Θεωρίες Υπερσυμμετρίας και Υπερβαρύτητας στον Τετραδιάστατο $\mathcal{N}=1$ Υπερχώρο

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Aspects of Supersymmetry and Supergravity in $4D$, $\mathcal{N} = 1$ Superspace

PhD Thesis

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Στους γόνεις μου.

To my parents.

Ευχαριστίες

Ευχαριστώ θερμά τον επιβλέποντα καθηγητή Αλέξανδρο Κεχαγιά που υπήρξε εξαίρετος δάσκαλος και συνεργάτης. Ευχαριστώ τα μέλη της τριμελούς επιτροπής καθηγητές Ulf Lindström και Κωνσταντίνο Σφέτσο για την καθοδήγηση κατά την διάρκεια εκπόνησης της εργασίας μου. Ευχαριστώ τα μέλη της επταμελούς επιτροπής καθηγητές Γεώργιο Ζουπάνο, Αθανάσιο Λαχανά, Ιωάννη Μπάκα και Νικόλαο Τράκα για την υποστήριξη κατά την διάρκεια του διδακτορικού μου. Ευχαριστώ για την συνεργασία τους καθηγητές Sergio Ferrara, Massimo Porrati, Antionio Riotto και Rikard von Unge και τους διδάκτορες Cristiano Germani , Ιωάννη Νταλιάνη και Εμμανουήλ Σαριδάκη. Ευχαριστώ τους καθηγητές Νικόλαο Ίργε και Γεώργιο Κουτσούμπα και τους διδάκτορες Δημήτριο Γιαταγάνα και Αθανάσιο Χατζησταυρακίδη για τις επιστημονικές μας συζητήσεις. Τέλος ευχαριστώ την οικογένεια και τούς φίλους μου.

Acknowledgements

It is a pleasure to thank my supervisor, professor Alexandros Kehagias, who has been an excellent advisor and collaborator. I am gratefull to the members of my advisor committee professors Ulf Lindström and Konstantinos Sfetsos for guidance throughout my PhD. I thank the members of the examinasion committee, professors George Zoupanos, Athanasios Lahanas, Ioannis Bakas and Nicolaos Tracas for discussions and support. I thank professors Sergio Ferrara, Massimo Porrati, Antionio Riotto, Rikard von Unge and doctors Cristiano Germani, Ioannis Dalianis and Emmanuel Saridakis for our collaborations. I thank professors Nicolaos Irges and George Koutsoumbas and doctors Dimitri Giataganas and Athanasios Chatzistavrakidis for our discussions. Finally I thank my family and friends.

Περίληψη στα Ελληνικά

Η παρούσα διδακτορική διατριβή αφορά τα παρακάτω θέματα:

- Αυθόρμητο σπάσιμο της υπερσυμμετρίας από όρους ανωτέρας τάξης.

  Η θεωρία της υπερσυμμετρίας είναι η πιο πολλά υποσχόμενη επέκταση στο καθιερωμένο πρότυπο των στοιχειωδών σωματιδίων. Αυτή η συμμετρία έχει αρκετές αξιόλογες θεωρητικές και φαινομενολογικές ιδιότητες. Εν τούτοις δεν έχει παρατηρηθεί μέχρι σήμερα κάποιο σήμα της στους επιταχυντές και αυτό σημαίνει ότι αν είναι πραγματικά μια συμμετρία της φύσης θα πρέπει να είναι σπασμένη. Τον κυρίαρχο ρόλο στο αυθόρμητο σπάσιμο της υπερσυμμετρίας παίζει το δυναμικό του βαθμωτού πεδίου που θα προκαλέσει αυτό το φαινόνενο διότι καθορίζει ποιες είναι οι θεμελιακές καταστάσεις της θεωρίας. Το βαθμωτό δυναμικό για θεωρίες με όρους μέχρι διάσταση τέσσερα έχει μελετηθεί εκτενώς. Είναι όμως γνωστό,ότι όροι ανωτέρας τάξης έχουν άμεση επίδραση σε αυτό το δυναμικό. Βρέθηκε οτι υπάρχουν συγκεκριμένα μοντέλα των οποίων οι όροι διάστασης τέσσερα δεν οδηγούν σε αυθόρμητο σπάσιμο της υπερσυμμετρίας ενώ λαμβάνοντας υπ΄όψη συγκεκριμένους όρους ανωτέρας διάστασης η θεωρία έχει πια την δυνατότητα να βρεθεί και σε θεμελιακές καταστάσεις όπου η υπερσυμμετρία είναι σπασμένη.

- Μη-γραμμικές αναπαραστάσεις θεωριών σπασμένης υπερσυμμετρίας.

  Στα πλαίσια των μη γραμμικών αναπαραστάσεων της υπερσυμμετρίας στο ελάσσον υπερσυμμετρικό καθιερωμένο πρότυπο μελετήθηκε η δυνατότητα ύπαρξης συνεπούς μοντέλου που χρησιμοποιεί μόνο ένα πεδίο Higgs. Οι διορθώσεις λόγω της μη-γραμμικής υπερσυμμετρίας στο δυναμικό του πεδίου Higgs βελτιώνουν την συμπεριφορά του σε σχέση με το αντίστοιχο δυναμικό στο ελάσσον υπερσυμμετρικό καθιερωμένο πρότυπο. Επίσης βρέθηκε ότι τέτοια μοντέλα γεννούν από μόνα τους μια ιεραρχία στις μάζες των βαριών φερμιονίων.Για την εξουδετέρωση των κβαντικών ανωμαλιών υπάρχουν διάφορες δυνατότητες ,μια εκ των οποίων είναι για παράδειγμα η υπόθεση ύπαρξης μιας επιπλέον μισής οικογένειας λεπτονίων. Σε πιο θεωρητικό επίπεδο μελετήθηκαν θεωρίες υπερβαρύτητας όπου η υπερσυμμετρία ήταν αυθόρμητα σπασμένη. Βρέθηκε, ότι στο όριο άπειρης μάζας του βαθμωτού σωματιδίου που σπάει την υπερσυμμετρία, αυτό αποσυζεύγνεται και οι εξισώσεις κίνησής του μετατρέπονται σε συνδέσμους που σηματοδοτούν μη γραμμική αναπαράσταση της υπερσυμμετρίας.

- Συνεπείς θεωρίες ανωτέρων παραγώγων στην υπερσυμμετρία και στην υπερβαρύτητα.

  Κατά την κυρίαρχη άποψη για την γέννηση του σύμπαντος υπήρξε μια περίοδος κατά την οποία αυτό επεκτεινόταν εκθετικά ,γνωστή ως "πληθωρισμός". Είναι γνωστό ότι θεωρίες με ανώτερες παραγώγους ευνοούν την ύπαρξη μιας τέτοιας περιόδου δια τούτο και υπάρχει κίνητρο να εμβαπτιστούν στην θεωρία της υπερβαρύτητας. Το ενδιαφέρον όμως δεν περιορίζεται μόνον για την εφαρμογή τους στην κοσμολογία. Από μόνη της η μελέτη των δυνατοτήτων που υπάρχουν να εισάγει κανείς τέτοιους όρους αποτελεί πρόκληση διότι συνεπείς θεωρίες ανωτέρων παραγώγων χωρίς αστάθειες είναι δύσκολο να κατασκευασθούν στην θεωρία της υπερβαρύτητας. Ένα επιπλέον κίνητρο για την κατασκευή τέτοιων όρων αποτελεί η ύπαρξή τους στις ενεργές θεωρίες χαμηλών ενεργειών της θεωρίας χορδών. Συγκεκριμένα τα μοντέλα που μελετήθηκαν στην υπερβαρύτητα ήταν τα εξής: Μη-ελάσσων σύζευξη παραγώγων βαθμωτού πεδίου με την υπερβαρύτητα, υπερσυμμετρικές θεωρίες Galileon συμμετρίας και εμβαπτίσεις του πληθωριστικού μοντέλου Starobinsky στις ελάσσωνες θεωρίες υπερβαρύτητας.

Abstract

This dissertation is concerned with the following topics:

- Spontaneous breaking of supersymmetry by higher dimension operators.

  The dominant role in the breaking of supersymmetry is taken over by the scalar potential which in theories with up to dimension four operators has been studied extensively. This work has showed that there are examples where theories with terms of up to dimension four do not lead to spontaneous breaking of supersymmetry while taking into account the contribution of the higher dimension operators may lead to ground states where supersymmetry is broken.

- Non-linear realizations in theories of broken supersymmetry.

  The minimal supersymmetric standard model includes two Higgs fields. In the framework of non-linear realizations of supersymmetry, the existence of a consistent model with a single Higgs field was studied. It was found that such models are equally promising: Corrections due to non-linear supersymmetry in the scalar potential of the Higgs field improve its behavior relative to the minimal supersymmetric standard model, moreover, such models seem to generate a hierarchy of masses for the heavy fermions. Supergravity theories where supersymmetry is spontaneously broken are also studied. It is found that in the limit of infinite mass of the scalar particle that breaks supersymmetry, it decouples and its equations of motion are converted into constraints that signal a non-linear realization of supersymmetry.

- Consistent higher derivative theories in supergravity.

  The dominant view for the birth of the universe is that there was a period in which it expanded exponentially, known as "Inflation." Theories with higher derivatives favor the existence of such a period and it is motivating to incorporate them in a supergravity theory. An additional incentive for the construction of such higher derivative terms is their existence in the effective low energy actions of string theory. The models studied were: Non-minimal derivative coupling of a scalar field to supergravity, supersymmetric theories with Galileon symmetry and the Starobinsky inflationary model in old- and new-minimal supergravity.

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Εκτενής Περίληψη στήν Ελληνική

1.1 Εισαγωγή

Η υπερσυμμετρία είναι μια συμμετρία βαθμίδας που συνδέει μποζονικούς και φερμιονικούς βαθμούς ελευθερίας. Είναι μια επέκταση της άλγεβρας Poincare με σπινοριακούς γεννήτορες η οποία πραγματώνεται ως εσωτερική συμμετρία όταν μελετηθεί στα πλαίσια μιας θεωρίας πεδίου και χαρακτηρίζεται συχνά ως η πιο όμορφη συμμετρία της φυσικής στοιχειωδών σωματιδίων.

Έχουν περάσει σαρράντα χρόνια από τότε που πρώτη φορά προτάθηκε σαν θεμελιακή συμμετρία της κβαντικής θεωρίας πεδίου, στις αρχές της δεκαετίας του 70. Από τότε δεν έχουν παρατηρηθεί άμεσες πειραματικές ενδείξεις για την σχέση της υπερσυμμετρίας με την φυσική στοιχειωδών σωματιδίων. Επιπλέον δε, έχουμε μόλις μπει σε μια νέα εποχή για την θεωρητική φυσική υψηλών ενεργειών· Είμαστε στην εποχή της ανακάλυψης του σωματιδίου Χιγγς, με μάζα 126 GeV. Με την ανακάλυψη του πολυαναμενώμενου αυτού βαθμωτού πεδίου όλα τα σωμάτια που προέβλεπε το καθιερωμένο πρότυπο έχουν πια ανακαλυφθεί. Η συγκεκριμένη μάζα που μετρήθηκε για το βαθμωτό πεδίο Χιγγς έχει προκαλέσει αμφιβολίες για το αν και πως η υπερσυμμετρία μπορεί να λύσει το πρόβλημα της ιεραρχίας, καθώς ένα πρόβλημα ιεραρχίας μέσα στο υπερσυμμετρικό καθιερωμένο πρότυπο φαίνεται να γεννιέται. Συχνά πια τίθεται το ερώτημα: Το καθιερωμένο πρότυπο των στοιχειωδών σωματιδίων λειτουργεί άριστα, γιατί να προσπαθήσουμε να διορθώσουμε κάτι που δεν είναι χαλασμένο;

Η απάντηση σε αυτό το ερώτημα έρχεται από διάφορους τομείς της θεωρητικής φυσικής, συγχρόνως δείχνοντας πως η υπερσυμμετρία είναι πραγματικά η πιο αυτοσυνεπής πρόταση για νέα φυσική. Είναι γνωστό ότι οι μάζες των βαθμωτών σωματιδίων είναι ιδιαίτερα ευαίσθητες σε κβαντομηχανικές διορθώσεις λόγω ύπαρξης βαρύτερων σωματιδίων ή ενεργειακών κλιμάκων. Ώς εκ τούτου μέσα στο καθιερωμένο πρότυπο υπάρχει το περίφημο πρόβλημα της ιεραρχίας · Η μάζα του μποζονίου Χιγγς είναι ιδιαίτερα ευαίσθητη στη φυσική υψηλών ενεργειακών κλιμάκων από αυτήν του καθιερωμένου προτύπου ώστε μία λεπτομερειακή ρύθμιση τών παραμέτρων είναι αυστηρά απαραίτητη για να δώσει την πειραματικά επιβεβαιωμένη τιμή των 126 GeV. Εάν παρόλα αυτά δεν υπάρχει νέα φυσική τότε δεν υπάρχει το πρόβλημα της λεπτομερειακής ρύθμισης των παραμέτρων , και κανένας λόγος να αμφισβητήσουμε την αξιοπιστία του καθιερωμένου πρότυπου ως απείρως υψηλές ενέργειες. Στην πραγματικότητα η ύπαρξη φυσικής πέραν του καθιερωμένου προτύπου είναι προφανής από μια καθημερινή μας εμπειρία,την βαρύτητα. Κανενός είδους βαρυτική αλληλεπίδραση δεν έχει περιγραφεί στα πλαίσια του καθιερωμένου προτύπου. Μια συνεπής εισαγωγή της βαρύτητας στη καθιερωμένη θεωρία στοιχειωδών σωματιδίων απαιτεί την απάντηση θεμελιακών ερωτημάτων όπως για παράδειγμα

- Κβαντική θεωρία βαρύτητας

- Το πρόβλημα της κοσμολογικής σταθεράς
- Μικροσκοπική περιγραφή σκοτεινής ύλης

Προβλήματα τα οποία απέχουν πολύ από το να έχουν λυθεί. Άρα νέα φυσική πρέπει να υπάρχει και θα πρέπει να απευθυνθεί προς αυτά τα ερωτήματα. Επιπλέον υπάρχει η ελπίδα της ενοποίησης των θεωριών βαθμίδας υπό μίαν μεγαλύτερη θεωρία. Μια τέτοια θεωρία δεν φαίνεται να μπορεί να πραγματοποιηθεί στα πλαίσια των συμμετριών μόνο του καθιερωμένου προτύπου.

Αποδεχόμενοι λοιπόν την ύπαρξη ενεργειακών κλιμάκων ανωτέρων της ηλεκτρασθενούς, ερχόμαστε αντιμέτωποι με το πρόβλημα της ιεραρχίας. Οι ηλεκτρασθενείς αλληλεπιδράσεις στο καθιερωμένο πρότυπο χαρακτηρίζονται από μια ενεργειακή κλίμακα, μια παράμετρο με διάσταση ενέργειας

$$v \sim 246\text{GeV} \tag{1.1}$$

όπου $v/\sqrt{2}$ είναι η αναμενόμενη τιμή του ουδέτερου Χίγγς πεδίου στο κενό. Το γεγονός ότι το πεδίο Χιγγς έχει μη μηδενική αναμενόμενη τιμή στο κενό και προκαλεί το αυθόρμητο σπάσιμο της συμμετρίας βαθμίδας, έχει ως αποτέλεσμα την δημιουργία μιας φυσικής ενεργειακής κλίμακας η οποία θα συνδέεται με όλες τις μάζες της θεωρίας. Για παράδειγμα η κλασσική μάζα των $W^{\pm}$ μποζονίων δίνεται από την σχέση

$$M_W = \frac{gv}{2} \sim 80\text{GeV} \tag{1.2}$$

όπου g ειναι η σταθερά σύζευξης της $SU(2)$. Το Χιγγς πεδίο είναι μια διπλέττα της $SU(2) \times U(1)_Y$

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \tag{1.3}$$

όπου h^0 είναι αφόρτιστο κάτω από την μη σπασμένη $U(1)$ του ηλεκτρομαγνητισμού. Το βαθμωτό δυναμικό έχει την διάσημη μορφή

$$V = -\mu^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 \tag{1.4}$$

όπου $\lambda > 0$ και $\mu^2 > 0$ το οποίο οδηγεί σε μια μάζα για το ουδέτερο Ηιγγς σωματίδιο

$$M_h = v\sqrt{\frac{\lambda}{2}} \sim 126\text{GeV}. \tag{1.5}$$

Πρέπει να τονίσουμε ότι το αρνητικό πρόσημο $-\mu^2$ στον τύπο (1.4) είναι σημαντικό ώστε να λάβει χώρα ο μηχανισμός αυθόρμητου σπασίματος της συμμετρίας και θα πρέπει εν πάσει περιπτώσει να διατηρηθεί , το ίδιο ισχύει και για το μέτρο του $-\mu^2$. Αν δηλαδή είχαμε $+\mu^2$ δεν θα υπήρχε σπάσιμο συμμετρίας και η αναμενόμενη τιμή του πεδίου Χιγγς στο κενό θα ήταν $v = 0$.

Μέχρι τώρα συζητούσαμε υπό μία έννοια αποτελέσματα μόνον της κλασσικής φυσικής. Το πρόβλημα της λεπτομερειακής ρύθμισης παραμέτρων αναδύεται όταν λάβουμε υπ όψιν μας κβαντομηχανικές διορθώσεις. Το καθιερωμένο πρότυπο είναι μια επανακανονικοποιήσημη θεωρία ,το οποίο σημαίνει ότι πεπερασμένα αποτελέσματα προκύπτουν ακόμη και αν λάβουμε υπ όψιν μας όλους τους όρους της θεωρίας διαταραχών και επιπλέον ακόμη και αν επιτρέψουμε τις ορμές των δυνητικών σωματιδίων να πάνε στο άπειρο. Το γεγονός αυτό εγγυάται την αξιοπιστία της θεωρίας και ότι είναι καλά θεμελιωμένη, αλλά δεν αποκλείει την ύπαρξη νέας φυσικής, μάλιστα το ακριβώς αντίθετο, η ευαισθησία των παραμέτρων αυτής της θεωρίας σε νέα φυσική πρέπει να μας βάζει σε προβληματισμούς όπως θα δούμε παρακάτω.

Στην κβαντική θεωρία πεδίου γενικά συναντάμε ολοκληρώματα της μορφής

$$\int^{\Lambda} d^4k f(k, \text{εξωτερικές ορμές}) \tag{1.6}$$

όπου Λ είναι η ενέργεια αποκοπής ,μια ενεργειακή κλίμακα η οποία σηματοδοτεί πότε η θεωρία μας παύει να έχει προβλεψιμότητα και πρέπει να βελτιωθεί. Από τεχνικής άποψης το καθιερωμένο πρότυπο στην απουσία νέας φυσικής παραμένει αξιόπιστο για

$$\Lambda \to \infty. \tag{1.7}$$

Παρόλα αυτά γνωρίζουμε τουλάχιστον μια ενεργειακή κλίμακα όπου το καθιερωμένο πρότυπο πρέπει να βελτιωθεί, η ενεργειακή περιοχή που γίνονται σημαντική η κβαντική βαρύτητα

$$M_P \sim 1.2 \times 10^{19}\text{GeV}. \tag{1.8}$$

Επιπλέον υπάρχει και σημαντική ένδειξη για μια ακόμη ενεργειακή κλίμακα , την κλίμακα της μεγάλης ενοποίησης

$$M_{GUT} \sim 10^{16}\text{GeV} \tag{1.9}$$

όπου μέσα στο καθιερωμένο πρότυπο οι μεταβλητές σύζευξης τείνουν να συναντηθούν. Αυτό όμως συμβαίνει μόνον στα πλαίσια του υπερσυμμετρικού καθιερωμένου προτύπου, δίνοντας άλλο ένα σημάδι για την υπερσυμμετρία

Συγκεκριμένα,ο όρος αυτοαλληλεπίδρασης του πεδίου Χιγγς

$$\frac{\lambda}{4}(H^\dagger H)^2 \tag{1.10}$$

στον τύπο (1.4) θα γεννήσει έναν κβαντομηχανικό όρο ενός -βρόγχου ανάλογο με

$$\lambda \int^{\Lambda} d^4k \frac{1}{k^2 - M_H^2} \tag{1.11}$$

που συνεισφέρει στον τετραγωνικό όρο $H^\dagger H$. Αυτό θα δώσει μια θετική διόρθωση στο κλασσικό δυναμικό

$$\sim \lambda \Lambda^2 H^\dagger H \tag{1.12}$$

που θα οδηγήσει σε ένα ενός -βρόγχου διορθωμένο τετραγωνικό όρο

$$-\mu^2_{\text{φυσ.}} = -\mu^2 + \lambda\Lambda^2. \tag{1.13}$$

Ώστε να λάβουμε πραγματικά τις κβαντικές διορθώσεις υπ όψιν μας πρέπει να ελαχιστοποιήσουμε το βαθμωτό δυναμικό (2.4), αλλά τώρα χρησιμοποιώντας το $\mu^2_{\text{φυσ.}}$ αντί για το μ^2. Ας θυμηθούμε ότι η μάζα του σωματιδίου Χιγγς συνδέεται με το $\mu_{\text{φυσ.}}$ μέσω της σχέσης

$$M_h = \sqrt{2}\mu_{\text{φυσ.}}. \tag{1.14}$$

Ας θεωρήσουμε λοιπόν ότι πραγματικά νέα φυσική εμφανίζεται στην κλίμακα Plank (10^{19}GeV) ενώ η μάζα του σωματιδίου Χιγγς έχει μετρηθεί στα 126 GeV. Οπότε η παρακάτω θαυματουργή αλληλοεξουδετέρωση θα πρέπει να συμβεί

$$-126\text{GeV} = -\mu^2 + 10^{19}\text{GeV} \tag{1.15}$$

υπονοώντας ότι η κλασσική τιμή μ^2 θα πρέπει να είναι της τάξης ενεργειων Πλανκ ώστε να αναιρεί τη συνεισφορά του Λ^2 το οποίο επίσης είναι της τάξης ενεργειών Πλανκ με μια απίστευτη ακριβεια μερικων GeV. Αυτό είναι το πρόβλημα λεπτομερειακής ρύθμισης παραμέτρων του καθιερωμένου προτύπου. Πρέπει να τονισθεί ότι το πρόβλημα της λεπτομερειακής ρύθμισης των παραμέτρων παρόλο που γεννιέται από την μάζα του πεδίου Χιγγς δεν είναι συνδεδεμένη μόνον με αυτή εν τέλει όλες οι μάζες των σωματιδίων του καθιερωμένου προτύπου επιρεάζονται από αυτό.

Να σημειώσουμε εδώ ότι το πρόβλημα της λεπτομερειακής ρύθμισης παραμέτρων στο καθιερωμένο πρότυπο δεν είναι μόνον θέμα προσωπικού γούστου του καθενός, συχνά στη φυσική λύσεις σε προβλήματα ακαδημαικού ενδιαφέροντος έχουν οδηγήσει σε ραγδαίες εξελίξεις στον κλάδο της θεωρητικής φυσικής. Για παράδειγμα ,όταν ο Dirac πρότεινε την θεωρία των ηλεκτρονίων και ποζιτρονίων ώστε να λύσει το πρόβλημα αρνητικών ενεργειών του Klein-Gordon πεδίου ήταν αδύνατον να μη διπλασιαστεί ο αριθμός των στοιχειωδών σωματιδίων. Η αντιύλη (όπως ονομάστηκαν αυτά τα σωματίδια) ανακαλύφθηκε μόλις δέκα χρόνια αργότερα. Ένα άλλο παράδειγμα αποτελεί η αλληλεπίδραση τεσσάρων φερμιονίων του Fermi. Αυτή η αλληλεπίδραση λειτουργούσε ιδιαίτερα καλά για μέχρι συγκεκριμένες ενέργειες, αλλά ο Heisenberg είχε ήδη εκφράσει ανησυξίες για το γεγονός ότι αυτή η θεωρία χάνει την αξιοπιστία της για ενέργειες από τα 300GeV και πάνω- ενέργειες που εκείνη την εποχή ήταν εξωπραγματικές για το πείραμα. Αργότερα έγινε κατανοητό ότι αυτό το πρόβλημα της θεωρίας πύγαζε από την μη -επανακανονικοποιησημότητα της θεωρίας ,ένα καθαρά θεωρητικό πρόβλημα. Τελικά βρέθηκε η επανακανονικοποίήσιμη θεωρία που περιγράφει αυτή την αλληλεπίδραση στις υψηλότερες ενέργειες και έχει πια επιβεβαιωθεί και πειραματικά. Είναι το πρόβλημα της ιεραρχίας άλλο ένα παράδειγμα όπως τα παραπάνω,που σηματοδοτεί την ανάγκη για νέα φυσική;

Είναι κοινώς αποδεκτό ότι η πιο φυσική λύση για το πρόβλημα της ιεραρχίας θα ήταν η ύπαρξη μιας νέας ενεργειακής κλίμακας η οποία να βρίσκεται σχετικά κοντά στην ηλεκτρασθενή. Έτσι βρισκόμαστε αντιμέτωποι με τα παρακάτω προφανή ερωτήματα

- Ποιά είναι αυτή η νέα φυσική;
- Έχει και η ίδια αντίστοιχο πρόβλημα ιεραρχίας;
- Μπορούμε να εισάγουμε στα πλαίσια αυτής της θεωρίας και την βαρύτητα;

Έχουν προταθεί διάφορες ενδιαφέρουσες θεωρίες για την νέα φυσική, λίγες όμως παρέχουν την απαραίτητη βαθειά θεωρητική διαίσθηση.

Άς είμαστε αισιόδοξοι και ας φανταστούμε την καλύτερη περίσταση για να λυθεί το πρόβλημα της ιεραρχίας: Οι Λ^2 διορθώσεις στην μάζα του Χιγγς μποζονίου αυτόματα αλληλοαναιρούνται. Αυτό σημαίνει ότι η θεωρία μας θα πρέπει να περιέχει πολύ συγκεκριμένες αλληλεπιδράσεις που θα εγγυώνται αυτές τις αναιρέσεις. Η λύση στην πραγματικότητα δεν είναι πολύ μακριά, τέτοιου είδους κβαντικές διορθώσεις προέρχονται από αλληλεπιδράσεις τύπου Ψυκαβα του Χιγγς πεδίου με ένα φερμιόνιο. Πραγματικά, αγνοώντας τις εξωτερικές ορμές η ενός βρόγχου συνεισφορά από τέτοιου τύπου Yukawa αλληλεπίδραση δίνεται από

$$\left(-4g_f^2 \int^{\Lambda} d^4k \frac{1}{(\not{k} - m_f)^2}\right) H^\dagger H \tag{1.16}$$

οδηγώντας σε μια συνολική συνεισφορά από φερμιονικούς και μποζονικούς βρόγχους

$$(\lambda - g_f^2)\Lambda^2 H^\dagger H. \tag{1.17}$$

Αν θέσουμε

$$\lambda = g_f^2 \tag{1.18}$$

τότε οι συνεισφορές από διαγράμματα ενός βρόνγχου αλληλοαναιρούνται με ακρίβεια. Μπορεί να υπάρχει ένας βαθύτερος λόγος για να συμβεί μια τέτοια αλληλοαναίρεση; Εδώ είναι που συνεισφέρει η υπερσυμμετρία: τέτοιου είδους συσχετισμοί μεταξύ σταθερών σύζευξης είναι χαρακτηριστική στις υπερσυμμετρικές θεωρίες. Παρατηρούμε ότι η συνεισφορά ενός βρόνγχου του μποζονίου θα μπορούσε να αναιρεθεί μόνο από έναν φερμιονικό,λόγω της αντιστροφής προσήμου στο φερμιονικό βρόνγχο. Συνεπάγεται λοιπόν ότι μια τέτοια συμμετρία θα απαιτούσε ζευγάρια φερμιονίων και μποζονίων, άλλο ένα χαρακτηριστικό της υπερσυμμετρίας. Σκοπός αυτής της διατριβής είναι η μελέτη θεωριών τέτοιας υπερ-συμμετρίας βαθμίδας.

Το πρώτο ερώτημα που έρχεται στο μυαλό ενός θεωρητικού φυσικού είναι: Μπορεί η υπερσυμμετρία να γίνει μια τοπική συμμετρία; Η απάντηση είναι ότι αυτό γίνεται, είναι η θεωρία της υπερβαρύτητας. Εφόσον η υπερσυμμετρία είναι μια κλειστή Άλγεβρα μαζί με την πουανκαρέ, κάνοντας την υπερσυμμετρία τοπική απαιτεί οι καθολικοί μετασχηματισμοί συντεταγμένων να γίνουν τοπικοί, δηλαδή γενικοί μετασχηματισμοί συντεταγμένων. Είναι λοιπόν αναπόφευκτο αν θέλουμε μια θεωρία να είναι αναλλοίωτη κάτω από τοπική υπερσυμμετρία να μην είναι αναλλοίωτη κάτω από γενικούς μετασχηματισμούς συντεταγμένων. Αυτό σημαίνει ότι μια θεωρία τοπικής υπερσυμμετρίας εισαγάγει την βαρύτητα. Το μόνο που παραμένει είναι να ταυτοποιήσουμε το πεδίο βαθμίδας της υπερσυμμετρίας, αυτό είναι το λεγόμενο *gravitino* , το οποίο είναι ο υπερσυμμετρικός σύντροφος του βαρυτονίου (το κβαντο του πεδίου βαρύτητας). Πραγματικά από την δουλειά των Rarita και Schwinger μπορεί κανείς να δει ότι το φερμιόνιο με ιδιοστροφορμή 3/2 έχει μια συμμετρία βαθμίδας της μορφής

$$\delta\psi_m^\alpha = D_m \xi^\alpha \tag{1.19}$$

το οποίο σηματοδοτεί την ύπαρξη μιας θεωρίας βαθμίδας και διευκρινίζει ποιό είναι το πεδίο βαθμίδας. Ειναι πραγματικά αξιόλογο ότι η θεωρία της απλής υπερβαρύτητας περιλαμβάνει μόνο την δράση Einstein-Hilbert και την δράση Ραριτα-Σςηωινγερ για το φερμιονικό πεδίο. Η έννοια της υπερβαρύτητας συνοψίζεται στην μετατροπή των συναλλοίωτων ποσοτήτων σε υπερ-συναλλοίωτες.

Τα πρώτα χρόνια της υπερβαρύτητας υπήρχε η ελπίδα ότι τα θεωρήματα μη-επανακανονικοποιησημότητας της υπερσυμμετρίας θα μπορούσαν να τιθασεύσουν τους απειρισμούς που συναντάει κανείς στην προσπάθεια κβάντησης της γενικής θεωρίας της σχετικότητας. Παρόλο που η υπερβαρύτητα έχει σαφώς καλύτερη κβαντομηχανική συμπεριφορά από την γενική σχετικότητα, και στήν ίδια εμφανίζονται αποκλείνοντα διαγράμματα σε αρκετά υψηλή τάξη της θεωρίας διαταραχών. Η επί το πλείστον αποδεκτή άποψη αυτό τον καιρό είναι ότι η υπερβαρύτητα είναι η χαμηλών ενεργειών πραγμάτωση μιας πιο θεμελιακής και κβαντομηχανικά συνεπούς θεωρίας - Η θεωρία υπερχορδών . Πραγματικά είναι δυνατόν να υπολογίσει κανείς πλάτη σκέδασης στην θεωρία υπερχορδών και να συνεπάγει την ενεργό θεωρία που τα περιγραφει στη γλώσσα των στοιχειωδών σωματιδίων και προκύπτει ότι προβλέπεται ακριβώς το φάσμα μιας δεκαδιάστατης θεωρίας υπερβαρύτητας! Οπόταν η μελέτη των θεωριών υπερβαρύτητας φαίνεται να περιγράφει τον κόσμο μας σε ενέργειες πολύ χαμηλώτερες από την ενεργειακή κλίμακα της υπερχορδής όπου η δράση της κβαντικής βαρύτητας και άλλα φαινόμενα γίνονται σημαντικά.

Σε αυτή την διατριβή θα μελετήσουμε σύγχρονα θέματα στην υπερσυμμετρία και την υπερβαρύτυτα . Για να το κάνουμε αυτό πρέπει να χρησιμοποιήσουμε ένα φορμαλισμό που θα μας επιτρέπει να χειριζόμαστε τις υπερσυμμετρικές θεωρίες με ευκολία . Το μαθηματικό αυτό κατασκεύασμα λέγεται υπερχώρος. Η διατριβή λοιπόν ξεκινάει με την εισαγωγή τεχνικών του υπερχώρου. Εισαγάγουμε την έννοια των υπερπεδίων και πώς από αυτά μπορεί κανείς να διαβάσει τις συνιστώσες τους , οι οποίες χρησιμοποιούνται για να γράψουμε υπερσυμμετρικές δράσεις. Ειναι σημαντικό το γεγονός ότι οι Λαγκρατζιανές που βρίσκουμε με αυτήν την μέθοδο είναι σε μορφή που περιέχει όλα τα βοηθητικά πεδία της θεωρίας. Η υπερσυμμετρία έχει ένα μεγάλο εύρος από υπερπολλαπλέτες, εδώ θα αναφερθούμε σε αυτές που χρησιμοποιούμε στο κυρίως κείμενο οι οποίες είναι και οι πιο συχνά χρησιμοποιούμενες. Αφού παρουσιάσουμε τα βασικά εργαλεία της υπερσυμμετρίας επεκτείνουμε την συζήτηση στις θεωρίες υπερβαρύτητας. Είναι αξιόλογο το γεγονός ότι υπάρχουν δύο ξεχωριστοί συνδυασμοί βοηθητικών πεδίων για την τετραδιάστατη υπερβαρύτητα, ο καθένας εκ των οποίων έχει τις δικές του ιδιαιτερότητες και ξεχωριστό ενδιαφέρον. Η πρώτη έκδοση της τετραδιάστατης υπερβαρύτητας με βοηθητικά πεδία που ανακαλύφθηκε πέραν του βαρυτονίου και του *gravitino* που είναι τα φυσικά πεδία, περιέχει ένα μη- διαδιδόμενο μιγαδικό βαθμωτό πεδίο και ένα μη διαδιδόμενο πραγματικό διανυσματικό πεδίο· Αυτή είναι η θεωρία που αναφέρεται ως παλαιά-ελλάσουσα υπερβαρύτητα. Η νέα-ελλάσουσα υπερβαρύτητα ανακαλύφθηκε λίγο καιρό αργότερα και ενώ περιέχει τα ίδια φυσικά πεδία με την παλαιά, διαφέρει στα βοηθητικά πεδία . Τα βοηθητικά πεδία της νέας υπερβαρύτητας είναι ένα μη-διαδιδόμενο διανυσματικό πεδίο βαθμίδας και ένα μη-διαδιδόμενο πεδίο βαθμίδας 2-μορφή.. Είναι αξιόλογο ότι οι δύο αυτές θεωρίες στο επίπεδο των δύο παραγώγων είναι ισοδύναμες . Όταν όμως κανείς εισάγει όρους ανωτέρων παραγώγων αυτή η ισοδυναμία φαίνεται να καταρρέει.

Κατά την διάρκεια του διδακτορικού δουλέψαμε στο θέμα του σπασίματος της υπερσυμμετρίας και το πώς αυτό μεταδίδεται στα σωματίδια του καθιερωμένου προτύπου Το θέμα αυτό είναι πολύ επίκαιρο διότι μέχρι στιγμής δεν έχουν παρατηρηθεί οι υπερσυμμετρικοί σύντροφοι των σωματιδίων του καθιερωμένου προτύπου. Επίσης μελετήσαμε την περιγραφή των πιο πολλά υποσχόμενων πληθωριστικών μοντέλων στα πλαίσια της υπερβαρύτητας. Παρακάτω θα αναφερθούμε περιληπτικά στο κάθε θέμα.

1.2 Σπάσιμο Υπερσυμμετρίας από Τελεστές Ανωτέρας Διάστασης

Η υπερσυμμετρία είναι η πιο πολλά υποσχόμενη υποψήφια θεωρία για να περιγραφεί νέα φυσική. Δεν έχει παρατηρηθεί έως σήμερα ,οπότε θα πρέπει να είναι σπασμένη σε κάποια υψηλότερη ενεργειακή κλίμακα· αν είναι πραγματικά μια συμμετρία της φύσης. Τον κυρίαρχο ρόλο στο σπάσιμο της υπερσυμμετρίας τον έχει το βαθμωτό δυναμικό του τμήματος της θεωρίας που σπάει την υπερσυμμετρία. Τα βαθμωτά δυναμικά στην υπερσυμμετρία και στην υπερβαρύτητα έχουν μελετηθεί εκτενώς για θεωρίες μέχρι δύο παραγώγους. Παρόλο που είναι γνωστό ότι η εισαγωγή όρων ανωτέρας διάστασης ή ανωτέρων παραγώγων δύναται να αλλοιώσει την μορφή του δυναμικού, υπάρχουν παραδείγματα όπου η θεωρία κάπως αυτοπροστατεύεται από μη συμβατικά μη υπερσυμμετρικά κενά· αυτό όμως δεν συμβαίνει πάντα. Ο σκοπός μας εδώ είναι να μελετήσουμε πως τα βαθμωτά δυναμικά τροποποιούνται και μπορούν να οδηγήσουν σε σπάσιμο της υπερσυμμετρίας όταν κανείς εισάγει όρους ανωτέρας διάστασης.

Το φερμιόνιο *goldstone* που συσχετίζεται με το σπάσιμο της υπερσυμμετρίας, το *goldstino*, περιγράφεται από την $Volkov - Akoulov$ δράση,όπου η υπερσυμμετρία είναι μη γραμμικά πραγματωμένη . Συγκεκριμένα ,η δυναμική του *goldstino* έχει συνδεθεί με την χειραλική πολλαπλέτα X που εμφανίζεται στην σχέση παραίασης της διατήρησης του υπερρεύματος $Ferrara - Zumino$. Αυτό το χειραλικό υπερπεδίο στις χαμηλές ενέργειες ικανοποιεί τον σύνδεσμο

$$X_{NL}^2 = 0. \tag{1.20}$$

Η δυναμική του *goldstino* είναι ανεξάρτητη μικροσκοπικής περιγραφής. Πραγματικά,έχουμε βρεθεί διάφοροι εναλλακτικοί τρόποι περιγραφής του *goldstino* υπερπεδίου. Για παράδειγμα μπορεί να περιγραφεί από ένα χειραλικό υπερπεδίο,ένα υπερπεδίο διανύσματος ένα φερμιονικό υπερπεδίο ή ένα μιγαδικό γραμμικό πεδίο, το καθένα εκ των οποίων ικανοποιεί τον αντίστοιχο σύνδεσμο. Υπερπεδία με συνδέσμους έχουν χρησιμοποιηθεί πρόσφατα στα πλαίσια του υπερσυμμετρικού καθιερωμένου προτύπου καθώς και σε μοντέλα πληθωρισμού όπου το $inflaton$ (πληθωριστικό βαθμωτό πεδίο) ταυτοποιείται με τον υπερσυμμετρικό σύντροφο του *goldstino*.

Υπερσυμμετρικές θεωρίες που περιέχουν όρους ανωτέρων διαστάσεων (με παραγώγους ή χωρίς) έχουν κάποια ιδιαίτερα χαρακτηριστικά ανάμεσα σε αυτά ένα ενδιαφέρον χαρακτηριστικό είναι ότι τελεστές ανωτέρας διάστασης συνεισφέρουν στο βαθμωτό δυναμικό. Αυτό έχει συζητηθεί νωρίτερα σε μια σειρά εργασιών όπου μερικά παραδείγματα έχουν δοθεί. Συγκεκριμένα θεωρίες χωρίς δυναμικό στούς κυρίαρχους όρους ,μπορούν να αποκτήσουν μη τετριμμένο δυναμικό όταν όροι ανωτέρας διάστασης ληφθούν υπόψιν και δύναται να οδηγήσουν στο σπάσιμο της υπερσυμμετρίας. Σε αυτό το σημείο υπάρχουν όμως δυο σημαντικά θέματα. Το πρώτο αφορά την εμφάνιση ασταθειών . Στις θεωρίες που εμείς συζητάμε δεν υπάρχει αστάθεια διότι οι όροι ανωτέρων παραγώγων που χρησιμοποιούμε δεν οδηγούν σε επικίνδυνα ταχυονικά πεδία. Το δεύτερο θέμα αφορά τα βοηθητικά πεδία. Εδώ είναι ακόμη δυνατόν να λύσουμε τις εξισώσεις κίνησης αυτών των πεδίων εφόσον αυτές παραμένουν αλγεβρικές.

Μελετήσαμε διάφορες θεωρίες όπου εμφανίζεται αυθόρμητο σπάσιμο της υπερσυμμετρίας υπό την παρουσία τελεστών ανωτέρας διάστασης. Ιδιαίτερο ενδιαφέρον έχει ένα μοντέλο μιγαδικής γραμμικής υπερπολλαπλέτας. Μια τέτοια μιγαδική γραμμική υπερπολλαπλέτα περιέχει τους βαθμούς ελευθερίας μιας χειραλικής πολλαπλέτας και επί πλέον δυο φερμιόνια και ένα μιγαδικό διανυσματικό πεδίο. Στο επίπεδο των δυο παραγώγων ,τα επιπλέον φερμιόνια και το μιγαδικό διανυσματικό πεδίο περιγράφουν βοηθητικούς βαθμούς ελευθερίας, ενώ οι διαδιδόμενοι βαθμοί ελευθερίας περιγράφουν ένα ελευθερο μιγαδικό βαθμωτό πεδίο και ένα ελεύθερο φερμιόνιο. Λόγω της δομής της μιγαδικής γραμμικής υπερπολλαπλέτας, δεν υπάρχει τρόπος να γράψει κανείς υπερδυναμικό ούτε μπορεί να εισάγει τους συνήθεις όρους για αλληλεπιδράσεις που δεν περιέχουν παραγώγους. Οπότε πρέπει κανείς να βασιστεί στην εισαγωγή όρων ανωτέρας διάστασης η παραγώγων, ώστε να εμφανιστούν μη τετριμμένες αλληλεπιδράσεις και αναδυόμενα δυναμικά. Υπό συγκεκριμένες συνθήκες φαίνεται πως αυτά τα νέα δυναμικά μπορούν να οδηγήσουν τα βοηθητικά πεδία σε καινούρια ακρότατα που σπάνε την υπερσυμμετρία. Συγκεκριμένα οι εξισώσεις κίνησης για το βοηθητικό πεδίο έχουν την μορφή

$$F\left(1-\frac{1}{2\Lambda^4}F\bar{F}\right)=0. \tag{1.21}$$

Παρατηρούμε ότι υπάρχουν δυο λύσεις για αυτήν την εξίσωση:

$$(i) \quad F = 0\,, \tag{1.22}$$

$$(ii) \quad F\bar{F} = 2\Lambda^4. \tag{1.23}$$

Η πρώτη λύση περιγράφει το σύνηθες υπερσυμμετρικό κενό, ενώ η δεύτερη πραγματικά σηματοδοτεί το σπάσιμο της υπερσυμμετρίας. Σε αυτήν την περίπτωση , νέες φάσεις αναδύονται μόνον μία εκ των οποίων συνδέεται με την αρχική θεωρία. Πρέπει να σημειώσουμε ότι αυτές οι νέες φάσεις δεν θα έπρεπε να θεωρηθούν ως διαφορετικές φάσεις της ίδιας θεωρίας, αλλά καλύτερα ως διαφορετικές θεωρίες. Προσπάθειες να κατασκευασθούν τέτοιου είδους υπερσυμμετρικές θεωρίες, στις οποίες να υπάρχουν περισσόρερες από μια λύσεις για τα βοηθητικά πεδία έχουν γίνει και παλαιότερα, χωρίς επιτυχία. Η ιδιαίτερη δομή της μιγαδικής γραμμικής υπερπολλαπλέτας επιτρέπει να δημιουργηθούν επιτυχώς τέτοιες θεωρίες.

Άλλο ένα πολύ ενδιαφέρον φαινόμενο λαμβάνει χώρα σε αυτό το μοντέλο. Το *goldstino* αυτής της θεωρίας προκύπτει να είναι ένα πρώην βοηθητικό πεδίο , το οποίο στο κενό που σπάει την υπερσυμμετρία αποκτάει κανονικό κινητικό όρο και γίνεται διαδιδόμενο. Αυτός ο κινητικός όρος βέβαια στο υπερσυμμετρικό κενό μηδενίζεται και το πεδίο παραμένει βοηθητικό, ενώ προφανώς δεν υπάρχει ούτε *goldstino*. Παρατηρούμε λοιπόν ότι στην σπασμένη φάση υπάρχει μια μη ισότητα διαδιδόμενων φερμιονικών και μποζονικών βαθμών ελευθερίας· ένα χαρχκτηριστικό μη γραμμικής πραγμάτωσης της υπερσυμμετρίας. Πραγματι, αυτό πού συμβαίνει στη σπασμένη φάση της θεωρία μας περιγράφεται από μια κανονική υπερσυμμετρική πολλαπλέτα και ένα $Volkov - Akulov$ πεδίο αποσαφηνίζοντας την διαφορά στους διαδιδόμενους βαθμούς ελευθερίας. Η λύση στις εξισώσεις κίνησης των υπερπεδίων δίνεται από την σχέση

$$\Sigma = X_{NL} + \bar{\Phi} \tag{1.24}$$

όπου X_{NL} συμβολίζει το $Volkov - Akulov$ πεδίο, $\bar{\Phi}$ περιγράφει μια ελεύθερη χειραλική υπερπολλαπλέτα, ενώ Σ είναι η μιγαδική γραμμική υπερπολλαπλέτα.

