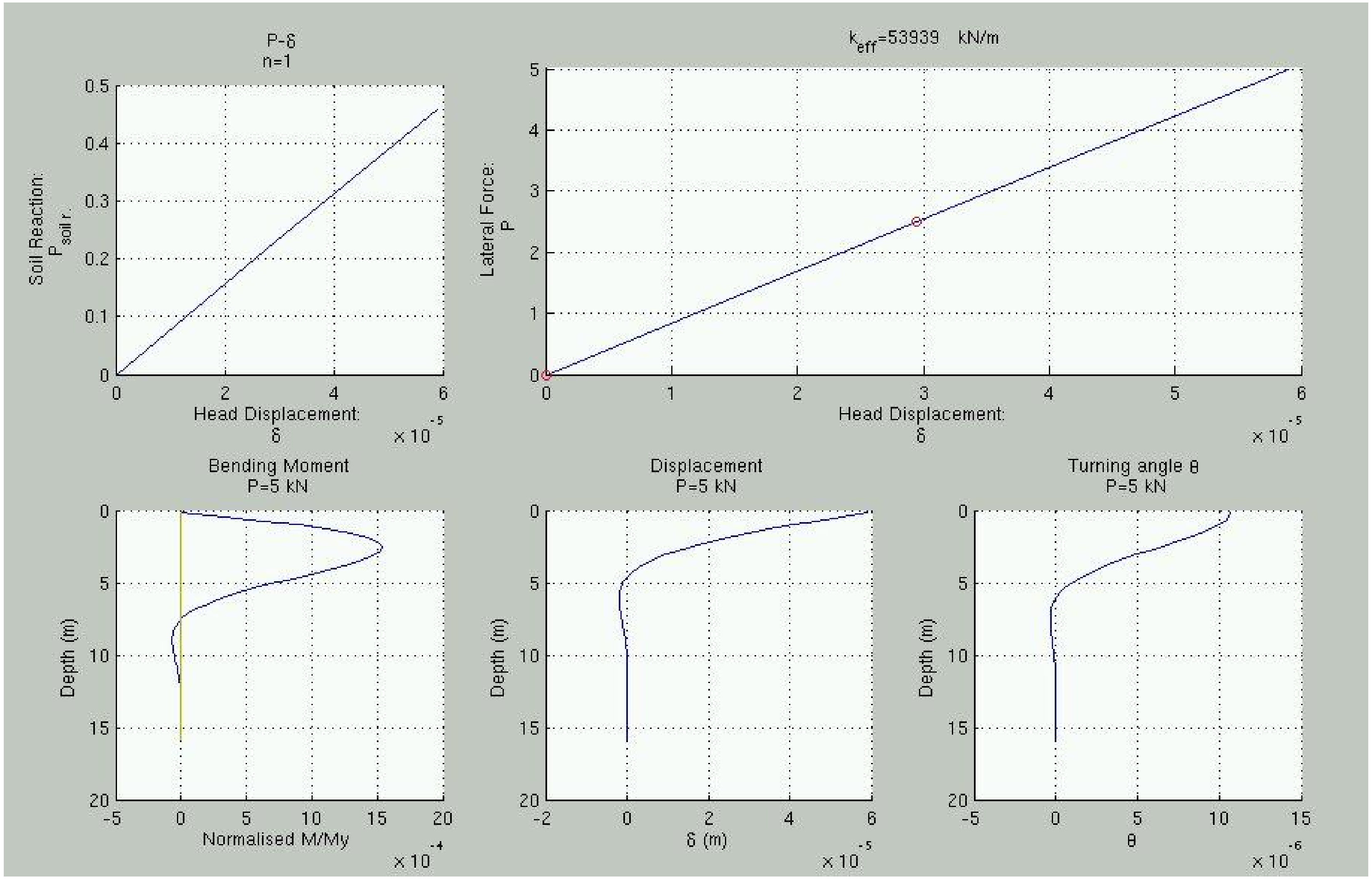


FIGURE 3.2.6: Validation case: optimum $k_{elastic}$

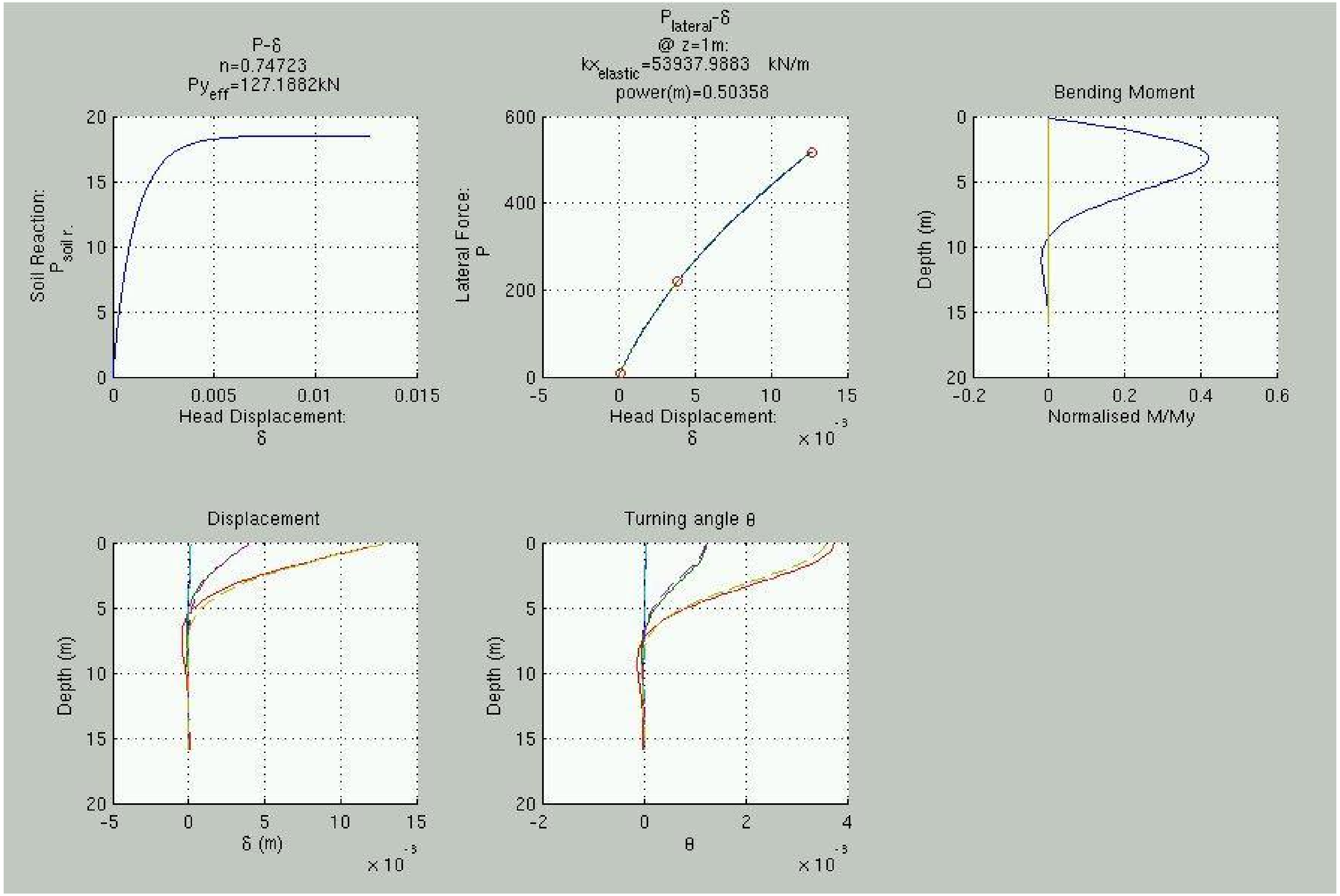


FIGURE 3.2.7: Validation case: optima n, P_y, m

Chapter 4

Conclusions

The calculation of soil parameters, required for designing pile foundation under lateral load - and specifically of the soil spring's initial stiffness k_o and the exponential power m, that defines the stiffness' distribution with depth, of the parameter n that governs the sharpness of transition from the linear to nonlinear range during initial loading and of the ultimate soil reaction P_{yo} - with speed and acceptable geotechnical accuracy, utilizing the respective incremental static in-situ load test, was achieved, within delimitations. The need of further analyses and more cases, is unquestionable, before more conclusions are made.