Τέλος, μελετήθηκαν και άλλα παραδείγματα με αντίστοιχες ιδιότητες στα οποία χρησιμοποιήσαμε πραγματικά γραμμικά υπερπεδία ή πραγματικά υπερπεδία διανύσματος ή και φερμιονικά χειραλικά υπερπεδία καταλήγοντας ότι και αυτές οι θεωρίες έχουν την δυνατότητα να παρέχουν ενδιαφέροντα αποτελέσματα όπως τα παραπάνω.

1.3 Σπάσιμο Υπερσυμμετρίας και Σωματιδιακή Φυσική

Από τον καιρό της ανακάλυψης της υπερσυμμετρίας το ερώτημα του προσδιορισμού της υπερσυμμετρικής θεωρίας που περιγράφει τις αλληλεπιδράσεις του καθιερωμένου προτύπου είναι ένα από τα πιο καίρια προβλήματα της φυσικής υψηλών ενεργειών. Επιπλέον είναι πια αδιαμφισβήτητο ότι το περίφημο μποζόνιο Χιγγς έχει βρεθεί στον επιταχυντή LHC. Αυτό το γεγονός έχει αναζωογονήσει το ενδιαφέρον στην σωματιδιακή φυσική αφού , η μάζα του και η σύζευξή του με τα υπόλοιπα σωματίδια του καθιερωμένου προτύπου ενδέχεται να αποκαλύψει που είναι κρυμμένη η νέα φυσική. Οι υπερσυμμετρικές επεκτάσεις του καθιερωμένου προτύπου έχουν , εκτός των άλλων, την δυνατότητα να δίνουν μια λύση στο πρόβλημα της ιεραρχίας , να επιτρέπουν την ενοποίηση των μεταβλητών σύζευξης,να παρέχουν υποψήφια σωματίδια για μικροσκοπική περιγραφή της σκοτεινής ύλης και τέλος να παρέχουν μια εξήγηση δυναμικού τύπου για την ηλεκτρασθενή ενεργειακή κλίμακα. Πραγματικά,είναι δύσκολο να φανταστούμε έναν υποψήφιο καλύτερα από την υπερσυμμετρία για την φυσική πέραν του καθιερωμένου προτύπου στην περίπτωση ενός θεμελιακού σωματιδίου Χιγγς.

Στην ελλάσονα υπερσυμμετρική επέκταση του καθιερωμένου προτύπου, ο τομέας Χιγγς αποτελείται από ένα ζευγάρι υπερπολλαπλέτων H_u καί H_d. Είναι πια κοινώς αποδεκτό ότι οποιαδήποτε υπερσυμμετρική επέκταση του καθιερωμένου προτύπου θα περιέχει απαραιτήτως και τα δυο υπερπεδία Χιγγς. Ο λόγος για αυτό είναι διπλός: Πρώτον δυο Χιγγς πεδία είναι απαραίτητα ώστε να δίνουν μάζες στα *up* και *doun quark* διότι η ολομορφικότητα του υπερδυναμικού δεν επιτρέπει την απαραίτητη σύζευξη τύπου $Yukawa$ ώστε να πάρουν μάζα και των δυο τύπων *quark* με την χρήση ενός και μόνον υπερπεδίου Χιγγς. Δεύτερον, από απλή μελέτη κβαντικών ανωμαλιών οδηγεί στην ανάγκη εισαγωγής και δεύτερου Χιγγς υπερπεδίου ώστε αυτές να εξαληφθούν. Οπότε,είτε μπορεί κανείς να θεωρήσει ακριβή υπερσυμμετρία με δυο υπερπεδία Χιγγς, είτε εναλλακτικά,μπορεί για παράδειγμα να αγνοήσει τελείως το δεύτερο Χιγγς, με το κόστος του σκληρού σπασίματος της υπερσυμμετρίας , το οποίο θα προέρχεται από την μη ολομορφικότητα του δυναμικού. Μια δυσκολία τέτοιων θεωριών είναι ότι με την απουσία του δεύτερου υπερπεδίου Χιγγς, το υπερσυμμετρικό ζευγάρι του καθιερωμένου πεδίου Χιγγς θα παραμείνει άμαζο μέχρι το σπάσιμο της ηλεκτρασθενούς συμμετρίας. Επιπλέον η αναίρεση των ανωμαλιών, οι οποίες προηγουμένως εξουδετερώνονταν από το δεύτερο Χιγγς, απαιτεί την εισαγωγή άλλων νέων πεδίων σε κατάλληλες αναπαραστάσεις. Αυτά τα νέα πεδία θα πρέπει να είναι και χειραλικά και αρκετά βαριά ώστε να μην αλλοιώνουν την γνωστή φυσική χαμηλών ενεργειών. Αυτή είναι επίσης και η περίπτωση σε μοντέλα με δυο Χιγγς πεδία, όπου το δεύτερο Χιγγς είναι απλά ένας παρατηρητής,χωρίς να παίρνει μη μηδενική αναμενόμενη τιμή στο κενό και ούτε έχει σύζευξη με τα φερμιόνια.

Όπως προαναφέραμε η υπερσυμμετρία πρέπει να είναι σπασμένη , υποννοώντας την ύπαρξη ενός φερμιονίου τύπου *goldstone* . Σε ένα γενικό σενάριο σπασίματος της υπερσυμμετρίας το *goldstino* έχει ένα μιγαδικό βαθμωτό πεδίο για υπερσύντροφο το οποίο γενικά θα έχει μάζα· Επειδή καμμιά συμμετρία δεν προστατεύει αυτήν την μάζα μπορεί να γίνει γενικά πολύ μεγάλη, και το σωματίδιο αυτό να αποσυζευχθεί από την φυσική χαμηλών ενεργειών. Μελετήθηκαν μοντέλα υπερσυμμετρίας και υπερβαρύτητας όπου υπάρχει αυθόρμητο σπάσιμο της υπερσυμμετρίας. Βρέθηκε ότι στο όριο της άπειρης μάζας για το βαθμωτό πεδίο οι εξισώσεις κίνησης του υπερπεδίου που σπάει την υπερσυμμετρία διαχωρίζονται σε δυο κομμάτια. Το ένα κομμάτι αφορά την αποσύζευξη του υπερβαρέως βαθμωτού σωματιδίου που έχει ως αποτέλεσμα αυτό το κομμάτι να μετατραπεί σε σύνδεσμο για το υπερπεδίο του *goldstino*

$$X^2 = 0. \tag{1.25}$$

Το υπερπεδίο τυ *goldstino* συνεχίζει να αλληλεπιδρά με τα υπόλοιπα υπερπεδία όμως τώρα ικανοποιεί τον προαναφερθέντα σύνδεσμο και συγχρόνως είναι υπεύθυνο για την μετάδοση του σπασίματος της υπερσυμμετρίας στον τομέα των υπερπεδίων του καθιερωμένου προτύπου. Σε τέτοιες περιπτώσεις η ενεργός θεωρία χαμηλών ενεργειών περιγράφεται από μη γραμμική υπερσυμμετρία.

Υπάρχουν στην βιβλιογραφία πλήθος μεθόδων για να περιγραφεί η σύζευξη του *goldstino* και η μη γραμμική υπερσυμμετρία. Μεταξύ αυτών μια πολύ ενδιαφέρουσα μεθοδολογία έγκειται στην χρήση υπερπεδιων που ικανοποιούν συγκεκριμένους συνδέσμους. Έχουμε μελετήσει την σύζευξη του μη γραμμικού (*goldstino*) τομέα με το υπερσυμμετρικό καθιερωμένο πρότυπο μέσω τελεστών ανώτερης διάστασης στον υπερχώρο. Τέτοιες συζεύξεις έχουν επίσης μελετηθεί και σε μια σειρά από διάφορες εργασίες για την σύζευξη του πεδίου *goldstino* με το μη-γραμμικό υπερσυμμετρικό καθιερωμένο πρότυπο. Για την χρήση του συγκεκριμένου φορμαλισμού πρέπει να υποθέσουμε ότι η υπερσυμμετρία είναι αυθόρμητα σπασμένη σε μια ενεργειακή κλίμακα $\sqrt{f}$, η οποία πρέπει να βρίσκεται στην ενεργειακή περιοχή μερικών TeV. Εδώ μελετήσαμε την ενεργειακή περιοχή γύρω από την ενεργειακή κλίμακα m_{soft} (μάζες των υπερσυντρόφων των σωματιδίων του καθιερωμένου προτύπου) αλλά κάτω από το $\sqrt{f}$ όπου η υπερσυμμετρία είναι μη γραμμικά πραγματωμένη στο πεδίο *goldstino*.

Στις θεωρίες που μας ενδιαφέρουν το πεδίο H_d δεν θα χρησιμοποιηθεί πια για την κατασκευή συζεύξεων τύπου $Yukawa$. Όσον αφορά τον μηχανισμό γέννησης μάζας για τα $quarks$ (και αντίστοιχα για τα λεπτόνια), η $Yukawa$ σύζευξη

$$\int d^2\theta \bar{d} Q \cdot H_d \tag{1.26}$$

δεν είναι διαθέσιμη πια. Στα μοντέλα που μελετήσαμε η γέννηση μάζας για τα φερμιόνια επιτυγχάνεται μέσω της χρήσης του *goldstino* υπερπεδίου και μόνον του ενός Χιγγς πεδίου H_u,και της σύζευξης

$$\frac{m_{soft}}{f\,\Lambda}\int d^2\theta d^2\bar{\theta}\bar{X}\bar{H}_u e^V Q\bar{d} \tag{1.27}$$

όπου Λ είναι η ενέ́ργεια αποκοπής της θεωρίας. Χρησιμοποιώντας τέτοιου είδους αλληλεπιδράσεις κατασκευάσαμε συνεπείς μη γραμμικές υπερσυμμετρικές επεκτάσεις του καθιερωμένου προτύπου που περιέχουν:

- Ένα μοναδικό πεδίο Χιγγς H_u όπου το δεύτερο πεδίο Χιγγς H_d έχει αντικατασταθεί από μια μισή οικογένεια, και
- ένα καθιερωμένο πεδίο Χιγγς H_u όπου το δεύτερο πεδίο Χιγγς H_d είναι πια μόνο ένας παρατηρητητής.

Στις συγκεκριμένες υπερσυμμετρικές επεκτάσεις του καθιερωμένου προτύπου δεν υπάρχει το σύνηθες μ-πρόβλημα αφού δεν υπάρχει δεύτερο Χιγγς πεδίο. Τα συγκεκριμένα μοντέλα πέραν του ότι κάνουν χρήση μόνο ενός Χιχχς πεδίου έχουν τις ακόλουθες δυο πολύ ενδιαφέρουσες ιδιότητες: Πρώτον, αναδύεται μια φυσική ιεραρχία για τις μάζες των βαρέων φερμιονίων της μορφής

$$m_b \sim m_\tau \sim \frac{m_{soft}}{\Lambda}m_t. \tag{1.28}$$

Δεύτερον,το δυναμικό του πεδίου Χιγγς έχει την μορφή

$$\mathcal{V} \;=\; f^2 + m_u^2|H_u|^2 + \frac{1}{f^2}m_u^4|H_u|^4 + \frac{g_1^2+g_2^2}{8}|H_u|^4 + \mathcal{O}(\frac{1}{f^3}) \tag{1.29}$$

στην οποία παρατηρούμε ότι συνεισφέρουν και διορθώσεις ($\frac{1}{f^2}m_u^4|H_u|^4$) που οφείλονται μόνο και μόνο στην μη γραμμική πραγμάτωση της υπερσυμμετρίας. Στα συνήθη υπερσυμμετρικά καθιερωμένα μοντέλα υπάρχει ένα πάνω όριο στην κλασσική μάζα του πεδίου Χιγγς το οποίο είναι $92GeV$. Στα συγκεκριμένα μοντέλα όμως λόγω της συνεισφοράς στο δυναμικό από την μη γραμμική υπερσυμμετρία αυτό το όριο μπορεί να ανεβεί αισθητά διότι εξαρτάται πια και από την ενεργειακή κλίμακα του σπασίματος της υπερσυμμετρίας.

1.4 Θεωρίες Υπερβαρύτητας Ανωτέρων Παραγώγων με Εφαρμογή στην Κοσμολογία

Η πιο γενική θεωρία που περιγράφει την διάδοση ενός άμαζου βαθμού ελευθερίας ιδιοστροφορμής δυο,και ενός βαθμωτού βαθμού ελευθερίας *δεν* είναι η γενική θεωρία της σχετικότητας συζευγμένη με τον ελλάσονα τρόπο με ένα βαθμωτό πεδίο. Πράγματι, ο *Horndeski* απέδειξε ότι θεωρίες αλληλεπίδρασης βαθμωτού πεδίου με την βαρύτητα που έχουν εξισώσεις κίνησης μέξρι δεύτερης τάξης δεν είναι περιορισμένες στις προαναφερθείσες θεωρίες ελλάσουσας σύζευξης. Μέχρι όρους τετραγωνικούς στα βαθμωτά πεδία και για τέσσερεις διαστάσεις , ο *Horndeski* έδειξε ότι οι πιο γενικές θεωρίες που περιγράφουν την διάδοση του πεδίου βαρύτητας και ενός βαυμωτού πεδίου είναι

$$\mathcal{L} = \mathcal{L}_{\rm GRM} \pm \frac{1}{M_I^2}\mathcal{L}_I \;\pm \frac{1}{M_{II}^2}\mathcal{L}_{II} + \xi\mathcal{L}_{III}\ , \tag{1.30}$$

όπου

$$
\begin{aligned}
\mathcal{L}_{\mathrm{GRM}} &= \frac{1}{2}\left[M_P^2 R - \partial_a\phi\partial^a\phi\right], &(1.31)\\
\mathcal{L}_I &= \left(M_\phi^I\phi+\phi^2\right) R_{GB}^2\,, &(1.32)\\
\mathcal{L}_{II} &= G^{mn}\partial_m\phi\partial_n\phi\,, &(1.33)\\
\mathcal{L}_{III} &= \left(M_\phi^{III}\phi+\phi^2\right) R\,, &(1.34)
\end{aligned}
$$

και

$$
G_{mn} = R_{mn} - \frac{1}{2}g_{mn}R\,,\quad R_{GB}^2 = R_{mn\gamma\delta}R^{mn\gamma\delta} - 4R_{mn}R^{mn} + R^2 \tag{1.35}
$$

είναι οι $Einstein$ και $Gauss - Bonnet$ τανηστές ,αντίστοιξα, $M_{(I,II)}, M_\phi^{I,II}$ είναι ενεργειακές κλίμακες, ξ είναι μια σταθερά και M_P είναι η ενεργειακή κλίμακα Πλανκ. Το ότι η Λαγκρατζιανή πυκνότητα $\mathcal{L}_I$ οδηγεί σε εξισώσεις κίνησης δεύτερης τάξης φαίνεται εύκολα από το γεγονός ότι ο συνδυασμός $Gauss - Bonnet$ είναι ολική παράγωγος στις τέσσερεις διαστάσεις και είναι γραμμικός στις παραγώγους δεύτερης τάξης. Αντίστοιχα είναι εύκολο κανείς να δει ότι η Λαγκραντζιανή πυκνότητα $\mathcal{L}_{II}$ οδηγεί επίσης σε εξισώσεις κίνησης κάνοντας την ανάλυση των επιμέρους συνιστωσών στον ADM φορμαλισμό.

Η Λαγκρατζιανή πυκνότητα $\mathcal{L}_{\mathrm{GRM}}$ δεν είναι παρά η καθιερωμένη θεωρία της υπερβαρύτητας. Η σύζευξη με την υπερβαρύτητα της θεωρίας $\mathcal{L}_I$ έχει πραγματοποιηθεί από άλλους. Η σύζευξη της θεωρίας $\mathcal{L}_{III}$ με την τετραδιάστατη υπερβαρύτητα διαφέρει από την ελλάσουσα σύζεψξη $\mathcal{L}_{\mathrm{GRM}}$ κατά έναν $Weyl$ μετασχηματισμό της μετρικής. Η μόνη λοιπόν θεωρία που δεν είχε μέχρι πρότινος συζευξθεί με την βαρύτητα ήταν η $\mathcal{L}_{II}$. Η επιδίωξή μας ήταν η συνεπής σύζευξη της θεωρίας $\mathcal{L}_{II}$ με την υπερβαρύτητα.

Πέραν του προφανούς ενδιαφέροντος που έχει η μελέτη κατασκευής των πιο γενικών υπερσυμμετρικών θεωριών που δεν υπόκεινται σε αστάθειες λόγω $ghost$ πεδίων είναι σημαντικό να παρατηρήσουμε ότι η παραπάνω συνεπείς θεωρίες ανωτέρων παραγώγων εμφανίζονται συχνά στην θεωρία υπερχορδών. Συγκεκριμένα ο όρος $\mathcal{L}_{II}$ εμφανίζεται στην 10-διαστατη ενεργό θεωρία της ετεροτικής χορδής. Να επισημάνουμε ότι τέτοιοι όροι συνδέονται μεταξύ τους στις ενεργές θεωρίες των υπερχορδών μέσω επαναπροσδιορισμού των πεδίων.

Από μια πιο φαινομενολογική σκοπιά η θεωρία $\mathcal{L}_{II}$ παίζει τον κυρίαρχο ρόλο στον λεγόμενο μηχανισμό ¨Ενδυναμωμένη Βαρυτικά Τριβή' (ΕΒΤ). Λόγω αυτού του μηχανισμού ΕΒΤ ένα απόκρημνο (ή και όχι) βαθμωτό δυναμικό, δύναται έν γένει να προκαλέσει κοσμικό πληθωρισμό για μια σχετικά μικρή ενεργειακή κλίμακα M_{II}. Αυτό οφείλεται στην δημιουργία ενός φαινομένου τριβής που προκαλείται από την επέκταση του σύμπαντος δρώντας στο βαθμωτό πεδίο που προκαλεί τον πληθωρισμό ($inflaton$). Είναι προφανές ότι η υλοποίηση ενός τέτοιου μηχανισμού στην θεωρία της υπερβαρύτητας πραγματικά αυξάνει την δυνατότητα να βρει κανείς πληθωριστικά σενάρια μέσα στην θεωρία της υπερβαρύτητας και κατ΄ έπέκταση στην θεωρία υπερχορδών.

Όλες οι προσπάθειες κατασκευής θεωριών υπερβαρύτητας ανωτέρων παραγώγων στις τέσσερεις διαστάσεις βασίζονται σε μεθοδολογίες που χρησιμοποιούν και τα λεγόμενα βοηθητικά πεδία. Η διαφορά θεωριών υπερβαρύτητας με βοηθητικά πεδία ,από αυτές που δεν έχουν τέτοια, έγκειται στο ότι οι πρώτες δεν είναι μοναδικές. Αυτό το φαινόμενο παρατηρείται επίσης στις θεωρίες υπερσυμμετρίας όπου υπάρχουν περισσότεροι του ενός τρόποι να μοιράσει κανείς τους απαραίτητους βοηθητικούς βαθμούς ελευθερίας σε διαφορετικού είδους μποζονικά πεδία Για παράδειγμα είναι γνωστόν ότι υπάρχουν περισσότερες από μια υπερπολλαπλέτες οι οποίες έχουν την δυνατότητα να περιγράψουν ένα ελεύθερο διαδιδόμενο μιγαδικό βαθμωτό πεδίο και ένα ελεύθερο διαδιδόμενο φερμιόνιο. Αυτές είναι οι υπερπολλαπλέτες γνωστές ως χειραλική, πραγματική γραμμική και μιγαδική γραμμική, οι οποίες είναι γνωστό ότι συνδέονται μεταξύ των με δυαδικούς μετασχηματισμούς. Προφανώς αυτά τα υπερπεδία όταν είναι ελεύθερα ταυτίζονται.

Το ίδιο φαινόμενο επιμένει να παρουσιάζεται και στις θεωρίες τοπικής υπερσυμμετρίας όπου υπάρχουν περισσότεροι του ενός τρόποι να μοιράσει κανείς τούς απαραίτητους βοηθητικούς βαθμούς ελευθερίας σε μη διαδιδόμενα πεδία. Υπάρχει λοιπόν μια κάποια ποικιλία από θεωρίες υπερβαρύτητας οι οποίες διαφέρουν μόνο στον μη διαδιδόμενο τομέα. Ενώ και πάλι ελεύθερες αυτές οι θεωρίες ταυτίζονται. Αυτό συμβαίνει διότι τα υπερπεδία της $\mathcal{N} = 1$ υπερσυμμετρίας περιέχουν αναγωγήσιμες αναπαραστάσεις της υπερσυμμετρίας και πρέπει κανείς να χρησιμοποιήσει επιπλέον περιορισμούς ώστε να ξεχωρίσει την μη αναγώγιμη αναπαράσταση που τον ενδιαφέρει. Στην θεωρία λοιπόν της υπερβαρύτητας οι ταυτότητες *Bianchi* στον υπερχώρο λύνονται με την χρήση επιπλέον περιορισμών οδηγώντας σε διαφορετικές θεωρίες υπρβαρύτητας λόγω της διαφοράς τους στα βοηθητικά πεδία. Αυτές οι θεωρίες είναι για παράδειγμα η 12+12 παλαιά-ελλάσουσα θεωρία υπερβαρύτητας και η 12+12 νέα-ελλάσουσα θεωρία υπερβαρύτητας. Υπάρχει στην πραγματικότητα τουλάχιστον άλλη μια γνωστή 12+12 ελλάσουσα θεωρία υπερβαρύτητας, η οποία σπανίως έχει χρησιμιποιηθεί γιατί τα βοηθητικά της πεδία ειναι *Hodge* δυαδικά με αυτά της παλαιάς-ελλάσουσας. Για τον ίδιο λόγο δεν έχει χρησιμοποιηθεί και τόσο συχνά η νέα-ελλάσουσα υπερβαρύτητα διότι υπάρχει δυαδικός μετασχηματισμός που μας μεταφέρει από την μια θεωρία στην άλλη. Ακόμη περισσότερο αυτή η δυαδικότητα μπορεί να εφαρμοστεί ανάμεσα σε αυτές τις θεωρίες ακόμη και αν αυτές είναι συζευγμένες με ύλη. Γιαυτόν τον λόγο συχνά κανείς περιορίζεται στην μελέτη μόνον της παλαιάς-ελλάσουσας υπερβαρύτητας, η οποία και αναφέρεται και ως συνήθης υπερβαρύτητα. Είναι όμως γνωστό ότι αυτή η δυαδικότητα παύει να ισχύει με την εισαγωγή υπερβαρυτικών αλληλεπιδράσεων ανωτέρων παραγώγων. Πρέπει να αναφερθεί ότι γι αυτόν τον λόγο μόνο μια εκ των παραπάνω ελλάσουσα θεωρία υπερβαρύτητας πραγματικά δύναται να περιγράψει την φυσική της ετεροτικής χορδής· Αυτή θεωρείται ότι είναι η νέα-ελλάσουσα υπερβαρύτητα.

Ο σκοπός μας λοιπόν ήταν η σύζευξη της θεωρίας $\mathcal{L}_{II}$ με την υπερβαρύτητα . Αυτό πραγματικά επιτεύξθηκε στα πλαίσια της νέας- ελάσσουσας υπερβαρύτητας και η Λαγκρατζιανή πυκνότητα που περιγράφει μια τέτοια θεωρία στον συγκεκριμένο υπερχώρο είναι

$$\mathcal{L} = \int d^4\theta \, E \left\{ M_P^2 V_{\mathrm{P}} + \Phi\bar{\Phi} + w^2 \left[i\bar{\Phi}E^a \nabla_a^- \Phi + h.c. \right] \right\} \tag{1.36}$$

η οποία όταν γραφεί σε μορφή συνιστωσών και αφού ολοκληρώσουμε τις εξισώσεις τών βοηθητικών πεδίων, παίρνει την μορφή

$$e^{-1}\mathcal{L} \;\; = \;\; \frac{1}{2\kappa^2}\mathcal{R} + A\Box A^* + w^2\, G^{mn}\, \partial_m A^*\, \partial_n A. \tag{1.37}$$

Πρέπει να αναφέρουμε ότι το χειραλικό υπερπεδίο Φ πρέπει να έχει ουδέτερο R-φορτίο για την αποφυγή ασταθειών στην θεωρία. Αυτό συνεπάγεται ότι η R-συμμετρία απαγορεύει την εισαγωγή υπερδυναμικού στην θεωρία μας· δηλαδή δεν υπάρχει F-τύπου δυναμικό. Επίσης η μελέτη που κάναμε έδειχε ότι το χειραλικό υπερπεδίο Φ πρέπει επίσης να είναι ουδέτερα φορτισμένο και κάτω από αβελιανές ή μη συμμετρίες· δηλαδή δεν υπάρχει ούτε D-τύπου δυναμικό. Συνεπάγεται λοιπόν ότι είναι αδύνατον αυτή η θεωρία να αποκτήσει υπαρσυμμετρικό δυναμικό. Αυτό όμως δεν επηρεάζει καθόλου την χρησιμότητα της θεωρίας στην κοσμολογία αφού είναι γνωστό ότι κατά την διάρκεια του πληθωρισμού η υπερσυμμετρία είναι σπαχσμένη. Επίσης το σπάσιμο της υπερσυμμετρίας συνδέεται άμεσα με το σπάσιμο της R-συμμετρίας στην νέα-ελάσσουσα υπερβαρύτητα. Οπότε οποιοσδήποτε μηχανισμός σπασίματος της υπερσυμμετρίας θα παραβιάσει και την R-συμμετρία οδηγώντας εύκολα στην δημιουργία δυναμικού για το βαθμωτό πεδίο μας, το οποίο χάριν του μηχανισμού ΓΕΒ ενεργοποιέι με ευκολία μια περίοδο πληθωρισμού.

Έχουν πρόσφατα ανακαλυφθεί μια σειρά από μη-επανακανονικοποιήσημες θεωρίες βαθμωτών πεδίων με την ιδιότητα η ενεργειακή σκάλα που τις χαρακτηρίζει να παραμένει σταθερή ενεργειακά. Επιπλέον αυτές οι θεωρίες στον επίπεδο χώρο έχουν μια συμμετρία η οποία αποκαλείται *Galileon* μετάθεση

$$\pi \to \pi + c + b_m x^m \ , \tag{1.38}$$

όπου s,b_m,είναι μια σταθερά και ένα σταθερό διάνυσμα αντίστοιχα και π είναι το λεγόμενο πεδίο *Galileon*. Η απαίτηση αυτή η θεωρία να έχει μόνον δεύτερης τάξης εξισώσεις κίνησης, ώστε να αποφευχθούν αστάθειες , περιορίζει τις θεωρίες τέτοιου τύπου κατά πολύ.

Από τότε που ανακαλύφθηκαν αυτές οι θεωρίες έχουν γίνει αρκετές προσπάθειες υπερσυμμετροποίησής τους. Μέχρι πρότινος καμμιά προσπάθεια δεν είχε πετύχει ,διότι τα συγκεκριμένα μοντέλα που είχαν προταθεί μόνο σε συγκεκριμένα υπόβαθρα μοιάζαν να δίνουν θεωρίες τύπου *Galileon* ενώ όταν κανείς μελετούσει την γενική θεωρία ή ξέφευγε λιγάκι από τα συγκεκριμένα υπόβαθρα καταστροφικές αστάθειες εμφανίζονταν. Έχει λοιπόν προταθεί από κάποιους ερευνητές ότι κυβικές θεωρίες *Galileon* τύπου δεν μπορούν να συζευχθούν με την υπερσυμμετρία.

Σκοπός μας είναι η μελέτη κατά πόσο ε·ιναι εφικτό να κατασκευάσει κανείς θεωρίες *Galileon* συζευγμένες με την υπερσυμμετρία. Αν και τα αποτελέσματά μας επιβεβαιώνουν την εικασία ότι δεν υπάρχουν υπερσυμμετρικές κυβικές *Galileon* θεωρίες, αυτό δεν μπορεί να αποδειχθεί γενικά. Ακολουθώντας όμως έναν έμεσο δρόμο καταφέραμε να κατασκευάσουμε την πρώτη υπερσυμμετρική *Galileon* θεωρία της οποίας η μορφή στον υπερχώρο είναι η εξής

$$\mathcal{L}_2 = \int d^4\theta (\bar{\Phi}\Phi - \frac{1}{\Lambda^6}\Phi(\bar{D}_{\dot{\alpha}}\partial_m\bar{\Phi}\bar{\sigma}_n^{\dot{\alpha}\alpha}D_\alpha\partial_r\Phi)\epsilon^{mnrs}\partial_s\bar{\Phi}). \tag{1.39}$$

Ο μποζονικός τομέας τέτοιων θεωριών ταυτίζεται με το μιγαδικό τετραπλό *Galileon*

$$\mathcal{L}_{\rm gal} = \pi\partial^2\bar{\pi} - \frac{4}{\Lambda^6}\pi(\partial_{[k}\partial^k\bar{\pi})(\partial_l\partial^l\pi)(\partial_{\zeta]}\partial^\zeta\bar{\pi}) \tag{1.40}$$

επιβεβαιώνοντας ότι πραγματικά έχουμε στο χέρι μια *Galileon* θεωρία.

Τους μετασχηματισμούς (1.38) τους επεκτείναμε στον υπερχώρο. Συγκεκριμένα,για ένα χειραλικό υπερπεδίο προτείναμε ότι ο συνεπής τρόπος επέκτασής τους είναι

$$\Phi \to \Phi + a + b_m y^m. \tag{1.41}$$

όπου

$$y^m = x^m + i\theta\sigma^m\bar{\theta} \tag{1.42}$$

να σημειωθεί ότι ο *Galileon*,μετασχηματισμός που προτείνουμε πραγματικά αναπαράγει μια *Galileon* μετάθεση για την βαθμωτή χαμηλώτερη συνιστώσα του χειραλικού πεδίου Φ, ενώ συγχρόνως διατηρεί την χειραλικότητά του. Η θεωρία (1.40) είναι αναλλοίωτη κάτω από την μετάθεση (1.41), και πάλιν επιβεβαιώνοντας ότι έχουμε μια *Galileon* θεωρία. Να σημειωθεί ότι όλες οι προαναφερθείσες προβληματικές θεωρίες δεν έχουν αυτή την συμμετρία . Η επέκταση λοιπόν της *Galileon* σψμμετρίας στον υπερχώρο αποτελεί πραγματικά ένα αξιόπισο κριτήριο που μπορεί να χρησιμοποιηθεί κατευθείαν στις υπερσυμμετρικές θεωρίες γραμμένες συναρτήσει υπερπεδίων ώστε να διευκρινισθεί εάν τελικά πρόκειται για μια *Galileon* τύπου θεωρία ή οχι.

Η μέθοδος που ακολουθήσαμε για την ανακάλυψη του υπερσυμμετρικού τετραπλού μιγαδικό *Galileon* ήταν όπως προαναφέραμε έμεση. Σε πρόσφατη δουλειά ενός συνεργάτη είχε επισημανθεί ότι οι *Galileon* τύπου θεωρίες μπορούν να περιγραφούν από θεωρίες βαρύτητας με ανώτερες παραγώγους, όταν αυτές μελετηθούν σε συγκεκριμένα όρια. Συγκεκριμένα η θεωρία βαρύτητας που θα έπρεπε να χρησιμοποιηθεί για να κατασκευάσουμε το τετραπλό μιγαδικό *Galileon* είναι η $\mathcal{L}_{II}$. Χρησιμοποιώντας λοιπόν μεθόδους του υπερχώρου και την $\mathcal{L}_{II}$ σε σύζευξη με την υπερβαρύτητα επιβεβαιώσαμε ότι η μέθοδος αυτή λειτουργεί και στην υπερβαρύτητα εφόσον τελικά μας υπέδειξε ποιά είναι η μορφή του τετραπλού μιγαδικού *Galileon* στον υπερχώρο.

Εάν οι διαταραχές κατά την διάρκεια του πληθωρισμού προέρχονται από το ίδιο πεδίο που οδηγεί τον πληθωρισμό, το $inflaton$, τα πρόσφατα δεδομένα από τον δορυφόρο Πλανκ για τις ανισοτροπίες στην κοσμική ακτινοβολία υποβάθρου έχουν περιορίσει κατ΄ πολύ τα υποψήφια πληθωριστικά μοντέλα ενός πεδίου. Το μοντέλο που είναι πιο πολύ συμβατό με τα δεδομένα είναι το ανωτέρων παραγώγων πληθωριστικό μοντέλο του Σταροβινσκι . Αυτό περιγράφεται από την Λαγκρατζιανή

$$\mathcal{L} = R + a\,R^2\,,\ a > 0 \tag{1.43}$$

και περιέχει εκτός από το βαρυτόνιο,έναν επιπλέον βαθμό ελευθερίας. Η σταθερά σύζευξης α είναι θετική ώστε να αποφευχθούν αστάθειες. Πραγματικά, μπορεί κάποιος να ξαναγράψει την Λαγκρατζιανή ως

$$\mathcal{L} = (1 + 2a\phi)\,R - a\phi^2 \tag{1.44}$$

από την οποία ολοκληρώνοντας το πεδίο φ μπορούμε να πάρουμε πίσω την αρχική θεωρία (1.43). Πρέπει να παρατηρήσουμε ότι αυτή είναι μια κλασσική αναλογία. Μπορεί λοιπόν τώρα κανείς νά γράψει την παραπάνω θεωρία στο Εινστειν σύστημα με την χρήση του μετασχηματισμού

$$g_{mn} \to (1 +\, 2a\phi)\,g_{mn} \tag{1.45}$$

και να ανακαλύψει ότι η ανάλογη βαθμωτή-τανυστική θεωρία με το αρχικό μοντέλο Σταροβινσκι είναι η

$$\mathcal{L} = \sqrt{-g}\left[R - 6\partial_m\phi\partial^m\phi - \frac{1}{4a}(1 - e^{-2\phi})^2\right] \tag{1.46}$$

όπου και γίνεται φανερό γιατί πρέπει η σταθερά α να είναι θετική. Ο πληθωρισμός λαμβάνει χώραν όποτε το βαθμωρό πεδίο κυλάει αργά κατά μήκος της πεδιάδας του δυναμικού που επιτυγχάνεται για φ>>1.

Σκοπός μας είναι να μελετήσουμε την δψνατότητα εμβαπτισμού της θεωρίας Σταροβινσκι στην ελάσσουσα τετραδιάστατη N=1 υπερβαρύτητα. Ο τρόπος να γίνει αυτό δεν είναι μοναδικός. Ο λόγος για αυτό είναι όπως έχουμε προαναφέρει ότι υπάρχουν δύο 12+12 ελάσσουσες θεωρίες υπερβαρύτητας. Όταν δεν υπάρχουν ανώτερες παράγωγοι οι δυο αυτές θεωρίες είναι ίδιες, καθότι συνδέονται με μετασχηματισμό δυαδικότητας μέσω της συναλλοίωτης θεωρίας υπερβαρύτητας. Η παλαιά-ελάσσουσα υπερβαρύτητα προκύπτει όταν κανείς καθορίσει την βαθμίδα της συναλλοίωτης θεωρίας μέσω ενός χειραλικού αντισταθμιστικύ υπερπεδίου. Η νέα ελάσσουσα θεωρία υπερβαρύτητας προκύπτει όταν κανείς καθορίσει την βαθμίδα της συναλλοίωτης θεωρίας μέσω ενός πραγματικού γραμμικού αντισταθμιστικού υπερπεδίου. Όταν όμως εισάγουμε ανώτερες παραγώγους η δυαδικότητα μεταξύ των θεωριών παύει να ισχύει.

Σε σχετική εργασία ερευνήσαμε τον εμβαπτισμό του Σταροβινσκι μοντέλου στις δυο 12+12 ελάσσουσες υπερβαρύτητες και μελετήσαμε ποια είναι η ανάλογη βαθμωτή-τανυστική θεωρία της καθεμιάς. Όπως είναι αναμενόμενο, επειδή η θεωρία Σταροβινσκι είναι θεωρία ανωτέρων παραγώγων, η δυαδικότητα παύει να ισχύει και για αψτό τον λόγο οι δυο ελάσσουσες θεωρίες έχουν διαφορετική ανάλογη βαθμωτή-τανυστική Λαγρατζιανή περιγραφή. Αυτό που ισχύει για την περίπτωση της παλαιάς-ελάσσουσας υπερβαρύτητας είναι ότι η ανάλογη βαθμωτή-τανυστική περιγραφή της δίδεται από μια συνήθη υπερβαρύτητα σε σύζευξη με δυο χειραλικές υπερπολλαπλέτες οι οποίες έχουν δομή παρόμοια με τα μοντέλα τύπου $no - scale$. Σε αυτήν την περίπτωση προκύπτει F-τύπου δυναμικό για το $inflaton$. Αντίστοιχα, αυτό που ισχύει για την περίπτωση της νέας-ελάσσουσας υπερβαρύτητας είναι ότι η ανάλογη βαθμωτή-τανυστική περιγραφή της δίδεται από μια συνήθη υπερβαρύτητα σε σύζευξη με ένα πραγματικό υπερπεδίο διανύσματος με μάζα. Σε αυτήν την περίπτωση προκύπτει D-τύπου δυναμικό για το $inflaton$.

Τέλος μελετήσαμε και πιθανές ανωτέρας τάξης διορθώσεις στο $inflaton$ δυναμικό. Η συγκεκριμένες διορθώσεις που θεωρήσαμε αντιστοιχούν σε υπερβαρύτητες τύπου

$$\mathcal{L} = R + R^2 + R^4. \tag{1.47}$$

Και για τις δυο υπερβαρύτητες οι διορθώσεις οδηγούν σε βαθμωτές-τανυστικές ανάλογες περιγραφές συνήθους υπερβαρύτητας που περιέχουν ανωτέρες παραγώγους υπερπεδίων και όπως έχουμε προαναφέρει συνεισφέρουν στο βαθμωτό δυναμικό· εδώ δηλαδή στο δυναμικό του $inflaton$. Βρέθηκε ότι υπάρχουν περιοχές του πεδίου τιμών των παραμέτρων της θερίας στις οποίες ο πληθωρισμός απηλείται από αυτές τις ανωτώρας τάξης διορθώσεις. Υπάρχει όμως και ένα εύρος τιμών στο οποίο ο πληθωρισμός δεν απηλείται οπότε τα μοντέλα τύπου Σταροβινσκι παραμένουν πολύ καλά πρότυπα για την περιγραφή αυτής της περιόδου του σύμπαντος.

1.5 Επίλογος

Έχουν περάσει σαρράντα χρόνια από την πρώτη θεωρητική αναφορά στην θεωρία της υπερσυμμετρίσς. Εν τούτοις μέχρι και σήμερα δεν έχει βρεθεί κανένα σήμα που να επιβεβαιώνει την ύπαρξη αυτήω της συμμετρίας στην φύση. Ας μη βιαστούμε όμως να βγάλουμε συμπεράσματα, ακόμη και το μποζόνιο Χιγγς, ο ακρογωνιαίος λίθος του καθιερωμένου προτύπου, που έχει προβλεφθεί σχεδόν πριν απ'ο πενήντα χρόνια , μόλις τώρα επιβεβαιώθηκε πειραματικά στον επιταχυντή LHC. Είναι ο επόμενος γύρος από δεδομένα του επιταχυντή LHC που θα ρίξει περισσότερο φως στην αναζήτηση των υπερσυμμετρικών συντρόφων των σωματιδίων του καθιερωμένου προτύποου. Πρέπει όμως κανείς να έχει πάντα στον νου του ότι αν πράγματι η υπερβαρύτητα είναι η θεωρία χαμηλών ενεργειών των υπερωορδών, τότε δεν υπάρχει κανένας λόγος (τουλάξιστον δεν έχει βρεθεί μέχρι τώρα) για τον οποίο η ενεργειακή κλίμακα σπασίματος της υπερσυμμετρίας θα έπρεπε να είναι χαμηλή. Η έρευνα στην θεωρία της υπερσυμμετρίας και της υπερβαρύτητας συνεχίζεται, αν και οι σημαντικές θεωρητικές ανακαλύψεις έχουν γίνει, υπάρχουν θεμελιακά ερωτήματα που παραμένουν αναπάντητα. Πραγματικά, νέα δεδομένα παρατηρησιακά δεδομένα από τον δορυφόρο PLANK φέρνουν την κοσμολογία στο προσκήνιο.

Σε αυτήν την διατριβή παρουσιάσαμε την πρόοδο στα πιο σημαντικά θέματα της υπερσυμμετρίας και της υπερβαρύτητας· Αυθορμητο σπάσιμο υπερσυμμετρίας, υπερσυμμετρία και σωματιδιακή φυσική, υπερβαρύτητα και κοσμολογία. Αυτά τα θέματα τα μελετήσαμε από την μαθηματική τους σκοπιά, αναφερόμενοι όπου ήταν απαραίτητο στην φυσική τους σημασία. Πρώτα παρουσιάσαμε νέες μεθόδους για το σπάσιμο της υπερσυμμετρίας, οι οποίες βασίζονται σε τελεστές ανωτέρας διάστασης. Ύστερα παρουσιάσαμε τροποποιήσεις στο υπερσυμμετρικό καθιερωμένο πρότυπο με ένα και μόνο Χιγγς πεδίο όπου και βρήκαμε μια ιεραρχία για τις μάζες των βαριών φερμιονίων ενώ συγχρόνως λόγω της μη γραμμικής υπερσυμμετρίας αυτά τα μοντέλα παρέχουν μια ελαστικότητα όσον αφορά την προβλεπόμενη μάζα Χιγγς. Τέλος κατασκευάσαμε και μελετήσαμε συνεπή μοντέλα υπερβαρύτητας ανωτέρων παραγώγων που σχετίζονται με την σύγχρονη κοσμολογία. Βέβαια πολλά ερωτήματα παραμένουν ανοιχτά, στα οποία υα απευθυνθούμε στο μέλλον.

Εν κατακλείδι, όποια και να είναι τα πειραματικά δεδομένα, η μαθηματική αρτιότητα των θεωριών της υπερσυμμετρίας και της υπερβαρύτητας είναι που προσελκύει έναν θεωρητικό φυσικό, ή με τα λόγια του *P. van Nieuwenhuizen* για την υπερβαρύτητα: ¨Είναι η πιο ωραία θεωρία βαθμίδας που ξέρουμε, τόσο ωραία, που πραγματικά, θα πρέπει και η φύση να την γνωρίζει.'

Chapter 2

Introduction

Supersymmetry is a gauge symmetry that relates bosonic and fermionic degrees of freedom. It is an extension to the Poincaré algebra with spinorial generators but manifests as an internal symmetry in a field theory setup. For it's mathematical elegance, it is often characterized as the most beautiful symmetry of particle physics. It has been 40 years since it was first proposed as an underlying symmetry of quantum field theory, in the early 70's [80, 192]. Since then there has not been any direct evidence of the relevance of supersymmetry in particle physics. On top of that, we have just entered a new era of theoretical high energy physics: the discovery of the Higgs particle era, with a mass of 126 GeV. With the discovery of the long sought after scalar boson all particles the standard model predicted have now been found. The particular mass of the Higgs boson triggered a debate on if and how supersymmetry can solve the hierarchy problem, and the question of a hierarchy problem inside the minimal supersymmetric standard model has been raised. After all, the standard model of particle physics works extremely well; why try to fix something that's not broken?

The answer to this comes from many independent considerations [110]. Let us present some simple arguments. It is well known that scalar particle masses are sensitive to quantum corrections due to heavier particles. Therefore inside the standard model, there is a fine tuning problem: The Higgs boson mass is extremely sensitive to physics beyond the standard model, and a great amount of fine tuning is needed in order to have the present value of 126 GeV. If there is no new physics nevertheless, then there is no fine-tuning problem, and no reason to question the validity of the standard model up to infinite scales. In fact the need for new physics is most evident even from an everyday life experience: Gravity [189]. **No gravitational interaction has been introduced within the standard model.** The job of incorporating gravity into particle physics comes along with fundamental open questions such as:

- quantum gravity
- cosmological constant
- dark matter

which are far from being solved. Thus new physics should exist, and should address these questions. Moreover there is the long-standing hope of grand unification, which does not seem to have any luck inside the standard model as it stands.

Accepting the fact that there is higher scales than that of the electroweak scale, brings us to the standard model fine-tuning problem. The electroweak sector of the standard model contains within it the weak scale, a parameter with the dimensions of energy, namely

$$v \sim 246\text{GeV} \tag{2.1}$$

where $v/\sqrt{2}$ is the vacuum expectation value of the neutral Higgs field. The fact that the Higgs field aquires a vev and spontaneously breaks the local gauge symmetry, gives rise to a natural scale, which is connected with all the masses of the theory. For example, the tree level mass of the $W^{\pm}$ gauge bosons is given by

$$M_W = \frac{gv}{2} \sim 80\text{GeV} \tag{2.2}$$

where g is the $SU(2)$ coupling constant. The Higgs field is an $SU(2) \times U(1)_Y$ doublet

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \tag{2.3}$$

where the h^0 is neutral under the unbroken electromagnetism $U(1)$. The scalar potential has the famous form

$$V = -\mu^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 \tag{2.4}$$

where $\lambda > 0$ and $\mu^2 > 0$ which leads to a mass for the neutral Higgs particle

$$M_h = v\sqrt{\frac{\lambda}{2}} \sim 126\text{GeV}. \tag{2.5}$$

Note that the negative sign $-\mu^2$ in (2.4) is crucial for the symmetry breaking mechanism to take place, it should be, whatever the cost, preserved - the same goes for the magnitude of $-\mu^2$. If instead we had $+\mu^2$ there would be no symmetry breaking and the vacuum expectation value of the Higgs boson would be at $v = 0$.