Future Work Working with the same algorithm, more cases, using in-situ archived data or scaled experiments, could be tested, to determine the full range of its capability, as well as its boundaries. Otherwise, the elimination of the assumptions, deliminations and delimitations of this thesis, e.g. (re)designing an inverse algorithm that support multi-layered soil deposit and dynamic and cyclic lateral load, could be a productive study field.

Appendix A

Inverse Analysis Algorithm: Matlab Code

The Inverse Analysis Algorithm implemented into MATLAB® environment, version 8.0 (Release 2012b). The code consists of three *.m* files: `main1PL.m`, `fun1PL.m`, `fun2PL.m`. The first one, the script file, should be runned. Its execution cause the other two, function, files to be called.

A.1 Guidance on input format

As far as input is concerned, the following points should be cosidered:

- Pile's geometry, ultimate Bending Moment (M_y) and modulus of Elasticity, are gathering from the qraphical dialog box created by promt commands , see figure 3.1.1 on p.19.
- Pile's recorded responses are gathering in a matrix saved as a *.txt* script (see the `load_dispPLAXIS.txt`, B.1.1 on p.53). The script's name should be manually written in the functions *.m* files, `fun1PL.m` and `fun2PL.m`, completing the load command of lines 18 and 16, respectively. Matrix format (whereas nL and nR are the number of pile's layers and the response's records, respectively) is:

$$\begin{bmatrix} y_{1,1} & y_{1,2} & y_{1,3} & \cdots & y_{1,nR} \\ y_{2,1} & y_{2,2} & y_{2,3} & \cdots & y_{2,nR} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{nL,1} & y_{nL,2} & y_{nL,3} & \cdots & y_{nL,nR} \end{bmatrix}$$

- Pile's $p - y$ curve is gathering in a matrix, saved as a *.txt* script, (see the `load_headDispPLAXIS.txt`, B.1.2 on p.54). The script's name should be manually written in the functions $.m$ files, `fun1PL.m` and `fun2PL.m`, completing the load command of lines 17 and 15, respectively. Matrix format (whereas nR is the number of the records,) is:

$$\begin{bmatrix} P_1 & y_{head,1} \\ P_2 & y_{head,2} \\ \vdots & \vdots \\ P_{nR} & y_{head,nR} \end{bmatrix}$$

- *Default* values are given for the BWGG parameters a_s and a_p equal to 1‱(see 2.1.3 on p.9) and the number of iterations used in the forward analysis equal to 1000. These values can be altered changing the relative commands of the $m.$ functions. Moreover, default initial estimation is equal to $k_o^{ref} = 100\ MN/m^2$, $m = 0.5$, $Py = 500\ kN$ and $n = 0.5$, which define the start point for the optimization's functions, and optimization's options are, also, set as: toleternce for accuracy equal to ± 1‰and maximum number of function evaluations equal to 5000. These values can be altered changing the relative commands of the `main1PL.m`.

A.2 Script: main1PL.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%            NLPile , MAIN EDITOR            %%
%% Version:1PL {1 soil LAYER (4parameters)} %%
%%    optimum: ko & x=[n*10,Pyo/100,m*10]    %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc
%% Input: geometry & strength of  PILE
prompt={'Enter in m, the pile"s Length (e.g.L=16):', ...
'Enter in m, the pile"s Diameter (e.g.d=1):'...
'Enter in kPa the pile"s modulus of Elastisity (e.g.E=25 000
    000):',...
'Enter  in kNm, the pile"s yield Moment (e.g.My=2200):'} ;
title='NLpile inverse anlaysis: INPUT ';
answer=inputdlg(prompt,title);
%
tic % time is on
profile on
%
L = str2num(answer{1});
d= str2num(answer{2});
Epp=str2num(answer{3});
My=str2num(answer{4});
%
App=pi()*d^2/4;
Ipp=(pi()*d^4)/64;
pile=[L;Epp;App;Ipp;My;1];
%
%% optimization for Ko
version=1;
[x] = fminsearch(@(x) fun1PL(x,pile),10 );
ko=[0,10000*x];
%
%% optimization for n,py,power
% [kx=(depth^power) *Ko]
version=2;
options=optimset('TolX',0.01, 'MaxFunEvals',5000,'PlotFcns'
    ,@optimplotx);
```

```
[x]= fminsearch(@(x) fun2PL(x,pile,ko,version),[5,5,5],
    options);
%
%% create graphics
version=3;
f=fun2PL(x,pile,ko,version);
%
%% output in text format on the Command Window
fprintf('\n----------------------');
fprintf('\n OPTIMIZATION results:');
fprintf('\n The no.%d Soil Material has n = %.2f .',1,x(1)
    /10) ;
fprintf('\n The no.%d Soil Material has Pyo = %.f kN.',1,x
    (2)*100);
fprintf('\n The no.%d Soil Material has ko = %.f kN/m2.',1,
    ko(2));
fprintf('\n The no.%d Soil Material has power (kx=ko*depth^
    power) = %.2f.\n\n',1,x(3)/10);
%
toc % time out
profile viewer
```

A.3 Function: fun1PL.m

```
  function f=fun1PL(x,pile)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%       about fun1PL(x,pile):                 %
% Goal: Compute optimum elastic Ko,ref        %
%                                             %
% Input: x=ko*/10000, ko*=initial estimation, %
%   [pile]=[ L ; Epp ; App ; Ipp ; My ;1],    %
% as created from main1PL.m ,                 %
% and  P-y & pile's respnses (.txt) files,    %
%  which 1st respose is elastic               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% INPUT
%
%  Load's input : LATERAL load at
%  the head of the pile
load_headDisp=load('load_headDispPLAXIS.txt');
load_disp=load('load_dispPLAXIS.txt');
NumLayers=size(load_disp,1)-1;
 ascale=1;  % scale factor=ascale
P=load_headDisp(:,1);
dispHeadField=load_headDisp(:,2);
%
% Interpolation  of P-y values
dispHeadField_elastic=dispHeadField(1:2);
dispHeadField=interp1([0 1],dispHeadField_elastic
   ,[0,0.25,0.5,0.75,1]);
Pelastic=P(1:2); P=interp1([0 1],Pelastic
   ,[0,0.25,0.5,0.75,1]);
mm=3; %number of points used in  analysis
Pmax=P(mm);step=Pmax/(mm-1) ;
%
% Materials' input:
materials=[1,   0.5,   0.001,  0,      2,      0
        2,      0.5,   0.001,  10000*x(1), 1, 100];
NumMaterials=size(materials,1);
%
```

```
%  define  materials' properties
bl=repmat(0.00001,[NumMaterials,1]);mat_n=bl;nl=bl;a=bl;ko=
   bl;Pyo=bl;power=bl;
for  i=1:NumMaterials,
 mat_n(i)=materials(i,1);  bl(i)=materials(i,2); a(i)=
   materials(i,3);
  nl(i)=materials(i,5); ko(i)=materials(i,4); Pyo(i)=
   materials(i,6);
end
%
% define x
 for i=2:NumMaterials,
     x(i-1)=ko(i)/10000;
 end
%
% Pile's input
L=pile(1);
  nL=NumLayers;
  pile=[L/nL;pile(2:end)]';
  pile= [(1:nL)',repmat(pile,[nL,1])]; % ( NumLayers x 7)
  nn=nL+4; % plus 4 pseudo-nodes;
%
% [Pile]= extended matrix of [pile], (nn x 7)
%  (IMAGINARY nodes INCLUDED)
% {#3=head of pile, #(nn-2)=pinpoint of pile=deeper point},
Pile=zeros(nn,size(pile,2));
for i=1:2, Pile(i,:)=pile(1,:);
end
for i=nn-1:nn, Pile(i,:)=pile(nn-4,:);
end
Pile(3:nn-2,:)=pile;
%
h=zeros(nn,1); Epp=h; App=h;Ipp=h;My=h;
depth=h;lay_matp=ones(nn,1); lay_n=h; wyp=h;
for i=1:nn,
   lay_n(i)=Pile(i,1); h(i)=Pile(i,2); Epp(i)=Pile(i,3);
    App(i)=Pile(i,4); Ipp(i)=Pile(i,5);
     My(i)=Pile(i,6);lay_matp(i)=Pile(i,7);
    wyp(i)=My(i)/(Epp(i)*Ipp(i));depth(i+1)=depth(i)+h(i);
```

```
end
depth=abs(depth-h(1)-h(2)/2); depth(1)=0.01;
 EI=Epp.*Ipp;
 EIo=EI;
%
% SOIL profile
% soil=[layer #,soil material,OCR], (NumLayers x 3)
soil=[1:nL;repmat(2,1,nL);ones(1,round(nL/20)),repmat(1,1,nL
    -round(nL/20))]';
Soil_Input=size(soil,2);
Soil=zeros(nn,Soil_Input);
%
% [Soil]= extended matrix of [soil]
% (IMAGINARY nodes INCLUDED), (nn x 3)
for i=1:2, Soil(i,:)=soil(1,:);
end
for i=nn-1:nn, Soil(i,:)=soil(nn-4,:);
end
Soil(3:nn-2,:)=soil;
%
kx=zeros(nn,1);lay_mats=kx; py=kx; % define size
for i=1:nn
lay_mats(i)=Soil(i,2);  OCR=Soil(i,3); ims=lay_mats(i);
         if OCR ==1,
        kx(i)=ko(ims)*(depth(i)); %^power(ims), searching
    only the kref;
        py(i)=Pyo(ims)*depth(i) ;
        else  kx(i)=kx(i-1); py(i)=py(i-1); % if OCR>1;
         end
end
wys=py./kx;
kxo=kx;
%
% materials properties of each  layer
bp=zeros(nn,1);bs=bp;gp=bp;gs=bp;
np=bp;Ap=bp;As=bp;ap=bp;as=bp;cm=bp;ns=bp;
%
for i=1:nn
im=lay_matp(i); ims=lay_mats(i);
```

```
bp(i)=bl(im); bs(i)=bl(ims);
gp(i)=1-bl(im); gs(i)=1-bl(ims);
 np(i)=nl(im); ns(i)=nl(ims);
ap(i)=a(im); as(i)=a(ims);
end
%
%% w(node,repetition)=w(i,j): DISPLACEMENTS of each node of the PILE
%
w=zeros(nn,mm); curv=w; theta=w; Mom=w;
Shear=w; EIsaved=w; kxsaved=w;
KK=zeros(nn,nn); F=zeros(nn,1); dw=F;
 d2EIsaved=w; zetapSaved=w;
zetasSaved=w;zetas=F;dzetas=F;
 zetap=F;dzetap=F; Soil_Reaction=zeros(nn,mm);
Ksaved=ones(nn,mm);EIsaved(:,1)=EI;go=1;
%
    for j=1:mm-1
     % BOUNDARY CONDITIONS
        % head of pile (node 3),  [ Shear=P;Mom=0]
        dEI=(EI(4)-EI(2))/EI(3);
      KK(1:2,1:5)=[-1,2+dEI,-2*dEI,-2+dEI,1;  0,1,-2,1,0]; %
        % pinpoint of pile (node nn-2), [ Mom=0;Shear=0]
      KK(nn-1:nn,nn-4:nn)=[0,1,-2,1,0; -1,2,0,-2,1];
      %
      %  q= d2(Mom)/dx2= d2(EI* d2w/dx2)/dx2
          for i=3:nn-2
         Dx4=h(i)^4;
         d2EI=( EI(i-1)-2*EI(i)+EI(i+1))/EI(i);
         Ki= 6  + kx(i)*(Dx4) /EI(i)- 2*d2EI;
         KK(i,i-2:i+2)=[1,-4+d2EI,Ki,-4+d2EI,1];
           Ksaved(i,j+1)=Ki;
           d2EIsaved(i,j)=d2EI;
          end
  %
  F(1)=ascale*step *2*(h(3)^3)/EI(3); % Shear @ node 3 = lateral load
  %
```

```
  dw=KK\F;
  w(:,j+1) =w(:,j)+dw;
  %
     for n=3:nn-2,
    theta(n,j+1)=(w(n-1,j)-w(n+1,j))/ 2/(h(n));
    curv(n,j+1)=(w(n+1,j)-2*w(n,j)+w(n-1,j))/h(n)/h(n);
     end
  %
  % NonLinearity  of PILE:
  dcurv=curv(:,j+1)-curv(:,j);
  dzetap=(1-(bp+gp.*sign(dcurv.*zetap)).*(abs(zetap)).^np).*
   dcurv./wyp;
  zetap=zetap +dzetap;
  EI=ap.*EIo + ...
      (1-ap).*EIo.*(1-(bp+gp.*sign(dcurv.*zetap)).*(abs(
   zetap)).^np);
  EIsaved(:,j+1)=EI;
  zetapSaved(:,j+1)=zetap;
  %
  % NonLinearity  of GROUND:
  dzetas=(1-(bs+gs.*sign(dw.*zetas)).*(abs(zetas)).^ns).*dw
   ./wys;
  zetas=zetas +dzetas;
  kx=as.*kxo +...
      (1-as).*kxo.*(1-(bs+gs.*sign(dw.*zetas)).*(abs(zetas))
   .^ns);
  kxsaved(:,j+1)=kx;
  zetasSaved(:,j+1)=zetas;
  Soil_Reaction(:,j+1)=zetas.*py.*(1-as) +as.*kxo.* w(:,j+1)
   ;
  %
  % Pile bending MOMENT & SHEAR force
  Mom(:,j+1)=ap.*EIo.*curv(:,j+1) +(1-ap).*My.*zetap ;
     for n=3:nn-2, Shear(n,j+1)=Mom(n,j+1)-Mom(n-1,j+1)/h(n)
   ;
     end
  %
    end
%
```

```
%
%% GRAPHICS
%
% soil Reaction -displacement @ head of pile
figure(1);clf
subplot(2,3,1);hold on;grid on;
xlabel({'Head Displacement:';'{\delta}'});
ylabel({' Soil Reaction:';' P_{soil r.} '});
plot(w(3,1:mm),Soil_Reaction(3,1:mm))
title({' P-{\delta}';['n=',num2str(ns(3))]})
%
%Lateral Force- displacement @head of pile
subplot(2,3,[2 3]);hold on;grid on;
xlabel({;'Head Displacement:';'{\delta}'});
ylabel({' Lateral Force:';' P  '});
plot(w(3,1:mm),P(1:mm),dispHeadField(1:mm),P(1:mm),'or')
title({' P_{lateral}-{\delta}';[];[' k_{eff}=',num2str(ko(2)
    ,'% .f'),'   kN/m' ]})
%
% Bending Moment  - depth
subplot(2,3,4);hold on;grid on;set(gca,'YDir','reverse');
plot(Mom(3:nn-2,mm)/My(3:nn-2),depth(3:nn-2));
title({'Bending Moment';['P=',num2str(P(mm),'% .f'),' kN']})
xlabel('Normalised M/My'); ylabel('Depth (m)');
% Pile's displacemnt- depth
subplot(2,3,5);hold on;grid on;set(gca,'YDir','reverse');
plot(w(3:nn-2,mm),depth(3:nn-2));
title({'Displacement';['P=',num2str(P(mm),'% .f'),' kN']}),
xlabel('{\delta (m)}');
ylabel('Depth (m)');
% Pile's Turning angle- depth
subplot(2,3,6);hold on;grid on;set(gca,'YDir','reverse');
plot(theta(3:nn-2,mm),depth(3:nn-2));
title({'Turning angle {\theta }';['P=',num2str(P(mm),'% .f')
    ,' kN']}),
xlabel('{\theta }');
ylabel('Depth (m)');
%
%
```