Until now we have been discussing only tree level physics. The fine tuning problem arises as soon as we start taking into account radiative corrections [171]. It so happens that the standard model of particle physics is a renormalizable theory, this meaning that finite results will be obtained for all orders in loop corrections and for allowing the loop momenta go to infinity. This certainly guaranties the validity of the results obtained and the fact that the theory is indeed well defined, but does not exclude new physics, quite the contrary, this theory is suspiciously sensitive to new physics as we will demonstrate.

In quantum field theory one generically encounters integrals of the form

$$\int^{\Lambda} d^4k f(k, \text{external momenta}) \tag{2.6}$$

where Λ is the cut-off, it is an energy scale indicating that our theory stops being predictive, and it should be modified. Technically, the standard model in the presence of no new physics, is viable for

$$\Lambda \to \infty. \tag{2.7}$$

Nevertheless we know of at least one scale where the standard model has to be modified, the quantum gravity regime

$$M_P \sim 1.2 \times 10^{19}\text{GeV}. \tag{2.8}$$

Moreover there is the indication for another energy scale, the grand unification scale

$$M_{GUT} \sim 10^{16}\text{GeV} \tag{2.9}$$

where inside the standard model the running couplings tend to meet. Eventually this only happens in the minimal supersymmetric standard model, giving another hint towards supersymmetry [5, 161].

In particular, the Higgs field self interaction term

$$\frac{\lambda}{4}(H^\dagger H)^2 \tag{2.10}$$

in (2.4) will give rise to a one-loop self interaction diagram proportional to

$$\lambda \int^\Lambda d^4k \frac{1}{k^2 - M_H^2} \tag{2.11}$$

which contributes to the $H^\dagger H$ term. This gives a positive correction to the tree level potential

$$\sim \lambda \Lambda^2 H^\dagger H \tag{2.12}$$

leading to the one-loop corrected quadratic term

$$-\mu^2_{\text{phys}} = -\mu^2 + \lambda \Lambda^2. \tag{2.13}$$

In fact in order to take the quantum corrections into account one has to minimize the scalar potential (2.4), but now using μ^2_{phys} instead of μ^2. Let us recall that the Higgs mass is connected to μ_{phys} via

$$M_h = \sqrt{2}\mu_{\text{phys}}. \tag{2.14}$$

Let us now consider new physics indeed appearing in the Planck-scale 10^{19}GeV whereas the Higgs mass has been measured to be 126 GeV. Thus the following miraculous cancellation must occur

$$-126\text{GeV} = -\mu^2 + 10^{19}\text{GeV} \tag{2.15}$$

implying that the tree level μ^2 is of the order of the Planck-scale and then they cancel with Λ^2 up to a precision of a few GeV. This is the standard model fine-tuning problem. Note that the standard model fine-tuning problem even though stems from the Higgs mass is not only related to that; ultimately, all masses in the standard model are affected by this.

Let us mention in passing that the standard model fine-tuning problem is not only a matter of taste, it is common in physics that considerations for academic theoretical problems have led to breakthrough new predictions. For example, when Dirac proposed the theory of electrons and positrons in order to solve the negative energy eigenvalue problem of the Klein-Gordon field, it was inevitable not to double the spectrum of the observed matter particles. Antimatter was discovered only 10 years latter. Another example is the four-fermion interaction which even though worked very well, it was realized by Heisenberg that its predictability breaks down at the, unimaginably at that time, high energy scale of 300 GeV. Later this was traced to the non-renormalizability of the theory, a pure theoretical problem at the time. Is the standard model fine-tuning problem another example as the ones above, signaling the need for new physics?

It is widely believed that the most natural situation for solving the fine-tuning problem would be the existence of a new scale, within an order of magnitude from the weak scale. Now we are faced with the evident questions:

- What is this new physics?
- Does it have a fine-tuning problem of its own?
- Can we incorporate gravity?

There have been various proposals for the possible nature of new physics, but only few can provide a deep physical insight.

Let us be optimistic and imagine the best situation for solving the fine-tuning problem: the Λ^2 correction to the Higgs mass naturally cancels. This means our theory should contain specific tree level interactions to guarantee this cancellation. The answer is not too far away, all one has to realize is that this is exactly the loop contribution due to a Yukawa coupling type of interaction of a fermion with the Higgs field. At zero external momentum it is given by

$$\left(-4g_f^2 \int^{\Lambda} d^4k \frac{1}{(\not{k} - m_f)^2}\right) H^\dagger H \tag{2.16}$$

leading to a total contribution from both fermionic and bosonic loops

$$(\lambda - g_f^2)\Lambda^2 H^\dagger H. \tag{2.17}$$

One can now postulate

$$\lambda = g_f^2 \tag{2.18}$$

such that the loop corrections exactly cancel. Can there be a deeper reason behind such a miraculous cancellation? This is where supersymmetry steps in; these kind of relations between coupling constants is characteristic for supersymmetric theories. Note that the bosonic loop correction could only be canceled by a fermionic one, due to the opposite sign one has from the closed fermion loop. Thus, this symmetry would also require the pairing of fermions and bosons. The topic of this dissertation is to further study the aspects of this super gauge symmetry.

The first question that comes to a theorists mind is then: can supersymmetry be a local symmetry as well? The answer is affirmative, this would be the theory of Supergravity [70, 97]. Since supersymmetry is a closed algebra with the Poincaré group, making supersymmetry local demands rigid space-time transformations to become local i.e. general coordinate transformations. It is then inevitable if one wants to turn supersymmetry into a local symmetry not to introduce gravity. The only thing remaning is to identify the gauge field of supersymmetry, this is the so-called gravitino, it is the supersymmetric partner of the graviton. Indeed, from the work of Rarita and Schwinger [173] we can see that the spin-$\frac{3}{2}$ fermion has a gauge invariance of the form

$$\delta\psi_m^\alpha = D_m \xi^\alpha \tag{2.19}$$

which is nothing but the gauging of a local supersymmetry. In fact the free pure supergravity theory contains only the Einstein-Hilbert action and the action for the Rarita-Swinger field. Eventually, the theory of supergravity is the super covariantization of the field theories we are interested in [60].

In the first years of supergravity it was believed that the non-renormalization properties of supersymmetry could control the divergencies encountered in the quantum theory of general relativity [165]. Even though supergravity has a better quantum behavior than general relativity, still it becomes divergent, at higher loops. The common belief in the present day is that supergravity is the effective low energy incarnation of a more fundamental and quantum mechanically consistent theory - the superstring theory. It is indeed possible by calculating superstring scattering amplitudes to recover an effective theory in the language of particle physics and surprisingly enough: it is exactly the spectrum of an on-shell 10 dimensional supergravity theory! Thus it is believed that the study of supergravity theories indeed describes our world in energies far lower than the string scale where quantum gravity and other effects become important.

In this dissertation we are concerned with modern subjects on supersymmetry and supergravity. In order to do this, it is demanding that we use a formalism that allows us to built supersymmetric Lagrangians. This formalism is the superspace [39, 72, 99, 193]. A short introduction of superspace techniques is given in the first chapter. We give the notion of superfields and how to read their component form, which then is used to write down supersymmetric Lagrangians. It is important that the Lagrangians we find from this method are off-shell, i.e., they contain all the auxiliary field sector of the theory, which is eventually integrated out. Supersymmetry has a wide range of supermultiplets, we mention here the ones we will employ in the body of the dissertation which are in fact the ones commonly used. After presenting basic tools on supersymmetry we turn to supergravity and extend our discussion. It is quite interesting that there exist two off-shell versions of the minimal supergravity, both having their own interesting properties. The old minimal [95,183] version was the one first to be discovered, along with the graviton and the gravitino it contains as auxiliary fields a complex scalar and a real vector. The new minimal [98, 179] supergravity was later discovered and on top of the gravitino and the graviton it contains two more gauge fields, an auxiliary vector that gauges the R-symmetry and a two form. Let us note that at the two derivative level the two minimal off-shell supergravities are equivalent. When higher order corrections are taken into account the duality breaks down [92].

Supersymmetry might be a beautifull theory, but it has not been observed so far in the colliders. Thus, one of the most important subjects is supersymmetry breaking. In chapter 3 we discuss new methods for supersymmetry breaking. We show that contrary to a common lore that wanted supersymmetry broken only by leading terms, and preserved by higher order corrections [43], that it is possible for the opposite to occur [89]. We have found specific examples where supersymmetry indeed is broken by higher order correction when there was no supersymmetry breaking in the leading terms. Theories of this sort have two branches, a supersymmetric and a non-supersymmetric one. It does not seem conceptually correct to consider these branches as two phases of the same theory, rather they are two independent theories.

In chapter 4 we revisit the MSSM. The ultimate goal of supersymmetry is to be incorporated within the standard model. Nevertheless, due to technical reasons, it is widely believed that the minimal supersymmetric standard model should contain two Higgs doublet supermultiplets. Moreover supersymmetry breaking is introduced as soft terms, which can be equivalently written in the superspace language as contact terms of the hidden SUSY breaking sector with the standard model sector. The low energy effective theories of broken supersymmetry can be described by non-linear supersymmetry [40, 155, 174, 188], and models which incorporate non-linear realizations of supersymmetry into the MSSM have been built [11]. These models have some very interesting properties. Our work was concerned with the fact that contrary to the common belief, it is equally motivating to have a single-Higgs supersymmetric standard model [86]. We also point out that due to introducing non-linear supersymmetry for the breaking sector some very interesting properties have been found. We also discuss the decoupling limit of the sgoldstino and how it leads to non-linear supersymmetry realizations, in a supergravity setup [85].

We then turn to cosmological applications of supergravity in chapter 5, and more specifically we are interested in higher derivative theories. Introducing higher derivative theories in supersymmetry and supergravity is highly non-trivial. In fact there exist very few known examples of consistent supersymmetric and supergravitational higher derivative theories. In this part we discuss how they are constructed and what is their relevance to inflationary cosmology. We discuss the supersymmetrization of the non-minimal derivative coupling [84], the quartic galileon [87] and finally the Starobinski model of inflation ($R + R^2$) [88], which seems to be favored by the recent PLANCK satellite data.

Finally, in chapter 6, short concluding remarks are given.

Chapter 3

Techniques of $4D$, $\mathcal{N}=1$ Superspace

3.1 Global Supersymmetry

We give a technical review of the basic formalism and tools concerning the $4D$, $\mathcal{N}=1$ supersymmetry algebra. We discuss the manifestation of supersymmetry on field theory via the superfield method; a comonly used technique to study $4D$, $\mathcal{N}=1$ supersymmetric field theories. Superspace is an elegant way to tidy up the properties of supersymmetric theories. More specificaly we present the definition via projection method, and show how all the properties of a supersymmetric theory are derived from this.

3.1.1 Supersymmetry Algebra

The $4D$, $\mathcal{N}=1$ supersymmetry algebra is

$$\begin{aligned} \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2\sigma^a_{\alpha\dot{\alpha}} P_a \\ \{Q_\alpha, Q_\beta\} &= \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \\ [P_a, Q_\alpha] &= [P_a, \bar{Q}_{\dot{\alpha}}] = 0 \\ [P_a, P_b] &= 0. \end{aligned} \tag{3.1}$$

The supersymmetry algebra is the coset of the super-Poincaré over the Lorentz algebra. Indeed, this algebra can be viewed as a Lie algebra with anticommuting parameters, such that

$$\begin{aligned} \{\xi Q, \bar{\xi}\bar{Q}\} &= 2\xi\sigma^a\bar{\xi} P_a \\ \{\xi Q, \xi Q\} &= \{\bar{\xi}\bar{Q}, \bar{\xi}\bar{Q}\} = 0 \\ [P_a, \xi Q] &= [P_a, \bar{\xi}\bar{Q}] = 0 \end{aligned} \tag{3.2}$$

with the summation convention

$$\xi Q = \xi^\alpha Q_\alpha \tag{3.3}$$

$$\bar{\xi}\bar{Q} = \bar{\xi}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}. \tag{3.4}$$

Thus we can define the corresponding group element

$$G(x, \theta, \bar{\theta}) = e^{i\{-x_a P^a + \theta Q + \bar{\theta}\bar{Q}\}} \tag{3.5}$$

and with the use of the Hausdorff formula

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]+\frac{1}{12}[[A,B],B]+\frac{1}{12}[[B,A],A]+\ldots} \tag{3.6}$$

we will identify the apropriate generators. For two group elements we find

$$G(0,\xi,\bar{\xi})G(x^a,\theta,\bar{\theta}) = G(x^a + i\theta\sigma^a\bar{\xi} - i\xi\sigma^a\bar{\theta}, \theta+\xi, \bar{\theta}+\bar{\xi}). \tag{3.7}$$

To find the differential operators one interprets (3.7) as a motion on the parameter space of the group induced by the appropriate generators. By convention, in the case of left multiplication (that is Eq. (3.7)) we find the supersymmetry generators

$$\begin{aligned} P_a &= -i\partial_a \\ Q_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i\sigma^a_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_a \\ \bar{Q}^{\dot{\alpha}} &= \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} - i\theta_\alpha\bar{\sigma}^{a\dot{\alpha}\alpha}\partial_a \end{aligned} \tag{3.8}$$

which are a differential representation of (3.1). For the case of right multiplication one finds the superspace derivatives

$$\begin{aligned} P_a &= -i\partial_a \\ D_\alpha &= \frac{\partial}{\partial\theta^\alpha} + i\sigma^a_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_a \\ \bar{D}^{\dot{\alpha}} &= \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + i\theta_\alpha\bar{\sigma}^{a\dot{\alpha}\alpha}\partial_a. \end{aligned} \tag{3.9}$$

These operators realize the following flat superspace geometry

$$\begin{aligned} \{D_\alpha, \bar{D}_{\dot{\alpha}}\} &= -2i\sigma^a_{\alpha\dot{\alpha}}\partial_a \\ \{D_\alpha, D_\beta\} &= 0 \\ \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} &= 0. \end{aligned} \tag{3.10}$$

In fact Eq. (3.10) describes torsion, this serves as a constraint when one solves the Bianchi identities of the supergravity geometry. Moreover, the two sets of differential operators (3.8) and (3.9) completely commute-anticommute with each other. Let us note that the $\mathcal{N}=1$ supersymmetry can be naturally enlarged by a chiral $U(1)$ symmetry, refered to as R-symmetry, with the following commutation relations

$$\begin{aligned} [\mathrm{R}, D_\alpha] &= -\frac{1}{2}D_\alpha \\ [\mathrm{R}, \bar{D}_{\dot{\alpha}}] &= \frac{1}{2}\bar{D}_{\dot{\alpha}}. \end{aligned} \tag{3.11}$$

A conventional road to find representations of supersymmetry on fields is by the expansion in the anticommuting variables. We are not going to follow this formalism here. Nevertheless, it is important to first mention it, because it will give a good insight to the more formal "definition via projection" which we are going to use throughout this dissertation. Since now we are working with a superspace spanned by x^a, θ, $\bar{\theta}$, a general field on this space will be

$$\mathcal{V}(x^a, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}). \tag{3.12}$$

These fields (3.12) are called superfields. One can always expand (3.12) in the grassmann variables, and the series will always terminate due to their anticommuting nature. This representation nevertheless will be highly reducible. In order to extract a non-reducible representation constraints are imposed. Let us now expand in the anticommuting space

$$\begin{aligned}\mathcal{V}(x,\theta,\bar\theta) &= f(x)+\theta\phi(x)+\bar\theta\chi(x)\\ &+\theta\theta m(x)+\bar\theta\bar\theta n(x)+\theta\sigma^a\bar\theta v_a(x)\\ &+\theta\theta\bar\theta\bar\lambda(x)+\bar\theta\bar\theta\theta\psi(x)+\theta\theta\bar\theta\bar\theta d(x)\end{aligned} \tag{3.13}$$

and all higher powers of θ and $\bar\theta$ vanish. Note that by simple dimensional analysis, not all the fields inside (3.13) can be physical; some will be gauge degrees of freedom and some will be auxiliary fields and some will be solved in terms of the physical ones. Auxiliary fields are components of supermultiplets that guarantee the closure of the supersymmetry algebra of shell. This is needed for a well defined symmetry in a quantum mechanical theory. Auxiliary fields are usualy gaussian in the Lagrangian and non-propagating thus can be integrated out in terms of the other fields. Their role in supersymmetry and supergravity is central. The transformation law of the superfield is defined as follows

$$\begin{aligned}\delta_\xi\mathcal{V}(x,\theta,\bar\theta) &= \delta_\xi f(x)+\theta\delta_\xi\phi(x)+\bar\theta\delta_\xi\chi(x)\\ &+\theta\theta\delta_\xi m(x)+\bar\theta\bar\theta\delta_\xi n(x)+\theta\sigma^a\bar\theta\delta_\xi v_a(x)\\ &+\theta\theta\bar\theta\delta_\xi\bar\lambda(x)+\bar\theta\bar\theta\theta\delta_\xi\psi(x)+\theta\theta\bar\theta\bar\theta\delta_\xi d(x)\\ &\equiv (\xi Q+\bar\xi\bar Q)\mathcal{V}.\end{aligned} \tag{3.14}$$

From Eq. (3.14) one reads the supersymmetry transformations of the components of this general superfield. The discussion up to now gives us a good insight of the Grassmann nature of supersymmetry, nevertheless, we will depart from the above considerations in two ways

- We will use the method of projections to define the component fields of (3.13)
- We will use specific supersymmetric conditions to reduce the degrees of freedom inside (3.13)

Let us now see how this comes about.

3.1.2 The Definition via Projection Method

It is easy to see that hitting the superfield (3.13) with the derivative D_α of (3.9) and then setting the thetas to zero all we find is the leading fermion component of (3.13). This observation will be the guide to the definition via projection method. Let us note in advance that this method of defining the component fields or the Grassmann expansion method are completelly equivalent, up to irrelevant redefinitions. In fact by a correct definition of the projections no redefinition is needed. We will also now present the basic multiplets of supersymmetric field theory, and at the same time give their projection definitions.

Chiral Multiplet

We will now introduce the chiral superfield Φ

$$\bar D_{\dot\alpha}\Phi = 0. \tag{3.15}$$

This multiplet is widelly used for the simple fact that it can accomondate chiral fermion, the only fermion found so far in nature. The second reason for its wide use is its simplicity, in fact it was the first supersymmetric multiplet discovered by J. Wess and B. Zumino. The degrees of freedom inside the chiral multiplet are a physical complex scalar A with 2 degrees of freedom, a physical complex Weyl spinor χ_α with 4 degrees of freedom and a complex auxiliary scalar field F with 2 auxiliary degrees of freedom. Altogether 4 fermionic and 4 bosonic degrees of freedom. Note that fermionic fields anticommute

$$\chi_1 \, \chi_2 = -\chi_2 \, \chi_1 \quad , \quad \bar{\chi}_{\dot{1}} \, \bar{\chi}_{\dot{2}} = -\bar{\chi}_{\dot{2}} \, \bar{\chi}_{\dot{1}}. \tag{3.16}$$

The definition of the component fields is

$$\begin{aligned} \Phi| &= A \\ D_\alpha \Phi| &= \sqrt{2}\chi_\alpha \\ D^2 \Phi| &= -4F. \end{aligned} \tag{3.17}$$

In the same way one defines the anti-chiral multiplet as

$$D_\alpha \bar{\Phi} = 0 \tag{3.18}$$

and components

$$\begin{aligned} \bar{\Phi}| &= \bar{A} \\ \bar{D}_{\dot{\alpha}} \bar{\Phi}| &= \sqrt{2}\bar{\chi}_{\dot{\alpha}} \\ \bar{D}^2 \bar{\Phi}| &= -4\bar{F}. \end{aligned} \tag{3.19}$$

Chiral Projection

Let us now consider a generic superfield $\mathcal{U}$. From the anticommuting properties of the D's

$$\{D_\alpha, D_\beta\} = 0$$

we have that

$$D_\alpha (D^2 \mathcal{U}) = 0.$$

This implies that

$$-\frac{1}{4} D^2 \mathcal{U}$$

is a chiral superfield. The operator

$$-\frac{1}{4} D^2 \tag{3.20}$$

is called chiral projection. In the same way one defines the anti-chiral projection as

$$-\frac{1}{4} \bar{D}^2. \tag{3.21}$$

Real Linear Multiplet

A real linear superfield is defined to satisfy the following two conditions

$$\begin{aligned} D^2L &= 0 \\ \bar{D}^2L &= 0 \\ L &= \bar{L}. \end{aligned} \tag{3.22}$$

The component fields of this multiplet are defined as follows

$$\begin{aligned} L| &= \phi \\ D_\alpha L| &= \sqrt{2}\psi_\alpha \\ \bar{D}_{\dot{\alpha}} L| &= \sqrt{2}\bar{\psi}_{\dot{\alpha}} \\ -\frac{1}{2}\{D_\alpha, \bar{D}_{\dot{\alpha}}\}L| &= \sigma^a_{\alpha\dot{\alpha}} h_a \end{aligned} \tag{3.23}$$

with

$$h^a = \frac{1}{3!}\epsilon^{abcd}(\partial_b b_{cd} + \partial_c b_{db} + \partial_d b_{bc}) \tag{3.24}$$

and

$$b_{ab} = -b_{ba} \tag{3.25}$$

a real two form field.

Abelian Field Strength Multiplet

Let us now consider the following real superfield

$$V = \bar{V}. \tag{3.26}$$

The superfield (3.26) is usualy reffered as vector superfield, since it contains a real vector, in the following projection

$$-\frac{1}{2}[D_\alpha, \bar{D}_{\dot{\alpha}}]V| = \sigma^a_{\alpha\dot{\alpha}} v_a. \tag{3.27}$$

Nevertheless the constraint (3.26) leads a reducible multiplet. There is two ways to reduce it. First one can impose further constraint like (3.22) which will render the lower components physical while the higher will be solved for the lower ones; this will lead to the real linear superfield (3.22), (3.23). The second way is to build gauge invariant quantities out of V and thus, reduce the degrees of freedom it carries by gauging some of them away. Note that the degrees of freedom we will gauge away are exactly the ones inside the real linear multiplet. It is easy to show that the following chiral superfield

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V \tag{3.28}$$

is invariant under the following transformation

$$V \to V + \Lambda + \bar{\Lambda} \tag{3.29}$$

where Λ is a chiral superfield (3.15) and $\bar{\Lambda}$ an anti-chiral (3.18). Note that the transformation (3.29) has the following effects on the components of the vector superfield V

- It acts as a field dependent shift on the components of V lower than v_a
- It acts as a $U(1)$ gauge transformation on v_a
- It does not affect the components of W_α.

The final effect is related to the fact that a $U(1)$ group has a vanishing adjoint representation. We will see later on how the situation changes in the case of non-abelian transformations on v_a. Thus, in a theory invariant under (3.29) we can set

$$\begin{aligned} V| &= 0 \\ D_\alpha V| &= 0 \\ \bar{D}_{\dot{\alpha}} V| &= 0 \\ D^2 V| &= 0 \\ \bar{D}^2 V| &= 0. \end{aligned} \tag{3.30}$$

The higher components of V are in fact expressed as the components of W_α. This choise (3.30) is called Wess-Zumino gauge. The components of W_α are defined via projection as

$$\begin{aligned} W_\alpha| &= -i\lambda_\alpha \\ \bar{W}_{\dot{\alpha}}| &= i\bar{\lambda}_{\dot{\alpha}} \\ D^\alpha W_\alpha| &= \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}| = -2\,\mathrm{D}. \end{aligned} \tag{3.31}$$

Here λ_α is a complex Weyl spinor and D is a real auxiliary scalar. Using the flat superspace geometry (3.10), the definition (3.27) and the definitions (3.31) one finds for the rest of the W_α components

$$\begin{aligned} D_\alpha W_\beta| &= -i\sigma^{ab}_{\alpha}{}^{\gamma}\epsilon_{\gamma\beta} F_{ab} - \epsilon_{\alpha\beta}\mathrm{D} \\ D_\alpha W_\beta| + D_\beta W_\alpha| &= -2i\sigma^{ab}_{\alpha}{}^{\gamma}\epsilon_{\gamma\beta} F_{ab}. \end{aligned} \tag{3.32}$$

We see that (3.28) contains the field strength of the gauge vector

$$F_{ab} = \partial_a v_b - \partial_b v_a \tag{3.33}$$

this is the reason it is called field strength multiplet.

Non-Abelian Field Strength Multiplet

The above vector multiplet will help us gauge the $U(1)$ symmetry, but to gauge non-Abelian symmetries we have to modify the fields strenght into something more general. We define the field strenght chiral multiplet to have the form

$$W_\alpha = -\frac{1}{4}\bar{D}^2(e^{-V} D_\alpha e^V) \tag{3.34}$$

where

$$V = V^{(a)} T^{(a)} \tag{3.35}$$

and the $T^{(a)}$ are a matrix representation of the non-Abelian group we want to study. Here the gauge transformation is defined as

$$e^V \to e^{-i\bar{\Lambda}} e^V e^{i\Lambda} \tag{3.36}$$

and the W_α transform as

$$W_\alpha \to e^{-i\Lambda} W_\alpha e^{i\Lambda} \tag{3.37}$$

with

$$\Lambda = \Lambda^{(a)} T^{(a)}. \tag{3.38}$$

Turning to the Wess-Zumino gauge it is easy to show that

$$e^{-V} D_\alpha e^V = D_\alpha V - \frac{1}{2}[V, D_\alpha V]. \tag{3.39}$$

3.1.3 Supersymmetry Transformations and Lagrangians

A supersymmetry transformation on a unconstrained general superfield is defined as

$$\delta_\xi \mathcal{U} = \xi^\alpha D_\alpha \mathcal{U} + \bar{\xi}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} \mathcal{U}. \tag{3.40}$$

The supersymmetry variations for the components of the supermultiplets are derived from this formula. Supersymmetric Lagrangians are now easy to built, they are (hermitian) superfields invariant under (3.40) up to a space-time derivative. The most general form of a supersymmetric Lagrangian is

$$\mathcal{L} = D^2 \bar{D}^2 (\mathcal{U} + \bar{\mathcal{U}})|. \tag{3.41}$$

The variation reads

$$\begin{aligned}
\delta_\xi \mathcal{L} &= \xi^\alpha D_\alpha D^2 \bar{D}^2 (\mathcal{U} + \bar{\mathcal{U}}) + \bar{\xi}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} D^2 \bar{D}^2 (\mathcal{U} + \bar{\mathcal{U}}) \\
&= 0 + \bar{\xi}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} D^\alpha D_\alpha \bar{D}^2 (\mathcal{U} + \bar{\mathcal{U}}) \\
&= \bar{\xi}_{\dot{\alpha}} (-2i) \bar{\sigma}^{b\dot{\alpha}\alpha} \partial_b D_\alpha \bar{D}^2 (\mathcal{U} + \bar{\mathcal{U}}) - \bar{\xi}_{\dot{\alpha}} D^\alpha \bar{D}^{\dot{\alpha}} D_\alpha \bar{D}^2 (\mathcal{U} + \bar{\mathcal{U}}) \\
&= \partial_b (-2i \bar{\xi}_{\dot{\alpha}} \bar{\sigma}^{b\dot{\alpha}\alpha} D_\alpha \bar{D}^2 (\mathcal{U} + \bar{\mathcal{U}})) + \bar{\xi}_{\dot{\alpha}} D_\alpha \bar{D}^{\dot{\alpha}} D^\alpha \bar{D}^2 (\mathcal{U} + \bar{\mathcal{U}}) \\
&= \partial_b (-2i \bar{\xi}_{\dot{\alpha}} \bar{\sigma}^{b\dot{\alpha}\alpha} D_\alpha \bar{D}^2 (\mathcal{U} + \bar{\mathcal{U}})) + \bar{\xi}_{\dot{\alpha}} D_\alpha (-2i) \bar{\sigma}^{b\dot{\alpha}\alpha} \partial_b \bar{D}^2 (\mathcal{U} + \bar{\mathcal{U}}) - \bar{\xi}_{\dot{\alpha}} D_\alpha D^\alpha \bar{D}^{\dot{\alpha}} \bar{D}^2 (\mathcal{U} + \bar{\mathcal{U}}) \\
&= \partial_b (-4i \bar{\xi}_{\dot{\alpha}} \bar{\sigma}^{b\dot{\alpha}\alpha} D_\alpha \bar{D}^2 (\mathcal{U} + \bar{\mathcal{U}})) + 0.
\end{aligned} \tag{3.42}$$

In supersymmetric theories it is common to reformulate the general Lagrangian (3.41) into the following two

$$\mathcal{L}_{\mathrm{D}} = \int d^2\theta d^2\bar{\theta} \mathcal{M} = \frac{1}{16} D^2 \bar{D}^2 \mathcal{M}| \tag{3.43}$$

$$\mathcal{L}_{\mathrm{F}} + h.c. = \int d^2\theta \mathcal{N} + h.c. = -\frac{1}{4} D^2 \mathcal{N}| + h.c. \tag{3.44}$$

with $\mathcal{M}$ a generic hermitian superfield and $\mathcal{N}$ a generic chiral superfield.

Chiral models

The supersymmetry transformations for the chiral multiplet are found by using the definition (3.40), the chiral condition (3.15), the component definition (3.17), and the superspace geometry (3.10), to find

$$\begin{aligned}\delta_\xi A &= \delta_\xi \Phi| = \xi^\alpha D_\alpha \Phi| + \bar{\xi}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} \Phi| = \sqrt{2}\xi\chi \\ \delta_\xi \chi_\beta &= \frac{1}{\sqrt{2}}\delta_\xi D_\beta \Phi| = \frac{1}{\sqrt{2}}\xi^\alpha D_\alpha D_\beta \Phi| + \frac{1}{\sqrt{2}}\bar{\xi}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} D_\beta \Phi| = \sqrt{2}\xi_\beta F + i\sqrt{2}\sigma^a_{\beta\dot{\beta}}\bar{\xi}^{\dot{\beta}}\partial_a A \\ \delta_\xi F &= -\frac{1}{4}\delta_\xi D^2\Phi| = -\frac{1}{4}\xi^\alpha D_\alpha D^2 \Phi| - \frac{1}{4}\bar{\xi}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} D^2\Phi| = i\sqrt{2}\bar{\xi}_{\dot{\alpha}}\bar{\sigma}^{a\dot{\alpha}\alpha}\partial_a\chi_\alpha. \end{aligned} \tag{3.45}$$

The simplest supersymmetric Lagrangian that can be built is the following

$$\mathcal{L}_0 = \int d^2\theta d^2\bar{\theta}\Phi\bar{\Phi} = \frac{1}{16}D^2\bar{D}^2\Phi\bar{\Phi}|. \tag{3.46}$$

To find the component form we have

$$\begin{aligned}\mathcal{L}_0 &= \frac{1}{16}D^2\bar{D}^2\Phi\bar{\Phi}| = \frac{1}{16}D^\alpha D_\alpha \bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}(\Phi\bar{\Phi})| = \frac{1}{16}D^\alpha D_\alpha(\Phi\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}\bar{\Phi})| \\ &= \frac{1}{16}(D^\alpha D_\alpha\Phi)|(\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}\bar{\Phi})| + \frac{1}{8}D^\alpha\Phi|D_\alpha\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}\bar{\Phi}| + \frac{1}{16}\Phi|D^\alpha D_\alpha\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}\bar{\Phi}|. \end{aligned} \tag{3.47}$$

The various contributions in (3.47) are

$$\begin{aligned} D^\alpha D_\alpha \Phi| &= -4F \\ \bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}\bar{\Phi}| &= -4\bar{F} \\ D^\alpha\Phi| &= \sqrt{2}\chi^\alpha \\ D_\alpha\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}\bar{\Phi}| &= -4i\sqrt{2}\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{\chi}^{\dot{\alpha}} \qquad (3.48) \\ \Phi| &= A \\ D^2\bar{D}^2\bar{\Phi}| &= 16\ \partial^2\bar{A}. \qquad (3.49) \end{aligned}$$

Some of the above component forms are just definitions while (3.48) and (3.49) have to be calculated. For example, for (3.48) we have

$$\begin{aligned} D_\alpha\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}\bar{\Phi}| &= -\bar{D}_{\dot{\alpha}}D_\alpha\bar{D}^{\dot{\alpha}}\bar{\Phi}| - 2i\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{D}^{\dot{\alpha}}\bar{\Phi}| \\ &= \bar{D}^{\dot{\alpha}}D_\alpha\bar{D}_{\dot{\alpha}}\bar{\Phi}| - 2i\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{D}^{\dot{\alpha}}\bar{\Phi}| \\ &= \bar{D}^{\dot{\alpha}}(-2i)\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{\Phi}| - 2i\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{D}^{\dot{\alpha}}\bar{\Phi}| \\ &= -4i\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{D}^{\dot{\alpha}}\bar{\Phi}| \\ &= -4i\sqrt{2}\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{\chi}^{\dot{\alpha}}. \end{aligned} \tag{3.50}$$

The Lagrangian (3.47) then reads

$$\mathcal{L}_0 = A\partial^2\bar{A} - i\chi^\alpha\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{\chi}^{\dot{\alpha}} + F\bar{F}. \tag{3.51}$$

The equations for the auxiliary field F imply

$$\bar{F} = 0 \tag{3.52}$$

and the on-shell theory reads

$$\mathcal{L}_0 = A\partial^2\bar{A} - i\chi^\alpha\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{\chi}^{\dot{\alpha}}. \tag{3.53}$$

One can employ the following superspace Lagrangian in order to find the standard mass terms

$$\begin{aligned}
\mathcal{L}_m + h.c. &= \frac{m}{2}\int d^2\theta\ \Phi^2 + h.c. = -\frac{m}{8}D^2\Phi^2| + h.c. \\
&= -m\Phi|\ \frac{1}{4}D^2\Phi| - \frac{1}{2}m\ \frac{1}{\sqrt{2}}D^\alpha\Phi|\ \frac{1}{\sqrt{2}}D_\alpha\Phi| + h.c. \\
&= m(AF + \bar{A}\bar{F}) - \frac{1}{2}m(\chi^\alpha\chi_\alpha + \bar{\chi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}).
\end{aligned} \tag{3.54}$$

Now (3.46) together with (3.54) read

$$\mathcal{L}_0 + (\mathcal{L}_m + h.c.) = A\partial^2\bar{A} - i\chi^\alpha\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{\chi}^{\dot{\alpha}} + F\bar{F} + m(AF + \bar{A}\bar{F}) - \frac{1}{2}m(\chi^\alpha\chi_\alpha + \bar{\chi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}). \tag{3.55}$$

The auxiliary equations of motion now give

$$\bar{F} = -mA \tag{3.56}$$

and when (3.56) is inserted into (3.55) we find

$$\mathcal{L}_0 + (\mathcal{L}_m + h.c.) = A\partial^2\bar{A} - i\chi^\alpha\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{\chi}^{\dot{\alpha}} - m^2A\bar{A} - \frac{1}{2}m(\chi^\alpha\chi_\alpha + \bar{\chi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}) \tag{3.57}$$

which describes a complex massive scalar and a complex massive fermion both with mass

$$m_A = m_\chi = m. \tag{3.58}$$

The most general chiral Lagrangian has the form

$$\begin{aligned}
\mathcal{L} &= \int d^2\theta d^2\bar{\theta} K(\Phi^i, \bar{\Phi}^{\bar{j}}) + \left[\int d^2\theta W(\Phi^i) + h.c.\right] \\
&= \frac{1}{16}D^2\bar{D}^2K(\Phi^i, \bar{\Phi}^{\bar{j}})| + \left[-\frac{1}{4}D^2W(\Phi^i)| + h.c.\right].
\end{aligned} \tag{3.59}$$

Where we have introduced more than one chiral superfields, labeled as Φ^i, $i = 1, ..n$. Note that the Lagrangian (3.59) has a symmetry

$$K(\Phi^i, \bar{\Phi}^{\bar{j}}) \to K(\Phi^i, \bar{\Phi}^{\bar{j}}) + H(\Phi^i) + \bar{H}(\bar{\Phi}^{\bar{j}}) \tag{3.60}$$

for a holomorphic function $H(\Phi^i)$, as was first pointed out by B. Zumino. The component form of the Lagrangian (3.59) is obtained using the methods described above, one finds

$$\begin{aligned}
\mathcal{L} = &-K_{i\bar{j}}\partial_a A^i\partial^a\bar{A}^{\bar{j}} - iK_{i\bar{j}}\bar{\chi}^{\bar{j}}\bar{\sigma}^a D_a\chi^i \\
&+K_{i\bar{j}}F^i\bar{F}^{\bar{j}} + \frac{1}{4}K_{i\bar{j}k\bar{l}}\chi^i\chi^k\bar{\chi}^{\bar{j}}\bar{\chi}^{\bar{l}} \\
&-F^i\left\{\frac{1}{2}K_{i\bar{l}}\Gamma^{\bar{l}}_{\bar{j}\bar{k}}\bar{\chi}^{\bar{j}}\bar{\chi}^{\bar{k}} - \frac{\partial W}{\partial A^i}\right\} \\
&-\bar{F}^{\bar{l}}\left\{\frac{1}{2}K_{i\bar{l}}\Gamma^i_{jk}\chi^j\chi^k - \frac{\partial\bar{W}}{\partial\bar{A}^{\bar{i}}}\right\} \\
&-\frac{1}{2}\frac{\partial^2W}{\partial A^i\partial A^j}\chi^i\chi^j - \frac{1}{2}\frac{\partial^2\bar{W}}{\partial\bar{A}^{\bar{i}}\partial\bar{A}^{\bar{j}}}\bar{\chi}^{\bar{i}}\bar{\chi}^{\bar{j}}.
\end{aligned} \tag{3.61}$$

Where

$$D_a\chi^i_\alpha = \partial_a\chi^i_\alpha + \Gamma^i_{jk}\partial_a A^j\chi^k_\alpha \tag{3.62}$$

the Kähler metric is

$$K_{i\bar{j}} = \frac{\partial^2 K}{\partial A^i \partial \bar{A}^{\bar{j}}} \tag{3.63}$$

and the Kähler connections are defined as

$$K_{j\bar{l}k} = K_{i\bar{l}}\,\Gamma^i_{jk} \tag{3.64}$$
$$K_{\bar{j}i\bar{k}} = K_{i\bar{l}}\,\Gamma^{\bar{l}}_{\bar{j}\bar{k}}. \tag{3.65}$$

We stress that the Lagrangian (3.61) is invariant under the supersymmetry transformations derived in (3.45), but now for the individual multiplets

$$\begin{aligned}
\delta_\xi A^i &= \sqrt{2}\xi\chi^i \\
\delta_\xi \chi^i_\beta &= \sqrt{2}\xi_\beta F^i + i\sqrt{2}\sigma^a_{\beta\dot{\beta}}\bar{\xi}^{\dot{\beta}}\partial_a A^i \\
\delta_\xi F^i &= i\sqrt{2}\bar{\xi}_{\dot{\alpha}}\bar{\sigma}^{a\dot{\alpha}\alpha}\partial_a\chi^i_\alpha.
\end{aligned} \tag{3.66}$$

The final step is to integrate out the auxiliary field sector, from the equations of motion for $\bar{F}^{\bar{j}}$ we have

$$K_{i\bar{j}}F^i - \frac{1}{2}K_{l\bar{j}}\Gamma^l_{ik}\chi^i\chi^k + \frac{\partial\bar{W}}{\partial\bar{A}^{\bar{j}}} = 0 \tag{3.67}$$

and when (3.67) is inserted into the Lagrangian (3.61) we find

$$\begin{aligned}
\mathcal{L} &= -K_{i\bar{j}}\partial_a A^i\partial^a\bar{A}^{\bar{j}} - iK_{i\bar{j}}\bar{\chi}^{\bar{j}}\bar{\sigma}^a D_a\chi^i + \frac{1}{4}R_{i\bar{j}k\bar{l}}\chi^i\chi^k\bar{\chi}^{\bar{j}}\bar{\chi}^{\bar{l}} \\
&\quad -\frac{1}{2}D_iD_jW\chi^i\chi^j - \frac{1}{2}D_{\bar{i}}D_{\bar{j}}\bar{W}\bar{\chi}^{\bar{i}}\bar{\chi}^{\bar{j}} - K^{i\bar{j}}D_iWD_{\bar{j}}\bar{W}.
\end{aligned} \tag{3.68}$$

Where

$$D_iW = \frac{\partial W}{\partial A^i} \tag{3.69}$$
$$D_iD_jW = \frac{\partial^2 W}{\partial A^i\partial A^j} - \Gamma^k_{ij}\frac{\partial W}{\partial A^k} \tag{3.70}$$

and $K^{i\bar{j}}$ is the inverse metric

$$K^{i\bar{j}}K_{k\bar{j}} = \delta^i_k. \tag{3.71}$$

We emphasize that the Lagrangians (3.61) and (3.68) are by construction supersymmetric; they are derived from (3.59) which can be written in the general form (3.41) and thus as we show in (3.42) is invariant up to a total derivative under (3.40). In a sence, the supersymmetry transformations are embedded inside (3.59).

Gauge invariant models

The supersymmetry transformations for the components of W_α are again found by applying the general supersymmetry transformation rule. In fact we will create Lagrangians invariant under the supersymmetry transformations by construction, since we will use the superfield method. For the Abelian W_α the supersymmetric Lagrangian reads

$$\mathcal{L} = \frac{1}{4}\int d^2\theta W^\alpha W_\alpha + h.c. = -\frac{1}{16} D^2\, W^\alpha W_\alpha| + h.c. \tag{3.72}$$

Expanding (3.72) in component form by using the projection definitions (3.31) and (3.32) we find

$$\mathcal{L} = -\frac{1}{4}F^{ab}F_{ab} - i\bar{\lambda}\bar{\sigma}^a\partial_a\lambda + \frac{1}{2}\mathrm{D}^2. \tag{3.73}$$

The equations of motion for D are

$$\mathrm{D} = 0 \tag{3.74}$$

and the on-shell theory becomes

$$\mathcal{L} = -\frac{1}{4}F^{ab}F_{ab} - i\bar{\lambda}\bar{\sigma}^a\partial_a\lambda. \tag{3.75}$$

Let us now see how the vector multiplet is coupled to a chiral model in a gauge invariant way. The superspace Lagrangian for a simple model is

$$\begin{aligned}\mathcal{L}_0 &= \int d^2\theta d^2\bar{\theta}\bar{\Phi}e^{2gV}\Phi + \frac{1}{4}(\int d^2\theta W^\alpha W_\alpha + h.c.) \\ &= \frac{1}{16}D^2\bar{D}^2\bar{\Phi}e^{2gV}\Phi| - \frac{1}{16}(D^2\, W^\alpha W_\alpha| + h.c.).\end{aligned} \tag{3.76}$$

We have already defined a gauge transformation as

$$V \to V - i\bar{\Lambda} + i\Lambda \tag{3.77}$$

for the vector multiplet where Λ is chiral. For the chiral multiplet we have

$$\Phi \to e^{-2ig\Lambda}\Phi \tag{3.78}$$

and

$$\bar{\Phi} \to e^{2ig\bar{\Lambda}}\Phi \tag{3.79}$$

where g is a coupling. It is easy to see that (3.76) is invariant under the combined transformations (3.77), (3.78) and (3.79). Note that the lowest component of Φ transforms as

$$A \to e^{-2ig\phi}A \tag{3.80}$$

exactly a gauge transformation. In fact this happens for all the components of Φ, this stems from the fact that Φ transforms covariantly, as defined by (3.78). Here another magnificent property of superspace is just revealed! Gauge invariance of the theory is easily seen from (3.76), without turning to component form calculations. We see that by implementing the desired symmetries in superspace they are guaranteed to be preserved by the complicated component form Lagrangian as well. To calculate the component form we turn to the Wess-Zumino gauge. Our Lagrangian becomes

$$\begin{aligned}\mathcal{L}_0 &= -\frac{1}{4}F^{ab}F_{ab} - i\bar{\lambda}\bar{\sigma}^a\partial_a\lambda + \frac{1}{2}\mathrm{D}^2 \\ &\quad -D^aAD_a\bar{A} - i\chi^\alpha\sigma^a_{\alpha\dot{\alpha}}D_a\bar{\chi}^{\dot{\alpha}} + F\bar{F} \\ &\quad +\sqrt{2}ig(\bar{A}\chi\lambda - \bar{\lambda}\bar{\chi}A) + g\mathrm{D}\bar{A}A. \end{aligned} \tag{3.81}$$

We have used the definitions

$$D_aA = \partial_aA + igv_aA \tag{3.82}$$

$$D_a\chi_\alpha = \partial_a\chi_\alpha + igv_a\chi_\alpha. \tag{3.83}$$

An important comment is in order. The Wess-Zumino gauge choice will now break supersymmetry in the sense that the supersymmetry transformations we derived earlier are not a symmetry of the Lagrangian any more. The solution to this is implemented by the problem it-self

- Supersymmetry transformations are not gauge covariant - they need improvement.
- Our Lagrangian is gauge invariant, thus does not need improvement.

In fact one replaces the partial derivatives in the old transformations with covariant derivatives and curvatures with respect to the gauge theory and finds the new transformations. This procedure is not at all *ad hoc*, it is rooted in the geometric nature of the gauge theories and can be derived from a superspace formalism where one defines the D^A $(A = a, \alpha, \dot{\alpha})$ supersymmetric derivatives to be gauge covariant. Then following the same procedure of definition via projections, all derivatives will be gauge covariant, thus supersymmetry transformations will be gauge covariant, and gauge fixing will be straightforward. See for example [30, 99, 156]. We can now integrate out D to find that there is a potential due to the gauging. The equation for D is

$$\mathrm{D} = -gA\bar{A} \tag{3.84}$$

for F we have

$$F = 0 \tag{3.85}$$

and the on-shell theory is

$$\begin{aligned}\mathcal{L}_0 &= -\frac{1}{4}F^{ab}F_{ab} - i\bar{\lambda}\bar{\sigma}^a\partial_a\lambda - D^aAD_a\bar{A} - i\chi^\alpha\sigma^a_{\alpha\dot{\alpha}}D_a\bar{\chi}^{\dot{\alpha}} \\ &\quad +\sqrt{2}ig(\bar{A}\chi\lambda - \bar{\lambda}\bar{\chi}A) - \frac{1}{2}g^2(A\bar{A})^2. \end{aligned} \tag{3.86}$$

Real linear models

The standard kinematic Lagrangian is

$$\mathcal{L}_0 = -\int d^4\theta L^2 \tag{3.87}$$

but one can consider more general models as well

$$\mathcal{L} = -\int d^4\theta G(L). \tag{3.88}$$

Using the definitions it is straightforward to find the component form of the linear multiplet kinetic Lagrangian

$$\mathcal{L}_0 = \frac{1}{2}h^a h_a - \frac{1}{2}\partial^a\phi\partial_a\phi - \frac{i}{2}\sigma^a_{\alpha\dot{\alpha}}(\psi^\alpha\partial_a\bar{\psi}^{\dot{\alpha}} + \bar{\psi}^{\dot{\alpha}}\partial_a\psi^\alpha). \tag{3.89}$$

Note that the real linear multiplet is dual to the chiral only when there is an isometry, for example, if the superspace Lagrangian for the chiral depends on $(\Phi + \bar{\Phi})$.

Complex linear models

The complex linear or nonminimal multiplet is defined as

$$\bar{D}^2\Sigma = 0. \tag{3.90}$$

The constraint (3.90) above is just the field equation for a free chiral multiplet. Note that if the further constraint $\Sigma = \bar{\Sigma}$ is imposed, the complex linear multiplet turns into a linear one. The standard kinetic Lagrangian for the complex linear superfield in superspace reads

$$\mathcal{L}_0 = -\int d^4\theta\, \Sigma\bar{\Sigma}. \tag{3.91}$$

Note the relative minus sign compared to the kinetic Lagrangian of a chiral multiplet. This is necessary for the theory to contain no ghosts. The relative minus sign of the complex linear multiplet Σ compared to the standard kinetic term for a chiral multiplet Φ can be understood in terms of a duality transformation. Indeed, consider the action

$$\mathcal{L}_D = -\int d^4\theta\big(\Sigma\bar{\Sigma} + \Phi\Sigma + \bar{\Phi}\bar{\Sigma}\big), \tag{3.92}$$

where Φ is chiral and Σ is unconstrained. Integrating out Φ we get both eq. (3.91) and the constraint (3.90). However, by integrating out Σ, we get $\Sigma = -\bar{\Phi}$. Plugging back this equality into (3.92), we get the standard kinetic term of a chiral multiplet

$$\mathcal{L}_0 = \int d^4\theta\Phi\bar{\Phi}. \tag{3.93}$$

As announced, the overall sign in Lagrangian (3.93) is opposite to that of (3.91).

To find the superspace equation of motion, we should express Σ in terms of an unconstrained superfield. This can be done by introducing a general spinor superfield Ψ^α with gauge transformation

$$\delta\Psi_\alpha = D^\beta\Lambda_{(\alpha\beta)} \tag{3.94}$$

where $\Lambda_{(\alpha\beta)}$ is arbitrary. It is easy to see that by defining

$$\Sigma = \bar{D}_{\dot{\alpha}}\bar{\Psi}^{\dot{\alpha}}, \tag{3.95}$$

Σ satisfies the constraint (3.90). Then the field equation following from eq. (3.91) is

$$D_\alpha \Sigma = 0\,. \tag{3.96}$$

Therefore, the field equation of a complex linear multiplet is just the constraint of a chiral multiplet and, as noticed above, the constraint on a linear is the field equation of a chiral. This indicated the duality between the two kind of multiplets, at least in the free case. The field content of the complex linear multiplet Σ is revealed via the projection over components as

$$\begin{aligned}
A &= \Sigma|, \\
\psi_\alpha &= \frac{1}{\sqrt{2}} D_\alpha \bar{\Sigma}|, \\
F &= -\frac{1}{4} D^2 \Sigma|, \\
\lambda_\alpha &= \frac{1}{\sqrt{2}} D_\alpha \Sigma|, \\
P_{\alpha\dot{\beta}} &= \bar{D}_{\dot{\beta}} D_\alpha \Sigma|\,, \qquad\qquad \bar{P}_{\alpha\dot{\beta}} = -D_\beta \bar{D}_{\dot{\alpha}} \bar{\Sigma}|, \\
\chi_\alpha &= \frac{1}{2} \bar{D}_{\dot{\alpha}} D_\alpha \bar{D}^{\dot{\alpha}} \bar{\Sigma}|\,, \qquad \bar{\chi}_{\dot{\alpha}} = \frac{1}{2} D^\alpha \bar{D}_{\dot{\alpha}} D_\alpha \Sigma|.
\end{aligned} \tag{3.97}$$

In other words, a complex linear multiplet contains a chiral multiplet (A, λ_α, F) and an antichiral spinor superfield $(\psi_\alpha, P_{\alpha\dot{\beta}}, \chi_\alpha)$. Therefore, the complex linear multiplet is a reducible 12 + 12 dimensional representation of the $\mathcal{N} = 1$ supersymmetry. It should be noted that since Σ is not chiral, there is no superpotential and there are no supersymmetric non-derivative interactions. However, the complex linear multiplet can still be consistently coupled to ordinary vector multiplets of the $\mathcal{N} = 1$ theory.

The supersymmetry transformations of the fermionic components of Σ

$$\delta\psi_\alpha = \sqrt{2} i \sigma^a_{\alpha\dot{\beta}} \bar{\xi}^{\dot{\beta}} \partial_a \bar{A} - \frac{1}{\sqrt{2}} \bar{\xi}^{\dot{\beta}} \bar{P}_{\alpha\dot{\beta}} \tag{3.98}$$

$$\delta\chi_\alpha = 2i\sigma^n_{\alpha\dot{\alpha}} \bar{\sigma}^{a\dot{\alpha}\beta} \xi_\beta \partial_a \bar{P}_n + i\sigma^a_{\alpha\dot{\alpha}} \bar{\sigma}^{n\dot{\alpha}\beta} \xi_\beta \partial_a \bar{P}_n - 4\xi_\alpha \partial^2 \bar{A} + 2i\sigma^a_{\alpha\dot{\alpha}} \bar{\xi}^{\dot{\alpha}} \partial_a \bar{F} \tag{3.99}$$

$$\delta\lambda_\alpha = \sqrt{2} \xi_\alpha F - \frac{1}{\sqrt{2}} \bar{\xi}^{\dot{\beta}} P_{\alpha\dot{\beta}}. \tag{3.100}$$

The transformation rulles of the bosonic sector of the complex linear multiplet is

$$\delta A \;=\; \sqrt{2}\bar{\xi}\bar{\psi} + \sqrt{2}\xi\lambda, \tag{3.101}$$

$$\delta F \;=\; \frac{i}{\sqrt{2}} \bar{\xi} \bar{\sigma}^a \partial_a \lambda + \frac{1}{2} \bar{\xi} \bar{\chi}, \tag{3.102}$$

$$\delta P_{\alpha\dot{\beta}} \;=\; -2\sqrt{2} i \xi^\gamma \sigma^a_{\gamma\dot{\beta}} \partial_a \lambda_\alpha + \sqrt{2} i \xi_\alpha \sigma^a_{\beta\dot{\beta}} \partial_a \lambda_\beta - \xi_\alpha \bar{\lambda}_{\dot{\beta}} - 2\sqrt{2} i \bar{\xi}_{\dot{\beta}} \sigma^a_{\alpha\dot{\rho}} \partial_a \bar{\psi}^{\dot{\rho}}. \tag{3.103}$$

In terms of the components of Σ, Lagrangian (3.91) is explicitly written as

$$\mathcal{L}_0 = A\partial^2 \bar{A} - F\bar{F} + i\partial_a \bar{\psi} \bar{\sigma}^a \psi + \frac{1}{2} P_a \bar{P}^a + \frac{1}{2\sqrt{2}} \big(\chi\lambda + \bar{\chi}\bar{\lambda} \big). \tag{3.104}$$

The complex vector P_a, the complex scalar F and the spinors λ, χ are auxiliary fields. Note that the minus sign in front of the superspace action (3.91) guarantees that the scalar A is a normal field and not a ghost. However, this choice of sign has flipped the sign of the $F\bar{F}$ relative to the action for a chiral multiplet.

3.1.4 Superfield Equations of Motion

The component field equations are easily derived from (3.61). Nevertheless, the superspace formulation has again more to offer. One can use simple superfield variation methods to find the equations of motion for the chiral superfields straight from (3.61). These superfield equations of motion will contain in fact the individual equations of motion for the component fields. Let us start with the simple example of the Lagrangian

$$\mathcal{L}_0 = \int d^2\theta d^2\bar{\theta}\Phi\bar{\Phi}. \tag{3.105}$$

A variation for the superfield Φ will read

$$\Phi + \delta\Phi = \Phi - \frac{1}{4}\bar{D}^2(\delta\mathcal{J}) \tag{3.106}$$

for $\delta\mathcal{J}$ an infinitesimal generic superfield. Note that in (3.106) we have used the chiral projection in order to maintain the superspace chirality of Φ. Inserting (3.106) into (3.105) we find

$$\delta\mathcal{L}_0 = \int d^2\theta d^2\bar{\theta}(-\frac{1}{4}\bar{D}^2(\delta\mathcal{J})\bar{\Phi}) \tag{3.107}$$

and via a superspace integration by parts we have

$$\delta\mathcal{L}_0 = \int d^2\theta d^2\bar{\theta}(\delta\mathcal{J}(-\frac{1}{4}\bar{D}^2\bar{\Phi})). \tag{3.108}$$

Now (3.108) describes a general superfield variation $\delta\mathcal{J}$ in the full superspace measure $\int d^2\theta d^2\bar{\theta}$, thus for the variation of the Lagrangian $\delta\mathcal{L}_0$ to vanish for all $\delta\mathcal{J}$ we have

$$-\frac{1}{4}\bar{D}^2\bar{\Phi} = 0. \tag{3.109}$$

Note that Eq. (3.109) is in fact a chiral superfield, and thus the only independent components of (3.109) will be the ones equivalent to the standard definition via projection method. The lowest component of (3.109) reads

$$-\frac{1}{4}\bar{D}^2\bar{\Phi}| = 0 \tag{3.110}$$

which leads to

$$\bar{F} = 0. \tag{3.111}$$

This is exactly the equation of motion (3.52) for the component F as was found from (3.51). The next component of (3.109) leads to the fermionic equation of motion

$$-\frac{1}{4\sqrt{2}}D_\alpha\bar{D}^2\bar{\Phi}| = 0 \ \rightarrow \ i\,\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{\chi}^{\dot{\alpha}} = 0. \tag{3.112}$$

Finally the highest component gives

$$\frac{1}{16}D^2\bar{D}^2\bar{\Phi}| = 0 \ \rightarrow \ \partial^2\bar{A} = 0. \tag{3.113}$$

Equations (3.111), (3.112) and (3.113) are the equations of motion for the massless chiral supermultiplet i.e., a vanishing auxiliary field, a massless complex scalar and a massless complex Weyl spinor.

3.2 Old-minimal Supergravity

We will not present here an introduction to $\mathcal{N}=1$ old-minimal supergravity, instead we will demonstrate how Lagrangian densities invariant under local supergauge transformations can be built. The approach we follow in doing so is based on the approach of J. Wess and B. Zumino in the formulation of the $\mathcal{N}=1$ local superspace. For an introduction and an extensive review we refer to [193].

3.2.1 Simple old-minimal supergravity

The first off-shell supergravity theory discovered was the so-called *old-minimal*. The field spectrum of this theory is

- The vielbein e_a^m with 6 propagating degrees of freedom
- The gravitino ψ_m^α with 12 (6 propagating) degrees of freedom
- The complex scalar auxiliary M with 2 auxiliary degrees of freedom
- The real vector auxiliary b_m with 4 auxiliary degrees of freedom

The transformation rules are

$$\begin{aligned}
\delta e_m{}^a &= i(\psi_m \sigma^a \bar{\zeta} - \zeta \sigma^a \bar{\psi}_m) \\
\delta \psi_m{}^\alpha &= -2\mathcal{D}_m \zeta^\alpha + i e_m{}^c \left\{ \frac{1}{3} M(\epsilon \sigma_c \bar{\zeta}) + b_c \xi^\alpha + \frac{1}{3} b^d (\zeta \sigma_d \bar{\sigma}_c)^\alpha \right\} \\
\delta M &= -\zeta(\sigma^a \bar{\sigma}^b \psi_{ab} + i b^a \psi_a - i \sigma^a \bar{\psi}_a M) \\
\delta b_{\alpha\dot{\alpha}} &= \zeta^\delta \left\{ \frac{3}{4} \bar{\psi}_\alpha{}^{\dot{\gamma}}{}_{\delta\dot{\gamma}\dot{\alpha}} + \frac{1}{4} \epsilon_{\delta\alpha} \bar{\psi}^{\gamma\dot{\gamma}}{}_{\gamma\dot{\alpha}\dot{\gamma}} - \frac{i}{2} \bar{M} \psi_{\alpha\dot{\alpha}\delta} \right. \\
&\qquad \left. + \frac{i}{4} (\bar{\psi}_{\alpha\dot{\rho}}{}^{\dot{\rho}} b_{\delta\dot{\alpha}} + \bar{\psi}_{\delta\dot{\rho}}{}^{\dot{\rho}} b_{\alpha\dot{\alpha}} - \bar{\psi}^{\dot{\rho}}_{\delta\ \dot{\alpha}} b_{\alpha\dot{\rho}}) \right\} + h.c. \qquad (3.114)
\end{aligned}$$

an the Lagrangian of this theory is

$$e^{-1}\mathcal{L}_{OM} = -\frac{1}{2}R - \frac{1}{3}M\bar{M} + \frac{1}{3}b^a b_a + \frac{1}{2}\epsilon^{klmn}(\bar{\psi}_k \bar{\sigma}_l \tilde{\mathcal{D}}_m \psi_n - \psi_k \sigma_l \tilde{\mathcal{D}}_m \bar{\psi}_n). \qquad (3.115)$$

It is interesting to note that the scalar auxiliary field M in this Lagrangian has opposite sign compared to the scalar auxiliary of the chiral multiplet. This is connected to the chiral compensator breaking the underlying superconformal theory. By the equation of motion for the auxiliary field one finds

$$M = 0 \qquad (3.116)$$

$$b_a = 0 \qquad (3.117)$$

and when thse are plugged back, we recover the on-shell $\mathcal{N}=1$ minimal supergravity

$$e^{-1}\mathcal{L} = -\frac{1}{2}R + \frac{1}{2}\epsilon^{klmn}(\bar{\psi}_k \bar{\sigma}_l \tilde{\mathcal{D}}_m \psi_n - \psi_k \sigma_l \tilde{\mathcal{D}}_m \bar{\psi}_n). \qquad (3.118)$$

The supersymmetry variations of the component fields M and b_a should vanish on-shell as well, this is indeed the case since they are in fact the equations of motion of a free gravitino. Finally, from the supersymmetry transformations of the gravitino one can observe one very interesting fact: the gravitino is the gauge field of supersymmetry. Thus, when supersymmetry becomes local, all derivatives are promoted into supercovariant derivatives much the same way as in other gauge theories, but now the gauge field is the gravitino.

3.2.2 Old-minimal Superspace Geometry

The supergravity theory was first discovered by brute force calculation. Since then, the powerfull formalism of superspace has been promoted to curved superspace as well, giving us deep insight and huge calculational power. Let us now present the *master formula* for the old-minimal supergravity geometry

$$(\mathcal{D}_C\mathcal{D}_B - (-1)^{bc}\mathcal{D}_B\mathcal{D}_C)V^A = -\mathcal{T}^D_{CB}\mathcal{D}_D V_A + (-1)^{d(b+c)}V^D\mathcal{R}_{CBD}{}^A \tag{3.119}$$

where for fermions $b = 1$ and for bosons $b = 0$ and $A = a, \alpha, \dot{\alpha}$. Here we need to fix the notation.

- The Greek indices will refer to spinors while the Latin will refer to vectors.
- Letters from the start of the alphabet represent flat indices, while letters from the middle refer to curved indices.

For example, μ is a curved spinor index, α is a flat spinor index, m is a curved space vector index and, a is a flat space vector index. Inside formula (3.119) there is three important quantities

- The superspace covariant derivatives $\mathcal{D}_C$
- The super-torsion $\mathcal{T}^D_{CB}$
- The Riemman tensor of superspace $\mathcal{R}_{CBD}{}^A$.

It is interesting to note that when gravity decouples the torsion component $\mathcal{T}^d_{\alpha\dot{\alpha}}$ should become

$$\mathcal{T}^d_{\alpha\dot{\alpha}} = 2i\sigma^d_{\alpha\dot{\alpha}} \tag{3.120}$$

since the generic formula (3.119) should reduce to supersymmetry. In fact the relation (3.120) is used as a constraint on the torsions of superspace in order to reduce the many components to those that are independent; constraints like (3.120) are refered to as conventional constraints. Thus, along with the constraints that guarantee we will recover supersymmetry in the $M_P \to \infty$ limit, one also takes into account the self consistency of the supergravity algebra, i.e., the Bianchi identities. Solving the Bianchi identities (with the conventional constraints) is not a trivial proceedure, in fact there is a variety of solutions with only two representing a minimal supergravity. The solution to the Bianchi identities first discovered was called Old-minimal supergravity, and it can be found in [193].

After the prosedure of solving the Bianchi identities one can express all the superspace curvatures and torsions in terms of a small number of superfields. These are the curvature superfields of old-minimal supergravity. Their components are again defined via projection but now one has to use the superspace covariant derivatives. Thus, the gravitational curvature tensors, the gravitino field strength and curvature and the auxiliary fields of supergravity all settle inside the curvature superfields.

The Ricci superfield $\mathcal{R}$

This superfield plays the central role in old-minimal supergravity since it is used in order to construct the simple $\mathcal{N}=1$ supergravity action. The Ricci superfield $\mathcal{R}$ is a chiral superfield

$$\bar{\mathcal{D}}_{\dot{\alpha}}\mathcal{R}=0 \tag{3.121}$$

with lowest component the auxiliary field M

$$\mathcal{R}|=-\frac{1}{6}M. \tag{3.122}$$

The next component is

$$\mathcal{D}_{\alpha}\mathcal{R}|=-\frac{1}{6}(\sigma^a\bar{\sigma}^b\psi_{ab}+ib^a\psi_a-i\sigma^a\bar{\psi}_aM)_{\alpha} \tag{3.123}$$

while the highest component of this curvature chiral superfield is

$$\begin{aligned}\mathcal{D}^2\mathcal{R}| &= -\frac{1}{3}R+\frac{4}{9}M\bar{M}+\frac{2}{9}b^ab_a-\frac{2i}{3}e_a{}^m\mathcal{D}_mb^a+\frac{1}{3}\bar{\psi}\bar{\psi}M-\frac{1}{3}\psi_m\sigma^m\bar{\psi}_nb^n \\ &+ \frac{2i}{3}\bar{\psi}^m\bar{\sigma}^n\psi_{mn}+\frac{1}{12}\epsilon^{klmn}[\bar{\psi}_k\bar{\sigma}_l\psi_{mn}+\psi_k\sigma_l\bar{\psi}_{mn}].\end{aligned} \tag{3.124}$$

The superfield $\mathcal{G}_a$

The real vector auxiliary field b_a of old-minimal supergravity resides inside the real superfield $\mathcal{G}_a$ as its lowest component

$$\mathcal{G}_a|=-\frac{1}{3}b_a. \tag{3.125}$$

This superfield satisfies the following Bianchi identities

$$\mathcal{D}^{\alpha}\mathcal{G}_{\alpha\dot{\beta}}=\bar{\mathcal{D}}_{\dot{\beta}}\bar{\mathcal{R}}\ ,\ \bar{\mathcal{D}}^{\dot{\beta}}\mathcal{G}_{\alpha\dot{\beta}}=\mathcal{D}_{\alpha}\mathcal{R}. \tag{3.126}$$

It is interesting to mention that in its vector component it contains the Riemann tensor, the bosonic part is

$$\bar{\sigma}_b^{\dot{\alpha}\alpha}\mathcal{D}_{\alpha}\bar{\mathcal{D}}_{\dot{\alpha}}\mathcal{G}_a|=(\frac{1}{6}R+\frac{1}{9}M\bar{M}+\frac{1}{9}b^2)\eta_{ab}-R_{ab}-\frac{2i}{3}\mathcal{D}_bb_a-\frac{1}{3}\epsilon_{ab}^{cd}\mathcal{D}_cb_d-\frac{2}{9}b_ab_b. \tag{3.127}$$

The Weyl superfield $\mathcal{W}_{\alpha\beta\gamma}$

The third basic superfield of Old-minimal supergravity is a curvature superfield which contains the Weyl tensor in its lowest fermionic component. It is a chiral superfield

$$\bar{\mathcal{D}}_{\dot{\alpha}}\mathcal{W}_{\alpha\beta\gamma}=0 \tag{3.128}$$

which is also completely symmetric in its indices. Finally it satisfies the following identity

$$\mathcal{D}^{\alpha}\mathcal{W}_{\alpha\beta\gamma}+\frac{i}{2}(\mathcal{D}_{\beta\dot{\beta}}\mathcal{G}_{\gamma}{}^{\dot{\beta}}+\mathcal{D}_{\gamma\dot{\beta}}\mathcal{G}_{\beta}{}^{\dot{\beta}})=0. \tag{3.129}$$

3.2.3 Superfields in Curved Space

Superfields in curved space are defined essentially again via projection but using the curved superspace covariant derivatives.

Chiral superfields

The chiral superfield satisfies the covariant constraint

$$\bar{\mathcal{D}}_{\dot{\alpha}}\Phi = 0. \tag{3.130}$$

The definition of the components of the curved superspace chiral superfield is

$$\begin{aligned} \Phi| &= A \\ \mathcal{D}_\alpha\Phi| &= \sqrt{2}\chi_\alpha \\ \mathcal{D}^2\Phi| &= -4F. \end{aligned} \tag{3.131}$$

To find the transformation rules we proceed as in the global case and we have

$$\delta A = \sqrt{2}\zeta\chi \tag{3.132}$$

$$\delta\chi_\alpha = \sqrt{2}F\zeta_\alpha + i\sqrt{2}\sigma^a_{\alpha\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}}\hat{D}_a A \tag{3.133}$$

$$\delta F = \frac{\sqrt{2}}{3}\bar{M}\zeta\chi + \bar{\zeta}^{\dot{\alpha}}(i\sqrt{2}\hat{D}_{\alpha\dot{\alpha}}\chi^\alpha - \frac{\sqrt{2}}{6}b_{\alpha\dot{\alpha}}\chi^\alpha) \tag{3.134}$$

where we have made use of the supercovariant derivatives

$$\hat{D}_a A = e_a{}^m\left(\partial_m A - \frac{1}{\sqrt{2}}\psi^\alpha_m\chi_\alpha\right) \tag{3.135}$$

$$\hat{D}_a\chi_\alpha = e_a{}^m\left(\partial_m\chi_\alpha - \omega_{m\alpha}{}^\beta\chi_\beta - \frac{1}{\sqrt{2}}\psi_{m\alpha}F - \frac{i}{\sqrt{2}}\bar{\psi}_m{}^{\dot{\beta}}\hat{D}_{\alpha\dot{\beta}}A\right). \tag{3.136}$$

Chiral projection

The chiral projection in old-minimal supergravity differs from the one we encounter in supersymmetry. This has to do with the structure of the theory. For a general superfield $\mathcal{U}$ it is straightforward to prove that the superfield

$$-\frac{1}{4}(\bar{\mathcal{D}}^2 - 8\mathcal{R})\mathcal{U} \tag{3.137}$$

satisfies

$$\bar{\mathcal{D}}_{\dot{\alpha}}[-\frac{1}{4}(\bar{\mathcal{D}}^2 - 8\mathcal{R})\mathcal{U}] = 0. \tag{3.138}$$

Gauge Superfields

The superfields that contain the gauge fields are defined as in global supersymmetry but with the use of curved superspace derivatives. We again make use of the following chiral superfield

$$W_\alpha = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 8\mathcal{R})\mathcal{D}_\alpha V \tag{3.139}$$

which is invariant under the gauge transformation

$$V \to V + \Lambda + \bar{\Lambda} \tag{3.140}$$

where Λ is a chiral superfield and $\bar{\Lambda}$ an anti-chiral. For the non-Abelian case we similarly have

$$W_\alpha = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 8\mathcal{R})e^{-V}\mathcal{D}_\alpha e^V. \tag{3.141}$$

3.2.4 Chiral Densities and Invariant Actions

Now that we have defined the covariant components of the superfields, it is convenient to introduce new theta variables such that the grassmann expansion of a chiral superfield in these variables is exactly the covariant components. This is simply postulating

$$\Phi = A + \sqrt{2}\Theta\chi + \Theta^2 F \tag{3.142}$$

where Θ are the so-called new theta variables. Superfields now transform covariantly, thus, in order to built actions that are invariant up to a total derivative we have to introduce the notion of superscace densities. Specifically, we will use the chiral density

$$2\mathcal{E} = e\left\{1 + i\Theta\sigma^a\bar{\psi}_a - \Theta\Theta\Big(M^* + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b\Big)\right\} \tag{3.143}$$

which guaranties that quantities of the form

$$\int d^4x\, 2\mathcal{E}\,[\text{Something Chiral}] \tag{3.144}$$

are invariant under supersymmetry. For example, the Lagrangian of simple supergravity is

$$\mathcal{L}_{OM} = -6\int d^2\Theta\, 2\mathcal{E}\,\mathcal{R} + h.c. \tag{3.145}$$

and by using the projection definition of $\mathcal{R}$ and the superspace geometry we recover the result (3.115). The most general gauge invariant supergravity Lagrangian that can be built has the form

$$\mathcal{L}_{tot} = \int d^2\Theta\; 2\mathcal{E}\left\{\frac{3}{8}\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)e^{-\tilde{K}/3} + \frac{1}{16g^2}\mathcal{H}_{(ab)}(\Phi)W^{(a)}W^{(b)} + P(\Phi)\right\} + h.c. \tag{3.146}$$

where

$$\tilde{K} = K(\Phi,\bar{\Phi}) + \Gamma(\Phi,\bar{\Phi},V), \tag{3.147}$$

and

$$\Gamma(\Phi,\bar{\Phi},V) = V^{(a)}\mathcal{D}^{(a)} + \frac{1}{2}g_{i\bar{r}}X^{i(a)}\bar{X}^{\bar{r}(b)}V^{(a)}V^{(b)}. \tag{3.148}$$

In addition, as usual, $V^{(a)}$ is the supersymmetric Yang-Mills vector multiplet and

$$W_\alpha = W_\alpha^{(a)}T^{(a)} = -\frac{1}{4}\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)e^{-V}\mathcal{D}_\alpha e^V \tag{3.149}$$

is the gauge invariant chiral superfield containing the gauge field strength. The holomorphic function $\mathcal{H}_{(ab)}$ is included for generality, but in what follows we will consider $\mathcal{H}_{(ab)} = \delta_{(ab)}$. Expression (3.148) is calculated in the Wess-Zumino gauge, $\mathcal{D}^{(a)}$ are the so-called Killing potentials whereas $X^{i(a)}$ and $\bar{X}^{\bar{r}(b)}$ are the components of the holomorphic Killing vectors that generate the isometries of the Kähler manifold. The Killing vectors and the Killing potential are connected via

$$g_{i\bar{r}}\bar{X}^{\bar{r}(a)} = i\frac{\partial}{\partial a^i}\mathcal{D}^{(a)}, \tag{3.150}$$

$$g_{i\bar{r}}X^{i(a)} = -i\frac{\partial}{\partial \bar{a}^{\bar{r}}}\mathcal{D}^{(a)} \tag{3.151}$$

where a^i and $\bar{a}^{\bar{r}}$ are the Kähler space complex co-ordinates. We note that the $\mathcal{D}^{(a)}$ that correspond to some $U(1)$ gauged symmetry are only determined up to a constant ξ, which is the analog for the Fayet-Iliopoulos D-term in supergravity. After following the standard procedure, the bosonic part of the Lagrangian (3.146) turns out to be

$$\begin{aligned} e^{-1}\mathcal{L}_{tot} = & - \frac{1}{2}R - g_{i\bar{r}}\tilde{D}_m A^i \tilde{D}^m \bar{A}^{\bar{r}} - \frac{1}{16g^2}F^{(a)}_{mn}F^{mn(a)} \\ & - e^K\Big(g^{i\bar{r}}D_iPD_{\bar{r}}\bar{P} - 3P\bar{P}\Big) - \frac{1}{2}g^2\big(\mathcal{D}^{(a)}\big)^2 \end{aligned} \tag{3.152}$$

where

$$D_iP = P_i + K_iP. \tag{3.153}$$

For the covariant derivative we have

$$\tilde{D}_cA^j = \partial_cA^j - \frac{1}{2}B^{(a)}_c X^j_{(a)} \tag{3.154}$$

and $B^{(a)}_c$ is a vector field (belonging to the $V^{(a)}$ vector multiplet) that corresponds to the gauged isometries, with field strength $F^{(a)}_{mn}$.

3.3 New-minimal Supergravity

There exist another minimal off-shell version of the $\mathcal{N} = 1$ supergravity, the so-called *new-minimal.*

3.3.1 Simple new-minimal supergravity

In the new minimal supergravity instead, the multiplet consists of the vierbein $e^a{}_m$ and its supersymmetric partner, the gravitino ψ^α_m. In order to implement supersymmetry off-shell and the propagation of the physical degrees of freedom only, one has to also add auxiliary fields, as in the old minimal supergravity. However, in this case, the auxiliary fields are no longer a vector and a scalar but a 2-form B_{mn} with gauge invariance (B-gauge)

$$\delta B_{mn} = \partial_m\xi_n - \partial_n\xi_m, \tag{3.155}$$

and a gauge vector A_m with associated R gauge invariance

$$\delta A_m = -\partial_m\phi\ . \tag{3.156}$$

Thus, to wrap it up, the off-shell new minimal supergravity is based on the gravitational multiplet

$$e_m^a\,,\quad \psi_m\,,\quad A_m\,,\quad B_{mn}\ . \tag{3.157}$$

For more specific details on the structure of this theory the reader should consult [90].

It has been argued that the natural superspace geometry for four-dimensional $\mathcal{N}=1$ heterotic superstring corresponds to the new minimal formulation of the $\mathcal{N}=1$ supergravity [53, 152, 168]. This R symmetry is however anomalous (actually it is a mixed superconformal-Weyl-$U(1)$ anomaly [100]). Nevertheless, by using the Green-Schwarz mechanism, the symmetry is restored at one loop thanks to the introduction of a matter linear multiplet together with supersymmetric Lorentz and Chern-Simons terms [26, 157]. Note that this R symmetry has interesting implications on the gravitino over-abundance problem [63, 64].

In the new minimal supergravity, there exist three sets of chiral and Lorentz connections

$$\begin{aligned} \omega^{\pm}_{abc} &= \omega_{abc} \pm H_{abc}\ , \\ A^+_m &= A_m - H_m\ , \\ A^-_m &= A_m - 3H_m, \end{aligned} \tag{3.158}$$

where the following notation has been used

$$\begin{aligned} H_{mnl} &= \partial_m B_{nl} + \partial_n B_{lm} + \partial_l B_{mn} \\ &\quad + \frac{i}{8}\bar{\psi}_m\gamma_n\psi_l + \frac{i}{8}\bar{\psi}_n\gamma_l\psi_m + \frac{i}{8}\bar{\psi}_l\gamma_m\psi_n\ , \\ H^m &= -\frac{1}{3!}\varepsilon^{mnkl}H_{nkl}. \end{aligned} \tag{3.159}$$

The covariant derivatives in this formulation are therefore defined as

$$\begin{aligned} \mathcal{D} &= d + \delta_L(\omega_{ab}) + \delta_A(\mathcal{A})\ , \\ \mathcal{D}^{\pm} &= d + \delta_L(\omega^{\pm}_{ab}) + \delta_A(\mathcal{A}^{\pm}), \end{aligned} \tag{3.160}$$

with

$$\begin{aligned} &\delta_A(\phi)\Phi = i\,n\,\phi\,\Phi\ , \\ &\delta_L(\Lambda)\Phi = \frac{i}{2}S_{ab}\Lambda^{ab}\Phi\ , \\ &\omega^{\pm}_{ab} = \omega^{\pm}_{abm}dx^m\,, \qquad \mathcal{A}^{\pm} = A^{\pm}_m dx^m\ . \end{aligned} \tag{3.161}$$

For the gravitino, for example, we have $S_{ab} = \sigma_{ab}/2$ and $n = -\gamma_5/2$. Here $\delta_A(\phi)$, $\delta_L(\Lambda)$ denote the $U(1)$ R-symmetry and Lorentz transformations with parameters ϕ and Λ, respectively. Supercovariant derivatives $\hat{\mathcal{D}}$ are defined as usual and it should be noted for future reference that $\hat{\mathcal{D}}_a^{\pm}H_b = \hat{\mathcal{D}}_a H_b$ and for any neutral vector $\hat{\mathcal{D}}_a^{\pm}V^a = \hat{\mathcal{D}}_a V^a$.

The transformations of the supergravity multiplet fields under supersymmetry are [90, 179, 180]

$$\begin{aligned} \delta e^a{}_m &= \frac{i}{2}\bar{\epsilon}\gamma^a\psi_m\ , \\ \delta\psi_m &= -\mathcal{D}_m^{+}\epsilon\ , \\ \delta B_{mn} &= \frac{i}{4}\bar{\epsilon}\left(\gamma_m\psi_n - \gamma_n\psi_m\right)\ , \\ \delta A^-_m &= \frac{i}{4}\bar{\epsilon}\gamma_m\gamma_5\sigma^{ab}\psi_{ab}\ , \end{aligned} \tag{3.162}$$

these transformations form an algebra along with general coordinate, Lorentz, chiral and B-gauge transformations. The supersymmetry parameter ϵ transforms as $\delta_A \epsilon = -(i\gamma_5/2)\phi\epsilon$ under chiral transformations so that in two component notation ψ_m, ϵ, θ have chiral weight $\frac{1}{2}$ and $\bar{\psi}_m, \bar{\epsilon}, \bar{\theta}$ have chiral weight $-\frac{1}{2}$. The chiral weight of the other components follows by these rules. The gravitino curvature used in (3.162) is defined in (3.164).

Throughout the work on new-minimal supergravity we use a Minkowski metric with signature (-,+,+,+), and the fully antisymmetric tensor is taken as $\varepsilon_{0123} = +1$. The Dirac matrix conventions are $\{\gamma_a, \gamma_b\} = -2g_{ab}$, $\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$, while we use $\sigma_{ab} = \frac{i}{2}[\gamma_a, \gamma_b]$, and $\bar{\psi} = \psi^\dagger C$.

In a Majorana representation $C = \gamma^0$ and the Majorana condition is $\psi = \psi^\star$. The two-component spinor formalism is derived from the following chiral representation of the Dirac matrices,

$$
\begin{aligned}
&\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \gamma_a = \begin{pmatrix} 0 & \sigma_a \\ \bar{\sigma}_a & 0 \end{pmatrix}, \\
&\sigma_a = (1, \vec{\sigma}), \quad \bar{\sigma}_a = (1, -\vec{\sigma}), \\
&\psi = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}, \quad \bar{\psi} = \begin{pmatrix} -\psi^\alpha, -\bar{\psi}_{\dot{\alpha}} \end{pmatrix} .
\end{aligned} \tag{3.163}
$$

The gravitino curvature is given by

$$
\psi_{mn} = \mathcal{D}^+_m \psi_n - \mathcal{D}^+_n \psi_m , \qquad \psi_{ab} = e^a_m e^b_n \psi_{mn} \tag{3.164}
$$

and the Rarita-Schwinger operator is

$$
r^a = \frac{1}{4}\gamma_5 \gamma_b \varepsilon^{bade} \psi_{de} . \tag{3.165}
$$

Finally, the Ricci scalar, the Ricci tensor and the Riemann curvature are given by

$$
\begin{aligned}
\mathcal{R} &= \eta^{ca} \mathcal{R}_{ca} , \\
\mathcal{R}_{ca} &= \mathcal{R}_{nma}{}^b e_b{}^m e_c{}^n , \\
\mathcal{R}_{nma}{}^b &= \partial_n \omega_{ma}{}^b - \partial_n \omega_{na}{}^b + \omega_{ma}{}^c \omega_{nc}{}^b - \omega_{na}{}^c \omega_{mc}{}^b .
\end{aligned} \tag{3.166}
$$

The action for Poincaré supergravity is obtained by the Fayet-Iliopoulos term of the chiral gauge multiplet (6.126) and reads

$$
\frac{1}{\kappa^2}\mathcal{L}_{sugra} = \frac{1}{\kappa^2}[V_{\mathrm{R}}]_D = \frac{1}{\kappa^2}\, e \left(\frac{1}{2}R + \bar{\psi}^a r_a + 2A_a H^a - 3H_a H^a \right) . \tag{3.167}
$$

Variation of the action (3.167) with respect to A_m and B_{mn} gives

$$
H_m = 0 = \epsilon^{mnrs} \partial_m A_n . \tag{3.168}
$$

Thus the vector H_m vanish and A_m reduces to a pure gauge and can therefore be set to zero by a gauge transformation. Finally then, the on-shell action of the new-minimal supergravity turns out to be

$$
S^{on-shell}_{sugra} = \frac{1}{\kappa^2} \int d^4x \, e \left(\frac{1}{2}R + \bar{\psi}^a r_a \right) , \tag{3.169}
$$

which matches the on-shell $\mathcal{N} = 1$ old minimal supergravity [66, 97].

3.3.2 New-minimal Superspace Geometry and Multiplets

The superspace derivatives are defined in the usual way [90, 99, 113] and the very structure of the new minimal supergravity is incarnated in their commutation and anti-commutation relations

$$
\begin{aligned}
\{\nabla, \bar{\nabla}\} &= 2i\nabla\!\!\!/^{-} \,, \\
\left[\nabla_a^-, \nabla\right] &= \gamma_a \left(\frac{1}{2} T^{bc} S_{bc} + Tn - i\gamma_5 E\!\!\!/ \nabla\right) \,, \\
\left[\nabla_a^-, \nabla_b^-\right] &= \frac{i}{2} S^{cd} R^-_{cdab} + inF^-_{ab} - 2E_{abc}\eta^{cd}\nabla_d^- + \frac{1}{2}\bar{T}_{ab}\nabla \,.
\end{aligned}
\tag{3.170}
$$

Here $F_{mn} = \partial_m A_n - \partial_n A_m$ is the field strength of the gauge field A_m, $E_{abc} = -\varepsilon_{abcd}E^d$ and the superfields E_a, T_{ab} and T will be defined in a moment.

A general multiplet of new minimal supergravity is

$$
V = (C, \chi, H, K, V_a, \lambda, D). \tag{3.171}
$$

It is specified by the spin and the chiral weight

$$
\delta_L C = \frac{i}{2}\Lambda_{ab} S^{ab} C \,, \tag{3.172}
$$

$$
\delta_A C = in\phi C \tag{3.173}
$$

of its lowest component C, respectively. Frequently the two real scalars H, K are traded for a complex $H + iK$ one. The supersymmetry transformations of this multiplet are

$$
\begin{aligned}
\delta C &= -\frac{1}{2}\bar{\epsilon}\chi \,, \\
\delta\chi &= \frac{1}{2}\left\{i\hat{\mathcal{D}\!\!\!/}^{-} C + \gamma_5 V\!\!\!/ + H - \gamma_5 K\right\}\epsilon(-)^{\mathcal{F}} \,, \\
\delta(H \pm iK) &= -i\bar{\epsilon}\,\frac{1 \pm \gamma_5}{2}\left\{\gamma_5\lambda + \hat{\mathcal{D}\!\!\!/}^{-}\chi - 2i\gamma_5 H\!\!\!/\chi - i\xi C\right\} \,, \\
\delta V_a &= -\frac{i}{2}\bar{\epsilon}\left\{\gamma_a\lambda - \gamma_5\hat{\mathcal{D}}_a^{\,-}\chi - i\gamma_a H\!\!\!/\chi\right\} \,, \\
\delta\lambda &= -\left\{\frac{i}{4}\sigma_{ab}\hat{P}^{ab} + \frac{i\gamma_5}{2}D\right\}\epsilon(-)^{\mathcal{F}} - \frac{i}{2}\xi(\bar{\epsilon}\gamma_5\chi) \,, \\
\delta D &= -\frac{1}{2}\bar{\epsilon}\gamma_5\left\{\hat{\mathcal{D}\!\!\!/}^{-}\lambda - \gamma_a\xi V^a + i\Delta\chi\right\} \,.
\end{aligned}
\tag{3.174}
$$

We have used the following definitions

$$
\begin{aligned}
\xi &= \frac{i}{2}\psi_{ab}S^{ab} - i\gamma_5\gamma\cdot rn \,, \\
\Delta &= -\frac{i}{2}\hat{F}^+_{ab}S^{ab} - \frac{i}{2}\hat{R}^- n \,, \\
\hat{P}_{ab} &= \hat{\mathcal{D}}_a^- V_b - \hat{\mathcal{D}}_b^- V_a - 2H_{abc}V^d + \frac{i}{2}\bar{\psi}_{ab}\gamma_5\chi \,,
\end{aligned}
\tag{3.175}
$$

and the factor $(-)^{\mathcal{F}}$ accounts for the Fermi or Bose statistics of the first component. Note that ξ and Δ only involve the spin and chiral generators of the first component. The properties of the general multiplet can be encoded in the following superfield representation

$$V = C - \bar{\theta}\chi - \frac{1}{2}\bar{\theta}\left\{H - i\gamma_5 K + \gamma_5 \not{V}\right\}\theta \ ,$$
$$+i(\bar{\theta}\theta)\bar{\theta}\left\{\gamma_5\lambda + \frac{1}{2}\hat{\not{\mathcal{D}}}^{-}\chi - \frac{3i\gamma_5}{2}\not{H}\chi - i\xi C\right\} + \frac{1}{4}\left(\bar{\theta}\theta\right)^2\left(D + \frac{1}{2}\hat{\Box}^{-}C\right). \qquad (3.176)$$

Constrained multiplets may be obtained by imposing appropriate constraints on the general multiplet V. Known representations include complex vector and real vector multiplets, gauge and chiral multiplets and, linear and real linear multiplets. We discuss below the multiplets relevant for our work.

The R-symmetry gauge superfield $V_{\mathbf{R}}$

In new-minimal supergravity there exists the gauge multiplet of the supersymmetry algebra, namely

$$V_{\mathrm{R}} = \left(A_m^{-}, -\gamma_5\gamma\cdot r, -\frac{1}{2}\hat{\mathcal{R}}^{-}\right), \qquad (3.177)$$

with $\hat{\mathcal{R}}^{-} = \hat{\mathcal{R}} + 6H_aH^a$, which we will use in the following.

The Einstein superfield E_a

The Einstein multiplet is a real linear multiplet (with chiral weight zero), which means that

$$E_a = E_a^{*}\,, \qquad \nabla^2 E_a = \bar{\nabla}^2 E_a = 0 \ , \qquad (3.178)$$

and moreover, it satisfies the Bianchi identity

$$\nabla_a E^a = 0 \ , \qquad (3.179)$$

a property that only appears in the new minimal supergravity and it is of crucial importance for our results. Indeed, one can see that the independent components of the Einstein multiplet contain the Einstein tensor as the highest component. Specifically

$$E_a = \left(H_a, i\gamma_5 r_a, \frac{1}{2}(\hat{G}^{+}_{ab} - {}^{*}\hat{F}^{+}_{ab})\right), \qquad (3.180)$$

where $\hat{G}^{+}_{ab} - {}^{*}\hat{F}^{+}_{ab} = \hat{G}_{ab} - {}^{*}\hat{F}_{ab} - g_{ab}H_dH^d - 2H_aH_b$ with ${}^{*}\hat{F}^{+}_{ab}$ the supercovariant dual of the field strength defined as ${}^{*}F_{mn} = \frac{1}{2}\varepsilon_{mnkl}F^{kl}$. Moreover, r_m is the Rarita-Schwinger operator and $\hat{G}_{ab}$ is the supercovariant Einstein tensor [90].