```
%% deviation:  field vs  computed
  f=0  + (1/sqrt(2)) *norm((w(3,1:2)-dispHeadField(1:2)))
   +(1/sqrt(2))*norm((w(nL,1:2)-zeros(1,2)));
    end
```

A.4 Function: fun2PL.m

```
       function f=fun2PL(x,pile,ko,version)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%       about fun2PL(x,pile):                 %
% Goal: Compute optimum elastic Ko,ref        %
%                                             %
% Input: x=ko*/10000, ko*=initial estimation, %
%   [pile]=[ L ; Epp ; App ; Ipp ; My ;1],    %
% as created from main1PL.m ,                 %
% and  P-y & pile's respnses (.txt) files,    %
%  which 1st respose is elastic               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%% FIELD VALUES/ INPUT
L=pile(1);
load_headDisp=load('load_headDispPLAXIS.txt');
load_disp=load('load_dispPLAXIS.txt');
NumLayers=size(load_disp,1)-1;thickness=L/NumLayers;
NumRecords=size(load_disp,2);
%
% field measured displacements
Pfield=load_disp(1,1:NumRecords);
dispField=load_disp(2:end,1:NumRecords);
dispHeadField=dispField(1,:);
thetaField=zeros(NumLayers,NumRecords);
for j=1:NumRecords
for i=1:NumLayers-1
    thetaField(i,j)=(dispField(i,j)-dispField(i+1,j))/
   thickness;
end
end
%
%  Interpolation
 wanted_Num_analysis_points=1000;
 mm=wanted_Num_analysis_points;
load_step=max(Pfield)/(mm-1) ;
%
 Pi=interp1(Pfield,Pfield,0:load_step:max(Pfield),'*cube')';
```

```
  step=load_step;
 indexField=zeros(NumRecords,1);
%
 for i=1:NumRecords
     p=Pfield(i);
index=find((abs(Pi-p))<step);
indexField(i)=index(1);
 end
 dispHeadFieldi=interp1(Pfield,dispHeadField,Pi,'*cube');
%
%
%% materials
materials=[1,0.5,0.001
        2,0.5,0.001];
NumMaterials=size(materials,1);
 nL=NumLayers;
 nn=nL+4;
%
%  materials' properties
bl=repmat(0.00001,[NumMaterials,1]);mat_n=bl;nl=bl;a=bl;Pyo=
   bl; power=bl;
for  i=1:NumMaterials,
 mat_n(i)=materials(i,1);  bl(i)=materials(i,2); a(i)=
   materials(i,3);
end
%
% define x
nl(1)=2; % concrete
 for i=2:NumMaterials,
nl(i)=x(i-1)/10;
 Pyo(i)=x(i-1+NumMaterials-1)*100;
 power(i)=x(i-1 +2*(NumMaterials-1))/10;
  end
%
% Pile
pile=[thickness;pile(2:end)]';
pile= [(1:NumLayers)',repmat(pile,[NumLayers,1])];
%
Pile=zeros(nn,size(pile,2));
```

```
% [Pile]= extended matrix of [pile] (IMAGINARY nodes
    INCLUDED)
% {#3=head of pile, #(nn-2)=pinpoint of pile=deeper point}
for i=1:2, Pile(i,:)=pile(1,:);
end
for i=nn-1:nn, Pile(i,:)=pile(nn-4,:);
end
Pile(3:nn-2,:)=pile;
%
h=zeros(nn,1); Epp=h; App=h;Ipp=h;My=h;
depth=h;lay_matp=ones(nn,1); lay_n=h; wyp=h;
for i=1:nn,
    lay_n(i)=Pile(i,1); h(i)=Pile(i,2); Epp(i)=Pile(i,3);
     App(i)=Pile(i,4); Ipp(i)=Pile(i,5);
      My(i)=Pile(i,6);lay_matp(i)=Pile(i,7);
     wyp(i)=My(i)/(Epp(i)*Ipp(i));depth(i+1)=depth(i)+h(i);
end
depth=abs(depth-(h(1)-h(2))/2); depth(1)=0.01;
 EI=Epp.*Ipp;
 EIo=EI;
%
% SOIL profile
% soil=[layer #,soil material, %OCR]
soil=[1:nL;repmat(2,1,nL);ones(1,nL)]';
Soil_Input=size(soil,2);
size(soil,1);
Soil=zeros(nn,Soil_Input);
% [Soil]= extended matrix of [soil] (IMAGINARY nodes
    INCLUDED)
for i=1:2, Soil(i,:)=soil(1,:);
end
for i=nn-1:nn, Soil(i,:)=soil(nn-4,:);
end
Soil(3:nn-2,:)=soil;
%
kx=zeros(nn,1);lay_mats=kx; py=kx; % define size
for i=1:nn
lay_mats(i)=Soil(i,2);
% OCR=Soil(i,3);
```

```
ims=lay_mats(i);
%           if OCR ==1,
        kx(i)=ko(ims)*(depth(i))^power(ims);
        py(i)=Pyo(ims)*depth(i) ;
%          else  kx(i)=kx(i-1); py(i)=py(i-1); % if OCR>1;
%           end
end
wys=py./kx;
kxo=kx;
%
% define properties of each node
bp=zeros(nn,1);bs=bp;gp=bp;gs=bp;
np=bp;ap=bp;as=bp;ns=bp;
for i=1:nn
im=lay_matp(i); ims=lay_mats(i);
bp(i)=bl(im); bs(i)=bl(ims);
gp(i)=1-bl(im); gs(i)=1-bl(ims);
 np(i)=nl(im); ns(i)=nl(ims);
ap(i)=a(im); as(i)=a(ims);
end
%
%
%% w(node,repetition)=w(i,j): DISPLACEMENTS of each node of
   the PILE
%
w=zeros(nn,mm); curv=w; theta=w; Mom=w; Shear=w;EIsaved=w;
   kxsaved=w;
KK=zeros(nn,nn); F=zeros(nn,1); dw=F; d2EIsaved=w;
   zetapSaved=w;aa=w;
zetasSaved=w;zetas=F;dzetas=F; zetap=F;dzetap=F;
   Soil_Reaction=zeros(nn,mm);
Ksaved=ones(nn,mm);EIsaved(:,1)=EI;
% go=1;
%
for j=1:mm-1
     %
     % BOUNDARY CONDITIONS
        % head of pile (node 3), [ Shear=P;Mom=0]
        dEI=(EI(4)-EI(2))/EI(3);
```

```
        KK(1:2,1:5)=[-1,2+dEI,-2*dEI,-2+dEI,1;  0,1,-2,1,0];
          % pinpoint of pile (node nn-2), [ Mom=0;Shear=0]
        KK(nn-1:nn,nn-4:nn)=[0,1,-2,1,0; -1,2,0,-2,1];
        %
        %  q= d2(Mom)/dx2= d2(EI* d2w/dx2)/dx2
             for i=3:nn-2
           Dx4=h(i)^4;
           d2EI=( EI(i-1)-2*EI(i)+EI(i+1))/EI(i);
           Ki= 6  + kx(i)*(Dx4) /EI(i)- 2*d2EI;
           KK(i,i-2:i+2)=[1,-4+d2EI,Ki,-4+d2EI,1];
             Ksaved(i,j+1)=Ki;
             d2EIsaved(i,j)=d2EI;
             end
          %
    % Shear @ node 3 = lateral load
    F(1)=step *2*(h(3)^3)/EI(3);
    %
    dw=KK\F;
    w(:,j+1) =w(:,j)+dw;
    %
       for n=3:nn-2,
           theta(n,j+1)=(w(n-1,j)-w(n+1,j))/ 2/(h(n));
        curv(n,j+1)= (w(n+1,j)-2*w(n,j)+w(n-1,j)) /h(n)/h(n);
     end
    %
    %
    % NonLinearity  of PILE:
    dcurv=curv(:,j+1)-curv(:,j);
    dzetap=(1-(bp+gp.*sign(dcurv.*zetap)).*(abs(zetap)).^np).*
     dcurv./wyp;
    zetap=zetap +dzetap;
    EI=ap.*EIo + ...
        (1-ap).*EIo.*(1-(bp+gp.*sign(dcurv.*zetap)).*(abs(
     zetap)).^np);
    EIsaved(:,j+1)=EI;
    zetapSaved(:,j+1)=zetap;
    %
    % NonLinearity  of GROUND:
```