The Riemann superfield $T_{ab\alpha}$

The irreducible pieces of the Riemann multiplet are the scalar curvature multiplet, T^{α}, and the Weyl multiplet, W^{α}_{ab}. The Riemann multiplet is chiral $\left(\bar{\nabla}_{\dot{\alpha}}T^{\alpha}_{ab} = 0\right)$ with components

$$T_{ab} = \psi_{ab} - \left(\frac{i}{2}\sigma^{cd}\hat{\mathcal{R}}^{+}_{cdab} + i\hat{F}^{+}_{ab}\right)\theta + i\hat{\not{\mathcal{D}}}^{-}\bar{\psi}_{ab}\theta^2. \qquad (3.181)$$

The rest curvature multiplets are defined as

$$
\begin{aligned}
T &= \frac{1}{2}\sigma_{ab}T^{ab} \ , \\
W_{ab} &= \frac{1}{24}\left(3\sigma_{cd}\sigma_{ab}+\sigma_{ab}\sigma_{cd}\right)T^{cd} \ ,
\end{aligned}
$$

that is the scalar curvature multiplet and Weyl multiplet respectively.

3.3.3 Chiral superfields in new-minimal supergravity

Chiral superfields

A chiral multiplet $\Phi(A,\chi,F)$ is defined by the constraint $\bar{\nabla}_{\dot{\alpha}}\Phi = 0$ and its embedding in the general multiplet is given by

$$
V(\Phi) = (A, \chi_L, F, -iF, -i\hat{\mathcal{D}}^{-}_{a}A, -i\xi A, -i\Delta A). \tag{3.182}
$$

The transformation rules are

$$
\begin{aligned}
\delta A &= \frac{1}{2}\epsilon\chi \ , \\
(-)^{\mathcal{F}}\delta\chi &= i\hat{\not{\mathcal{D}}}^{-}A\bar{\epsilon} + F\epsilon \ , \\
\delta F &= \frac{1}{2}\bar{\epsilon}(i\hat{\not{\mathcal{D}}}^{-} + 2\not{H})\chi + \bar{\epsilon}\bar{\xi}A \ ,
\end{aligned} \tag{3.183}
$$

and its chiral superfield representation is

$$
\Phi = A + \theta\chi + \theta^2 F. \tag{3.184}
$$

Up to field redefinitions one can always define the components of a superfield by projections. A common projection which we use throughout this work is [99, 113]

$$
\begin{aligned}
\Phi| &= A \ , \\
\nabla_\alpha\Phi| &= \chi_\alpha \ , \\
-\frac{1}{4}\nabla^2\Phi| &= F,
\end{aligned} \tag{3.185}
$$

where $\nabla^2 \equiv \nabla^\alpha\nabla_\alpha$ and similarly $\bar{\nabla}^2 \equiv \bar{\nabla}_{\dot{\alpha}}\bar{\nabla}^{\dot{\alpha}}$.

Chiral projection

From an arbitrary multiplet V of weight n, one can form a chiral multiplet with weight $n+1$ by the chiral projection operator

$$
\Pi(V) = -\frac{1}{4}\bar{\nabla}^2 V, \tag{3.186}
$$

with components

$$
\begin{aligned}
\Pi(V)| &= \bar{F} \ , \\
\nabla_\alpha\Pi(V)| &= i(\hat{\not{\mathcal{D}}}^{-}\bar{\chi} + 2i\not{H}\bar{\chi} - \lambda - i\xi C)_\alpha \ , \\
-\frac{1}{4}\nabla^2\Pi(V)| &= \frac{1}{2}\left\{D - i(\hat{D}^{-} - 2iH)\cdot(V + i\hat{D}^{-}C) + i\Delta C + \frac{i}{2}\bar{\psi}_{ab}\bar{\sigma}^{ab}\bar{\chi} + 2\bar{\xi}\bar{\chi}\right\} \ .
\end{aligned} \tag{3.187}
$$

3.3.4 Chiral Densities and Invariant Actions

Chiral multiplets with chiral weight $n = 1$ can be used to form invariant actions by the F-density formula [180]

$$[\Sigma]_F = e\left\{F + \frac{i}{2}\chi\sigma\cdot\bar{\psi} + \frac{i}{2}A\,\bar{\psi}^a\bar{\sigma}_{ab}\bar{\psi}^b\right\}. \tag{3.188}$$

In superfield notation this can be written as

$$[\Sigma]_F = \int d^2\theta\,\mathcal{E}\Sigma, \tag{3.189}$$

with

$$\mathcal{E} = e\left\{1 - i\theta\sigma\cdot\bar{\psi} + \frac{i}{2}\theta^2\bar{\psi}^a\bar{\sigma}_{ab}\bar{\psi}^b\right\}. \tag{3.190}$$

The restriction $n = 1$ follows as $d\theta$ has $n = -\frac{1}{2}$ ($d\theta$ has $n = \frac{1}{2}$). Furthermore, one can also build invariant actions from a multiplet with chiral weight zero, using the D-density formula

$$[V]_D = e\left\{D - \frac{1}{2}\bar{\psi}\cdot\gamma\gamma_5\,\lambda + \left(V_m + \frac{i}{2}\bar{\psi}_m\gamma_5\chi\right)\varepsilon^{mnrl}\partial_n B_{rl}\right\} + \text{surface terms}. \tag{3.191}$$

We mention here that the F and D density formulas are related by $[V]_D = 2[\Pi(V)]_F$+surface terms.

Since the kinetic term of a general chiral superfield is given by the F-term density formula (3.189), we will have for example

$$\mathcal{L}^{(0)}_{kin} = \int d^2\theta\,\mathcal{E}\Phi\left[-\frac{1}{4}\bar{\nabla}^2\Phi^\dagger\right] + \text{h.c.}\,, \tag{3.192}$$

where $-\frac{1}{4}\bar{\nabla}^2$ is the chiral projection operator for the new minimal supergravity. In component form, and recalling that Φ has a zero chiral weight $n = 0$, the bosonic part of the Lagrangian (3.192) is found to be

$$\mathcal{L}^{(0)}_{kin} = 2e\,A\Box A^* + 2eFF^* - 2ieH^c\left(A\,\partial_c A^* - A^*\partial_c A\right)\,. \tag{3.193}$$

The most general chiral model coupled to new-minimal supergravity, in superspace is

$$\mathcal{L}_{chiral} = \frac{1}{2}[\mathcal{F}(\Phi,\bar{\Phi})]_D + 2\text{Re}[P(\Phi)]_F - [V_{\text{R}}]_D \tag{3.194}$$

and the pure bosonic sector reads

$$\begin{aligned} e^{-1}\mathcal{L}_{chiral} \;=\; & \frac{1}{2}[1 - \mathcal{F}_j(nA)^j](R + 6H^2) \\ & +2\left\{A^-_m + \frac{i}{2}\mathcal{F}_j\mathcal{D}^-_m A^j - \frac{i}{2}\mathcal{F}_{\bar{j}}\mathcal{D}^-_m\bar{A}^{\bar{j}}\right\}H^m \\ & -\mathcal{F}_{i\bar{j}}\mathcal{D}^-_m A^i\mathcal{D}^-_m\bar{A}^{\bar{j}} + \mathcal{F}_{i\bar{j}}F^i\bar{F}^{\bar{j}} + P_iF^i + \bar{P}_{\bar{j}}\bar{F}^{\bar{j}}. \end{aligned} \tag{3.195}$$

What is then left to do is to integrate out the auxiliary sector, see for example [90]. Nevertheless, this theory, is the two-derivative new-minimal supergravity, it is thus equivalent to the old-minimal. Indeed, if one solves the auxiliary sector of (3.195) it is then a matter of redefinitions in order to bring the theory in the standard form (3.152) (ignoring gauge interactions).

Chapter 4

Supersymmetry Breaking by Higher Dimensional Operators

By employing consistent supersymmetric higher derivative and higher dimensional terms, we show that the supersymmetric theories may have a sector where the scalar potential does no longer have the conventional form. The theories under consideration contain consistent higher-derivative terms which do not give rise to instabilities and ghost states. The chiral auxiliaries are still not propagating and can be integrated out. Their elimination gives rise to emerging potentials even when there is not a superpotential to start with. This novel feature of higher derivative supersymmetric chiral models is also extended to vector multiplets both in global and local supersymmetry. We show that such operators for real linear and chiral spinor superfields that break superymmetry reduce to the Volkov-Akulov action. In these cases, there is no sgoldstino mode and thus the goldstino does not have a superpartner. The sgoldstino is decoupled since the goldstino is one of the auxiliaries, which is propagating only in the supersymmetry breaking vacuum. We also consider supersymmetry breaking induced by a higher dimensional operator of a nonminimal scalar (complex linear) multiplet. The latter differs from the standard chiral multiplet in its auxiliary sector, which contains, in addition to the complex scalar auxiliary of a chiral superfield, a complex vector and two spinors auxiliaries. By adding an appropriate higher dimension operator, the scalar auxiliary may acquire a nonzero vev triggering spontaneous supersymmetry breaking. We find that the spectrum of the theory in the supersymmetry breaking vacuum consists of a free chiral multiplet and a constrained chiral superfield describing the goldstino. Interestingly, the latter turns out to be one of the auxiliary fermions, which becomes dynamical in the supersymmetry breaking vacuum. We also point out how higher dimension operators introduce a potential for the propagating scalar of the theory. In particular, in supergravity, the emerging potentials give rise always to a de Sitter vacuum signaling supersymmetry breaking.

4.1 Emergent Potentials

Supersymmetry is an extension of the Poincare spacetime symmetry with the inclusion of fermionic generators. It has various remarkable properties concerning phenomenological and theoretical aspects of particle physics. In particular, supersymmetry is one of the most appealing candidates for new physics. It has not been observed so far and thus, it should be broken at some high energy scale if it is realised at all. The central role on how supersymmetry is broken is played by the scalar potential of the supersymmetry breaking sector. Scalar potentials in supersymmetry and supergravity have been extensively studied for theories with up to two derivatives. Even though it is known that introducing higher derivatives will spoil the form of the scalar potential, the self-consistency of the theory protects it from unconventional non-supersymmetric vacua [43]. Our task here is to discuss how scalar potentials are modified when higher derivatives are introduced. However, the higher derivatives we are interested in, are those which do not introduce instabilities and/or ghost states. This is a known drawback of such kind of interactions, connected with the so-called Ostrogradski [167] instability in classical physics. We will see that such "safe" higher derivatives may consistently be introduced in supergravity and we will determine the form of the potential for the scalars of the theory they produce. We will also see that such potentias are sustained by background fluxes and have de Sitter vacua indicating that supersymmetry is broken.

In this work we are discussing the bosonic sector of supersymmetric interactions that belong to a specific class of higher derivative theories with the following two properties

- they do not introduce ghost states
- they introduce a scalar potential without a superpotential or gauging.

These theories involve chiral and vector multiplets.

In $N = 1$ superspace there is a number of conventional methods to introduce a scalar potential for a chiral superfield. The *superpotential* is the most widely used, in which case one employes a holomorphic function of the chiral superfield and after integrating out the auxiliary sector, a scalar potential appears. More specificaly, the free Wess-Zumino Lagrangian is given by [193]

$$L_0 = A\partial^2\bar{A} + i\partial_a\bar{\psi}_{\dot{\alpha}}\bar{\sigma}^{a\dot{\alpha}\alpha}\psi_\alpha + F\bar{F}. \tag{4.1}$$

It is straightforward to integrate out the auxiliary field via its equations of motion

$$F = 0 \tag{4.2}$$

which for the massless and free theory (4.1) vanishes, leading to

$$L_0 = A\partial^2\bar{A} + i\partial_a\bar{\psi}_{\dot{\alpha}}\bar{\sigma}^{a\dot{\alpha}\alpha}\psi_\alpha. \tag{4.3}$$

A standard mass term contribution is given by employing the following Lagrangian

$$\begin{aligned} L_0 + \frac{m}{2}(L_m + h.c.) &= A\partial^2\bar{A} + i\partial_a\bar{\psi}_{\dot{\alpha}}\bar{\sigma}^{a\dot{\alpha}\alpha}\psi_\alpha + F\bar{F} \\ &+ mFA - \frac{1}{2}m\psi^\alpha\psi_\alpha + m\bar{F}\bar{A} - \frac{1}{2}m\bar{\psi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}. \end{aligned} \tag{4.4}$$

A naive inspection of (4.4) would tell us that there is massive fermions, but no mass for the scalar fields has appeared. The equations of motion for the auxiliary field F read

$$\bar{F} = -mA \tag{4.5}$$

and eventually, the on-shell form of (4.4) becomes

$$L_0 + \frac{m}{2}(L_m + h.c.) = A\partial^2\bar{A} + i\partial_a\bar{\psi}_{\dot{\alpha}}\bar{\sigma}^{a\dot{\alpha}\alpha}\psi_\alpha - m^2A\bar{A} - \frac{1}{2}m\psi^\alpha\psi_\alpha - \frac{1}{2}m\bar{\psi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} \tag{4.6}$$

where now we can see that supersymmetric masses have been raised. The lesson from the above discussion is that, until integrating out the auxiliary sector, it is not obvious if there exists a mass term, and in a more general context, what is the form of the scalar potential.

Turning to supergravity, the above discussion is straightforwardly generalised and the same prosedure is followed. The most general (two-derivative) superspace Lagrangian of chiral superfields coupled to supergravity is in superspace formalism [1]

$$\mathcal{L}_0 = \frac{1}{\kappa^2}\int d^2\Theta\ 2\mathcal{E}\left[\frac{3}{8}\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)\, e^{-\frac{\kappa^2}{3}K(\Phi^i,\bar{\Phi}^{\bar{j}})} + \kappa^2 P(\Phi)\right] + h.c. \tag{4.7}$$

The hermitian function $K(\Phi^i, \bar{\Phi}^{\bar{j}})$ is the Kähler potential, $P(\Phi^i)$ is the superpotential (a holomorphic function of the chiral superfields Φ^i) and κ is proportional to the Planck length, which from now on will be set equal to 1. From the supergravity multiplet sector, $2\mathcal{E}$ is the usual chiral density employed to create supersymmetric Lagrangians, which in the new Θ variables has the expansion

$$2\mathcal{E} = e\left\{1 + i\Theta\sigma^a\bar{\psi}_a - \Theta\Theta\Big(M^* + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b\Big)\right\} \tag{4.8}$$

in terms of the vielbein (e_m^a), the gravitino (ψ_m) and the complex scalar auxiliary field M. In addition, $\mathcal{R}$, the superspace curvature, is a chiral superfield which contains the Ricci scalar in its highest component. In the matter sector, Φ^i and $\bar{\Phi}^{\bar{j}}$ denote a set on chiral and anti-chiral superfields ($\bar{\mathcal{D}}_{\dot{\alpha}}\Phi^i = 0$, $\mathcal{D}_\alpha\bar{\Phi}^{\bar{j}} = 0$) whose components are defined via projection

$$\begin{aligned} A^i &= \Phi^i|_{\theta=\bar{\theta}=0}, \\ \chi^i_\alpha &= \frac{1}{\sqrt{2}}\mathcal{D}_\alpha\Phi^i|_{\theta=\bar{\theta}=0}, \\ F^i &= -\frac{1}{4}\mathcal{D}\mathcal{D}\Phi^i|_{\theta=\bar{\theta}=0}. \end{aligned} \tag{4.9}$$

After calculating the component form of (4.7), integrating out the auxiliary fields and performing a Weyl rescaling of the gravitational field (accompanied by a redefinition of the fermionic fields), the pure bosonic Lagrangian reads

$$e^{-1}\mathcal{L}_0 = -\frac{1}{2}R - g_{i\bar{j}}\partial_a A^i\partial^a\bar{A}^{\bar{j}} - \ e^K\left[g^{i\bar{j}}(D_iP)(D_{\bar{j}}\bar{P}) - 3P\bar{P}\right]. \tag{4.10}$$

Further details maybe found for example in [193]. Here

$$g_{i\bar{j}} = \frac{\partial^2 K(A,\bar{A})}{\partial A^i\partial\bar{A}^{\bar{r}}} \tag{4.11}$$

is the positive definite Kähler metric, on the manifold parametrized by A^i and $\bar{A}^{\bar{j}}$. Moreover, the Kähler space covariant derivatives are defined as follows

$$D_iP = P_i + K_iP \tag{4.12}$$

where in general we denote $f_i = \frac{\partial f}{\partial A^i}$. The Lagrangian (4.10) is Kähler invariant as long as the superpotential scales as

$$P(A^i) \to e^{-S(A^i)}P(A^i) \tag{4.13}$$

[1] Our framework and conventions are those of Wess and Bagger [193].

under a Kähler transformation

$$K(A^i, \bar{A}^{\bar{j}}) \rightarrow K(A^i, \bar{A}^{\bar{j}}) + S(A^i) + \bar{S}(\bar{A}^{\bar{j}}). \tag{4.14}$$

$S(A^i)$ and $\bar{S}(\bar{A}^{\bar{j}})$ are holomorphic functions of the complex coordinates.

Equaly important conventional methods for introducing scalar potentials is by gauging the chiral models or by *D-terms*, the interested reader should consult [187].

4.1.1 F-Emergent Potential

The idea of the emergent potentials is essentialy a generalization of the standard methods discused above. The theory we are interested in, has a superspace Lagrangian of the form

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{HD} \tag{4.15}$$

where $\mathcal{L}_0$ is the standard superspace supergravity Lagrangian given in eq.(4.7) and [82, 135, 136]

$$\mathcal{L}_{HD} = \int d^2\Theta\ 2\mathcal{E}\left\{\frac{1}{8}\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)\Lambda^{\bar{r}i\bar{n}j}\Big[\bar{\mathcal{D}}_{\dot{\alpha}}K_i\mathcal{D}_\alpha K_{\bar{r}}\bar{\mathcal{D}}^{\dot{\alpha}}K_j\mathcal{D}^\alpha K_{\bar{n}}\Big]\right\} + h.c. \tag{4.16}$$

This Lagrangian was initially studied in global supersymmetry in [133]. It is important that $\mathcal{L}$ is manifestly both Kähler and (independently) super-Weyl invariant as has been shown in [82]. These two symmetry properties, although obviously they do not specify the form of the action, they are essential in the consistency of the model as well as for the supergravity theory that it describes. As we will see, (4.16) does not involve derivatives of the auxiliary fields, which are not propagating and can be integrated out. Equivalently, (4.16) can be expressed in terms of the chiral superfields Φ^i as

$$\mathcal{L}_{HD} = \int d^2\Theta\ 2\mathcal{E}\left\{\frac{1}{8}\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)\Lambda_{i\bar{r}j\bar{n}}\Big[\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\Phi}^{\bar{r}}\mathcal{D}_\alpha\Phi^i\bar{\mathcal{D}}^{\dot{\alpha}}\bar{\Phi}^{\bar{n}}\mathcal{D}^\alpha\Phi^j\Big]\right\} + h.c. \tag{4.17}$$

where

$$K_{i\bar{r}} = \frac{\partial^2 K(\Phi,\bar{\Phi})}{\partial\Phi^i\partial\bar{\Phi}^{\bar{r}}} \tag{4.18}$$

is the Kähler metric on the complex space spanned by the chiral and anti-chiral superfields and $\Lambda_{i\bar{r}j\bar{n}}$ represents a Kähler tensor. For example, one may choose

$$\Lambda_{i\bar{r}j\bar{n}} = \mathcal{G}(\Phi,\bar{\Phi})K_{i\bar{r}}K_{j\bar{n}} + \mathcal{H}(\Phi,\bar{\Phi})\mathcal{R}_{i\bar{r}j\bar{n}} \tag{4.19}$$

with $\mathcal{G}(\Phi,\bar{\Phi})$ and $\mathcal{H}(\Phi,\bar{\Phi})$ being some Kähler invariant hermitian functions and $\mathcal{R}_{i\bar{r}j\bar{n}}$ the Kähler space Riemann tensor defined as

$$\mathcal{R}_{i\bar{j}k\bar{l}} = \frac{\partial}{\partial\Phi^i}\frac{\partial}{\partial\bar{\Phi}^{\bar{j}}}K_{k\bar{l}} - K^{m\bar{n}}\left(\frac{\partial}{\partial\bar{\Phi}^{\bar{j}}}K_{m\bar{l}}\right)\left(\frac{\partial}{\partial\Phi^i}K_{k\bar{n}}\right). \tag{4.20}$$

The form (4.19) implies some symmetries for the Kähler indices which, without loss of further generality, we will assume to be possessed by all the $\Lambda_{i\bar{r}j\bar{n}}$ to be considered in this work. Our next task is to extract the component field expression for the Lagrangian (4.17), which after superspace integration turns out to be

$$e^{-1}\mathcal{L}_{HD} = -16\,\mathcal{U}_{i\bar{r}j\bar{n}} \quad \Big(F^i F^j \bar{F}^{\bar{r}} \bar{F}^{\bar{n}} + \partial_a A^i \partial^a A^j \partial_b \bar{A}^{\bar{r}} \partial^b \bar{A}^{\bar{n}} \\ -F^i \bar{F}^{\bar{r}} \partial_a A^j \partial^a \bar{A}^{\bar{n}} - F^i \bar{F}^{\bar{n}} \partial_a A^j \partial^a \bar{A}^{\bar{r}}\Big). \tag{4.21}$$

for the pure bosonic sector. In (4.21) we have used the notation

$$\mathcal{U}_{i\bar{r}j\bar{n}}(A,\bar{A}) = \Lambda_{i\bar{r}j\bar{n}}(\Phi,\bar{\Phi})\Big|_{\theta=\bar{\theta}=0} \tag{4.22}$$

Again it is easy to see that (4.21) is manifestly Kähler invariant.

In order to make the effect of the new coupling (4.16) more transparent we will consider now a theory with only *one chiral multiplet and no superpotential.* In this case, the Lagrangian (4.15) is explicitly written as

$$\mathcal{L} = \int d^2\Theta\; 2\mathcal{E}\left\{\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)\left[\frac{3}{8}e^{-\frac{1}{3}K} + \frac{1}{8}\Lambda\; \bar{\mathcal{D}}_{\dot{\alpha}}\bar{\Phi}\mathcal{D}_{\alpha}\Phi\bar{\mathcal{D}}^{\dot{\alpha}}\bar{\Phi}\mathcal{D}^{\alpha}\Phi\right]\right\} + h.c. \tag{4.23}$$

with Λ being an abbreviation for $\Lambda_{\Phi\bar{\Phi}\Phi\bar{\Phi}}$, a hermitian and Kähler invariant function of Φ and $\bar{\Phi}$. In component form, the bosonic sector of the Lagrangian (4.23) turns out to be (after integrating out the auxiliary fields of the supergravity sector and subsequently appropriately rescaling)

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{bos}} \;&=\; -\frac{1}{2}R - g_{A\bar{A}}\partial_a A\partial^a\bar{A} + g_{A\bar{A}}\; e^{\frac{K}{3}}F\bar{F} \\ &\quad -16\,\mathcal{U}\;\left\{e^{\frac{2K}{3}}(F\bar{F})^2 + \partial_a A\partial^a A\partial_b\bar{A}\partial^b\bar{A} - 2e^{\frac{K}{3}}F\bar{F}\partial_a A\partial^a\bar{A}\right\} \end{aligned} \tag{4.24}$$

where $\mathcal{U}$ is a hermitian Kähler invariant function of the scalar field (it is the lowest component of Λ, eq.(4.22)). The equation of motion for F is

$$\bar{F}\Big(g_{A\bar{A}} - 32\,\mathcal{U}e^{\frac{K}{3}}F\bar{F} + 32\,\mathcal{U}\partial_a A\partial^a\bar{A}\Big) = 0 \tag{4.25}$$

which can be easily solved for

- Standard solution:

$$F = 0, \tag{4.26}$$

- New solution:

$$F\bar{F} = e^{\frac{-K}{3}}\left(\frac{g_{A\bar{A}}}{32\,\mathcal{U}} + \partial_a A\partial^a\bar{A}\right). \tag{4.27}$$

Here we should discuss the difference between the two solutions. To make the point clear we first stress that the stability of the theory demands

$$g_{A\bar{A}} > 0 \tag{4.28}$$

$$\mathcal{U} < 0. \tag{4.29}$$

Thus the standard solution (4.26) can always be realized, while the new solution (4.27) can only be realized in the presence of fluxes so that

$$F\bar{F} = e^{\frac{-K}{3}} \left(\frac{g_{A\bar{A}}}{32\,\mathcal{U}} + \partial_a A \partial^a \bar{A} \right) > 0. \tag{4.30}$$

The on-shell Lagrangian for the conventional branch is

$$e^{-1}\mathcal{L}_{\text{bos}} = -\frac{1}{2}R - g_{A\bar{A}}\partial_a A \partial^a \bar{A} - 16\,\mathcal{U}\ \partial_a A \partial^a A \partial_b \bar{A} \partial^b \bar{A} \tag{4.31}$$

where there is no scalar potential, as expected, since no superpotential was introduced. The on-shell Lagrangian for the new branch is

$$e^{-1}\mathcal{L}_{\text{bos}} = -\frac{1}{2}R + \frac{(g_{A\bar{A}})^2}{64\,\mathcal{U}} - 16\,\mathcal{U}\partial_a A \partial^a A \partial_b \bar{A} \partial^b \bar{A} + 16\,\mathcal{U}\partial_a A \partial^a \bar{A} \partial_b A \partial^b \bar{A}. \tag{4.32}$$

What has happened here has completely changed the dynamics of the theory. The minimal kinematic term for the scalar is lost and a scalar potential has *emerged*

$$\mathcal{V} = -\frac{1}{64}\frac{(g_{A\bar{A}})^2}{\mathcal{U}}. \tag{4.33}$$

From (4.29) we see that the potential (4.33) is positive defined

$$\mathcal{V} > 0 \tag{4.34}$$

and therefore the theory may only have de Sitter vacua. Another important property of the emerging potential is that it is not built from a holomorphic function. Moreover, the function $\mathcal{U}$ governs now the kinetic terms and in fact it was shown in [135] that it has to be negative to avoid tachionic states. In the framework of new-minimal supergravity, consistent higher derivative terms which satsify the above restrictions have been considered [84], but no scalar potential emerged in that case.

Super-Weyl Invariance

At this point its is crucial to make a comment on a subtlety concerning the hermitian vector superfield

$$V = \Lambda_{i\bar{r}j\bar{n}} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Phi}^{\bar{r}} \mathcal{D}_{\alpha} \Phi^i \bar{\mathcal{D}}^{\dot{\alpha}} \bar{\Phi}^{\bar{n}} \mathcal{D}^{\alpha} \Phi^j, \tag{4.35}$$

namely, its scaling properties under super-Weyl transformations. We emphasize that V is defined through its components. For example, its lowest component will be

$$V\Big|_{\theta=\bar{\theta}=0} \quad = \quad 4\,\mathcal{U}_{i\bar{r}j\bar{n}}\ \bar{\chi}^{\bar{r}}_{\dot{\alpha}} \chi^i_{\alpha} \bar{\chi}^{\dot{\alpha}\bar{n}} \chi^{\alpha j}. \tag{4.36}$$

Moreover, all components of V should be understood as those of a hermitian vector superfield defined via projection and will eventually be related to (6.22). This definition will gives Weyl weight -2 to the vector superfield V, as is required so that (4.17) is indeed Kähler and super-Weyl invariant. These symmetries are crucial for consistency of the supergravity Lagrangian on curved superspace. Under a super-Weyl transformation, the superspace covariant derivatives change as [72]

$$\begin{aligned}\delta_\Sigma \mathcal{D}_\alpha &= (\Sigma - 2\bar{\Sigma})\mathcal{D}_\alpha - (\mathcal{D}^\gamma\Sigma) l_{\alpha\gamma}\\ \delta_\Sigma \bar{\mathcal{D}}_{\dot\alpha} &= (\bar{\Sigma} - 2\Sigma)\bar{\mathcal{D}}_{\dot\alpha} - (\bar{\mathcal{D}}^{\dot\gamma}\bar{\Sigma}) l_{\dot\alpha\dot\gamma}\end{aligned} \tag{4.37}$$

where the $l_{\alpha\gamma}$ and $l_{\dot\alpha\dot\gamma}$ stand for the (anti)self-dual parts of infinitesimal Lorentz transformations. Moreover, by choosing Φ^i and the tensor $\Lambda_{i\bar r j\bar n}$ to have vanishing Weyl weights, i.e.

$$\delta_\Sigma \Lambda_{i\bar r j\bar n} = \delta_\Sigma \Phi^i = \delta_\Sigma \bar{\Phi}^{\bar r} = 0, \tag{4.38}$$

and by using (4.37), one may straightforwardly check that under a super-Weyl transformation, the vector superfield (4.35), scales as

$$\delta_\Sigma(\Lambda_{i\bar r j\bar n}\bar{\mathcal{D}}_{\dot\alpha}\bar{\Phi}^{\bar r}\mathcal{D}_\alpha\Phi^i\bar{\mathcal{D}}^{\dot\alpha}\bar{\Phi}^{\bar n}\mathcal{D}^\alpha\Phi^j) = -2(\Sigma+\bar{\Sigma})(\Lambda_{i\bar r j\bar n}\bar{\mathcal{D}}_{\dot\alpha}\bar{\Phi}^{\bar r}\mathcal{D}_\alpha\Phi^i\bar{\mathcal{D}}^{\dot\alpha}\bar{\Phi}^{\bar n}\mathcal{D}^\alpha\Phi^j). \tag{4.39}$$

Of course, when we perform the super-Weyl rescaling to our Lagrangian (4.17), we have to consider the variation of the involved superfields in the new Θ variables [193].

4.1.2 Gauge Invariant F-Emergent Potential

The Lagrangian (4.23) can be straightforwardly be generalized to include gauge invariant interactions [24]. In this case, the gauge invariant superspace Lagrangian is

$$\begin{aligned}\mathcal{L}_{tot} = \int d^2\Theta\, 2\mathcal{E}\Bigg\{&\frac{3}{8}\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)e^{-\tilde{K}/3} + \frac{1}{16g^2}\mathcal{H}_{(ab)}(\Phi)W^{(a)}W^{(b)} + P(\Phi)\\ &+\frac{1}{8}\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)\Big[\tilde{\Lambda}^{\bar r i\bar n j}\ \bar{\mathcal{D}}_{\dot\alpha}\tilde{K}_i\mathcal{D}_\alpha\tilde{K}_{\bar r}\bar{\mathcal{D}}^{\dot\alpha}\tilde{K}_j\mathcal{D}^\alpha\tilde{K}_{\bar n}\Big]\Bigg\} + h.c.\end{aligned} \tag{4.40}$$

where

$$\tilde{K} = K(\Phi,\bar{\Phi}) + \Gamma(\Phi,\bar{\Phi},V), \tag{4.41}$$

and

$$\Gamma(\Phi,\bar{\Phi},V) = V^{(a)}\mathcal{D}^{(a)} + \frac{1}{2}g_{i\bar r}X^{i(a)}\bar{X}^{\bar r(b)}V^{(a)}V^{(b)}. \tag{4.42}$$

In addition, as usual, $V^{(a)}$ is the supersymmetric Yang-Mills vector multiplet and

$$W_\alpha = W_\alpha^{(a)}T^{(a)} = -\frac{1}{4}\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)e^{-V}\mathcal{D}_\alpha e^V \tag{4.43}$$

is the gauge invariant chiral superfield containing the gauge field strength. The holomorphic function $\mathcal{H}_{(ab)}$ is included for generality, but in what follows we will consider $\mathcal{H}_{(ab)} = \delta_{(ab)}$. Expression (4.42) is calculated in the Wess-Zumino gauge, $\mathcal{D}^{(a)}$ are the so-called Killing potentials whereas $X^{i(a)}$ and $\bar{X}^{\bar r(b)}$ are the components of the holomorphic Killing vectors that generate the isometries of the Kähler manifold. The Killing vectors and the Killing potential are connected via

$$g_{i\bar r}\bar{X}^{\bar r(a)} = i\frac{\partial}{\partial a^i}\mathcal{D}^{(a)}, \tag{4.44}$$

$$g_{i\bar r}X^{i(a)} = -i\frac{\partial}{\partial \bar{a}^{\bar r}}\mathcal{D}^{(a)} \tag{4.45}$$

where a^i and $\bar{a}^{\bar{r}}$ are the Kähler space complex co-ordinates. We note that the $\mathcal{D}^{(a)}$ that correspond to some $U(1)$ gauged symmetry are only determined up to a constant ξ, which is the analog for the Fayet-Iliopoulos D-term in supergravity. Now $\tilde{\Lambda}^{\bar{r}i\bar{n}j}$ has to respect all the isometries of the Kähler manifold. Again, following the standard procedure, the bosonic part of the Lagrangian (4.40) turns out to be

$$\begin{aligned} e^{-1}\mathcal{L}_{tot} = \quad & \frac{1}{2}R - g_{i\bar{r}}\tilde{D}_m A^i \tilde{D}^m \bar{A}^{\bar{r}} + e^{\frac{K}{3}} g_{i\bar{r}} F^i \bar{F}^{\bar{r}} \\ & -\frac{1}{16g^2} F^{(a)}_{mn} F^{mn(a)} - \frac{1}{2} g^2 \left(\mathcal{D}^{(a)}\right)^2 \\ & - e^{\frac{2K}{3}} \left(F^i D_i P + \bar{F}^{\bar{r}} D_{\bar{r}} \bar{P} \right) + 3 e^K P \bar{P} \\ & -16\, \tilde{\mathcal{U}}_{i\bar{r}j\bar{n}} \Big(e^{\frac{2K}{3}} F^i F^j \bar{F}^{\bar{r}} \bar{F}^{\bar{n}} + \tilde{D}_a A^i \tilde{D}^a A^j \tilde{D}_b \bar{A}^{\bar{r}} \tilde{D}^b \bar{A}^{\bar{n}} \\ & \qquad - e^{\frac{K}{3}} F^i \bar{F}^{\bar{r}} \tilde{D}_a A^j \tilde{D}^a \bar{A}^{\bar{n}} - e^{\frac{K}{3}} F^i \bar{F}^{\bar{n}} \tilde{D}_a A^j \tilde{D}^a \bar{A}^{\bar{r}} \Big). \end{aligned} \tag{4.46}$$

We note that

$$\tilde{D}_c A^j = \partial_c A^j - \frac{1}{2} B^{(a)}_c X^j_{(a)} \tag{4.47}$$

is the covariant derivative and $B^{(a)}_c$ is a vector field (belonging to the $V^{(a)}$ vector multiplet) that corresponds to the gauged isometries, with field strength $F^{(a)}_{mn}$.

In order to illustrate the properties of the *emergent potential* in the case of gauged models, our example will be a single chiral multiplet with *no* superpotential. In this case the Lagrangian (4.46) is

$$\begin{aligned} e^{-1}\mathcal{L}_{tot} = \quad & \frac{1}{2}R - g_{A\bar{A}}\tilde{D}_m A \tilde{D}^m \bar{A} + e^{\frac{K}{3}} g_{A\bar{A}} F \bar{F} \\ & -\frac{1}{16g^2} F^{(a)}_{mn} F^{mn(a)} - \frac{1}{2} g^2 \left(\mathcal{D}^{(a)}\right)^2 \\ & -16\, \tilde{\mathcal{U}} \Big(e^{\frac{2K}{3}} (F\bar{F})^2 + \tilde{D}_a A \tilde{D}^a A \tilde{D}_b \bar{A} \tilde{D}^b \bar{A} - 2\, e^{\frac{K}{3}} F \bar{F} \tilde{D}_a A \tilde{D}^a \bar{A} \Big). \end{aligned} \tag{4.48}$$

The single auxiliary field F can now be eliminated from (4.48) by its equations of motion, leading to

$$F\bar{F} = e^{\frac{-K}{3}} \left(\frac{g_{A\bar{A}}}{32\, \tilde{\mathcal{U}}} + \tilde{D}_a A \tilde{D}^a \bar{A} \right). \tag{4.49}$$

Plugging (4.49) back in (4.48), we can easily read-off the potential for the gauged model which turns out to be

$$\mathcal{V} = \frac{1}{2} g^2 \left(\mathcal{D}^{(a)}\right)^2 - \frac{(g_{A\bar{A}})^2}{64\, \tilde{\mathcal{U}}} \tag{4.50}$$

with $\tilde{\mathcal{U}} = \tilde{\mathcal{U}}_{A\bar{A}A\bar{A}}$, a Kähler-space tensor that respects all the isometries of the gauged group. For a first example we will take a flat model with Kähler potential

$$K = a\bar{a} + d \tag{4.51}$$

which leads to

$$g_{a\bar{a}} = 1\,, \qquad \mathcal{R}_{a\bar{a}a\bar{a}} = 0 \tag{4.52}$$

The $U(1)$ Killing potential is

$$D^{(1)} = a\bar{a} + \xi \tag{4.53}$$

where the parameter ξ corresponds to the aforementioned freedom to shift the $U(1)$ Killing potential. When we promote a and $\bar{a}$ to the superfields Φ and $\bar{\Phi}$, our Kähler potential K together with the counter term Γ become

$$\tilde{K}_{U(1)} = \Phi\bar{\Phi} + V\Phi\bar{\Phi} + \frac{1}{2}V^2\Phi\bar{\Phi} + d + V\xi. \tag{4.54}$$

The bosonic part of our Lagrangian in component form then turns out to be

$$\begin{aligned} e^{-1}\mathcal{L}_{U(1)} = \quad & -\frac{1}{2}R - \frac{1}{16g^2}F_{cd}F^{cd} \\ & -16\,\tilde{\mathcal{U}}\tilde{D}_a A\tilde{D}^a A\tilde{D}_b\bar{A}\tilde{D}^b\bar{A} + 16\,\tilde{\mathcal{U}}\tilde{D}_a A\tilde{D}^a\bar{A}\tilde{D}_b A\tilde{D}^b\bar{A} \\ & -\frac{1}{2}g^2\left(A\bar{A} + \xi\right)^2 + \frac{1}{64\,\tilde{\mathcal{U}}}, \end{aligned} \tag{4.55}$$

with $\tilde{D}_m A = \partial_m A + \frac{i}{2}B_m A$. Then the scalar potential is

$$\mathcal{V} = \frac{1}{2}g^2\big(D^{(a)}\big)^2 - \frac{1}{64\tilde{\mathcal{U}}}. \tag{4.56}$$

A simple choice for $\tilde{\mathcal{U}}$ could be

$$\tilde{\mathcal{U}} = m g_{A\bar{A}} g_{A\bar{A}} = m < 0\,, \tag{4.57}$$

where m is a negative constant. It is again important to emphasise that m now governs the kinematics of the scalar fields, and that the condition

$$F\bar{F} = e^{\frac{-K}{3}}\left(\frac{g_{A\bar{A}}}{32\,\tilde{\mathcal{U}}} + \tilde{D}_a A\tilde{D}^a\bar{A}\right) > 0 \tag{4.58}$$

has to hold for the theory to be consistent.

4.1.3 D-Emergent Potential

Higher derivative interactions are not restricted only to scalar fields. In fact we will show that an equivalent method as before can be followed which again leads to a scalar potential. Now the auxiliary fields that are integrated out are the ones of the vector multiplet, the "D" fields.

The higher derivative term we want to discuss is (in superspace)

$$\mathcal{L}_{gHD} = \int d^2\Theta\; 2\mathcal{E}\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)\big(-\frac{1}{4}\mathcal{J}_{ab}(\Phi,\bar{\Phi})W^{(a)}W^{(b)}\mathcal{Y}_{cd}(\Phi,\bar{\Phi})\bar{W}^{(c)}\bar{W}^{(d)}\big) + h.c. \tag{4.59}$$

The superfields $\mathcal{J}_{ab}(\Phi,\bar{\Phi})$ and $\mathcal{Y}_{cd}(\Phi,\bar{\Phi})$ are functions of the various chiral superfields that are present in our theory, the only restriction is that they should transform correctly under the gauge group. The bosonic sector of Lagrangian (4.59) after performing the superspace integration is

$$\begin{aligned} e^{-1}\mathcal{L}_{gHD} &= [J_{ab}\bar{Y}_{cd} + \bar{J}_{ab}Y_{cd}] \times \\ &\{ \quad \frac{1}{4}F^{dc(a)}F^{(b)}_{dc}F^{ab(c)}F^{(d)}_{ab} - \frac{1}{2}F^{dc(a)}F^{(b)}_{dc}D^{(c)}D^{(d)} - \frac{1}{2}D^{(a)}D^{(b)}F^{ab(c)}F^{(d)}_{ab} \\ &\quad + D^{(a)}D^{(b)}D^{(c)}D^{(d)} + \frac{1}{16}\epsilon^{abcd}F^{(a)}_{ab}F^{(b)}_{cd}\epsilon^{efgh}F^{(c)}_{ef}F^{(d)}_{gh} \ \}. \end{aligned} \tag{4.60}$$

Here $J_{ab} = \mathcal{J}_{ab}|$ and $Y_{ab} = \mathcal{Y}_{ab}|$. Moreover for the gauge sector we will consider a more general coupling allowing for a kinetic gauge function as well. The standard kinetic term for the gauge fields is

$$\mathcal{L}_{g0} = \int d^2\Theta \ 2\mathcal{E}\mathcal{H}_{(ab)}(\Phi)W^{(a)}W^{(b)} + h.c. \tag{4.61}$$

and the bosonic sector in components reads

$$e^{-1}\mathcal{L}_{g0} = [H_{(ab)} + \bar{H}_{(ab)}]\{-\frac{1}{2}F^{dc(a)}F^{(b)}_{dc} + \frac{i}{4}\epsilon^{abcd}F^{(a)}_{ab}F^{(b)}_{cd} + D^{(a)}D^{(b)}\} \tag{4.62}$$

with $H_{ab} = \mathcal{H}_{ab}|$. Up to now the most general Lagrangian in superspace reads

$$\begin{aligned} \mathcal{L}_{tot} = \int d^2\Theta \ 2\mathcal{E}\Big\{ &\frac{3}{8}\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)e^{-\tilde{K}/3} + \mathcal{H}_{(ab)}(\Phi)W^{(a)}W^{(b)} + P(\Phi) \\ &+ \frac{1}{8}\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)\Big[\tilde{\Lambda}^{\bar{r}i\bar{n}j} \ \bar{\mathcal{D}}_{\dot{\alpha}}\tilde{K}_i\mathcal{D}_{\alpha}\tilde{K}_{\bar{r}}\bar{\mathcal{D}}^{\dot{\alpha}}\tilde{K}_j\mathcal{D}^{\alpha}\tilde{K}_{\bar{n}}\Big] \\ &- \frac{1}{4}\Big(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}\Big)[\mathcal{J}_{ab}(\Phi,\bar{\Phi})W^{(a)}W^{(b)}\mathcal{Y}_{cd}(\Phi,\bar{\Phi})\bar{W}^{(c)}\bar{W}^{(d)}]\Big\} + h.c. \end{aligned} \tag{4.63}$$

Finally, in order to study the properties of this new term, let us consider a very simple example of a single $U(1)$ group and a single uncharged (under this $U(1)$) chiral multiplet. The higher derivative terms will be only for the gauge sector. Our Lagrangian, in component form reads

$$\begin{aligned} e^{-1}\mathcal{L}_{ex} = \quad & \frac{1}{2}R - g_{A\bar{A}}\partial_m A\partial^m\bar{A} + [H(A) + \bar{H}(\bar{A})]\{-\frac{1}{2}F^{dc}F_{dc} + \frac{i}{4}\epsilon^{abcd}F_{ab}F_{cd} + D^2\} \\ & + e^{-2K/3}[J\bar{Y} + Y\bar{J}]\{\frac{1}{4}(F^{dc}F_{dc})^2 - F^{dc}F_{dc}D^2 + \frac{1}{16}(\epsilon^{abcd}F_{ab}F_{cd})^2 + D^4\}. \end{aligned} \tag{4.64}$$

Here J and Y are positive definite gauge invariant functions of A and $\bar{A}$. Now we can easily solve the auxiliary D equations of motion to find two solutions

- Standard solution:

$$D = 0, \tag{4.65}$$

- New solution:

$$D^2 = \frac{1}{2}F^{dc}F_{dc} - \frac{1}{2}e^{2K/3}\frac{H + \bar{H}}{J\bar{Y} + Y\bar{J}}. \tag{4.66}$$

The first one is the standard supersymmetric solution and has been also studied in [43] in the presence of higher derivatives. The new solution can only be consistently realized in the presence of magnetic fluxes so that

$$D^2 = \frac{1}{2}F^{dc}F_{dc} - \frac{1}{2}e^{2K/3}\frac{H+\bar{H}}{J\bar{Y}+Y\bar{J}} > 0. \tag{4.67}$$

Eventually the on-shell theory will be

$$\begin{aligned} e^{-1}\mathcal{L}_{ex} = & \frac{1}{2}R - g_{A\bar{A}}\partial_m A\partial^m \bar{A} - \frac{1}{4}e^{2K/3}\frac{(H+\bar{H})^2}{J\bar{Y}+Y\bar{J}} + \frac{i}{4}[H(A)+\bar{H}(\bar{A})]\epsilon^{abcd}F_{ab}F_{cd} \\ & + \frac{1}{16}[J\bar{Y}+Y\bar{J}](\epsilon^{abcd}F_{ab}F_{cd})^2 \\ = & \frac{1}{2}R - g_{A\bar{A}}\partial_m A\partial^m \bar{A} - \frac{1}{4}e^{2K/3}\frac{(H+\bar{H})^2}{J\bar{Y}+Y\bar{J}} + \frac{i}{4}[H(A)+\bar{H}(\bar{A})]\epsilon^{abcd}F_{ab}F_{cd} \\ & + [J\bar{Y}+Y\bar{J}]\{-\frac{1}{2}(F^{dc}F_{dc})^2 + F_{ab}F^{bc}F_{cd}F^{da}\}. \end{aligned} \tag{4.68}$$

It is easy to see that there is a positive definite emergent potential due to integrating out of the D auxiliary field

$$\mathcal{V}(A,\bar{A}) = \frac{1}{4}e^{2K/3}\frac{(H+\bar{H})^2}{J\bar{Y}+Y\bar{J}}. \tag{4.69}$$

A simple example can be given by a gauge kinetic function

$$H = A^2 \tag{4.70}$$

with a, b two real positive constants

$$J = a > 0, \quad Y = b > 0.$$

The potential will be

$$\mathcal{V}(A,\bar{A}) = e^{2K/3}\frac{(A^2+\bar{A}^2)^2}{8ab}. \tag{4.71}$$

This novel feature of gauge fields higher derivatives has not been studied before and deserves further investigation.