```
  dzetas=(1-(bs+gs.*sign(dw.*zetas)).*(abs(zetas)).^ns).*dw
   ./wys;
  zetas=zetas +dzetas;
  kx=as.*kxo +...
  (1-as).*kxo.*(1-(bs+gs.*sign(dw.*zetas)).*(abs(zetas)).^ns
   );
    kxsaved(:,j+1)=kx;
  zetasSaved(:,j+1)=zetas;
  Soil_Reaction(:,j+1)=zetas.*py.*(1-as) +as.*kxo.* w(:,j+1)
   ;
  %
  % Pile bending MOMENT & SHEAR force
  Mom(:,j+1)=ap.*EIo.*curv(:,j+1) +(1-ap).*My.*zetap ;
     for n=3:nn-2, Shear(n,j+1)=Mom(n,j+1)-Mom(n-1,j+1)/h(n)
   ;
     end
end
%
%
%% GRAPHICS
%
 if version ==3,
 fprintf('\n----------------------');
fprintf('\n INPUT evaluaton:');
fprintf('\n %.1f m is the length of the pile.',L);
fprintf('\n %.3g m is the average thickness of layers.',L/(
   NumLayers));
 fprintf('\n %d materials have been read.',NumMaterials);
 fprintf('\n %d layers have been read.',NumLayers);
 fprintf('\n %.1f kN have been laterally loaded (head of
   pile).',max(Pfield));
 fprintf('\n %.2f kN is the used load-step.\n',step);
  end
%
% Soil Reaction- displacement @head of pile
 figure(2);clf
subplot(2,3,1);hold on;grid on;
xlabel({' Head Displacement:';'{\delta (m)}'});
ylabel({' Soil Reaction:';' P_{soil} (kN)'});
```

```
plot(w(3,1:mm),Soil_Reaction(3,1:mm))
title({' P_{soil}- {\delta}';...
    ['n=',num2str(ns(3),'% .2f')];...
    ['Py_{eff}=',num2str(Pyo(2),'% .f'),'kN']})
%
% Lateral Force- displacement @head of pile
subplot(2,3,[2 3]);hold on;grid on;
xlabel({;'Head Displacement {\delta (m)} :'});
ylabel({' Lateral Force (P, kN):'   });
 plot(w(3,1:mm),Pi,dispHeadFieldi,Pi,'--r',dispHeadField,
   Pfield,'or');
title({' P_{lateral}-{\delta}';...
    [' kx_{elastic}=',num2str(ko(2),'% .f'),'   kN/m' ...
       ,'    ,','power(m)= ',num2str(power(2),'% .2f')];})
%
% Bending Moment - depth
subplot(2,3,4);hold on;grid on;set(gca,'YDir','reverse');
plot(Mom(3:nn-2,mm)/My(3:nn-2),depth(3:nn-2));
title({'Bending Moment';['P=',num2str(Pi(mm),'% .f'),' kN'
   ]}),
xlabel('Normalised M/My');ylabel('Depth (m)');
%
% Pile's displacemnt -depth
subplot(2,3,5);hold on;grid on;set(gca,'YDir','reverse');
plot(w(3:nn-2,indexField),depth(3:nn-2),'b',dispField,depth
   (3:nn-2),'-.r');
title('Displacement'),
xlabel('{\delta (m)}');
ylabel('Depth (m)');
%
% Pile's Turning angle -depth
subplot(2,3,6);hold on;grid on;set(gca,'YDir','reverse');
plot(theta(3:nn-2,indexField),depth(3:nn-2),'b',thetaField,
   depth(3:nn-2),'-.r');
title('Turning angle {\theta }'),
xlabel('{\theta }');
ylabel('Depth (m)');
%
%
```

```
%% deviation: field VS computed arguments
h=0;g=0;h1=0;
for i=1:max(size(x))
    if x(i)<0, h=1000;
    end
end
%
 for i=1:NumRecords
m=indexField(i);
%
h=h +(1/sqrt(NumLayers))*norm(w(3:nn-2,m)-dispField(:,i)) +
   (1/sqrt(NumLayers))*(max(dispField(:,i))/max(thetaField
   (:,i)))*norm(theta(3:nn-2,m)-thetaField(:,i));
%
g=g+norm((w(3,m)-dispHeadField(i)))/sqrt(NumRecords);
 end
f=(h+g+h1);
%
     end
```

Appendix B

Finite Elements 3D analysis

Finite Elements Method (FEM) software, PLAXIS 3D© was used in order to produce single pile response to lateral load, so as to validatate the developed algorithm.

In this chapter results of this analysis are presented. First, in the format compatible with the MATLAB code of the inverse analysis, afterwards, in the report PLAXIS produce to synopsize the main output and, also, the materials used. Finally, additional figures created from FEM analysis are placed in the last subsection.

B.1 Input used in Validation Case

Results of 3D FEM analysis are presented in the format compatible with the MATLAB code of the inverse analysis, as explained in the ch.A.1 on p.33. Important is that the first load should be small enough not to provoke inelastic response of the pile.

B.1.1 P-y curve

load_headDispPLAXIS.txt

```
10          0.000117642
220          0.0037269628
518          0.0121313528
```

B.1.2 Displacement's distribution

load_dispPLAXIS.txt

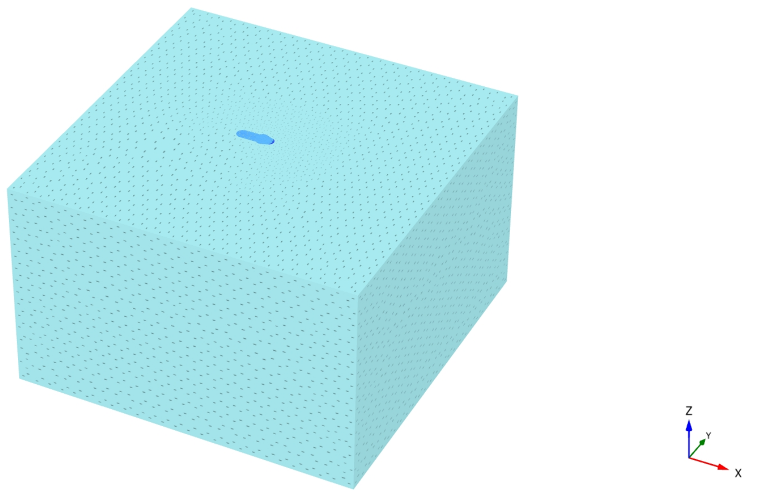
```
10          220          518
0.0001233553          0.003904409          0.0126515678
0.0001119286          0.0035495166          0.0116111378
0.0001007172          0.0031993882          0.0105815264
0.000089901          0.0028585867          0.0095732079
7.9611296030022E-005          0.0025310845          0.0085954683
6.99554282879493E-005          0.0022205082          0.0076569439
0.000060991          0.0019293405          0.0067639509
0.000052767          0.0016599251          0.0059227423
4.52947748674577E-005          0.00141349          0.0051374713
3.8577718755863E-005          0.0011909963          0.0044120448
3.25948405679969E-005          0.0009924292          0.003748595
2.7319176014695E-005          0.0008174579          0.0031486457
2.27097401427826E-005          0.0006650313          0.0026119682
1.87205242860784E-005          0.0005338173          0.0021374554
1.5303205311975E-005          0.0004222683          0.0017228194
1.24010644665075E-005          0.0003284809          0.0013647077
0.000009966          0.0002508106          0.0010600055
7.94040804856772E-006          0.0001872308          0.000803847
6.27689693926466E-006          0.0001360572          0.0005920808
4.92612459566748E-006          9.55317277760663E-005          0.000419645
3.8435931299679E-006          6.40662721584517E-005          0.0002817381
2.98839323854935E-006          4.02033727836609E-005          0.0001735792
2.32272290301809E-006          0.000022598          9.06437749256691E-005
1.81649274816065E-006          0.000010184          2.91855265504849E-005
1.43749644193348E-006          0.000001825          -1.4871294821593E-005
0.000001163          -3.272593769223E-006          -4.4501711314042E-005
9.69130888929538E-007          -5.93948636195215E-006          -6.28702567418048E-005
8.39617208808309E-007          -6.73278727919543E-006          -7.2192407521437E-005
7.57752894190005E-007          -6.22016798294316E-006          -7.4767330582904E-005
0.000000711          -4.81834061286833E-006          -7.23668439612574E-005
0.000000689          -0.000002867          -6.64709049072734E-005
6.83288120245687E-007          -6.33825662805782E-007          -5.83123804536071E-005
6.876570281985E-007          1.68641373429192E-006          -4.88142853786525E-005
0.000000697          3.9427453498096E-006          -3.87527046281134E-005
7.07344719993175E-007          6.02494348217307E-006          -0.000028724
7.16158089768108E-007          7.8663615775678E-006          -1.91444464426885E-005
7.21545961930317E-007          9.42517289651543E-006          -1.03177342723633E-005
7.22519944024101E-007          1.06885099403724E-005          -2.4123577864859E-006
7.18292382890172E-007          1.16488608812464E-005          4.43328055172048E-006
7.08920210971597E-007          0.000012325          0.000010215
6.94344648486205E-007          1.2731284294993E-005          1.49107005587346E-005
6.75091876628355E-007          1.28978427155829E-005          1.85974519693464E-005
6.51609755724687E-007          1.28532154386249E-005          2.13373354550553E-005
```