4.2 Supersymmetry Breaking by Higher Dimension Operators and Non-Linear Realizations

Supersymmetry is one of the most appealing candidates for new physics. It has not been observed so far; thus, it should be broken at some high energy scale if it is realised at all. The central role on how supersymmetry is broken is usually played by the scalar potential of the supersymmetry breaking sector. Scalar potentials in supersymmetry and supergravity have extensively been studied for two-derivative theories. Even though it is known that introducing higher dimension operators spoils the form of the scalar potential, it seems that the theory somehow protects itself from unconventional non-supersymmetric vacua [43]. Our task here is to discuss how scalar potentials are modified and may lead to supersymmetry breaking when higher dimension operators are introduced. The goldstone fermion associated with the supersymmetry breaking, the goldstino, is described by the Volkov-Akulov action [188], in which supersymmetry is non-linearly realized. In particular, the goldstino dynamics has been related in [139] to the superconformal anomaly multiplet X corresponding to the FZ supercurrent [94]. The multiplet of anomalies X, defined in the UV flows in the IR, under renormalization group, to a chiral superfield X_{NL} which obeys the constraint $X_{NL}^2 = 0$. This constrained superfield is the realization of the goldstino given in [40]. Since the dynamics of the goldstino is universal, the IR action in [139] is the same as in [40]. Constrained superfields have been used before to accomodate the goldstino. Indeed, there are alternative formulations in which the goldstino sits in a constrained superfield, such as a constrained chiral multiplet [174], a constrained vector multiplet [155], a spinor superfield [111], or a complex linear superfield [148, 149]. Constrained superfields have also been used recently in the MSSM context [11, 18, 29, 85, 86, 172] and in inflationary cosmology, where the inflaton is identified with the sgoldstino [8–10]. In addition their interaction with matter has been worked out in [19].

Supersymmetric theories that contains higher dimension operators (derivative or non-derivative ones) have some novel features [45, 48–52, 82, 83, 135]. Among these, an interesting aspect is that higher dimension operators can contribute to the scalar potential. This has been discussed earlier in [43] where a few examples have been given. In particular, theories with no potential at the leading two-derivative level, may develop a nontrivial potential when higher dimension operators are taken into account and may even lead to supersymmetry breaking, as already mentioned above. At this point there are however, two dangerous aspects. The first one concerns the appearance of ghost instabilities. In the type of theories we are discussing, this instability is not present as the theory does not have those higher derivatives terms which might give rise to such dangerous states. The second issue concerns the auxiliary fields. Here, we are still able to eliminate the auxiliaries of the multiplet since they appeared algebraically in the supersymmetric Lagrangian.

We will consider various theories exhibiting supersymmetry breaking in the presence of higher dimension operators. Special attention will be devoted to a globally supersymmetric model for a complex linear multiplet. As we will explain in one of the following sections, the complex linear multiplet, or nonminimal multiplet, contains the degrees of freedom of a chiral multiplet and in addition, two fermions and a complex vector. At the two derivative level, both the extra fermions and the complex vector are auxiliaries and can be integrated out, giving on-shell just a free complex scalar and a fermion. Due to the constraints the complex linear satisfies, there is no superpotential one can write down and the introduction of an F-term for non-derivative interactions is not possible. So, one relies on modifying the D-term in order to get some non-trivial interactions and an emerging potential induced by higher dimension operators [43, 82, 83, 135]. Under certain conditions, it may happen that the new potential develops another extremum for the auxiliaries which break supersymmetry. In this case, new phases will emerge, only one of which will be realized when the higher dimension operators interactions are turned off. It should be noted however, that these new phases are not different phases of the same theory, but rather different theories. The examples studied in [43] were not successful in this respect, basically because the auxiliaries appeared in the higher derivative terms with the same sign as in the leading two-derivative term. This has the effect that the minimum of the potential is stable with respect to the addition of the higher dimension term. However, in the case of the complex linear multiplet, the auxiliary in the two derivative term and in the higher derivative term appear with opposite sign. This has the effect of introducing now a new minimum for a non zero value of the auxiliary, thereby breaking supersymmetry. The interesting phenomenon that appears here is that the goldstino turns out to be one of the auxiliary fermions of the multiplet, which in the new vacuum acquires a kinetic term, but vanishes in the supersymmetric vacuum of the theory. After integrating out the auxiliaries, we are left with a complex scalar, a fermion and a goldstino without supersymmetric partner, as supersymmetry is broken. Therefore, there is a mismatch of bosonic and fermion degrees of freedom as for example in Volkov-Akulov type of models where supersymmetry is non-linearly realised [188].

This part is organized as follows. In the next section we present theories with higher dimensional operators that exhibit susy breaking and the corresponding Volkov-Akulov actions. In section 3 we describe the complex linear multiplet. In section 4 we show how higher dimensional operators of the complex linear multiplet may lead to susy breaking and we prove the equivalence to non-linear realizations. Finally, we conclude in the last section 5.

4.2.1 SUSY Breaking and Volkov-Akulov Actions

One of the explicit examples considered in [43] to demonstrate that the scalar potential is sensitive to the addition of higher dimension terms, is a supersymmetric σ-model with four-derivative coupling. Its standard Lagrangian is[2]

$$\mathcal{L}_\sigma = \int d^4\theta K(\Phi, \bar{\Phi}), \tag{4.72}$$

where $K(\Phi,\bar{\Phi})$ is the Kähler potential. The latter can be considered as a composite vector multiplet possessing an effective gauge (Kähler) invariance

$$K \to K + i(\Lambda - \bar{\Lambda}), \tag{4.73}$$

where Λ is a chiral superfield. As we are going to keep this invariance for the higher dimension operators as well, we will construct the latter in terms of the superfield field strength

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha K \tag{4.74}$$

for the composite vector $K(\Phi,\bar{\Phi})$. Then, clearly, the most general Kähler invariant Lagrangian up to four-derivative terms is

$$\mathcal{L}_\sigma = \int d^4\theta\, K(\Phi,\bar{\Phi}) + \left(\int d^2\theta\, g(\Phi) + \lambda \int d^2\theta\, W^2(K) + h.c.,\right) \tag{4.75}$$

where $g(\Phi)$ is the superpotential and $\lambda > 0$. Without loss of generality, let us consider the simplest case of a single chiral multiplet with $K = \Phi\bar{\Phi}$ and $g(\Phi) = 0$. Then eq. (4.75) turns out to be

$$\mathcal{L}_\sigma = \int d^4\theta \left(\Phi\bar{\Phi} + \frac{\lambda}{2} D^\alpha\Phi D_\alpha\Phi \bar{D}_{\dot{\alpha}}\bar{\Phi}\bar{D}^{\dot{\alpha}}\bar{\Phi}\right) \tag{4.76}$$

and the scalar potential turns out to be [43]

$$-V_F = |F|^2 + 8\lambda|F|^4. \tag{4.77}$$

[2]Our superspace conventions can be found in [193].

The minimum of the potential is at $F = 0$, which is also the minimum of the theory in the $\lambda \to 0$ limit. Nevertheless there exists another vacuum supported by a background flux for the scalar component of the chiral multiplet leading to $F \sim |\partial A|$ which may lead to supersymmetry breaking [83, 135] and is not continuously connected to the standard branch $F = 0$ as we saw earlier. This vacuum nevertheless breaks Lorentz invariance.

Chiral Spinor Superfield

There are other possibilities one may wish to consider which do not lead to Lorentz symmetry breaking. For example, let us consider the Lagrangian [cfr. [40, 139]]

$$\mathcal{L}_W = \frac{1}{4}\left(\int d^2\theta\, W^\alpha W_\alpha + h.c\right) + \frac{1}{\Lambda^4}\int d^4\theta\, W^\alpha W_\alpha \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}, \tag{4.78}$$

where

$$W_\alpha \;=\; \lambda_\alpha + \theta_\alpha D + \theta^\beta F_{\alpha\beta} + \theta^2 \chi_\alpha, \tag{4.79}$$

so that W_α is chiral but otherwise unconstrained and $F_{\alpha\beta} = F_{\beta\alpha}$.

The component form of the Lagrangian (4.78) is

$$\begin{aligned}
\mathcal{L}_W \;&=\; \frac{1}{4}(D^2 + 2\chi\lambda + \frac{1}{2}F^{\alpha\beta}F_{\alpha\beta} + h.c.) \\
&+\; \frac{1}{\Lambda^4}[\lambda^2\partial^2\bar{\lambda}^2 + (D^2 + 2\chi\lambda + \frac{1}{2}F^2)(\bar{D}^2 + 2\bar{\chi}\bar{\lambda} + \frac{1}{2}\bar{F}^2)] \\
&-\; i\frac{1}{\Lambda^4}(\lambda^\alpha D - F^{\alpha\beta}\lambda_\beta)\sigma^m_{\alpha\dot{\alpha}}\partial_m(\bar{\lambda}^{\dot{\alpha}}\bar{D} - \bar{F}^{\alpha\beta}\bar{\lambda}_{\dot{\beta}})
\end{aligned} \tag{4.80}$$

where

$$F^{\alpha\beta} = \epsilon^{\alpha\sigma}\epsilon^{\beta\rho}F_{\sigma\rho}, \tag{4.81}$$

In the particular case that W_α is the field-strength superfield and satisfies $D^\alpha W_\alpha = \bar{D}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}$, the Lagrangian has been worked out in [43, 83]. The Lagrangian (4.78) is of the form [40, 139]

$$\mathcal{L}_W = \int d^4\theta\, X\bar{X} + \frac{\Lambda^4}{4}\left(\int d^2\theta\, X + h.c\right) \tag{4.82}$$

where $X = W^\alpha W_\alpha$ satisfies

$$X^2 = 0\,. \tag{4.83}$$

The explicit form of X is

$$X \;=\; W^\alpha W_\alpha = \lambda^2 + 2\theta^\beta(\epsilon_{\beta\alpha}D - F_{\beta\alpha})\lambda^\alpha + (\frac{1}{2}F^{\alpha\beta}F_{\alpha\beta} + D^2 + 2\chi\lambda)\theta^2 \tag{4.84}$$

with $F^{\alpha\beta} = \epsilon^{\alpha\rho}\epsilon^{\beta\sigma}F_{\rho\sigma}$. By defining

$$G_\beta = 2\lambda_\beta D - 2F_{\beta\alpha}\lambda^\alpha \tag{4.85}$$

and noticing that, because $\lambda^2\lambda_\alpha = 0$,

$$G^2 = \lambda^2(4D^2 + 2F^{\alpha\beta}F_{\alpha\beta}) = \lambda^2(4D^2 + 2F^{\alpha\beta}F_{\alpha\beta} + 8\chi\lambda) \equiv 4\lambda^2\mathcal{F}, \tag{4.86}$$

we get the parametrization of X in chiral coordinates [40, 139]

$$X = \frac{\tilde{G}^2}{2\mathcal{F}} + \sqrt{2}\theta\tilde{G} + \theta^2\mathcal{F}. \tag{4.87}$$

Here we have rescaled $G = \sqrt{2}\tilde{G}$. In a sense, W_α is the square root of the goldstino. If the above form of X is plugged back in eq. (4.82), the Volkov-Akulov Lagrangian for the goldstino G is obtained [40, 139].

We should note here that the resulting Lagrangian is written entirely in terms of the goldstino G_α. One would expect the theory to propagate also its supersymmetric partner, the sgoldstino to fill together a multiplet of the (broken) susy. However, it seems that the sgoldstino has been integrated out from the theory. This is due to the fact that the original multiplet didn't have any propagating fields as both fermions χ, λ and bosons $D, F_{\alpha\beta}$ were auxiliaries. In a sense, the original theory can be considered as the zero-momentum limit (or infinite mass limit) of a theory were all fields were propagating. This is equivalent to sgoldstino decoupling [11, 18, 19, 40, 85, 86, 139] and we correctly find here that the goldstino is the only propagating mode in the susy broken branch.

A way to find the vev of $\mathcal{F}$ is from the bosonic part of (4.78), which turns out to be

$$\mathcal{L}_W^B = \left(\frac{1}{8}F^{\alpha\beta}F_{\alpha\beta} + \frac{1}{4}D^2 + h.c\right) + \frac{1}{\Lambda^4}\left(D^2 + \frac{1}{2}F^{\alpha\beta}F_{\alpha\beta}\right)\left(\bar{D}^2 + \frac{1}{2}\bar{F}^{\dot{\alpha}\dot{\beta}}\bar{F}_{\dot{\alpha}\dot{\beta}}\right). \tag{4.88}$$

The are now two solutions for D,

$$i)\;\; D = 0\,, \tag{4.89}$$

$$ii)\;\; D^2 = -\frac{1}{2}F^{\alpha\beta}F_{\alpha\beta} - \frac{\Lambda^4}{4}, \quad \bar{D}^2 = -\frac{1}{2}\bar{F}^{\dot{\alpha}\dot{\beta}}\bar{F}_{\dot{\alpha}\dot{\beta}} - \frac{\Lambda^4}{4}. \tag{4.90}$$

The first solution is the supersymmetric Lorentz-invariant vacuum, provided $F_{\alpha\beta} = 0$, whereas the second solution gives

$$\mathcal{F} = -\frac{\Lambda^4}{4}. \tag{4.91}$$

Then $\langle F_{\alpha\beta}\rangle \neq 0$ clearly breaks supersymmetry but also Lorentz invariance at the same time. However, it is possible to preserve Lorentz invariance if $\langle F_{\alpha\beta}\rangle = 0$ and $\langle F^{\alpha\beta}F_{\alpha\beta}\rangle \neq 0$ as required by (4.90).

In the particular case in which W_α is the field strength superfield, the bosonic part of (4.78) turns out to be [83]

$$\begin{aligned}\mathcal{L}_W^B = &-\frac{1}{4}F^{mn}F_{mn} - \frac{i}{8}\epsilon^{mnkl}F_{mn}F_{kl} + \frac{1}{2}D^2 \\ &+ \frac{1}{\Lambda^4}\left\{\frac{1}{4}(F^{mn}F_{mn})^2 - F^{mn}F_{mn}D^2 + \frac{1}{16}(\epsilon^{mnkl}F_{mn}F_{kl})^2 + D^4\right\}.\end{aligned} \tag{4.92}$$

There are two solutions for D,

$$i)\;\; D = 0\,, \tag{4.93}$$

$$ii)\;\; D^2 = \frac{1}{2}F^{mn}F_{mn} - \frac{\Lambda^4}{4}. \tag{4.94}$$

The first solution corresponds to the supersymmetric branch, whereas the second solution gives the possibility $< D^2 > \neq 0$ and may break supersymmetry. However, this is not a Lorentz-invariant vacuum, since (4.94) requires a non-vanishing $F^{mn}F_{mn}$ for supsersymmetry breaking. In particular, since D^2 is positive, this vacuum can only be sustained with a non-zero background magnetic field.

Real Linear Multiplet

Another interesting example is provided by the Lagrangian

$$L = \int d^4\theta \left(-L^2 + \frac{1}{64\Lambda^4} D^\alpha L D_\alpha L \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L \right), \tag{4.95}$$

where L is a real linear multiplet. The grassmann expansion of the latter may be written as

$$L = \phi + \theta\psi + \bar{\theta}\bar{\psi} - \theta\sigma_m\bar{\theta}H^m - \frac{i}{2}\theta^2\bar{\theta}\bar{\sigma}^m\partial_m\psi + \frac{i}{2}\bar{\theta}^2\theta\sigma^m\partial_m\bar{\psi} - \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\phi \tag{4.96}$$

and satisfies

$$L = \bar{L}\,, \qquad D^2 L = 0. \tag{4.97}$$

This implies that the vector H_m is divergeneless

$$\partial^m H_m = 0. \tag{4.98}$$

The action (4.95) can be written as

$$L = \int d^4\theta \left(-L^2 + \frac{1}{64\Lambda^4} X\bar{X} \right) = \int d^4\theta \left(\frac{1}{64\Lambda^4} X\bar{X} \right) + \left(\frac{1}{4} \int d^2\theta X + h.c. \right), \tag{4.99}$$

with

$$\bar{X} \equiv D^\alpha L D_\alpha L = \frac{1}{2} D^2 L^2. \tag{4.100}$$

Note that $\bar{X}$ is antichiral, so X is chiral and obeys $X^2 = 0$. Then the Lagrangian (4.99) is the same as in [40, 139] (modulo normalization factors). In particular, X is explicitly written in chiral coordinates as

$$X = \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L = \bar{\psi}^2 - 2\theta\sigma_m\bar{\psi}(i\partial^m\phi + H^m) + \theta^2[2i\partial^m\psi\sigma_m\bar{\psi} + (i\partial^m\phi + H^m)^2] \tag{4.101}$$

therefore, it is chiral with auxiliary field $\mathcal{F}$

$$\mathcal{F} = (i\partial_m\phi + H_m)(i\partial^m\phi + H^m). \tag{4.102}$$

The goldstino now is given by

$$G_\alpha = -2\sigma_{m\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}(i\partial^m\phi + H^m). \tag{4.103}$$

It is easy to see that the bosonic part of (4.99) is

$$\mathcal{L}^B = \frac{1}{2}H_mH^m - \frac{1}{2}\partial_m\phi\partial^m\phi + \frac{1}{64\Lambda^4}|(i\partial_m\phi + H_m)^2|^2 \tag{4.104}$$

There is a supersymmetric vacuum $H_m = 0$, $\phi = const.$ and a supersymmetry breaking one (with $\phi = const.$)

$$H_mH^m = -16\Lambda^4. \tag{4.105}$$

In this case, supersymmetry is broken and the theory reduces to the standard Volkov-Akulov for the goldstino G. In spite of appearances, the vacuum solution (4.105) does not breaks Lorentz invariance, since the divergenceless vector H_m and $\partial_m\phi$ combine into the unconstrained vector A_m, which does not propagate, because it has algebraic equations of motion. Therefore, a nonzero constant vev for A_m does not affect the dynamics since it either disappears from the Lagrangian or it arranges itself into Lorentz-invariant composite quantitites. We also note that, after using (4.101), the action (4.99) is written entirely in terms of the goldstino field G_α. Again here, similarly to the spinor superfield case above, there is no superpartner of the goldstino. The sgoldstino is decoupled as all fields before susy breaking were auxiliaries and therefore (4.99) may be consider as the zero-momentum limit of a theory were these were propagating. In this limit, the sgoldstino decouples and the theory describes a Volkov-Akulov model.

Validity of the Volkov-Akulov Description

The theories above, as well as the one we will examine later, must be understood as effective IR theories. If a supersymmetric UV completion existed, then the sgoldstino φ would have a large but finite mass m_s. It would interact with the goldstino through terms of the schematic form

$$\kappa G_\alpha G^\alpha\varphi + (m_s^2/2)\varphi^2 + ..., \tag{4.106}$$

with a coupling constant $\kappa = O(m_s^2/f)$. At energies below m_s, the sgoldstino fields can be integrated out, producing additional irrelevant operators weighted by inverse powers of the new scale $\Lambda' = f/m_s$. Curiously, these additional interactions become negligible when the sgoldstino is massive but *lighter* than $\sqrt{f}$: $\Lambda' \gg \sqrt{f} \to m_s \ll \sqrt{f}$. We will explicitly demonstrate this in the case of supersymmetric theories with chiral multiplets.

Let us recall that in globally supersymmetry theory with $n+1$ chiral multiplets Φ^i, the Yukawa couplings arise from the term

$$\mathcal{L} \supset W_{ij}(\phi)\chi^i\chi^j + h.c\,, \quad i,j = 0,1,...,n, \tag{4.107}$$

where ϕ^i, χ^i are the scalars and fermions of the chirals and $W_{ij} = \partial^2W/\partial\phi^i\partial\phi^j$. The potential is

$$V = W_iW^i, \tag{4.108}$$

where the notation $W^i = (W_i)^\dagger$ is used and let us assume for the moment that the Kähler metric is flat. The values of the fields in the ground state are $\langle\phi^i\rangle = a^i$, $\langle F^i\rangle = f^i$, $\langle\psi_i\rangle = 0$ and the equation of motions give

$$\bar{f}_i = -w_i\,, \qquad w_{ij}f^j = 0, \tag{4.109}$$

where

$$w_i = W_i(a^i)\,, \quad w_{ij} = W_{ij}(a^i)\,, \qquad ... \tag{4.110}$$

The term (4.107) gives then rise to the interaction

$$\mathcal{L} \supset w_{ijk}\delta\phi^k\chi^i\chi^j + h.c, \tag{4.111}$$

where $\delta\phi^i = \phi^i - a^i$. Since supersymmetry is broken, the fermionic shifts will not vanish in the vacuum

$$< \delta\chi_i >= -f_i\epsilon. \tag{4.112}$$

By an appropriate rotation of χ_i, we can define new fermionic fields $\tilde{\chi}_i$

$$\tilde{\chi}_i = R_i{}^j\chi_j, \tag{4.113}$$

where $R_i{}^j$ is an appropriate matrix such that the non-zero fermionic shift are along a specific direction, which we will call it ("0")

$$< \delta\tilde{\chi}_0 >= -f\epsilon\,, \qquad < \delta\tilde{\chi}_a >= 0\,, \quad a = 1, \ldots, n, \tag{4.114}$$

with $|f|^2 = f_if^i$. Clearly $\tilde{\chi}_0$ is the goldstino, which is defined then as

$$\tilde{\chi}_0 = R_0{}^i\delta\chi_i \tag{4.115}$$

and the rest of the modes are given by

$$\delta\tilde{\chi}_a = R_a{}^i\delta\chi_i. \tag{4.116}$$

The matrix R_{ij} is orthogonal and chosen to satisfy

$$R_a{}^if_i = 0\,. \tag{4.117}$$

When this equation is satisfied, then $R_0{}^i = f_i/|f|$ so that the goldstino is

$$\delta\tilde{\chi}_0 = \frac{f_i}{|f|}\delta\chi_i\,. \tag{4.118}$$

Note that instead of rotating χ_i's, we could have rotated the original superfields Φ^i so that the goldstino belongs to the $\tilde{\Phi}^0$ goldstino superfield, which is a linear combination of the original fields. According to (4.118), $\tilde{\Phi}^0$ is

$$\tilde{\Phi}_0 = \frac{f_i}{|f|}\Phi^i. \tag{4.119}$$

The rest of the superfields are given by

$$\tilde{\Phi}_a = R_a{}^i \Phi^i ; \tag{4.120}$$

therefore, the sgoldstino is

$$\phi^0 = \frac{f_i}{|f|}\phi^i . \tag{4.121}$$

The interaction (4.111) is written then in terms of the new fields as

$$\mathcal{L} \supset R^i{}_n R^j{}_m R^k{}_l w_{ijk} \delta\tilde{\phi}^n \tilde{\chi}^m \tilde{\chi}^l . \tag{4.122}$$

The possible Yukawa coupling of the golstino are

$$\mathcal{L}_1 \supset R^i{}_0 R^j{}_0 R^k{}_0 w_{ijk} \delta\tilde{\phi}^0 \tilde{\chi}^0 \tilde{\chi}^0 = |f|^{-3} f^i f^j f^k w_{ijk} \delta\tilde{\phi}^0 \tilde{\chi}^0 \tilde{\chi}^0 = |f|^{-3}\, s\, \delta\tilde{\phi}^0 \tilde{\chi}^0 \tilde{\chi}^0 \tag{4.123}$$

$$\mathcal{L}_2 \supset R^i{}_a R^j{}_0 R^k{}_0 w_{ijk} \delta\tilde{\phi}^a \tilde{\chi}^0 \tilde{\chi}^0 = |f|^{-2} R^i{}_a f^j f^k w_{ijk} \delta\tilde{\phi}^a \tilde{\chi}^0 \tilde{\chi}^0 = |f|^{-2} R^i{}_a\, s_i\, \delta\tilde{\phi}^a \tilde{\chi}^0 \tilde{\chi}^0 \tag{4.124}$$

$$\mathcal{L}_2 \supset R^i{}_a R^j{}_b R^k{}_0 w_{ijk} \delta\tilde{\phi}^a \tilde{\chi}^b \tilde{\chi}^0 , \tag{4.125}$$

where

$$s = f^i f^j f^k w_{ijk} \,, \qquad s_k = f^i f^j w_{ijk} . \tag{4.126}$$

We will show now that

$$s = 0 \,, \qquad s_i = 0 \tag{4.127}$$

so that a globally supersymmetric theory the only trilinear Yukawa coupling is the one that contains only one goldstino or one sgoldstino. For this, we need to recall that the fermionic mass matrix $m_F = w_{ij}$ has a zero eigenvalue

$$m_{F\,ij} f^j = 0 \,, \tag{4.128}$$

and the bosonic mass matrix

$$M_B^2 = \begin{pmatrix} m_F^\dagger m_F & \sigma \\ \sigma^\dagger & m_F m_F^\dagger \end{pmatrix} , \qquad \sigma_{ij} = w_{ijk} f^k \tag{4.129}$$

is positive definite

$$\langle \Psi | M_B^2 | \Psi \rangle \geq 0 . \tag{4.130}$$

For

$$|\Psi\rangle = \begin{pmatrix} f_i \\ f^i \end{pmatrix} \tag{4.131}$$

we get, since m_F annihilates f^i,

$$\mathrm{Re}(f^i f^j s_{ij}) \geq 0 . \tag{4.132}$$

Moreover, since m_F annihilates also $e^{i\varphi}f^i$, where ϕ is an arbitrary phase, we get in general

$$\mathrm{Re}(e^{2i\varphi}f^i f^j \sigma_{ij}) \geq 0 \tag{4.133}$$

which leads to

$$s = f^i f^j \sigma_{ij} = f^i f^j f^k w_{ijk} = 0. \tag{4.134}$$

Therefore, the coupling $\mathcal{L}_1$ vanishes and there is no (goldstino^2 sgoldstino) coupling.

We can also prove that there is no (goldstino^2 scalar) Yukawa coupling by showing that $s_i = 0$, which means that $\mathcal{L}_2$ vanishes as well. By using (4.134), it is easy to see that in fact

$$\big\langle \Psi | M_B^2 | \Psi \big\rangle = 0 \tag{4.135}$$

and since M_B^2 is positive definite, M_B^2 annihilates $|\Psi\big\rangle$

$$M_b^2 |\Psi\big\rangle = 0. \tag{4.136}$$

Then, by using (4.128,4.134), we find

$$\sigma_{ij} f^j = w_{ijk} f^j f^k = 0. \tag{4.137}$$

Therefore, $s_i = 0$ and the interaction $\mathcal{L}_2$ similarly vanish. As a result, in a globally supersymmetric theory, the only Yukawa coupling that is allowed, is only $\mathcal{L}_3$, i.e., a single goldstino interacting with a scalar and a fermion of the matter scalar multiplet or a single sgoldstino interacting with two fermions of the matter scalar multiplet. In particular, this means that there is no way to break supersymmetry just with a single chiral multiplet.

Let us now turn to the general case of a non-flat Kähler metric $g_{i\bar{j}}$. In this case, the bosonic mass matrix is

$$M_B^2 = \begin{pmatrix} -K^j{}_i + (m_F^\dagger m_F)^j{}_i & \sigma \\ \sigma^\dagger & -K_i{}^j + (m_F^\dagger m_F)_i{}^j \end{pmatrix}. \tag{4.138}$$

where

$$K^j{}_i = K_{\bar{j}i} = K_{\bar{j}i\bar{m}n} \bar{f}^{\bar{m}} f^k \tag{4.139}$$

and $K_{\bar{j}i\bar{m}n} = R_{\bar{j}i\bar{m}n}$ in normal coordinates. Now, the corresponding relation (4.130) for the positivity of M_B^2 does not lead to any conclusive relation. The Yukawa couplings originate from the term

$$\mathcal{L} \supset \Big(W_{ij} - \Gamma^k_{ij} W_k\Big)\chi^i \chi^j + h.c. \tag{4.140}$$

which gives rise to

$$\mathcal{L} \supset \Big(W_{ijk} - \partial_k \Gamma^l_{ij} W_l - \Gamma^l_{ij} W_{lk}\Big)\delta\phi^k \chi^i \chi^j + h.c.. \tag{4.141}$$

Rotating the fields such that again the goldstino is in the 0-direction as before, we get the interaction

$$\mathcal{L} \supset \tilde{s}\, \delta\phi^0 \chi^0 \chi^0 + h.c. \tag{4.142}$$

where

$$\tilde{s} = (W_{ijk} - \partial_k \Gamma^l_{ij} W_l - \Gamma^l_{ij} W_{lk}) f^i f^j f^k. \tag{4.143}$$

Clearly now $\tilde{s} \neq 0$ as can easily be checked for the simplest case of a linear superpotential $W = f\Phi$. In fact it is easy to see that if the scale of the Kähler manifold is Λ then the sgoldstino mass is

$$m_s \sim \frac{f}{\Lambda} \tag{4.144}$$

and $\tilde{s}$ is of the order of

$$\tilde{s} \sim \frac{f}{\Lambda^2} \sim \frac{m_s^2}{f}. \tag{4.145}$$

Therefore, the effective coupling in the IR will be schematically of the form

$$\frac{m_s^2}{f} \chi^0 \chi^0 \phi^0 - \frac{1}{2} m_s^2 \phi_0^2 + \cdots + h.c \tag{4.146}$$

which gives rise to a term of the form

$$\mathcal{L} \supset \frac{m_s^2}{f^2} (\chi^0 \bar{\chi}^0)^2 \tag{4.147}$$

after integrating out the sgoldstino. Such a term is supressed by the scale $\Lambda' = f/m_s$ and therefore it can be ignored as long as it is much larger than the Volkov-Akulov scale $\sqrt{f}$ ($\Lambda' >> \sqrt{f}$). In this case, interactions like (4.147) can safely be ignored and the theory will be described by Volkov-Akulov for

$$\frac{f}{m_s} >> \sqrt{f}. \tag{4.148}$$

In other words, the Volkov-Akulov description is valid for

$$m_s << \sqrt{f} << \Lambda. \tag{4.149}$$

This limit is the one considered in the models with constraint superfields in which the sgoldstino can be safely integrated out resulting in a non-linearly realized supersymmetric Volkov-Akulov theory for the goldstino mode. The V-A description is then valid only up to a UV cutoff equal to the mass $m_{lightest}$ of the lightest particle mixing with the goldstino. This particle can be the sgoldstino or one of the fermions orthogonal to the goldstino. Of course, as in all effective Lagrangians, the V-A scale f must obey $f > m^2_{lightest}$.

4.2.2 The Complex Linear Multiplet

We have explicitly demonstrated in the previous section that higher dimensional operators contribute to the vacuum structure and may lead to supersymmetry breaking.

Here we will see that it is possible to break supersymmetry without intorducing any Lorentz non-invariant vev.

The reason that the potential (4.77) cannot break superymmetry is that the two terms in (4.77), coming from the two- and four- derivative terms of (4.76) have the same sign. Clearly, new extrema can emerge only if these terms have opposite sign, i.e. if the first contribution coming form the leading term in (4.76) flips sign. This can happen for the complex linear multiplet [99, 101].

The complex linear or nonminimal multiplet is defined as

$$\bar{D}^2\Sigma = 0. \tag{4.150}$$

The constraint (4.150) above is just the field equation for a free chiral multiplet. Note that if the further constraint $\Sigma = \bar{\Sigma}$ is imposed, the complex linear multiplet turns into a linear one. The standard kinetic Lagrangian for the complex linear superfield in superspace reads

$$\mathcal{L}_0 = -\int d^4\theta\,\Sigma\bar{\Sigma}. \tag{4.151}$$

Note the relative minus sign compared to the kinetic Lagrangian of a chiral multiplet. This is necessary for the theory to contain no ghosts. The relative minus sign of the complex linear multiplet Σ compared to the standard kinetic term for a chiral multiplet Φ can be understood in terms of a duality transformation. Indeed, consider the action

$$\mathcal{L}_D = -\int d^4\theta\big(\Sigma\bar{\Sigma} + \Phi\Sigma + \bar{\Phi}\bar{\Sigma}\big), \tag{4.152}$$

where Φ is chiral and Σ is unconstrained. Integrating out Φ we get both eq. (4.151) and the constraint (4.150). However, by integrating out Σ, we get $\Sigma = -\bar{\Phi}$. Plugging back this equality into (4.152), we get the standard kinetic term of a chiral multiplet

$$\mathcal{L}_0 = \int d^4\theta\Phi\bar{\Phi}. \tag{4.153}$$

As announced, the overall sign in Lagrangian (4.153) is opposite to that of (4.151).

To find the superspace equation of motion, we should express Σ in terms of an unconstrained superfield. This can be done by introducing a general spinor superfield Ψ^α with gauge transformation

$$\delta\Psi_\alpha = D^\beta\Lambda_{(\alpha\beta)} \tag{4.154}$$

where $\Lambda_{(\alpha\beta)}$ is arbitrary. It is easy to see that by defining

$$\Sigma = \bar{D}_{\dot{\alpha}}\bar{\Psi}^{\dot{\alpha}}, \tag{4.155}$$

Σ satisfies the constraint (4.150). Then the field equation following from eq. (4.151) is

$$D_\alpha\Sigma = 0\,. \tag{4.156}$$

Therefore, the field equation of a complex linear multiplet is just the constraint of a chiral multiplet and, as noticed above, the constraint on a linear is the field equation of a chiral. This indicated the duality between the two kind of multiplets, at least in the free case. The field content of the complex linear multiplet Σ is revealed via the projection over components as

$$
\begin{aligned}
A &= \Sigma|, \\
\psi_\alpha &= \frac{1}{\sqrt{2}} D_\alpha \bar{\Sigma}|, \\
F &= -\frac{1}{4} D^2 \Sigma|, \\
\lambda_\alpha &= \frac{1}{\sqrt{2}} D_\alpha \Sigma|, \\
P_{\alpha\dot{\beta}} &= \bar{D}_{\dot{\beta}} D_\alpha \Sigma|\,, \qquad \bar{P}_{\alpha\dot{\beta}} = -D_\beta \bar{D}_{\dot{\alpha}} \bar{\Sigma}|, \\
\chi_\alpha &= \frac{1}{2} \bar{D}_{\dot{\alpha}} D_\alpha \bar{D}^{\dot{\alpha}} \bar{\Sigma}|\,, \qquad \bar{\chi}_{\dot{\alpha}} = \frac{1}{2} D^\alpha \bar{D}_{\dot{\alpha}} D_\alpha \Sigma|.
\end{aligned} \tag{4.157}
$$

In other words, a complex linear multiplet contains a chiral multiplet (A, λ_α, F) and an antichiral spinor superfield $(\psi_\alpha, P_{\alpha\dot{\beta}}, \chi_\alpha)$. Therefore, the complex linear multiplet is a reducible 12 + 12 dimensional representation of the $\mathcal{N} = 1$ supersymmetry. It should be noted that since Σ is not chiral, there is no superpotential and there are no supersymmetric non-derivative interactions. However, the complex linear multiplet can still be consistently coupled to ordinary vector multiplets of the $\mathcal{N} = 1$ theory.

We give for later use the supersymmetry transformations of the fermionic components of Σ

$$
\delta\psi_\alpha = \sqrt{2} i \sigma^m_{\alpha\dot{\beta}} \bar{\xi}^{\dot{\beta}} \partial_m \bar{A} - \frac{1}{\sqrt{2}} \bar{\xi}^{\dot{\beta}} \bar{P}_{\alpha\dot{\beta}} \tag{4.158}
$$

$$
\delta\chi_\alpha = 2i\sigma^n_{\alpha\dot{\alpha}} \bar{\sigma}^{m\dot{\alpha}\beta} \xi_\beta \partial_m \bar{P}_n + i\sigma^m_{\alpha\dot{\alpha}} \bar{\sigma}^{n\dot{\alpha}\beta} \xi_\beta \partial_m \bar{P}_n - 4\xi_\alpha \partial^2 \bar{A} + 2i\sigma^m_{\alpha\dot{\alpha}} \bar{\xi}^{\dot{\alpha}} \partial_m \bar{F} \tag{4.159}
$$

$$
\delta\lambda_\alpha = \sqrt{2} \xi_\alpha F - \frac{1}{\sqrt{2}} \bar{\xi}^{\dot{\beta}} P_{\alpha\dot{\beta}}. \tag{4.160}
$$

The transformation rules of the bosonic sector of the complex linear multiplet are

$$
\delta A \;=\; \sqrt{2}\bar{\xi}\bar{\psi} + \sqrt{2}\xi\lambda, \tag{4.161}
$$

$$
\delta F \;=\; \frac{i}{\sqrt{2}} \bar{\xi} \bar{\sigma}^m \partial_m \lambda + \frac{1}{2} \bar{\xi} \bar{\chi}, \tag{4.162}
$$

$$
\delta P_{\alpha\dot{\beta}} \;=\; -2\sqrt{2} i \xi^\gamma \sigma^m_{\gamma\dot{\beta}} \partial_m \lambda_\alpha + \sqrt{2} i \xi_\alpha \sigma^m_{\beta\dot{\beta}} \partial_m \lambda_\beta - \xi_\alpha \bar{\lambda}_{\dot{\beta}} - 2\sqrt{2} i \bar{\xi}_{\dot{\beta}} \sigma^m_{\alpha\dot{\rho}} \partial_m \bar{\psi}^{\dot{\rho}}. \tag{4.163}
$$

In terms of the components of Σ, Lagrangian (4.151) is explicitly written as

$$
\mathcal{L}_0 = A\partial^2 \bar{A} - F\bar{F} + i\partial_m \bar{\psi} \bar{\sigma}^m \psi + \frac{1}{2} P_m \bar{P}^m + \frac{1}{2\sqrt{2}} \big(\chi\lambda + \bar{\chi}\bar{\lambda}\big). \tag{4.164}
$$

The complex vector P_m, the complex scalar F and the spinors λ, χ are auxiliary fields. Note that the minus sign in front of the superspace action (4.151) guarantees that the scalar A is a normal field and not a ghost. However, this choice of sign has flipped the sign of the $F\bar{F}$ relative to the action for a chiral multiplet. This flip of sign is of fundamental importance for what follows and leads to supersymmetry breaking.

4.2.3 SUSY Breaking by Complex Linear Multiplets

As we have noticed before, although one can couple the linear multiplet to gauge fields [67, 112, 169, 170, 184], one cannot write down mass terms or non-derivative interactions as in the chiral multiplet case by means of a superpotential. So, the best we can hope for is to introduce a potential indirectly by using the higher dimensional operators first discussed in [43]. The idea of [43] has been recently revisited and the emergent potential for chiral and vector multiplets has been discussed in [82, 83, 135].

To achieve this, we introduce the following Lagrangian in superspace

$$\mathcal{L}_{EP} = \int d^4\theta \, \frac{1}{64\,\Lambda^4} \; D^\alpha \Sigma D_\alpha \Sigma \bar{D}_{\dot\alpha} \bar{\Sigma} \bar{D}^{\dot\alpha} \bar{\Sigma}, \tag{4.165}$$

where Λ is a mass scale. Then, the theory is described by

$$\begin{aligned} \mathcal{L}_\Sigma &= \mathcal{L}_0 + \mathcal{L}_{EP} \\ &= \int d^4\theta \, \left(-\Sigma\bar{\Sigma} + \frac{1}{64\Lambda^4} \; D^\alpha \Sigma D_\alpha \Sigma \bar{D}_{\dot\alpha} \bar{\Sigma} \bar{D}^{\dot\alpha} \bar{\Sigma} \right). \end{aligned} \tag{4.166}$$

By using the unconstrained superfield Φ_α, we find that the field equations are

$$D_\alpha \Sigma + \frac{1}{32\Lambda^4} D_\alpha \bar{D}_{\dot\alpha} \left(D^\beta \Sigma D_\beta \Sigma \bar{D}^{\dot\alpha} \bar{\Sigma} \right) = 0. \tag{4.167}$$

Clearly, the above equation always admits the supersummetry preserving solution

$$D_\alpha \Sigma = 0\,. \tag{4.168}$$

We are interested to investigate if another, supersymmetry breaking solution to (4.167) exists.

The component form of the bosonic part of eq. (4.165) is

$$\mathcal{L}^B_{EP} = \frac{1}{64\Lambda^4} \Big(P^m P_m \bar{P}^n \bar{P}_n + 4 P_m \bar{P}^m F \bar{F} + 16 F^2 \bar{F}^2 \Big), \tag{4.169}$$

so that the bosonic part of the full Lagrangian (4.166) turns out to be

$$\begin{aligned} \mathcal{L}^B =& - F\bar{F} + A \partial^2 \bar{A} + \frac{1}{2} P_m \bar{P}^m \\ &+ \frac{1}{64\Lambda^4} \Big(P^m P_m \bar{P}^n \bar{P}_n + 4 P_m \bar{P}^m F \bar{F} + 16 F^2 \bar{F}^2 \Big). \end{aligned} \tag{4.170}$$

From the equations of motion for the complex auxiliary vector we find that

$$P_m = 0, \tag{4.171}$$

whereas the equations of motion for the auxiliary scalar turns out to be

$$F \left(1 - \frac{1}{2\Lambda^4} F\bar{F} \right) = 0. \tag{4.172}$$

There are now two solutions:

$$(i) \quad F = 0\,, \tag{4.173}$$

$$(ii) \;\; F\bar{F} = 2\Lambda^4. \tag{4.174}$$

Clearly, as it follows from eqs. (4.158,4.159,4.160), the first vacuum $F = 0$ is the supersymmetric one, where supersymmetry is exact. However, the second vacuum, described by the solution (4.174), explicitly breaks supersymmetry. We note that the theories with $F = 0$ and $F \neq 0$ should not be thought as phases of the same theory but rather as two different theories. This can be illustrated by the following example. Consider a scalar A and an auxiliary field Y with Lagrangian:

$$\mathcal{L}_{AY} = -\frac{1}{2}\partial_m A\partial^m A - \frac{1}{2}Y^2(aA^2 + b) + \frac{1}{4}Y^4. \tag{4.175}$$

Solving for Y we get two solutions: $Y = 0$, which gives the free scalar Lagrangian

$$\mathcal{L}_A = -\frac{1}{2}\partial_m A\partial^m A, \tag{4.176}$$

and

$$Y^2 = aA^2 + b, \tag{4.177}$$

which gives the interacting Lagrangian

$$\mathcal{L}'_A = -\frac{1}{2}\partial_m A\partial^m A - \frac{1}{4}(aA^2 + b)^2. \tag{4.178}$$

No transition either perturbative or nonperturbative can occur between the two, precisely because the equations for Y are algebraic, so they are truly two different theories.