```
6.24537929426426E-007        0.000012628         2.32304918680934E-005
5.94549010448835E-007        1.2254319469929E-005         2.43838203413711E-005
5.62283771171139E-007        1.17606834551304E-005        2.49042894985048E-005
5.2838790607046E-007        0.000011175         2.49042975018907E-005
4.93412157218338E-007        0.000010521         2.44830614202188E-005
4.5784259751531E-007        9.8196087770393E-006        2.37357501100346E-005
4.22076852928449E-007        9.08851703357214E-006        2.27446573690005E-005
3.86425543243916E-007        8.34218448226832E-006        2.15853614784232E-005
3.51113130212172E-007        7.59157613457067E-006        2.03186880722826E-005
3.16265026685966E-007        0.000006845         1.90016539485918E-005
0.000000282        6.10792991314019E-006        1.76720945326672E-005
2.48062553195737E-007        5.38175728706447E-006        1.63786631018271E-005
```

B.2 PLAXIS Report

The report, that PLAXIS produce to synopsize the main output and the materials used, followes in the next page.

1.1.1.1 Calculation results, Phase_3 [Phase_3] (3/24), Materials plot



Materials plot

2

1.1.2.1.1 Materials - Soil and interfaces - Hardening soil

Identification		soil
Identification number		1
Drainage type		Drained
Colour		
Comments		
γ_{unsat}	kN/m^3	20.00
γ_{sat}	kN/m^3	20.00
Dilatancy cut-off		No
e_{init}		0.5000
e_{min}		0.000
e_{max}		999.0
Rayleigh α		0.000
Rayleigh β		0.000
E_{50}^{ref}	kN/m^2	50.00E3
E_{oed}^{ref}	kN/m^2	50.00E3
E_{ur}^{ref}	kN/m^2	150.0E3
power (m)		0.5000

Identification		soil
Use alternatives		No
C_c		6.900E-3
C_s		2.070E-3
e_{init}		0.5000
c_{ref}	kN/m^2	0.1000
φ (phi)	°	32.00
ψ (psi)	°	0.000
Set to default values		No
ν_{ur}		0.2000
p_{ref}	kN/m^2	100.0
K_0^{nc}		0.4701
c_{inc}	$kN/m^2/m$	0.000
z_{ref}	m	0.000
R_f		0.9000
Tension cut-off		No
Tensile strength	kN/m^2	10.00E6
Strength		Rigid
R_{inter}		1.000
δ_{inter}		0.000

Identification		soil
K_0 determination		Automatic
$K_{0,x} = K_{0,y}$		Yes
$K_{0,x}$		0.4701
$K_{0,y}$		0.4701
OCR		1.000
POP	kN/m^2	0.000
k_x	m/s	0.000
k_y	m/s	0.000
k_z	m/s	0.000
e_{init}		0.5000
c_k		1.000E15

1.1.2.1.2 Materials - Soil and interfaces - Mohr-Coulomb

Identification		RC
Identification number		2
Drainage type		Drained
Colour		
Comments		
γ_{unsat}	kN/m^3	20.00
γ_{sat}	kN/m^3	20.00
Dilatancy cut-off		No
e_{init}		0.5000
e_{min}		0.000
e_{max}		999.0
Rayleigh α		0.000
Rayleigh β		0.000
E	kN/m^2	25.00E6
ν (nu)		0.2000
G	kN/m^2	10.42E6
E_{oed}	kN/m^2	27.78E6

Identification		RC
c_{ref}	kN/m ^{2}	15.26E3
φ (phi)	°	0.000
ψ (psi)	°	0.000
V_s	m/s	2259
V_p	m/s	3689