It should also be noted that the susy-breaking vacuum is specified by the modulus of the auxiliary field F. So, F itself is specified only up to a phase. This is expected due to the invariance of Lagrangian (4.166) under the global $U(1)$ transformation

$$\Sigma \to e^{i\phi}\Sigma. \tag{4.179}$$

For completeness, we give the component form of Lagrangian (4.166)

$$\begin{aligned}
\mathcal{L}_\Sigma = {} & A\partial^2\bar{A} - F\bar{F} + i\partial_m\bar{\psi}\bar{\sigma}^m\psi + \frac{1}{2}P_m\bar{P}^m + \frac{1}{2\sqrt{2}}\left(\chi\lambda + \bar{\chi}\bar{\lambda}\right) \\
& + \frac{1}{64\Lambda^4}\Big\{4(\lambda^\alpha\partial^2\lambda_\alpha)\bar{\lambda}^2 + 2\sqrt{2}i(\partial_m\bar{\chi}\bar{\sigma}^m\lambda)\bar{\lambda}^2 \\
& - 16F\partial^2 A\bar{\lambda}^2 + 8iF\partial^m P_m\bar{\lambda}^2 \\
& + 8\partial^2 A\bar{\lambda}\bar{\sigma}^k\lambda\bar{P}_k + 4i\bar{\lambda}\bar{\sigma}^k\sigma^n\bar{\sigma}^m\lambda\bar{P}_k\partial_m P_n \\
& + 8i\bar{\lambda}\bar{\sigma}^k\sigma^m\partial_m\bar{\psi}F\bar{P}_k - 16\partial_m\bar{\psi}\bar{\sigma}^m\lambda\partial_n\psi\sigma^n\bar{\lambda} + 4i\partial_m\bar{\psi}\bar{\sigma}^m\lambda\bar{P}^2 \\
& + \frac{1}{2}\Omega^{\beta\dot{\beta}\alpha}\Omega_{\beta\dot{\beta}\alpha}\bar{\lambda}^2 - 8i\bar{\lambda}^2 P^k\partial_k F \\
& + \sqrt{2}\bar{P}_m\bar{\lambda}_{\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\beta}\Omega_{\beta\dot{\beta}\alpha}\bar{\sigma}^{k\dot{\beta}\alpha}P_k + 4iP^2\partial_m\psi\sigma^m\bar{\lambda} + P^2\bar{P}^2 \\
& - 8\sqrt{2}F\bar{\chi}\bar{\lambda}\bar{F} - 8F\bar{F}P_n\bar{P}^n - 2\sqrt{2}\chi\sigma^m\bar{\lambda}P_m F \\
& + 4iFP_m\bar{\lambda}\bar{\sigma}^m\sigma^n\partial_n\bar{\lambda} - 16i\lambda\sigma^n\bar{\lambda}\bar{F}\partial_n F \\
& + 2\sqrt{2}\bar{P}_n\bar{\sigma}^{n\dot{\beta}\beta}\Omega_{\beta\dot{\beta}\alpha}\lambda^\alpha\bar{F} - 2\Omega_{\beta\dot{\beta}\alpha}\chi^\beta\bar{\lambda}^{\dot{\beta}}\lambda^\alpha + 2\sqrt{2}i\partial_m\bar{\lambda}_{\dot{\rho}}\bar{\sigma}^{m\dot{\rho}\beta}\Omega_{\beta\dot{\beta}\alpha}\lambda^\alpha\bar{\lambda}^{\dot{\beta}}
\end{aligned} \tag{4.180}$$

$$- 8i\partial_n\psi\sigma^n\bar{\sigma}^m\lambda P_m\bar{F} - \sqrt{2}\lambda\sigma^m\bar{\sigma}^n\chi P_m\bar{P}_n - 2i\lambda\sigma^k\bar{\sigma}^m\sigma^n\partial_n\bar{\lambda}P_k\bar{P}_m$$
$$- 8\lambda\sigma^n\bar{\lambda}P_n\partial^2\bar{A} - 8i\lambda\sigma^n\bar{\lambda}P_n\partial_m\bar{P}^m$$
$$+ 16F^2\bar{F}^2 - 8\sqrt{2}\lambda\chi F\bar{F} - 16i\lambda\sigma^n\partial_n\bar{\lambda}F\bar{F}$$
$$- 16\lambda^2\bar{F}\partial^2\bar{A} - 16i\lambda^2\bar{F}\partial_m\bar{P}^m - \lambda^2\Xi^2\Big\},$$

where

$$\Omega^{\beta\dot{\beta}\alpha} = -2\sqrt{2}i\bar{\sigma}^{m\dot{\beta}\beta}\partial_m\lambda^\alpha - i\sqrt{2}\epsilon^{\beta\alpha}\bar{\sigma}^{m\dot{\beta}\gamma}\partial_m\lambda_\gamma - \epsilon^{\beta\alpha}\bar{\chi}^{\dot{\beta}} \quad , \quad \Omega_{\rho\dot{\rho}\sigma} = \epsilon_{\rho\beta}\epsilon_{\dot{\rho}\dot{\beta}}\epsilon_{\sigma\alpha}\Omega^{\beta\dot{\beta}\alpha} \tag{4.181}$$

and

$$\Xi_\beta = \chi_\beta + \sqrt{2}i\sigma^n_{\beta\dot{\beta}}\partial_n\bar{\lambda}^{\dot{\beta}}. \tag{4.182}$$

We should note that Lagrangian (4.180) contains also first derivatives of the auxiliaries F, P_m, χ. Therefore, one may question if these fields are really auxiliaries. However, it can easily be checked that these derivative terms are always multiplied by fermions. Therefore their equations of motion can be integrated by iteration in a power series of the fermions, which terminates due to the nilpotent nature of the latter.

To identify the goldstino mode, one should look at the supersymmetry transformations and, in particular, to the fermion shifts. It is then easy to recognize that since

$$\delta\lambda_\alpha = 2\,\xi_\alpha\Lambda^2 + \dots\,, \tag{4.183}$$

the goldstino of the broken supersymmetry is proportional to λ, i.e., one of the auxiliary fermions. Here something unusual has happened; namely, an auxiliary fermion has turned into a goldstino mode in the susy breaking vacuum. However, the latter is propagating and the vacuum (4.174) should definitely give rise to a kinetic term for λ. Indeed, it is straightforward to see that the higher dimensional operator Lagrangian gives rise to the following coupling for the auxiliary fermion λ

$$\mathcal{L}_{EP} \supset (\frac{1}{4\Lambda^4}F\bar{F})\; i\partial_m\bar{\lambda}\bar{\sigma}^m\lambda. \tag{4.184}$$

In the susy breaking vacuum obtained from eq. (4.172) we have

$$< F\bar{F} >= 2\Lambda^4, \tag{4.185}$$

leading to a standard fermionic kinetic term with the correct sign

$$\mathcal{L}_{EP} \supset \frac{i}{2}\partial_m\bar{\lambda}\bar{\sigma}^m\lambda. \tag{4.186}$$

Therefore, on the susy breaking vacuum (4.174), the auxiliary fermion λ is propagating and it is proportional to the goldstino mode of broken susy. Note that due to the model independent relation (4.185), the kinetic term (4.186) for the goldstino is also model independent. In fact what has happened here is that the susy breaking phase is a realization of non-linear supersymmetry.

We should also mention that the fermion bilinears $\chi\lambda$ and $\bar{\chi}\bar{\lambda}$ appear in the action as

$$\mathcal{L}_\Sigma \supset \frac{1}{2\sqrt{2}}\left(1 - \frac{F\bar{F}}{2\Lambda^4}\right)\left(\chi\lambda + \bar{\chi}\bar{\lambda}\right). \tag{4.187}$$

Such terms vanish on the non-supersymmetric vacuum and protect the theory from unwanted, dangerous terms. Moreover, as in the spinor superfield and real multiplet case, there is no superpartner of the goldstino. In fact, the propagating modes are the real scalar A, the fermion ψ and the golstino λ, which definitely do not form a multiplet of the (broken) susy. The reason again is that the rest of the fields of the complex linear multiplet are auxiliaries and therefore the sgoldstino decouples.

One could proceed and solve the field equations for the auxiliaries in (4.180). Although this is a formidable task, there is an indirect way to proceed in superspace. We will show below that the theory (4.180) describes a free chiral multiplet and a constraint chiral superfield which describes a Volkov-Akulov mode. To see how this happens, let us remind briefly some aspects of non-linear supersymmetry realizations. It is well known that the following Lagrangian [40]

$$\mathcal{L} = \int d^4\theta \, X_{NL}\bar{X}_{NL} + \sqrt{2}\Lambda^2 \left(\int d^2\theta \, X_{NL} + h.c \right) + \left(\int d^2\theta \, \Psi X_{NL}^2 + h.c \right) \tag{4.188}$$

is on-shell equivalent to the Akulov-Volkov theory. In fact, the Lagrange multiplier chiral superfield Ψ imposes the constraint

$$X_{NL}^2 = 0 \tag{4.189}$$

on the chiral superfield X_{NL}, leads to the non-linear realization of supersymmetry [40, 139, 174] and reproduces the Volkov-Akulov model. The Lagrangian (4.188) gives rise to the following two equations of motion in superspace

$$-\frac{1}{4}\bar{D}^2\bar{X}_{NL} + \sqrt{2}\Lambda^2 + 2\Psi X_{NL} \;\; = \;\; 0, \tag{4.190}$$

$$X_{NL}^2 = 0. \tag{4.191}$$

The theory we consider here is described by the Lagrangian

$$\mathcal{L} = -\int d^4\theta \, \Sigma\bar{\Sigma} + \int d^4\theta \, \frac{1}{64\,\Lambda^4} \;\; D^\alpha\Sigma D_\alpha\Sigma \bar{D}_{\dot\alpha}\bar{\Sigma}\bar{D}^{\dot\alpha}\bar{\Sigma} \tag{4.192}$$

and the superfield equations of motion are written as

$$D_\alpha\Sigma + \frac{1}{32\Lambda^4} D_\alpha \bar{D}_{\dot\alpha} \left(D^\beta\Sigma D_\beta\Sigma \bar{D}^{\dot\alpha}\bar{\Sigma} \right) = 0. \tag{4.193}$$

These equations can equivalently be expressed as

$$\Sigma = -\frac{1}{32\Lambda^4} \bar{D}_{\dot\alpha} \left(D^\beta\Sigma D_\beta\Sigma \bar{D}^{\dot\alpha}\bar{\Sigma} \right) + \bar{\Phi} \tag{4.194}$$

where Φ is a chiral superfield. Hitting the above equation with $\bar{D}^2$ leads to a consistency condition

$$\bar{D}^2\bar{\Phi} = 0, \tag{4.195}$$

which implies that Φ is a free chiral superfield. In fact, Σ can be written as

$$\Sigma = H + \bar{\Phi}, \tag{4.196}$$

where H satisfies the equations of motion

$$H = -\frac{1}{32\Lambda^4}\bar{D}_{\dot{\alpha}}\left(D^\beta H D_\beta H \bar{D}^{\dot{\alpha}}\bar{H}\right). \tag{4.197}$$

It is now straightforward to solve equation (4.197) in terms of a constrained chiral superfield subject to (4.190) and (4.191) by identifying H (up to a phase) with the goldstino chiral superfield X_{NL}

$$H = X_{NL}\,. \tag{4.198}$$

Let us verify that (4.198) indeed solves (4.197). From (4.191) one finds

$$D^\beta X_{NL} D_\beta X_{NL} = -X_{NL} D^2 X_{NL}, \tag{4.199}$$

whereas, (4.190) gives

$$X_{NL}\bar{D}^2\bar{X}_{NL} = 4\sqrt{2}\Lambda^2 X_{NL}, \tag{4.200}$$

$$X_{NL}D^2 X_{NL} = 4\sqrt{2}\Lambda^2 X_{NL} + 8X_{NL}\bar{X}_{NL}\bar{\Psi}. \tag{4.201}$$

For the right part of (4.197), by using (4.198) we have

$$\begin{aligned}
&- \frac{1}{32\Lambda^4}\bar{D}_{\dot{\alpha}}\left(D^\beta X_{NL} D_\beta X_{NL}\bar{D}^{\dot{\alpha}}\bar{X}_{NL}\right)\\
&= \frac{1}{32\Lambda^4}\bar{D}_{\dot{\alpha}}\left(X_{NL}D^2 X_{NL}\bar{D}^{\dot{\alpha}}\bar{X}_{NL}\right)\\
&= \frac{1}{32\Lambda^4}\bar{D}_{\dot{\alpha}}\left\{\left(4\sqrt{2}\Lambda^2 X_{NL} + 8X_{NL}\bar{X}_{NL}\bar{\Psi}\right)\bar{D}^{\dot{\alpha}}\bar{X}_{NL}\right\}\\
&= \frac{1}{32\Lambda^4}\bar{D}_{\dot{\alpha}}\left\{\left(4\sqrt{2}\Lambda^2 X_{NL}\right)\bar{D}^{\dot{\alpha}}\bar{X}_{NL}\right\}\\
&= \frac{1}{4\sqrt{2}\Lambda^2}X_{NL}\bar{D}^2\bar{X}_{NL}\\
&= X_{NL},
\end{aligned}$$

where we have used the identities (4.191), (4.199), (4.200) and (4.201). Thus, the equations of motion for the superfield Σ are solved in terms of a free chiral multiplet ($D^2\Phi = 0$), and a constrained chiral superfield ($H = X_{NL}$). Therefore, Σ describes on-shell a free chiral multiplet and a goldstino superfield. We should note however, that although (4.198) is a solution, we have not proven that it is unique.

The component fields of the Σ multiplet can be deduced from the relation

$$\Sigma = X_{NL} + \bar{\Phi}. \tag{4.202}$$

From eq. (4.202) the fields F and λ_α of Σ are identified as the appropriate component fields of the constrained chiral superfield X_{NL} since

$$\lambda_\alpha = \frac{1}{\sqrt{2}}D_\alpha\Sigma| = \frac{1}{\sqrt{2}}D_\alpha X_{NL}| \tag{4.203}$$

and

$$F = -\frac{1}{4}D^2\Sigma| = -\frac{1}{4}D^2X_{NL}|. \tag{4.204}$$

Thus, we can deduce their equations of motion just from the X_{NL}. On-shell we have

$$X_{NL} = \frac{\lambda^2}{2F} + \sqrt{2}\theta\lambda + \theta^2 F \tag{4.205}$$

with [139]

$$\begin{aligned} F &= -\sqrt{2}\Lambda^2\left(1 + \frac{\bar{\lambda}^2}{16\Lambda^8}\partial^2\lambda^2 - \frac{3}{256\Lambda^{16}}\lambda^2\bar{\lambda}^2\partial^2\lambda^2\partial^2\bar{\lambda}^2\right), & (4.206)\\ i\bar{\sigma}^{m\dot{\alpha}\alpha}\partial_m\lambda_\alpha &= \frac{1}{4\Lambda^4}\bar{\lambda}^{\dot{\alpha}}\partial^2\lambda^2 - \frac{1}{64\Lambda^{12}}\bar{\lambda}^{\dot{\alpha}}\lambda^2\partial^2\lambda^2\partial^2\bar{\lambda}^2 - \frac{1}{64\Lambda^{12}}\bar{\lambda}^{\dot{\alpha}}\partial^2(\lambda^2\bar{\lambda}^2\partial^2\lambda^2). & (4.207)\end{aligned}$$

Equation (4.207) is the equation of motion for the goldstino and eq.(4.206) is the solution for F in terms of the goldstino as anticipated. From the chiral multiplet we can easily identify ψ_α as the fermion of the chiral multiplet Φ, since

$$\psi_\alpha = \frac{1}{\sqrt{2}}D_\alpha\bar{\Sigma}| = \frac{1}{\sqrt{2}}D_\alpha\Phi|. \tag{4.208}$$

On-shell, Φ is a free chiral superfield so that

$$\Phi = A_\Phi + \sqrt{2}\theta\psi + \theta^2 F_\Phi \tag{4.209}$$

with

$$\begin{aligned} \partial^2 A_\Phi &= 0 & (4.210)\\ \bar{\sigma}^{m\dot{\alpha}\alpha}\partial_m\psi_\alpha &= 0 & (4.211)\\ F_\Phi &= 0. & (4.212)\end{aligned}$$

Thus, ψ_α is a free massless fermion. From (4.202) we have, for the scalar component A of Σ

$$A = \bar{A}_\Phi + \frac{\lambda^2}{2F}, \tag{4.213}$$

so that this component of Σ is solved in terms of the free scalar of the chiral multiplet and the goldstino. The last two auxiliary fields P_m and χ_α can be specified similarly. For the complex vector auxiliary P_m we have

$$P_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\alpha}}D_\alpha\Sigma| = \bar{D}_{\dot{\alpha}}D_\alpha X_{NL}| = -2i\sigma^m_{\alpha\dot{\alpha}}\partial_m\left(\frac{\lambda^2}{2F}\right) \tag{4.214}$$

whereas for χ_α we find

$$\chi_\alpha = \frac{1}{2}\bar{D}_{\dot{\alpha}}D_\alpha\bar{D}^{\dot{\alpha}}\bar{\Sigma}| = \frac{1}{2}\bar{D}_{\dot{\alpha}}D_\alpha\bar{D}^{\dot{\alpha}}\bar{X}_{NL}| = i\sigma^m_{\alpha\dot{\alpha}}\partial_m\bar{\lambda}^{\dot{\alpha}}. \tag{4.215}$$

Such a model of SUSY breaking can be considered as a hidden sector. Then, couplings to the visible sector can be introduced through the interactions

$$\mathcal{L}_{\text{int}} = -\frac{m_i^2}{2\Lambda^4}\int d^4\theta\Sigma\bar{\Sigma}\,\Phi^i\bar{\Phi}^i - \frac{m_g}{4\Lambda^4}\int d^4\theta\Sigma\bar{\Sigma}\left(W^\alpha W_\alpha + \bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}\right) \tag{4.216}$$

where Φ^i are chiral matter in the visible sector and W_α is the supersymmetric field strength of vectors. In the susy breaking vacuum, m_i, m_g are just soft masses for the scalars of the chiral multiplets of the visible sector and the gauginos, respectively.

4.2.4 Σ-Emergent potential

Instead of (4.165), one could consider the following more general Lagrangian

$$\mathcal{L}'_{EP} = \int d^4\theta \ \frac{1}{64}\mathcal{U}(\Sigma,\bar{\Sigma}) \ D^\alpha\Sigma D_\alpha\Sigma\bar{D}_{\dot{\alpha}}\bar{\Sigma}\bar{D}^{\dot{\alpha}}\bar{\Sigma}, \tag{4.217}$$

where, $\mathcal{U}$ is a real, strictly positive, but otherwise arbitrary function of Σ and $\bar{\Sigma}$ with mass dimension (-4). As we will see in the moment, a potential emerges for the complex scalar A of the complex linear multiplet Σ. The component form of the bosonic part of eq. (4.217) is

$$\mathcal{L}'^B_{EP} = \frac{1}{64}\mathcal{U} \ P^m P_m \bar{P}^n \bar{P}_n + \frac{1}{16}P_m\bar{P}^m\mathcal{U}F\bar{F} + \frac{1}{4} \ \mathcal{U}F^2\bar{F}^2, \tag{4.218}$$

where $\mathcal{U} = \mathcal{U}(A,\bar{A}) = \mathcal{U}(\Sigma,\bar{\Sigma})\Big|$. Then, the bosonic part of the Lagrangian

$$\begin{aligned}\mathcal{L}'_\Sigma &= \mathcal{L}_0 + \mathcal{L}'_{EP}\\ &= \int d^4\theta \ \left(-\Sigma\bar{\Sigma} + \frac{1}{64}\mathcal{U}(\Sigma,\bar{\Sigma}) \ D^\alpha\Sigma D_\alpha\Sigma\bar{D}_{\dot{\alpha}}\bar{\Sigma}\bar{D}^{\dot{\alpha}}\bar{\Sigma}\right),\end{aligned} \tag{4.219}$$

is

$$\begin{aligned}\mathcal{L}'^B =& - F\bar{F} + A\partial^2\bar{A} - \frac{1}{2}P_m\bar{P}^m\\ &+ \frac{1}{64}\mathcal{U} \ P^m P_m\bar{P}^n\bar{P}_n + \frac{1}{16}P_m\bar{P}^m\mathcal{U}F\bar{F} + \frac{1}{4} \ \mathcal{U}F^2\bar{F}^2.\end{aligned} \tag{4.220}$$

From the equations of motion for the complex auxiliary vector we find again that

$$P_m = 0, \tag{4.221}$$

whereas the equations of motion for the auxiliary scalar are now

$$F\left(1 - \frac{\mathcal{U}}{2}F\bar{F}\right) = 0. \tag{4.222}$$

There are again two solutions:

$$(i) \quad F = 0\,, \tag{4.223}$$

$$(ii) \quad F\bar{F} = \frac{2}{\mathcal{U}(A,\bar{A})}. \tag{4.224}$$

The first is the supersymmetric one while the second breaks supersymmetry. Plugging back eqs. (4.221) and (4.222) into (4.220) we find

$$\mathcal{L}^B = A\partial^2\bar{A} - \frac{1}{\mathcal{U}(A,\bar{A})}. \tag{4.225}$$

We see now that a potential has emerged

$$\mathcal{V}_{EP} = \frac{1}{\mathcal{U}(A,\bar{A})}. \tag{4.226}$$

For example one can have

$$\mathcal{U}(A,\bar{A}) = \frac{1}{\Lambda^4 + m_A^2 A\bar{A}} \tag{4.227}$$

where Λ is a mass scale. This case leads to a scalar potential

$$\mathcal{V} = \Lambda^4 + m_A^2 A\bar{A} \tag{4.228}$$

i.e to a mass for the scalar A. The minimum of potential (4.228) is at $A = 0$, which is a supersymmetry breaking vacuum since

$$< F\bar{F} >= 2\Lambda^4 \neq 0. \tag{4.229}$$

Another example is provided by

$$\mathcal{U}(A,\bar{A}) = \frac{1}{\Lambda^4 + \frac{\lambda}{4!}\left(A\bar{A} - m^2\right)^2}, \tag{4.230}$$

which gives rise to a potential

$$V = \Lambda^4 + \frac{\lambda}{4!}\left(A\bar{A} - m^2\right)^2. \tag{4.231}$$

In this case, the $U(1)$ global symmetry $A \to e^{i\alpha}A$ is broken at the vacuum $A\bar{A} = m^2$ where susy is also broken because

$$< F\bar{F} >= 2\Lambda^4 \neq 0. \tag{4.232}$$

In general, the complex scalar multiplet can have an arbitrary potential in the susy breaking vacuum, specified by the arbitrary real positive function $\mathcal{U}(A,\bar{A})$.

4.3 Summary

It has been advocated in [43] that the addition of higher dimension operators to a supersymmetric theory may lead to the appearance of new vacua, where only one of them is continuously connected to the standard theory in the limit of removing the higher dimension operators. This is possible, if the equations of motion for the auxiliaries have more than one solutions which satisfy the appropriate conditions. In [43], some examples were discussed, none of which however realized that proposal. Here we have provided an example, where the proposal works. The well-known standard form of the N = 1 scalar potential is restricted to the two-derivative level. Higher derivative interaction modify its form. In fact, when higher-derivatives are introduced, an emerging scalar potential appears even if there is no superpotential to start with. There are various types of emerging potential, F- Σ- and D-type. F-emerging potentials result by integrating out auxiliaries of chiral multiplets whereas, D-emerging potentials come from the integration of auxiliaries in vector multiplets. The Σ-emerging potential is obtained by integrating out the scalar auxiliary of the complex linear multiplet. As a general rule, emerging potentials are positive defined with de Sitter ground state, indicating supersymmetry breaking. In particular, for the complex linear multiplet, the quadratic term of its scalar auxiliary fields has opposite sign of the corresponding term in a chiral multiplet action. Therefore, by adding an appropriate ghost-free higher dimension operator, a potential is induced according to [43, 82, 83, 135]. This potential, has a second non-supersymmetric vacuum at a non-zero value of the scalar auxiliary besides the supersymmetric one. In the susy breaking vacuum, the propagating fields are the scalar and the fermion of the complex linear multiplet and the goldstino mode of the broken supersymmetry. Interesting enough, the goldstino mode turns out to be one of the auxiliary fermions of the complex linear multiplet, which now propagates in the new non-supersymmetric vacuum.

Chapter 5

Supersymmetry Breaking and Particle Physics

We present a non-linear MSSM with non-standard Higgs sector and goldstino field. Non-linear supersymmetry for the goldstino couplings is described by the constrained chiral superfield and, as usual, the Standard Model sector is encompassed in suitable chiral and vector supermultiplets. Two models are presented. In the first model (non-linear MSSM$3\frac{1}{2}$), the second Higgs is replaced by a new supermultiplet of half-family with only a new generation of leptons (or quarks). In the second model, for anomaly cancellation purposes, the second Higgs is retained as a spectator superfield by imposing a discrete symmetry. Both models do not have a μ-problem as a μ-term is forbidden by the discrete symmetry in the case of a spectator second Higgs or not existing at all in the case of a single Higgs. Moreover, the tree level relation between the Higgs mass and the hidden sector SUSY breaking scale $\sqrt{f}$ is derived. We also point out a relative suppression by m_{soft}/Λ of the bottom and tau Yukawa couplings with respect to those of the top quark. We then study the decoupling limit of a superheavy sgoldstino field in spontaneously broken $\mathcal{N}=1$ supergravity. Our approach is based on Kähler superspace, which, among others, allows direct formulation of $\mathcal{N}=1$ supergravity in the Einstein frame and correct identifications of mass parameters. Allowing for a non-renormalizable Kähler potential in the hidden sector, the decoupling limit of a superheavy sgoldstino is identified with an infinite negative Kähler curvature. Constraints that lead to non-linear realizations of supersymmetry emerge as consequence of the equations of motion of the goldstino superfield when considering the decoupling limit. Finally, by employing superspace Bianchi identities, we identify the real chiral superfield, which will be the superconformal symmetry breaking chiral superfield that enters the conservation of the Ferrra-Zumino multiplet in the field theory limit of $\mathcal{N}=1$ supergravity.

5.1 Non-Linear Single-Higgs MSSM

Since the invention of supersymmetry, the question of determing the supersymmetric theory that describes the Standard Model (SM) interactions has been at the forefront of High Energy Physics. Strong evidence of a new particle found at LHC, the Higgs boson, has renewed interest, since, the mass of this particle and its couplings to the rest SM particles will reveal where new physics might be hidden [71]. Supersymmetric extensions of the SM, have, among others, the potential to stabilize the weak scale, to allow gauge coupling unification, to provide dark matter candidates and to dynamically explain the hierarchy of weak and Planck scale. In fact, it is difficult to imagine a candidate better than supersymmetry for the physics beyond the SM in the case of a fundamental Higgs particle.

In the Minimal extension of the SM (MSSM), the Higgs sector is composed of a pair of multiplets H_u and H_d. It is by now a common belief that any supersymmetric extension of the Standard Model will necessarily include both Higgs fields. The reason is twofold: first two Higgs fields are required in order to give masses to up- and down quarks as holomorphicity of the superpotential does not allow appropriate Yukawa couplings for giving mass to both up- and down-type quarks by a single Higgs superfield. Second, simple anomaly arguments lead to an additional Higgs multiplet if quarks and leptons are organized in usual families. Therefore, either one considers exact supersymmetry with two Higgs multiplets, or, alternatively he gets rid of the down-type Higgs for example, at the cost of introducing hard supersymmetry breaking terms (arising basically from the non-holomorphicity of the superpotential) [123]. A difficulty with a chiral Higgs sector is that in the absence of H_d, the Higgsino is massless until electroweak symmetry breaking. Moreover, the cancellation of anomalies, previously canceled by H_d, requires the introduction of many new fields in various representations. These new fields should be chiral as well as heavy enough so that they do not mess low energy phenomenology. This is also the case in models with two Higgs fields and exact supersymmetry, where H_d is just a spectator with no vev and no coupling to fermions [65]. Such models, although challenging from the model building point of view, have a variety of new fields, which are needed to be introduced in order to take over the role of H_d, making the models less appealing.

When now gravity is taken into account, supersymmetry turns out to be a local symmetry with corresponding gauge field no other than the gravitino, a spin-$\frac{3}{2}$ massless Majorana fermion [70, 97]. If supersymmetry is a fundamental symmetry of nature, it should be broken. In fact the spontaneous breaking of the $\mathcal{N} = 1$ supersymmetry implies the existence of a pseudo-Goldstone fermion, the goldstino. The latter will serve as the longitudinal component of the gravitino when local SUSY is broken [68]. This is the super-Higgs mechanism which gives mass to gravitino. In a linearly realized supersymmetry, the superpartner of the goldstino is a complex scalar, the sgoldstino. As it is not protected by any symmetry, it gets a mass. If this mass is much larger than an energy scale, it can be integrated out. In this case, the spin-$\frac{3}{2}$ components of the gravitino are highly suppressed, and the phenomenological interesting part is the spin-$\frac{1}{2}$ [41, 42, 81], namely the goldstino, which possesses non-linear supersymmetry [16, 17, 23, 29, 40, 120, 122, 138, 153, 155, 159, 174, 186, 188]. In the opposite case of a light sgoldstino, the latter should be included in the low-energy effective theory.

There exist various formulations for goldstino couplings and non-linear supersymmetry. Among them, an interesting framework to discuss non-linear supersymmetry is the constrained superfield formalism [139]. We will consider couplings of the non-linear goldstino sector to the MSSM with the use of higher dimensional superspace operators. In fact, these couplings of the goldstino to the MSSM have been computed by Antoniadis *et. all* in a series of papers [11, 12] (see also [13–15] for higher dimensional effective operators in the MSSM). In the constrained superfield formulation we will employ here, we will assume that supersymmetry is spontaneously broken at a SUSY breaking scale $\sqrt{f}$, which will be taken to be at the multi-TeV region. Then at energy scales above $\sqrt{f}$, we have MSSM and the goldstino superfield. At lower scales below SUSY breaking scale $\sqrt{f}$ but above m_{soft} we have again MSSM but the goldstino now is non-linear (in the sense that supersymmetry transformations on goldstino are non-linear). Then at low energies below m_{soft} only the goldstino fermion couples to SM fields. Here, we will discuss energy regions around m_{soft} and below $\sqrt{f}$ where supersymmetry is non-linearly realized on the goldstino mode. We will see how the latter can be implemented such that to reduce the Higgs sector in non-linear MSSM.

As far as the mass generating mechanism for quarks (and appropriately for leptons) is concerned, the Yukawa couplings of H_d

$$\int d^2\theta \bar{d} Q \cdot H_d \tag{5.1}$$

are not available any more. In the models we will present here, mass generation is achieved by employing the constrained superfield X and the single Higgs H_u through the interaction

$$\frac{m_{soft}}{f\,\Lambda} \int d^2\theta d^2\bar{\theta} \bar{X} \bar{H}_u e^V Q \bar{d}. \tag{5.2}$$

The above coupling emerges from the coupling of the MSSM fields to the goldstino superfield (suppressed by the cutoff Λ) and originates from the replacement of the spurion $Y \to (m_{soft}/f)X$, where Y is the spurion $Y = \theta^2 m_{soft}$ and m_{soft} is a generic soft mass. For more details on this one may consult [139]. In particular, we will present consistent non-linear supersymmetric extensions to the SM that involve:

- A single Higgs field H_u where the second Higgs H_d has been replaced by a half family, and
- A standard Higgs H_u where the second Higgs H_d has been turned into a spectator.

We note that in these SUSY extensions of the SM there is no μ-problem due to symmetries or to the spectrum of the theory.

5.1.1 Non-Linear MSSM

By coupling the non-linear constrained superfield X to the MSSM [139], we get the "non-linear MSSM", details of which has been worked out in [11]. Here we will briefly recall its basic features. The chiral superfields spectrum of (the two-Higgs) non-linear MSSM is summarized in the following table

	spin 0	spin 1/2	$SU(3)_c,\ SU(2)_L,\ U(1)_Y$
Q (×3)	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$\mathbf{3},\ \mathbf{2},\ 1/3$
$\bar{u}$ (×3)	$\tilde{\bar{u}}_L$	$\bar{u}_L$	$\bar{\mathbf{3}},\ \mathbf{1},\ -4/3$
$\bar{d}$ (×3)	$\tilde{\bar{d}}_L$	$\bar{d}_L$	$\bar{\mathbf{3}},\ \mathbf{1},\ 2/3$
L (×3)	$(\tilde{n}_{eL}, \tilde{e}_L)$	(n_{eL}, e_L)	$\mathbf{1},\ \mathbf{2},\ -1$
$\bar{e}$ (×3)	$\tilde{\bar{e}}_L$	$\bar{e}_L$	$\mathbf{1},\ \mathbf{1},\ 2$
H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$\mathbf{1},\ \mathbf{2},\ 1$
H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$\mathbf{1},\ \mathbf{2},\ -1$
X	ϕ	G	$\mathbf{1},\ \mathbf{1},\ 0$

Table 5.1: MSSM chiral superfields spectrum

The theory is described by the superspace Lagrangian[1]

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_g \tag{5.3}$$

[1] Our superspace conventions are those of Wess and Bagger [193].

where

$$\mathcal{L}_0 = \mathcal{L}_K + \mathcal{L}_{Yu} + \mathcal{L}_{Yd} + +\mathcal{L}_\mu \tag{5.4}$$

is the MSSM superspace Lagrangian and

$$\mathcal{L}_g = \mathcal{L}_X + \mathcal{L}_s + \mathcal{L}_{tu} + \mathcal{L}_{td} + \mathcal{L}_B \tag{5.5}$$

describes collectively all the dynamics of the constrained superfield X. Note that $\mathcal{L}_g$ contains higher dimensional operators and hence it is defined with a cut off [11,139]. The Lagrangian (5.4) contains the kinematic terms $\mathcal{L}_K$, Yukawa couplings $\mathcal{L}_{Y_u}, \mathcal{L}_{Y_d}$ as well as the μ- and B-terms $\mathcal{L}_\mu$ and $\mathcal{L}_B$, respectively. In particular we have, in standard notation, the superspace form [5,161]

$$\mathcal{L}_K = \sum_\Phi \int d^4\theta \bar{\Phi} e^V \Phi + \int d^4\theta \bar{H}_d e^V H_d + \int d^4\theta \bar{H}_u e^V H_u + \left\{ \sum_{\text{gauge}} \frac{1}{16g^2\kappa} \int d^2\theta \text{Tr} W^\alpha W_\alpha + h.c. \right\} \tag{5.6}$$

where $\Phi = Q_i, \bar{u}_i, \bar{d}_i, L_i, \bar{e}_i$, denotes collectively the usual quark and lepton chiral superfields with $i = 1, ...3$ enumerating the three families. In the gauge sector, the sum is over the gauge group of the SM while κ is a constant to cancel the trace factor. The Yukawa couplings are described in superspace as

$$\mathcal{L}_{Yu} = \int d^2\theta y_u^{ij} \bar{u}_i Q_j \cdot H_u + h.c. \tag{5.7}$$

and

$$\mathcal{L}_{Yd} = \int d^2\theta \Big(- y_d^{ij} \bar{d}_i Q_j \cdot H_d - y_e^{ij} \bar{e}_i L_j \cdot H_d \Big) + h.c. \tag{5.8}$$

where y_s^{ij}, $(s = e, u, d)$ are the Yukawa matrices of the SM. The dot symbol above refers to the SU(2) invariant product of two doublets [2]. Finally, the last term of $\mathcal{L}_0$ is the μ-term, which describes a pure interaction between the two Higgses

$$\mathcal{L}_\mu = \mu \int d^2\theta H_u \cdot H_d + h.c. \tag{5.9}$$

Note that $\mathcal{L}_\mu$ involves the new parameter μ which does not have an analog in SM theory and no obvious origin. This term always appears even if it excluded at tree level as it will emerge through quantum corrections, except if a symmetry forbids it.
The constrained superfield (goldstino) Lagrangian has also various contributions. The first contribution $\mathcal{L}_g$ has the usual form [139]

$$\mathcal{L}_X = \int d^4\theta \bar{X} X + \left\{ \int d^2\theta f X + h.c. \right\} \tag{5.10}$$

with $\sqrt{f}$ the hidden sector SUSY breaking scale. The superfield X satisfies the constraint

$$X^2 = 0, \tag{5.11}$$

[2]For example, if A and B are two $SU(2)$ doublets, $A \cdot B = \epsilon^{ij} A_i B_j$.

and more on this can be found in the appendix. Soft masses are produced by the following Lagrangian [11]

$$\begin{aligned}\mathcal{L}_s &= \int d^4\theta \bar{X} X\Big(c_{H_u}\bar{H}_u e^V H_u + c_{H_d}\bar{H}_d e^V H_d\Big) + \sum_{\Phi} c_\Phi \int d^4\theta \bar{X} X \bar{\Phi} e^V \Phi \\ &- \left(\sum_{gauge} \frac{1}{16 g^2 \kappa}\frac{2m_\lambda}{f}\int d^2\theta X \,\mathrm{Tr} W^\alpha W_\alpha + h.c.\right)\end{aligned} \tag{5.12}$$

where

$$c_{H_{u,d}} = -\frac{m^2_{H_{u,d}}}{f^2}, \quad c_\Phi = -\frac{m^2_\Phi}{f^2}. \tag{5.13}$$

Moreover, the triple scalar coupling terms are given below in superspace form [5, 11]

$$\mathcal{L}_{tu} = \frac{a_u^{ij}}{f}\int d^2\theta X \bar{u}_i Q_j \cdot H_u + h.c. \tag{5.14}$$

and

$$\mathcal{L}_{td} = -\frac{a_d^{ij}}{f}\int d^2\theta X \bar{d}_i Q_j \cdot H_d - \frac{a_e^{ij}}{f}\int d^2\theta X \bar{e}_i L_j \cdot H_d + h.c. \tag{5.15}$$

The dimensionfull constants $a_u^{ij}, a_d^{ij}, a_e^{ij}$ are usually taken to be

$$a_u^{ij} = A_0\, y_u^{ij}, \quad a_d^{ij} = A_0\, y_d^{ij}, \quad a_e^{ij} = A_0\, y_e^{ij} \tag{5.16}$$

where A_0 is a mass parameter. The final contribution to $\mathcal{L}_g$ is the B-term

$$\mathcal{L}_B = \frac{B}{f}\int d^2\theta X H_u \cdot H_d + h.c. \tag{5.17}$$

We may proceed by integrating out the auxiliary fields, and in particular the auxiliary field of the constrained superfield X, which we will call it F. The resulting theory is the non-linear MSSM. Of course, to solve the equations of motion for F, an expansion in powers of the hidden sector SUSY breaking scale f is needed. The full Higgs potential then reads [11]

$$\begin{aligned}\mathcal{V} &= f^2 + (|\mu| + m_u^2)|H_u|^2 + (|\mu| + m_d^2)|H_d|^2 + (BH_u \cdot H_d + h.c.) \qquad (5.18)\\ &+ \frac{1}{f^2}|m_u^2|H_u|^2 + m_d^2|H_d|^2 + BH_u \cdot H_d|^2 + \frac{g_1^2 + g_2^2}{8}[|H_u|^2 - |H_d|^2] + \frac{g_2^2}{2}|H_u^\dagger H_d|^2 + \mathcal{O}(\frac{1}{f^3}).\end{aligned}$$

One exceptional property of any supersymmetric extension of the SM is that it can actually be used to make predictions for the Higgs mass. Given M_W, due to supersymmetry, the otherwise free Higgs self-coupling λ is now related to the $U(1)$ and $SU(2)$ couplings g_1, g_2 by the relation $\lambda \sim g_1^2 + g_2^2$ as can be seen from (5.18). Note that the Yukawa couplings in this theory are the same as in the MSSM

$$\begin{aligned}\mathcal{L}_{Yukawa} = &- y_u^{ij}\bar{u}^\alpha_{Li}(u_{Lj\alpha}, d_{Lj\alpha})\begin{pmatrix} H_u^0 \\ -H_u^+ \end{pmatrix} \\ &+ y_d^{ij}\bar{d}^\alpha_{Li}(u_{Lj\alpha}, d_{Lj\alpha})\begin{pmatrix} H_d^- \\ -H_d^0 \end{pmatrix} \\ &+ y_e^{ij}\bar{e}^\alpha_{Li}(n_{eLj\alpha}, e_{Lj\alpha})\begin{pmatrix} H_d^- \\ -H_d^0 \end{pmatrix} + h.c.\end{aligned} \tag{5.19}$$

5.1.2 Non-Linear MSSM3$\frac{1}{2}$

Let us recall at this point the two basic reasons for which a second Higgs field is needed in MSSM and in fact in most (if not all) of the supersymmetric extensions of the SM:

- A second Higgs is needed to cancel the gauge anomaly introduced by a single Higgs supermultiplet.
- Due to the holomorphicity of the superpotential, a second Higgs is necessary in order to write down Yukawa couplings and give masses to those fermions the first Higgs cannot.

Therefore, a theory with a single Higgs should be anomaly free and give masses to fermions. Mass generation by Yukawa couplings is crucial but before discussing this issue, we should make sure that the theory with a single Higgs makes sense, i.e., it is anomaly free. Therefore, the chiral spectrum should be such so there is no gauge anomaly. Anomaly cancellation can be achieved with an additional new "half-family" and deviate from standard MSSM. The resulting MSSM3$\frac{1}{2}$ deviations we will present here are presented in the following table

Higgs Multiplet:	Replaced with:
H_u	$Q,\ \bar{u},\ \bar{d},\ S$ (or)
	$\bar{L},\ e,\ S$
or	
H_d	$\bar{Q},\ u,\ d,\ \bar{S}$ (or)
	$L,\ \bar{e},\ \bar{S}$

Table 5.2: Possible Higgs superfields replacements

where S is a superfield that has the quantum numbers of $\bar{e}$ but no lepton number and it is necessary for anomaly cancellation. Here we will focus on the last possibility in the above table and replace H_d by a leptonic generation and S. We can equally adopt a half-family with only a quark generation, at least at the theoretical level, which, nevertheless will lead to different phenomenology.

The number of families is constrained by precision electroweak data [79]. Direct searches by CDF and D0 set strong limits $m_{t'} > 335$GeV [2] and $m_{b'} > 385$GeV at the 95% confidence level for a fourth generation of new t', b' quarks. LHC also puts more severe constraints in direct searches for extra quarks like short-lived b' quarks in the signature of trileptons and same-sign dileptons. CMS for example has ruled out $m_{b'} < 611$GeV at 95% confidence level by assuming exclusive decay of $b' \to t\,W$ [58]. Similarly, no excess over the SM expectations has been observed in CMS search for pair production of top-like quarks t', excluding a fourth generation t' quark with a mass $m_{t'} < 557$GeV [59]. Also for pair production of a bottom-like new quark b', ATLAS collaboration reported the exclusion at 95% confidence level of b' quarks with mass $m_{b'} < 400$GeV decaying via the channel $b' \to Z + b$ [1].