Set to default values		No
E_{inc}	kN/m ^{2} /m	0.000
z_{ref}	m	0.000
c_{inc}	kN/m ^{2} /m	0.000
z_{ref}	m	0.000
Tension cut-off		Yes
Tensile strength	kN/m ^{2}	7534
Strength		Rigid
R_{inter}		1.000
δ_{inter}		0.000
K_0 determination		Automatic
$K_{0,x} = K_{0,y}$		Yes
$K_{0,x}$		1.000
$K_{0,y}$		1.000

Identification		RC
k_x	m/s	0.000
k_y	m/s	0.000
k_z	m/s	0.000
e_{init}		0.5000
c_k		1.000E15

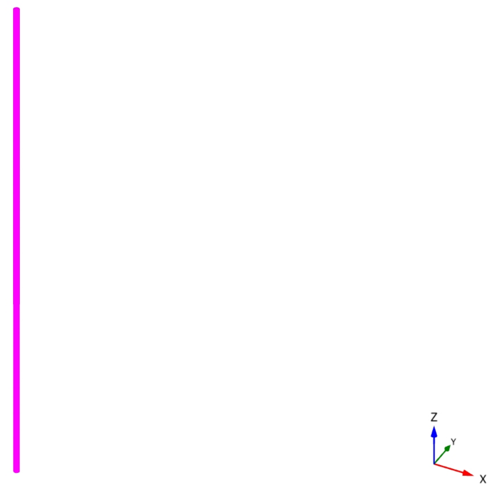
1.1.2.2 Materials - Plates -

Identification		head
Identification number		1
Comments		
Colour		
d	m	10.00
γ	kN/m^3	0.1000E-3
Linear		Yes
Isotropic		Yes
E_1	kN/m^2	25.00E6
E_2	kN/m^2	25.00E6
ν_{12}		0.2000
G_{12}	kN/m^2	10.42E6
G_{13}	kN/m^2	10.42E6
G_{23}	kN/m^2	10.42E6
Rayleigh α		0.000
Rayleigh β		0.000

1.1.2.3 Materials - Beams -

Identification		inclinometer
Identification number		1
Comments		
Colour		
A	m^2	0.7850
γ	kN/m^3	0.1000E-3
Linear		Yes
E	kN/m^2	2500
I_3	m^4	0.04908
I_2	m^4	0.04908
Rayleigh α		0.000
Rayleigh β		0.000

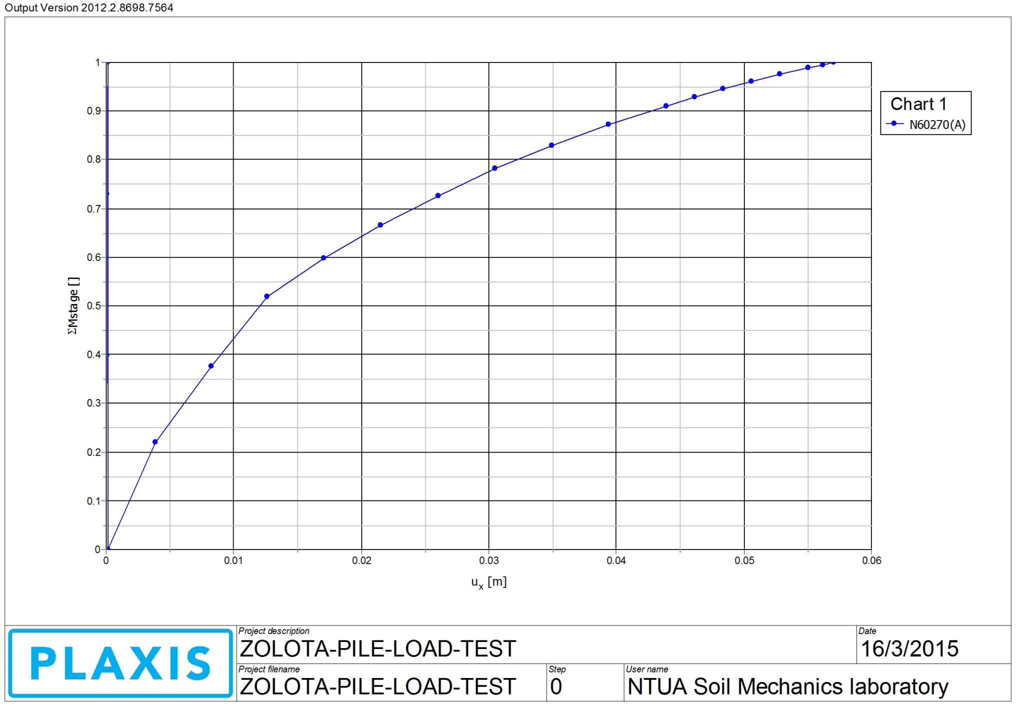
3.1.1.1.1 Calculation results, Beam, Phase_3 [Phase_3] (3/24), Connectivity plot



Connectivity plot

11

5.1 Chart 1


Output Version 2012.2.8698.7564
Chart 1
N60270(A)
ΣMstage []
u_x [m]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.01
0.02
0.03
0.04
0.05
0.06
PLAXIS
Project description
ZOLOTA-PILE-LOAD-TEST
Date
16/3/2015
Project filename
ZOLOTA-PILE-LOAD-TEST
Step
0
User name
NTUA Soil Mechanics laboratory

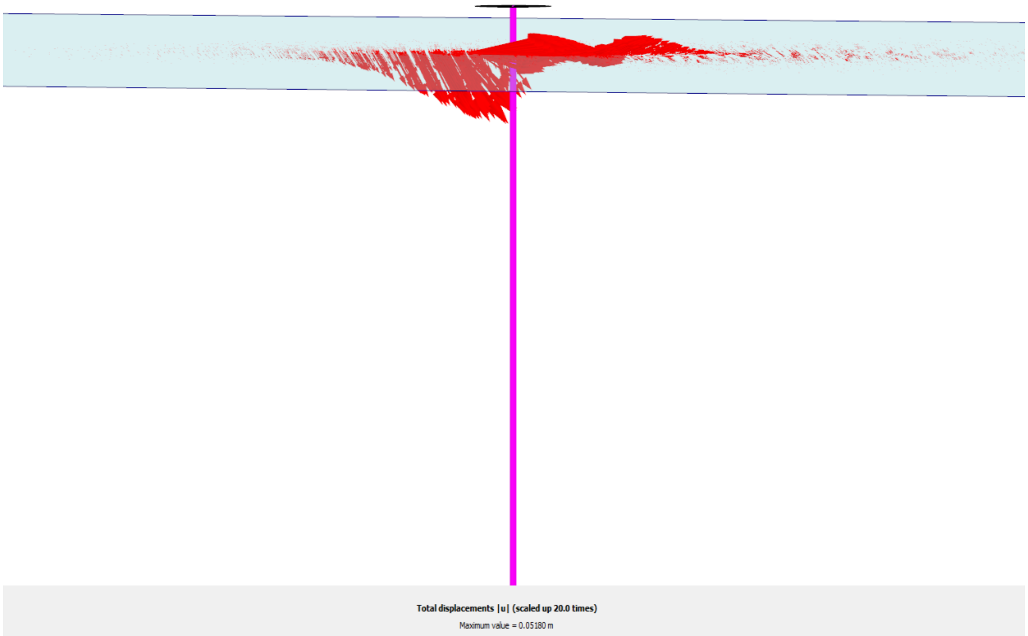
5.1.1 Chart 1(N60270(A))

Point	Step	u_x [10^{-3} m]	ΣMstage []
0	0	0.000	0.000
1	1	0.000	0.000
2	1	0.000	0.354
3	2	0.000	0.599
4	3	0.000	0.888
5	4	0.000	1.000
6	5	0.000	0.000
7	5	0.042	0.399
8	6	0.084	0.729
9	7	0.123	1.000
10	8	0.123	0.000
11	8	3.904	0.220
12	9	8.277	0.376
13	10	12.652	0.518
14	11	17.098	0.598
15	12	21.557	0.665

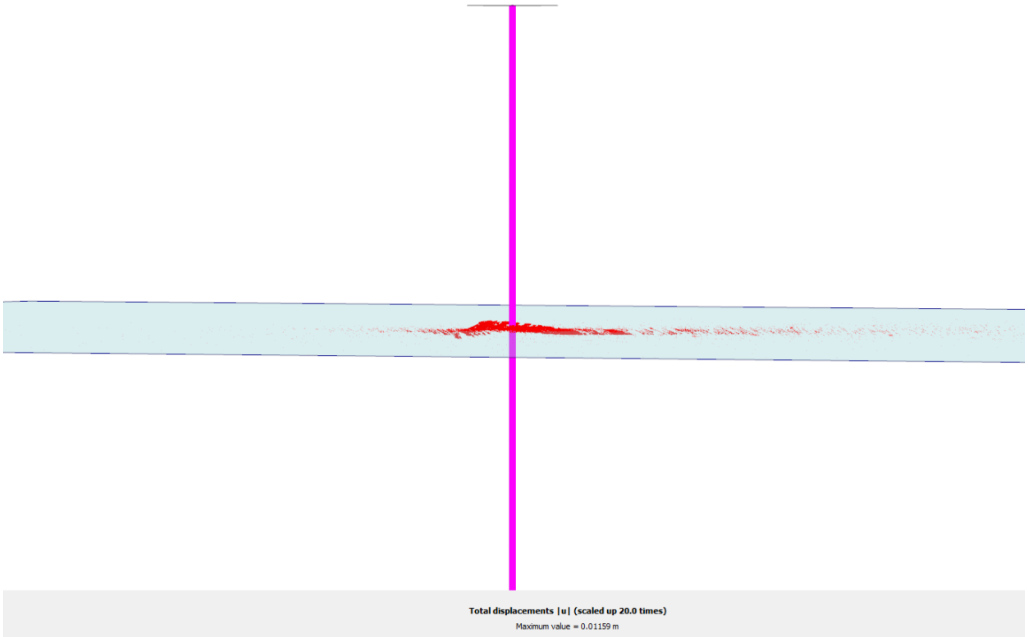
Point	Step	u_x [10^{-3} m]	ΣMstage []
16	13	26.019	0.726
17	14	30.484	0.781
18	15	34.953	0.829
19	16	39.423	0.872
20	17	43.893	0.910
21	18	46.128	0.928
22	19	48.363	0.945
23	20	50.598	0.960
24	21	52.834	0.975
25	22	55.069	0.988
26	23	56.186	0.995
27	24	57.085	1.000

B.3 Additional output figures

Figures representing results of the 3D FEM analysis, when the lateral load was ultimate ($\Sigma M_{stage} = 1$).



(A) $z = -0.5\,m$



(B) $z = -3,6\,m$

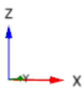


FIGURE B.3.1: Total Displacements, u

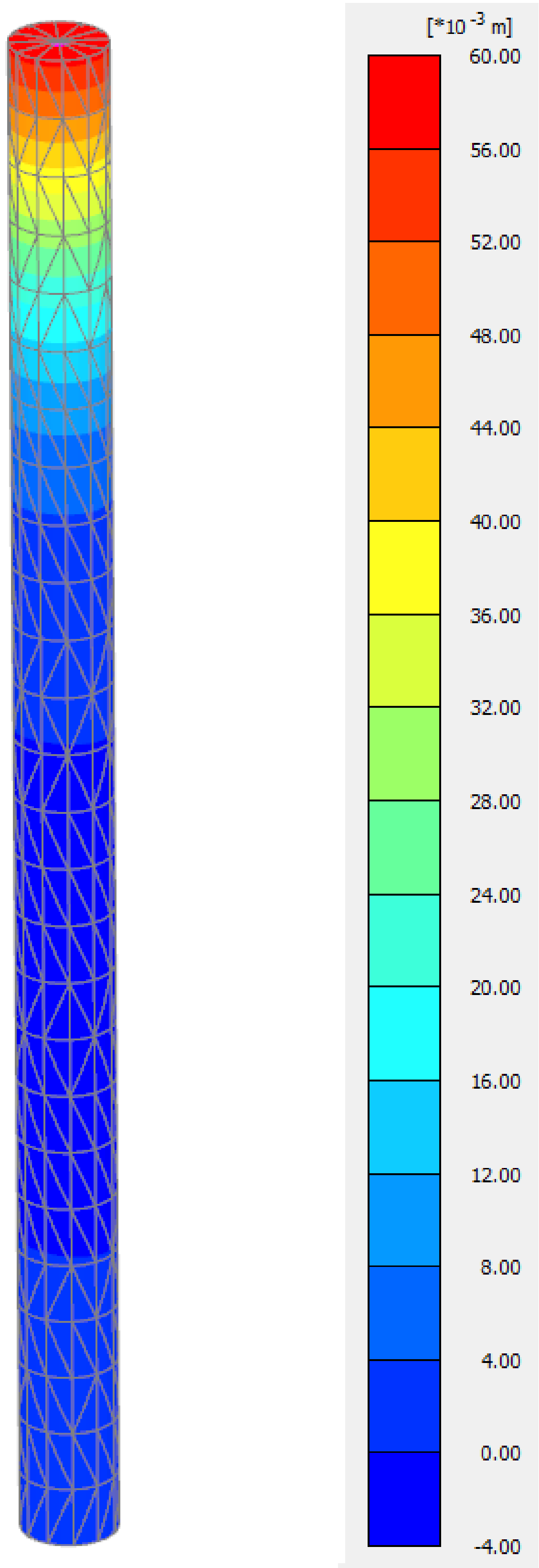


FIGURE B.3.2: Displacement u_x

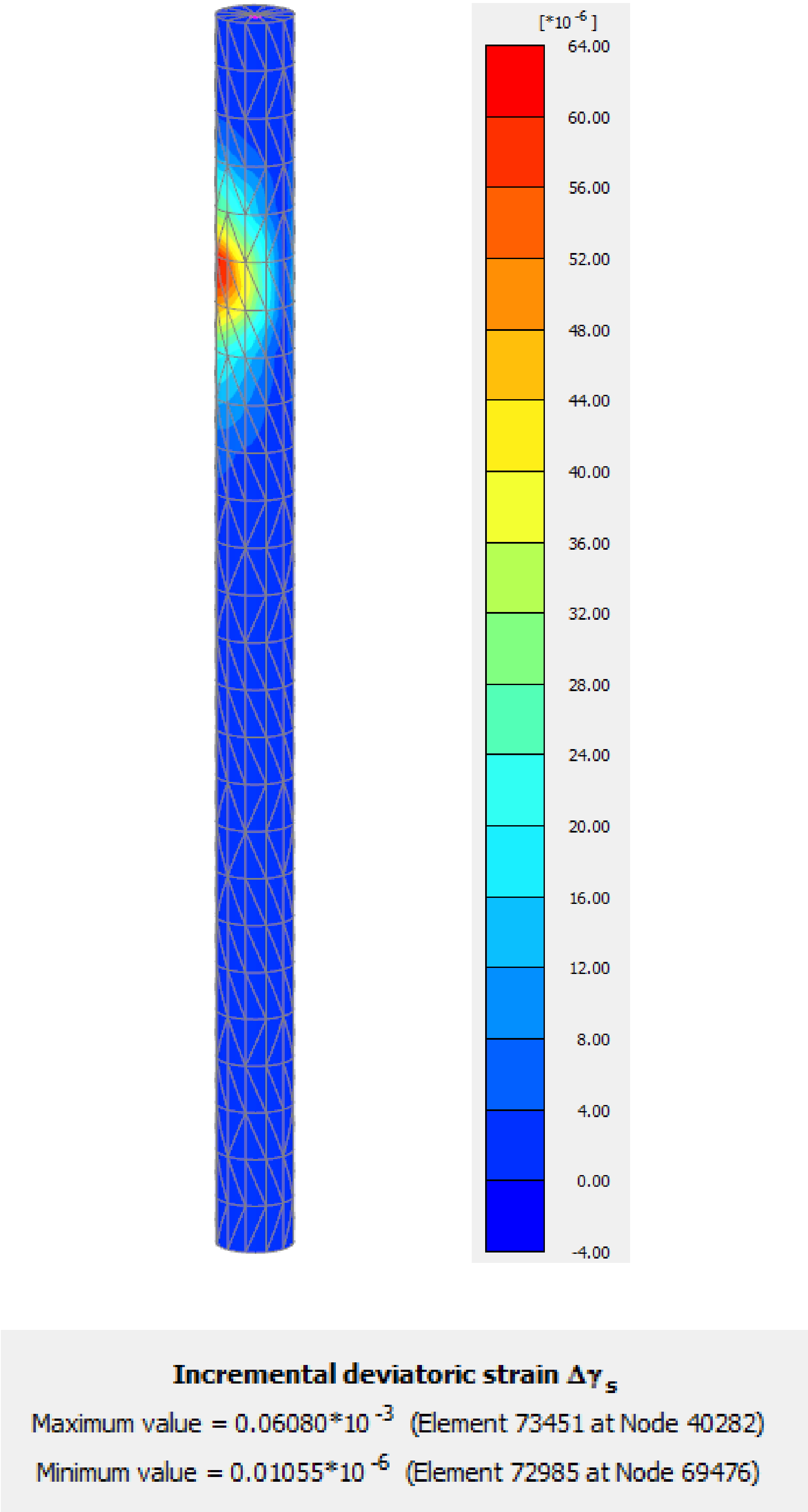


FIGURE B.3.3: Incremental deviatoric strain $\Delta\gamma_s$

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