Extra quarks and leptons are also severely constrained by Higgs production at LHC. For example, the dominant source of Higgs production is a single Higgs produced by gluon fusion through a heavy quark loop. The $gg \to h$ production cross section $\sigma(gg \to h)$ is proportional to the Higgs to gluon decay width $\Gamma(h \to gg)$ which is dominated by heavy quarks with the largest Yukawa

couplings. This decay width is for example increased by a factor of 5 to 6 relative to SM in fourth generation models [37, 116].
As far as a fourth generation of leptons is concerned, the LEP reported the lower bound for new heavy charged lepton τ', $m_{\tau'} > 100$GeV [3]. Similarly, the Z invisible width and the assumption of Dirac masses, set $m_{n'} > m_Z/2$ for new heavy stable neutrinos [119]. On the other hand, if such new neutral leptons are lighter than half the Higgs boson mass, a new invisible channel $H \to n'\bar{n}'$ is open up increasing the total Higgs width and overtakes the $H \to f\bar{f}$ rates for example with a significant branching ratio in the low mass region.
Returning to our model, the chiral superfields spectrum is

	spin 0	spin 1/2	$SU(3)_c,\ SU(2)_L,\ U(1)_Y$
$Q\ (\times 3)$	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$\mathbf{3},\ \mathbf{2},\ 1/3$
$\bar{u}\ (\times 3)$	$\tilde{\bar{u}}_L$	$\bar{u}_L$	$\bar{\mathbf{3}},\ \mathbf{1},\ -4/3$
$\bar{d}\ (\times 3)$	$\tilde{\bar{d}}_L$	$\bar{d}_L$	$\bar{\mathbf{3}},\ \mathbf{1},\ 2/3$
$L\ (\times 4)$	$(\tilde{n}_{eL}, \tilde{e}_L)$	(n_{eL}, e_L)	$\mathbf{1},\ \mathbf{2},\ -1$
$\bar{e}\ (\times 4)$	$\tilde{\bar{e}}_L$	$\bar{e}_L$	$\mathbf{1},\ \mathbf{1},\ 2$
H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$\mathbf{1},\ \mathbf{2},\ 1$
$\bar{S}$	$\bar{s}$	$\tilde{\bar{s}}$	$\mathbf{1},\ \mathbf{1},\ -2$
X	ϕ	G	$\mathbf{1},\ \mathbf{1},\ 0$

Table 5.3: Single Higgs Non-Linear MSSM$3\frac{1}{2}$

Even if the theory is anomaly free, we are still facing the problem of how to give masses to quarks and leptons while maintaining SUSY as the second Higgs H_d is missing. For this reason, we may introduce higher dimensional operators to replace the Yukawa couplings (5.8). The Lagrangian that will replace $\mathcal{L}_{Yd}$ in (5.8) is

$$\begin{aligned}\mathcal{L}_{Yd'} &= -\frac{m_{soft}}{f\,\Lambda}\int d^2\theta d^2\bar{\theta}\bar{X}\Big(y_d^{ij}\bar{H}_u e^V Q_j \bar{d}_i + y_e^{IJ}\bar{H}_u e^V L_J \bar{e}_I\Big) + h.c.\\ &= -\frac{m_{soft}}{16 f\,\Lambda} D^2\bar{D}^2\bar{X}\Big(y_d^{ij}\bar{H}_u e^V Q_j \bar{d}_i + y_e^{IJ}\bar{H}_u e^V L_J \bar{e}_I\Big)\Big| + h.c. \qquad (5.20)\end{aligned}$$

where now $I, J = 1, ...4$ run over the fourth lepton generation. We recall again that the factor m_{soft}/f emerges by the replacement of the spurion $Y = \theta^2 m_{soft}$ by $(m_{soft}/f)X$ as we have pointed out already in the introduction [139]. In component form (5.20) turns out to be

$$\begin{aligned}\mathcal{L}_{Yd'} \;=\; & \frac{m_{soft}}{f\,\Lambda}\, y_d^{ij}\Big\{\ \bar{F}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix} u_{Lj}^\alpha \\ d_{Lj}^\alpha\end{pmatrix}\bar{d}_{Li\alpha} - \bar{F}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix} \tilde{u}_{Lj} \\ \tilde{d}_{Lj}\end{pmatrix} F_{\bar{d}_{Li}}\\ & \quad - \bar{F}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix} F_{u_{Lj}} \\ F_{d_{Lj}}\end{pmatrix}\tilde{\bar{d}}_{Li}\Big\}\\ & + \frac{m_{soft}}{f\,\Lambda}\, y_e^{IJ}\Big\{\ \bar{F}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix} n_{eLJ}^\alpha \\ e_{LJ}^\alpha\end{pmatrix}\bar{e}_{LI\alpha} - \bar{F}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix} \tilde{n}_{eLJ} \\ \tilde{e}_{LJ}\end{pmatrix} F_{\bar{e}_{LI}}\\ & \quad - \bar{F}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix} F_{n_{eL}J} \\ F_{e_L J}\end{pmatrix}\tilde{\bar{e}}_{LI}\Big\} + h.c. \qquad (5.21)\end{aligned}$$

where we recall that F is the auxiliary field of the goldstino superfield. In the above equation (5.21) we have kept only the terms with no goldstino couplings. In the appendix, the higher dimensional operator that serves as a building block for the full Lagrangian is given in terms of the goldstino and its lowest component ϕ, which is integrated out to obtain the non-linear supersymmetric Lagrangian. In this framework a natural explanation of the scale f is proposed and our non-renormalizable operators (5.20) fit well to the general picture [11,12,139]. The Higgs triple scalar couplings Lagrangian to replace $\mathcal{L}_{td}$ in (5.15) is, in superspace form

$$\begin{aligned}\mathcal{L}_{td'} &= -\frac{m_{soft}^2}{f^2\,\Lambda^2}\int d^2\theta d^2\bar{\theta}\bar{X}X\left\{a_d^{ij}\bar{H}_u e^V Q_j\bar{d}_i + a_e^{IJ}\bar{H}_u e^V L_J\bar{e}_I\right\} + h.c.\\ &= -\frac{m_{soft}^2}{16f^2\,\Lambda^2}D^2\bar{D}^2\bar{X}X\left\{a_d^{ij}\bar{H}_u e^V Q_j\bar{d}_i + a_e^{IJ}\bar{H}_u e^V L_J\bar{e}_I\right\}\Big| + h.c.\end{aligned} \tag{5.22}$$

After performing the superspace integration we get

$$\mathcal{L}_{td'} = -\frac{m_{soft}^2}{f^2\,\Lambda^2}\bar{F}F\left\{a_d^{ij}(\bar{H}_u^+,\bar{H}_u^0)\begin{pmatrix}\tilde{u}_{Lj}\\ \tilde{d}_{Lj}\end{pmatrix}\tilde{\bar{d}}_{Li} + a_e^{IJ}(\bar{H}_u^+,\bar{H}_u^0)\begin{pmatrix}\tilde{n}_{eLJ}\\ \tilde{e}_{LJ}\end{pmatrix}\tilde{\bar{e}}_{LI}\right\} + h.c. \tag{5.23}$$

where goldstino couplings have been ignored. Then, it is clear that the replacements

$$\begin{aligned}&\mathcal{L}_{Yd}\to\mathcal{L}_{Yd'}\\ &\mathcal{L}_{td}\to\mathcal{L}_{td'}\end{aligned} \tag{5.24}$$

in (5.4) and (5.5) respectively give rise to (non-linear) MSSM with only one Higgs (the H_u).
We may proceed further and integrate out the auxiliary sector of the goldstino superfield. This will uncover the on-shell Lagrangian with Yukawa and triple scalar couplings. Since in this work we are only interested in the standard model sector, we will not write down any goldstino couplings when solving the equations of motion of the auxiliary fields. This greatly simplifies the results without spoiling the final answer. Nevertheless it is important to study the implications of these new terms that include the goldstino as well, but this is left for future work. The relevant terms in our total Lagrangian (5.3) are therefore

$$\begin{aligned}\mathcal{L}_F &= \frac{1}{2}F\bar{F} + f\bar{F} + \frac{1}{2}c_{H_u}\bar{F}F|H_u|^2 + \frac{1}{2}\sum_i c_i\bar{F}F|\tilde{\Phi}_i|^2\\ &+ \frac{m_{soft}}{f\,\Lambda}y_d^{ij}\bar{F}(\bar{H}_u^+,\bar{H}_u^0)\begin{pmatrix}u_{Lj}^\alpha\\ d_{Lj}^\alpha\end{pmatrix}\bar{d}_{Li\alpha} - \frac{m_{soft}}{f\,\Lambda}y_d^{ij}\bar{F}(\bar{H}_u^+,\bar{H}_u^0)\begin{pmatrix}\tilde{u}_{Lj}\\ \tilde{d}_{Lj}\end{pmatrix}F_{\bar{d}_{Li}}\\ &- \frac{m_{soft}}{f\,\Lambda}y_d^{ij}\bar{F}(\bar{H}_u^+,\bar{H}_u^0)\begin{pmatrix}F_{u_{Lj}}\\ F_{d_{Lj}}\end{pmatrix}\tilde{\bar{d}}_{Li} - \frac{m_{soft}}{f\,\Lambda}y_e^{IJ}\bar{F}(\bar{H}_u^+,\bar{H}_u^0)\begin{pmatrix}F_{n_{eL}J}\\ F_{e_LJ}\end{pmatrix}\tilde{\bar{e}}_{LI}\\ &+ \frac{m_{soft}}{f\,\Lambda}y_e^{IJ}\bar{F}(\bar{H}_u^+,\bar{H}_u^0)\begin{pmatrix}n_{eLJ}^\alpha\\ e_{LJ}^\alpha\end{pmatrix}\bar{e}_{LI\alpha} - \frac{m_{soft}}{f\,\Lambda}y_e^{IJ}\bar{F}(\bar{H}_u^+,\bar{H}_u^0)\begin{pmatrix}\tilde{n}_{eLJ}\\ \tilde{e}_{LJ}\end{pmatrix}F_{\bar{e}_LI}\\ &- \frac{m_{soft}^2}{f^2\,\Lambda^2}\bar{F}Fa_d^{ij}(\bar{H}_u^+,\bar{H}_u^0)\begin{pmatrix}\tilde{u}_{Lj}\\ \tilde{d}_{Lj}\end{pmatrix}\tilde{\bar{d}}_{Li} - \frac{m_{soft}^2}{f^2\,\Lambda^2}\bar{F}Fa_e^{IJ}(\bar{H}_u^+,\bar{H}_u^0)\begin{pmatrix}\tilde{n}_{eLJ}\\ \tilde{e}_{LJ}\end{pmatrix}\tilde{\bar{e}}_{LI}\\ &+ +\sum_i\frac{m_{\lambda_i}}{2f}F\lambda_i^2 + h.c.\end{aligned} \tag{5.25}$$

where by $\tilde{\Phi}_i$ we denote the lowest components of the various chiral superfields (the sparticles in our case). Assuming that f is large, we may use the expansion

$$\left(1 - \frac{m_{H_u}^2}{f^2}|H_u|^2 - \frac{m_{\Phi_i}^2}{f^2}|\tilde{\Phi}_i|^2 - \frac{m_{soft}^2}{f^2\,\Lambda^2}a_d^{ij}(\bar{H}_u^+,\bar{H}_u^0)\begin{pmatrix}\tilde{u}_{Lj}\\ \tilde{d}_{Lj}\end{pmatrix}\tilde{\bar{d}}_{Li} - \frac{m_{soft}^2}{f^2\,\Lambda^2}a_e^{IJ}(\bar{H}_u^+,\bar{H}_u^0)\begin{pmatrix}\tilde{n}_{eLJ}\\ \tilde{e}_{LJ}\end{pmatrix}\tilde{\bar{e}}_{LI}\right)^{-1}$$

$$\simeq 1+\frac{m_{H_u}^2}{f^2}|H_u|^2+\frac{m_{\Phi_i}^2}{f^2}|\tilde{\Phi}_i|^2+\frac{m_{soft}^2}{f^2\,\Lambda^2}\,a_d^{ij}(\bar{H}_u^+,\bar{H}_u^0)\left(\begin{array}{c}\tilde{u}_{Lj}\\ \tilde{d}_{Lj}\end{array}\right)\tilde{\bar{d}}_{Li}+\frac{m_{soft}^2}{f^2\,\Lambda^2}\,a_e^{IJ}(\bar{H}_u^+,\bar{H}_u^0)\left(\begin{array}{c}\tilde{n}_{eLJ}\\ \tilde{e}_{LJ}\end{array}\right)\tilde{\bar{e}}_{LI}.$$

in order to eliminate F from (4.2) so that

$$\begin{aligned}
\mathcal{L}_{\mathrm{F,on-shell}} = & -\frac{1}{2}f^2-\frac{1}{2}m_{H_u}^2|H_u|^2-\frac{1}{2}m_{\Phi_i}^2|\tilde{\Phi}_i|^2\\
& -\frac{m_{soft}}{\Lambda}\,y_d^{ij}(\bar{H}_u^+,\bar{H}_u^0)\left(\begin{array}{c}u_{Lj}^\alpha\\ d_{Lj}^\alpha\end{array}\right)\bar{d}_{Li\alpha}+\frac{m_{soft}}{\Lambda}\,y_d^{ij}(\bar{H}_u^+,\bar{H}_u^0)\left(\begin{array}{c}\tilde{u}_{Lj}\\ \tilde{d}_{Lj}\end{array}\right)F_{\bar{d}_{Li}}\\
& +\frac{m_{soft}}{\Lambda}\,y_d^{ij}(\bar{H}_u^+,\bar{H}_u^0)\left(\begin{array}{c}F_{u_{Lj}}\\ F_{d_{Lj}}\end{array}\right)\tilde{\bar{d}}_{Li}+\frac{m_{soft}}{\Lambda}\,y_e^{IJ}(\bar{H}_u^+,\bar{H}_u^0)\left(\begin{array}{c}F_{n_{eL}J}\\ F_{e_LJ}\end{array}\right)\tilde{\bar{e}}_{LI}\\
& -\frac{m_{soft}}{\Lambda}\,y_e^{IJ}(\bar{H}_u^+,\bar{H}_u^0)\left(\begin{array}{c}n_{eLJ}^\alpha\\ e_{LJ}^\alpha\end{array}\right)\bar{e}_{LI\alpha}+\frac{m_{soft}}{\Lambda}\,y_e^{IJ}(\bar{H}_u^+,\bar{H}_u^0)\left(\begin{array}{c}\tilde{n}_{eLJ}\\ \tilde{e}_{LJ}\end{array}\right)F_{\bar{e}_LI}\\
& -\frac{m_{soft}^2}{\Lambda^2}\,a_d^{ij}(\bar{H}_u^+,\bar{H}_u^0)\left(\begin{array}{c}\tilde{u}_{Lj}\\ \tilde{d}_{Lj}\end{array}\right)\tilde{\bar{d}}_{Li}-\frac{m_{soft}^2}{\Lambda^2}\,a_e^{IJ}(\bar{H}_u^+,\bar{H}_u^0)\left(\begin{array}{c}\tilde{n}_{eLJ}\\ \tilde{e}_{LJ}\end{array}\right)\tilde{\bar{e}}_{LI}\\
& -\frac{1}{2}m_{\lambda_i}\lambda_i^2+h.c.+\mathcal{O}(\frac{1}{f^2}).
\end{aligned} \tag{5.26}$$

Note that the larger the SUSY breaking scale the better the approximation. For a smaller SUSY breaking scale one has to include higher orders in the $\frac{1}{f}$ expansion, which leads to new interesting results as in the two-Higgs scenario [11].

Therefore, the Yukawa couplings in our theory (5.3) with the replacements (5.24) are

$$\begin{aligned}
\mathcal{L}_{Yukawa} = & -y_u^{ij}\bar{u}_{Li}^\alpha(u_{Lj\alpha},d_{Lj\alpha})\left(\begin{array}{c}H_u^0\\ -H_u^+\end{array}\right)\\
& -\frac{m_{soft}}{\Lambda}\,y_d^{ij}(\bar{H}_u^+,\bar{H}_u^0)\left(\begin{array}{c}u_{Lj}^\alpha\\ d_{Lj}^\alpha\end{array}\right)\bar{d}_{Li\alpha}\\
& -\frac{m_{soft}}{\Lambda}\,y_e^{IJ}(\bar{H}_u^+,\bar{H}_u^0)\left(\begin{array}{c}n_{eLJ}^\alpha\\ e_{LJ}^\alpha\end{array}\right)\bar{e}_{LI\alpha}+h.c.
\end{aligned} \tag{5.27}$$

Let us note that an interesting hierarchy has emerged. Namely, assuming the same order for y_u^{ij}, y_d^{ij}, y_e^{IJ} we see that the effective Yukawa couplings for the bottom and tau are suppressed by a factor m_{soft}/Λ. Thus, the bottom quark and τ lepton masses m_b and m_τ, respectively, should be of the same order and suppressed by m_{soft}/Λ with respect to the top quark mass m_t

$$m_b\sim m_\tau\sim\frac{m_{soft}}{\Lambda}m_t \tag{5.28}$$

This is indeed the case for a cutoff Λ of the order $\Lambda\sim 100\,m_{soft}$. With $\Lambda\sim\sqrt{f}$ we get that $\sqrt{f}\sim 100m_{soft}$, whereas a cutoff $\Lambda\sim f/m_{soft}$ gives rise to the $\sqrt{f}\sim m_{soft}$ estimate.

The Higgs potential is given by

$$\mathcal{V} \;=\; f^2+m_u^2|H_u|^2+\frac{1}{f^2}m_u^4|H_u|^4+\frac{g_1^2+g_2^2}{8}|H_u|^4+\mathcal{O}(\frac{1}{f^3}). \tag{5.29}$$

Radiative corrections to the Higgs potential are expected to drive the quadratic term negative and trigger electroweak symmetry breaking. Moreover, this effect is strengthened by the extra Yukawa coupling due to the new half-family. The explicit calculation of the 1-loop effective potential can place strong upper and lower bounds to the new leptonic family mass. The tree level prediction for the Higgs mass however is

$$M_{H_u}^2 = M_Z^2 + \frac{8M_W^2 m_u^4}{g_2^2 f^2} + \mathcal{O}(\frac{1}{f^4}) \tag{5.30}$$

Thus, as $f \to \infty$ we have $M_{H_u} \to M_Z$. Therefore, for very large SUSY breaking scale $\sqrt{f}$, the Higgs mass saturates the MSSM inequality $M_{H_u} \leq M_Z$. This saturation within MSSM corresponds to large $\tan\beta$. By adjusting $\sqrt{f}$, we may increase the tree-level Higgs mass so that quantum corrections may shift it to the measured value of around 126.5GeV. We plot below the dependence of the tree level Higgs mass to the supersymmetry breaking scale $\sqrt{f}$, for the single Higgs models.

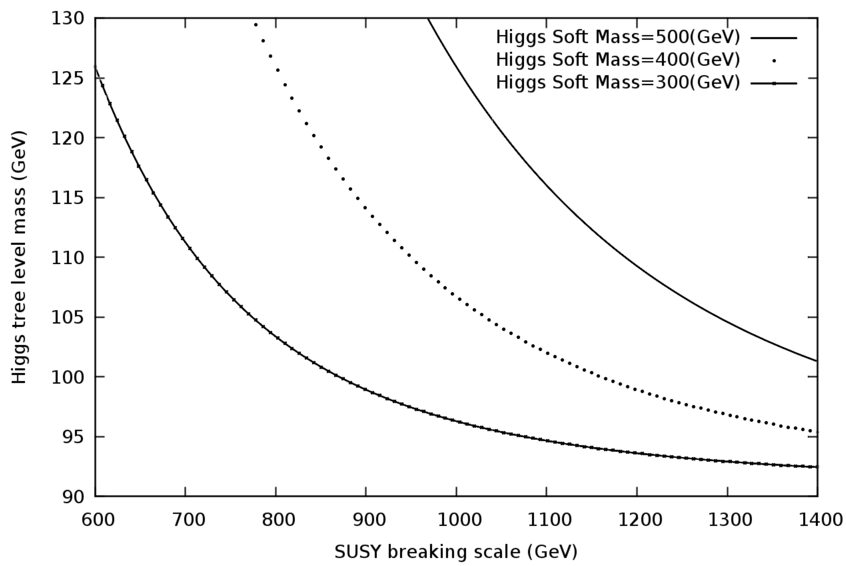

Figure 5.1: The dependence of the Higgs mass (M_{H_u}) on the hidden sector SUSY breaking scale $\sqrt{f}$, with the Higgs soft mass as a parameter.

5.1.3 Spectator H_d

As we have seen, Yukawa couplings of H_d can be replaced by the higher dimensional operators of the form (5.20) with the help of the constrained superfield X. Therefore, we can keep in the spectrum H_d just to cancel the anomalies but use (5.20) to generate fermion masses. This is possible as long as we can avoid couplings of H_d to matter. This can be achieved by imposing a $\mathbb{Z}_2$ symmetry. This symmetry will forbid interactions like (5.9) and (5.17). At the same time standard MSSM Yukawa couplings (5.7) of H_d will not be allowed as well, again due to the same Z_2 symmetry. Of course this is different from the case of wrong Higgs couplings of the MSSM where SUSY is hardly broken [117]. The chiral superfields spectrum and its $\mathbb{Z}_2$ assignment is

	spin 0	spin 1/2	$SU(3)_c,\ SU(2)_L,\ U(1)_Y$	$\mathbb{Z}_2$
Q (×3)	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$\mathbf{3},\ \mathbf{2},\ 1/3$	+1
$\bar{u}$ (×3)	$\tilde{\bar{u}}_L$	$\bar{u}_L$	$\bar{\mathbf{3}},\ \mathbf{1},\ -4/3$	+1
$\bar{d}$ (×3)	$\tilde{\bar{d}}_L$	$\bar{d}_L$	$\bar{\mathbf{3}},\ \mathbf{1},\ 2/3$	+1
L (×3)	$(\tilde{n}_{eL}, \tilde{e}_L)$	(n_{eL}, e_L)	$\mathbf{1},\ \mathbf{2},\ -1$	+1
$\bar{e}$ (×3)	$\tilde{\bar{e}}_L$	$\bar{e}_L$	$\mathbf{1},\ \mathbf{1},\ 2$	+1
H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$\mathbf{1},\ \mathbf{2},\ 1$	+1
H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$\mathbf{1},\ \mathbf{2},\ -1$	−1
X	ϕ	G	$\mathbf{1},\ \mathbf{1},\ 0$	+1

Table 5.4: Single-Higgs MSSM with a second spectator Higgs

We see that we keep here the second Higgs just for anomaly cancellation reasons and we have excluded any couplings to the MSSM, by imposing a $\mathbb{Z}_2$ symmetry. In other words, H_d is just a spectator and only H_u has Yukawa and triple scalar couplings. The Lagrangian that will take the place of $\mathcal{L}_{Yd}$ in (5.8) is then

$$\begin{aligned}\mathcal{L}_{Yd''} &= -\frac{m_{soft}}{f\,\Lambda}\int d^2\theta d^2\bar{\theta}\bar{X}\Big(y_d^{ij}\bar{H}_u e^V Q_j\bar{d}_i + y_e^{ij}\bar{H}_u e^V L_j\bar{e}_i\Big) + h.c.\\ &= -\frac{m_{soft}}{16f\,\Lambda}D^2\bar{D}^2\bar{X}\Big(y_d^{ij}\bar{H}_u e^V Q_j\bar{d}_i + y_e^{ij}\bar{H}_u e^V L_j\bar{e}_i\Big)\Big| + h.c.\end{aligned} \tag{5.31}$$

which in component form turns out to be

$$\begin{aligned}\mathcal{L}_{Yd''} \;=\; & \frac{m_{soft}}{f\,\Lambda}\,y_d^{ij}\Bigg\{\bar{F}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix} u_{Lj}^\alpha \\ d_{Lj}^\alpha\end{pmatrix}\bar{d}_{Li\alpha} - \bar{F}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix}\tilde{u}_{Lj}\\ \tilde{d}_{Lj}\end{pmatrix}F_{\bar{d}_{Li}}\\ & \quad - \bar{F}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix}F_{u_{Lj}}\\ F_{d_{Lj}}\end{pmatrix}\tilde{\bar{d}}_{Li}\Bigg\}\\ & + \frac{m_{soft}}{f\,\Lambda}\,y_e^{ij}\Bigg\{\bar{F}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix} n_{eLj}^\alpha \\ e_{Lj}^\alpha\end{pmatrix}\bar{e}_{Li\alpha} - \bar{F}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix}\tilde{n}_{eLj}\\ \tilde{e}_{Lj}\end{pmatrix}F_{\bar{e}_{Li}}\\ & \quad - \bar{F}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix}F_{n_{eLj}}\\ F_{e_{Lj}}\end{pmatrix}\tilde{\bar{e}}_{Li}\Bigg\} + h.c.\end{aligned} \tag{5.32}$$

In the above equation (5.32) we have kept only the terms with no goldstino couplings. The one-Higgs triple scalar couplings Lagrangian to replace $\mathcal{L}_{td}$ in (5.15) is, in superspace form

$$\begin{aligned}\mathcal{L}_{td''} &= -\frac{m_{soft}^2}{f^2\,\Lambda^2}\int d^2\theta d^2\bar{\theta}\bar{X}X\left\{a_d^{ij}\bar{H}_u e^V Q_j\bar{d}_i + a_e^{ij}\bar{H}_u e^V L_j\bar{e}_i\right\} + h.c.\\ &= -\frac{m_{soft}^2}{16f^2\,\Lambda^2}D^2\bar{D}^2\bar{X}X\left\{a_d^{ij}\bar{H}_u e^V Q_j\bar{d}_i + a_e^{ij}\bar{H}_u e^V L_j\bar{e}_i\right\}\Big| + h.c.\end{aligned} \tag{5.33}$$

After performing the superspace integration we get

$$\mathcal{L}_{td''} = -\frac{m_{soft}^2}{f^2\,\Lambda^2}\bar{F}F\left\{a_d^{ij}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix}\tilde{u}_{Lj}\\ \tilde{d}_{Lj}\end{pmatrix}\tilde{\bar{d}}_{Li} + a_e^{ij}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix}\tilde{n}_{eLj}\\ \tilde{e}_{Lj}\end{pmatrix}\tilde{\bar{e}}_{Li}\right\} + h.c. \tag{5.34}$$

where we have ignored any goldstino couplings. Then, it is clear that the replacements

$$\begin{aligned}\mathcal{L}_{Yd} &\to \mathcal{L}_{Yd''}\\ \mathcal{L}_{td} &\to \mathcal{L}_{td''}\end{aligned} \tag{5.35}$$

in (5.4) give rise to (non-linear) MSSM with only one Higgs (the H_u). For completeness, the Higgs potential of this theory is again

$$\mathcal{V} \;=\; f^2 + m_u^2|H_u|^2 + \frac{1}{f^2}m_u^4|H_u|^4 + \frac{g_1^2+g_2^2}{8}|H_u|^4 + \mathcal{O}(\frac{1}{f^3}) \tag{5.36}$$

while the Yukawa couplings are

$$\begin{aligned}\mathcal{L}_{Yukawa} = \;& - \; y_u^{ij}\bar{u}_{Li}^{\alpha}(u_{Lj\alpha}, d_{Lj\alpha})\begin{pmatrix} H_u^0 \\ -H_u^+ \end{pmatrix}\\ & - \; \frac{m_{soft}}{\Lambda}\, y_d^{ij}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix} u_{Lj}^{\alpha} \\ d_{Lj}^{\alpha} \end{pmatrix}\bar{d}_{Li\alpha}\\ & - \; \frac{m_{soft}}{\Lambda}\, y_e^{ij}(\bar{H}_u^+, \bar{H}_u^0)\begin{pmatrix} n_{eLj}^{\alpha} \\ e_{Lj}^{\alpha} \end{pmatrix}\bar{e}_{Li\alpha} + h.c.\end{aligned} \tag{5.37}$$

Note also that the $\mathbb{Z}_2$ symmetry does not allow μ- and B-terms.

5.1.4 Constrained Chiral Superfield

The constrained superfield (goldstino) Lagrangian has the usual form [139]

$$\mathcal{L}_X = \int d^4\theta \bar{X}X + \Big\{\int d^2\theta f X + h.c.\Big\} \tag{5.38}$$

with f the hidden sector SUSY breaking scale. The superfield X satisfies the constraint

$$X^2 = 0 \tag{5.39}$$

This constraint gives a relation among the component fields allowing to integrate out the sgoldstino in terms of the goldstino and the auxiliary field F, as

$$\phi = \frac{GG}{2F} \tag{5.40}$$

so that the component Lagrangian is written as

$$\mathcal{L}_X = i\partial\bar{G}\bar{\sigma}G + \bar{F}F + \frac{\bar{G}^2}{2\bar{F}}\partial^2\left(\frac{G^2}{2F}\right) + \Big\{fF + h.c.\Big\}. \tag{5.41}$$

The equations of motion for the auxiliary field F (and $\bar{F}$) read

$$\begin{aligned}F + f - \frac{\bar{G}^2}{2\bar{F}^2}\partial^2\left(\frac{G^2}{2F}\right) &= 0,\\ \bar{F} + f - \frac{G^2}{2F^2}\partial^2\left(\frac{\bar{G}^2}{2\bar{F}}\right) &= 0\end{aligned} \tag{5.42}$$

which are solved by

$$\begin{aligned} F &= -f\left(1+\frac{\bar{G}}{4f^4}\partial^2 G^2-\frac{3}{16f^8}G^2\bar{G}^2\partial^2 G^2\partial^2\bar{G}^2\right), \\ \bar{F} &= -f\left(1+\frac{G}{4f^4}\partial^2 \bar{G}^2-\frac{3}{16f^8}G^2\bar{G}^2\partial^2 G^2\partial^2\bar{G}^2\right) \end{aligned} \tag{5.43}$$

Inserting (5.43) back into (5.41) the on-shell Lagrangian

$$\mathcal{L}_X=-f^2+i\partial\bar{G}\bar{\sigma}G+\frac{1}{4f^2}\bar{G}^2\partial^2 G^2-\frac{1}{16f^6}G^2\bar{G}^2\partial^2 G^2\partial^2\bar{G}^2. \tag{5.44}$$

is recovered. Note that (5.44) is equivalent to the well known Akulov-Volkov Lagrangian [188].

5.1.5 Higher Dimensional Operators

We present the higher dimensional operators that serve as the building block for the component form of the Lagrangians (5.20) and (5.22). The component Lagrangian for the Yukawa couplings is

$$\begin{aligned} \mathcal{L}_Y &= -\frac{m_{soft}}{f\,\Lambda}\int d^2\theta d^2\bar{\theta}\bar{X}\bar{H}e^V Q\bar{d} \\ &= -\frac{m_{soft}}{f\,\Lambda}\Bigg\{-\bar{F}(\bar{h}^+,\bar{h}^0)\begin{pmatrix} u_L^\alpha \\ d_L^\alpha\end{pmatrix}\bar{d}_{L\alpha}+\bar{F}(\bar{h}^+,\bar{h}^0)\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}F_{\bar{d}_L} \\ &\quad+\bar{F}(\bar{h}^+,\bar{h}^0)\begin{pmatrix}F_{u_L}\\ F_{d_L}\end{pmatrix}\tilde{\bar{d}}_L+i\partial_a\bar{G}_{\dot{\rho}}\bar{\sigma}^{a\dot{\rho}\alpha}(\bar{h}^+,\bar{h}^0)\begin{pmatrix}u_{L\alpha}\\ d_{L\alpha}\end{pmatrix}\tilde{\bar{d}}_L \\ &\quad+i\partial_a\bar{G}_{\dot{\rho}}\bar{\sigma}^{a\dot{\rho}\alpha}(\bar{h}^+,\bar{h}^0)\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}\bar{d}_{L\alpha}+\Box\bar{\phi}(\bar{h}^+,\bar{h}^0)\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}\tilde{\bar{d}}_L \\ &\quad+2\partial_a\bar{\phi}[\partial^a(\bar{h}^+,\bar{h}^0)]\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}\tilde{\bar{d}}_L+i\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{\phi}(\tilde{\bar{h}}^{+\dot{\alpha}},\tilde{\bar{h}}^{0\dot{\alpha}})\begin{pmatrix}u_L^\alpha\\ d_L^\alpha\end{pmatrix}\tilde{\bar{d}}_L \\ &\quad+i\sigma^a_{\alpha\dot{\alpha}}\partial_a\bar{\phi}(\tilde{\bar{h}}^{+\dot{\alpha}},\tilde{\bar{h}}^{0\dot{\alpha}})\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}\bar{d}_L^\alpha+i\bar{G}^{\dot{\alpha}}\sigma^a_{\alpha\dot{\alpha}}[\partial_a(\bar{h}^+,\bar{h}^0)]\begin{pmatrix}u_L^\alpha\\ d_L^\alpha\end{pmatrix}\tilde{\bar{d}}_L \\ &\quad-\bar{G}_{\dot{\alpha}}(\tilde{\bar{h}}^{+\dot{\alpha}},\tilde{\bar{h}}^{0\dot{\alpha}})\begin{pmatrix}F_{u_L}\\ F_{d_L}\end{pmatrix}\tilde{\bar{d}}_L+\bar{G}_{\dot{\alpha}}(\tilde{\bar{h}}^{+\dot{\alpha}},\tilde{\bar{h}}^{0\dot{\alpha}})\begin{pmatrix}u_L^\alpha\\ d_L^\alpha\end{pmatrix}\bar{d}_{L\alpha} \\ &\quad+i\bar{G}^{\dot{\alpha}}\sigma^a_{\alpha\dot{\alpha}}[\partial_a(\bar{h}^+,\bar{h}^0)]\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}\bar{d}_L^\alpha-\bar{G}_{\dot{\alpha}}(\tilde{\bar{h}}^{+\dot{\alpha}},\tilde{\bar{h}}^{0\dot{\alpha}})\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}F_{\bar{d}_L} \\ &\quad-i\partial_a\bar{\phi}(\bar{h}^+,\bar{h}^0)\mathcal{V}^a\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}\tilde{\bar{d}}_L-\frac{i}{\sqrt{2}}\bar{G}_{\dot{\alpha}}(\bar{h}^+,\bar{h}^0)\bar{\lambda}^{\dot{\alpha}}\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}\tilde{\bar{d}}_L \\ &\quad+\frac{1}{2}\bar{\sigma}^{a\dot{\alpha}\alpha}\bar{G}_{\dot{\alpha}}(\bar{h}^+,\bar{h}^0)\mathcal{V}_a\begin{pmatrix}u_{L\alpha}\\ d_{L\alpha}\end{pmatrix}\tilde{\bar{d}}_L+\frac{1}{2}\bar{\sigma}^{a\dot{\alpha}\alpha}\bar{G}_{\dot{\alpha}}(\bar{h}^+,\bar{h}^0)\mathcal{V}_a\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}\bar{d}_{L\alpha} \\ &\quad+\bar{\phi}[\Box(\bar{h}^+,\bar{h}^0)]\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}\tilde{\bar{d}}_L+i\bar{\phi}\bar{\sigma}^{a\dot{\alpha}\alpha}[\partial_a(\tilde{\bar{h}}^+_{\dot{\alpha}},\tilde{\bar{h}}^0_{\dot{\alpha}})]\begin{pmatrix}u_{L\alpha}\\ d_{L\alpha}\end{pmatrix}\tilde{\bar{d}}_L \\ &\quad+i\bar{\phi}\bar{\sigma}^{a\dot{\alpha}\alpha}[\partial_a(\tilde{\bar{h}}^+_{\dot{\alpha}},\tilde{\bar{h}}^0_{\dot{\alpha}})]\begin{pmatrix}\tilde{u}_L\\ \tilde{d}_L\end{pmatrix}\bar{d}_{L\alpha}+\bar{\phi}(\bar{F}^+,\bar{F}^0)\begin{pmatrix}F_{u_L}\\ F_{d_L}\end{pmatrix}\tilde{\bar{d}}_L \end{aligned}$$

$$
\begin{aligned}
&+\bar{\phi}(\bar{F}^{+},\bar{F}^{0})\begin{pmatrix} u_{L\alpha} \\ d_{L\alpha}\end{pmatrix}\bar{d}^{\alpha}_{L}+\bar{\phi}(\bar{F}^{+},\bar{F}^{0})\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}F_{\bar{d}_L} \\
&-i\bar{\phi}[\partial_a(\bar{h}^{+},\bar{h}^{0})]\mathcal{V}^{a}\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L}-\frac{i}{\sqrt{2}}\bar{\phi}(\bar{\tilde{h}}^{+}_{\dot{\alpha}},\bar{\tilde{h}}^{0}_{\dot{\alpha}})\bar{\lambda}^{\dot{\alpha}}\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L} \\
&+\frac{1}{4}\bar{\phi}\bar{\sigma}^{a\dot{\alpha}\alpha}(\bar{\tilde{h}}^{+}_{\dot{\alpha}},\bar{\tilde{h}}^{0}_{\dot{\alpha}})\mathcal{V}_a\begin{pmatrix} u_{L\alpha} \\ d_{L\alpha}\end{pmatrix}\tilde{\bar{d}}_{L}+\frac{1}{4}\bar{\phi}\bar{\sigma}^{a\dot{\alpha}\alpha}(\bar{\tilde{h}}^{+}_{\dot{\alpha}},\bar{\tilde{h}}^{0}_{\dot{\alpha}})\mathcal{V}_a\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\bar{d}_{L\alpha} \\
&-\frac{i}{2}\bar{\phi}(\bar{h}^{+},\bar{h}^{0})\partial_a\mathcal{V}^{a}\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L}+\frac{i}{\sqrt{2}}\bar{\phi}(\bar{h}^{+},\bar{h}^{0})\lambda^{\alpha}\begin{pmatrix} u_{L\alpha} \\ d_{L\alpha}\end{pmatrix}\tilde{\bar{d}}_{L} \\
&+\frac{i}{\sqrt{2}}\bar{\phi}(\bar{h}^{+},\bar{h}^{0})\lambda^{\alpha}\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\bar{d}_{L\alpha}-\frac{1}{4}\bar{\phi}(\bar{h}^{+},\bar{h}^{0})[\mathcal{V}^{a}\mathcal{V}_a-2\mathcal{D}]\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L}\Big\} \qquad (5.45)
\end{aligned}
$$

where the gauge vector $\mathcal{V}^a$, the gaugino spinor λ^α and the auxiliary scalar $\mathcal{D}$ of the gauge vector multiplet are Lie algebra valued. The component Lagrangian for the triple scalar couplings is

$$
\begin{aligned}
\mathcal{L}_t &= -\frac{m^2_{soft}}{f^2\,\Lambda^2}\int d^2\theta d^2\bar{\theta}\bar{X}X\bar{H}e^{V}Q\bar{d} \\
&= -\frac{1}{4\sqrt{2}}\frac{m^2_{soft}}{f^2\,\Lambda^2}G^{\alpha}\Big\{4i\bar{\phi}(\bar{h}^{+},\bar{h}^{0})\lambda_{\alpha}\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L} \\
&-4\sqrt{2}\bar{F}(\bar{h}^{+},\bar{h}^{0})\begin{pmatrix} u_{L\alpha} \\ d_{L\alpha}\end{pmatrix}\tilde{\bar{d}}_{L}-4\sqrt{2}\bar{F}(\bar{h}^{+},\bar{h}^{0})\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\bar{d}_{L\alpha} \\
&+4i\sqrt{2}\epsilon_{\alpha\beta}(\partial_a\bar{G}_{\dot{\rho}})\bar{\sigma}^{a\dot{\rho}\beta}(\bar{h}^{+},\bar{h}^{0})\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L}-4i\sqrt{2}\sigma^{a}_{\alpha\dot{\alpha}}\partial_a\bar{\phi}(\bar{\tilde{h}}^{+\dot{\alpha}},\bar{\tilde{h}}^{0\dot{\alpha}})\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L} \\
&-4i\sqrt{2}\bar{G}^{\dot{\alpha}}\sigma^{a}_{\alpha\dot{\alpha}}\partial_a(\bar{h}^{+},\bar{h}^{0})\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L}-4\sqrt{2}\bar{G}^{\dot{\alpha}}(\bar{\tilde{h}}^{+}_{\dot{\alpha}},\bar{\tilde{h}}^{0}_{\dot{\alpha}})\begin{pmatrix} u_{L\alpha} \\ d_{L\alpha}\end{pmatrix}\tilde{\bar{d}}_{L} \\
&-4\sqrt{2}\bar{G}^{\dot{\alpha}}(\bar{\tilde{h}}^{+}_{\dot{\alpha}},\bar{\tilde{h}}^{0}_{\dot{\alpha}})\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\bar{d}_{L\alpha}-2\sqrt{2}\sigma^{a}_{\alpha\dot{\alpha}}\bar{G}^{\dot{\alpha}}(\bar{h}^{+},\bar{h}^{0})\mathcal{V}_a\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L} \\
&+4i\sqrt{2}\bar{\sigma}^{a\dot{\rho}\beta}\bar{\phi}\epsilon_{\alpha\beta}\partial_a(\bar{\tilde{h}}^{+}_{\dot{\rho}},\bar{\tilde{h}}^{0}_{\dot{\rho}})\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L}-4\sqrt{2}\bar{\phi}(\bar{F}^{+},\bar{F}^{0})\begin{pmatrix} u_{L\alpha} \\ d_{L\alpha}\end{pmatrix}\tilde{\bar{d}}_{L} \\
&-4\sqrt{2}\bar{\phi}(\bar{F}^{+},\bar{F}^{0})\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\bar{d}_{L\alpha}-2\sqrt{2}\sigma^{a}_{\alpha\dot{\alpha}}\bar{\phi}(\bar{\tilde{h}}^{+\dot{\alpha}},\bar{\tilde{h}}^{0\dot{\alpha}})\mathcal{V}_a\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L}\Big\} \\
&+\frac{m^2_{soft}}{f^2\,\Lambda^2}F\Big\{-\bar{F}(\bar{h}^{+},\bar{h}^{0})\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L}+\bar{G}_{\dot{\alpha}}(\bar{\tilde{h}}^{+}_{\dot{\alpha}},\bar{\tilde{h}}^{0}_{\dot{\alpha}})\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L}-\bar{\phi}(\bar{F}^{+},\bar{F}^{0})\begin{pmatrix} \tilde{u}_{L} \\ \tilde{d}_{L}\end{pmatrix}\tilde{\bar{d}}_{L}\Big\} \\
&+\frac{m_{soft}}{f\,\Lambda}\phi\;\mathcal{L}_Y. \qquad (5.46)
\end{aligned}
$$

In (5.45) and (5.46) the sgoldstino has not yet been integrated out. When this is done (by using $\phi=\frac{GG}{2F}$), a number of further goldstino couplings will appear.

5.2 sGoldstino Decoupling

Supersymmetry is one of the most appealing cadidates for new physics. It has not been observed so far and thus, it should be broken at some high energy scale if it is realised at all. However, supersymmetry breaking is not an easy task. In the MSSM for example, supersymmetry breaking is employed by introducing soft breaking terms. These terms are *ad hoc* masses for the superpartners of the SM particles, which nevertheless do not spoil the UV properties of the theory. In fact the MSSM includes all these soft breaking terms and one has to fit them into the observations. From a more theoretical point of view, the origin of these soft terms should be explored. The common lore is that supersymmetry should be broken in a sector of the theory, not directly connected to the SM particles, the hidden sector. For a review on soft terms, and other supersymmetry breaking mediation scenarios we refer to [96, 161, 191].

Whatever the nature of the mediation, the hidden sector should be studied on its own right. If it is a chiral multiplet that breaks supersymmetry, its highest component F will acquire a non-vanishing *vev*. There is a number of different scenarios for the origin of the supersymmetry breaking [96, 161]. Let us note that higher derivative operators [43, 82, 135, 137] may play an important role in hidden sector supersymmetry breaking. One of the most efficient methods for studying the phenomenology of the hidden sector is through the dynamics of the goldstino [16, 17, 23, 29, 40, 120, 122, 138, 148, 149, 153, 155, 159, 174, 186, 188]. The latter is the fermionic component of the superfield that breaks supersymmetry. If the supersymmetry breaking scale is low, goldstino dynamics become increasingly important for low energy phenomenology [8–12, 21, 27, 55, 62, 86, 139, 172]. In fact, if the SUSY breaking scale $\sqrt{f}$ is low with respect to Planck mass M_P ($\sqrt{f} \ll M_P$) as in gauge mediation, transverse gravitino couplings are of order M_P^{-1} and therefore are suppressed with respect to longitudinal gravitino couplings, which are of order $f^{-1/2}$. In this case, in the gravity decoupling limit, only the longitudinal gravitino component, i.e., the goldstino survives. Moreover, the highest component of the superfield to which the goldstino belongs, acquires a vev and breaks spontaneous the supersymmetry giving also mass to the sgoldstino (goldstino's superpartner). Therefore, at low energies, supersymmetry is spontaneous broken and after decoupling the sgoldstino (by making the latter superheavy) we are left with only the goldstino in the spectrum and a non-linear realised SUSY. In the case of local supersymmetry, non-linear realizations are less studied in the supergravity context [22, 99, 155].

Recently new methods have been proposed in order to study goldstino couplings, and MSSM extensions that incorporate them have been constructed [11, 12, 18, 19, 73, 74, 86, 139]. All this framework is based on the idea of constrained superfields [40, 155, 174] that introduce a non-linear supersymmetry representation for the goldstino when its massive scalar superpartner is heavy and can be integrated out. Moreover, when one studies physics much lower than the MSSM soft masses scale, non-linear supersymmetry is realized on the SM particles as well, via the appropriate constraints. The constraint that enforces a non-linear supersymmetry realization for the goldstino reads

$$\Phi_{NL}^2 = 0. \tag{5.47}$$

In addition, it has been proven in [139] that in fact Φ_{NL} is proportional in the IR limit to the chiral superfield X that sources the violation of the conservation of the Ferrara-Zumino supercurrent $J_{\alpha\dot{\alpha}}$ [61, 94]

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha X. \tag{5.48}$$

We extend this to the case of $\mathcal{N} = 1$ supergravity by identifying the superfield, which turns out to be the chiral superfield X of (5.48) in the gravity decoupling limit. Here, the conservation of the Ferrara-Zumino multiplet $J_{\alpha\dot{\alpha}}$ in (5.48) is replaced by the consistency conditions of the Bianchi identities [30]

$$X_\alpha = \mathcal{D}_\alpha \mathcal{R} - \bar{\mathcal{D}}^{\dot\alpha} G_{\alpha\dot\alpha} \tag{5.49}$$

where $G_{\alpha\dot\alpha}$ and $\mathcal{R}$ are the usual supergravity superfields and $X_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8\mathcal{R})\mathcal{D}_\alpha K$ is the matter sector contribution.

5.2.1 Supergravity in Einstein frame

In the standard $\mathcal{N}=1$ superspace forlmulation of supergravity, one is forced to perform a Weyl rescaling to the action in order to write the theory in the Einstein frame. Here, we should write the superspace action directly in the Einstein frame since we want to correctly identify the masses to be send to infinity. This will provide the superfield equations of motion in the correct frame as well. The appropriate framework for this is the Kähler superspace formalism which we will briefly present below. For a detailed description, one may consult for example [30,31,114]. An alternative method would be a super-Weyl invariant reformulation of the old minimal formulation for N=1 SUGRA [146].

In the conventional superspace approach to supergravity, the Lagrangian describing gravity coupled to matter would be (ignoring superpotential for the moment)

$$\mathcal{L}_{\mathrm{F}} = \int d^2\Theta 2\mathcal{E}\left\{\frac{3}{8}(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R})e^{-\frac{1}{3}K(\Phi,\bar\Phi)}\right\} + h.c. \tag{5.50}$$

where $2\mathcal{E}$ is the superspace chiral density and the new Θ variables span only the chiral superspace. An equivalent way to write the action (5.50) is

$$\mathcal{L}_{\mathrm{D}} = -3\int d^4\theta E e^{-\frac{1}{3}K(\Phi,\bar\Phi)}, \tag{5.51}$$

where now E is the full superspace density and θ are to be integrated over the full superspace. Both actions (5.50,5.51) can equavalently be used in order to build invariant theories in superspace. Note that $\mathcal{E}$ and E, both have the vierbein determinant in their lowest component. As usual $\mathcal{R}$ represents the supergravity chiral superfield which contains the Ricci scalar in its highest component. Direct calculation of (5.51) in component form shows that the theory is actually expressed in an unconventional Jordan frame. Of course a Weyl rescaling may be performed in order to bring the theory in the standard Einstein frame. Nevertheless, it is possible to perform this rescaling at the superspace level by considering

$$\begin{aligned}
{E'}_M^{\ a} &= e^{-\frac{1}{6}K(\Phi,\bar\Phi)}E_M^a\,, \quad {E'}_M^{\ \alpha} = e^{-\frac{1}{12}K(\Phi,\bar\Phi)}\left[E_M^{\ \alpha} - \frac{i}{12}E_M^b(\epsilon\sigma_b)^\alpha{}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}K(\Phi,\bar\Phi)\right], \\
{E'}_{M\dot\alpha} &= e^{-\frac{1}{12}K(\Phi,\bar\Phi)}\left[E_{M\dot\alpha} - \frac{i}{12}E_M^b(\epsilon\bar\sigma_b)_{\dot\alpha}{}^\alpha\mathcal{D}_\alpha K(\Phi,\bar\Phi)\right]
\end{aligned}$$

where E_A^M is the superspace frame, containing the gravitino and the vierbein in the appropriate lowest components. This redefinition will change the structure of the whole superspace including the Bianchi identity solutions and the superspace derivatives. Most importantly, the superspace geometry will receive contributions at the same time from the matter and supergravity fields in a unified way. The Lagrangian (5.51) now becomes in the new superspace frame (erasing the primes for convenience)

$$\mathcal{L}_{\text{Dnew}} = -3 \int d^4\theta E. \tag{5.52}$$

This form now contains the properly normalized supergravity action coupled to matter. The interested reader should consult an extensive review on the subject [30]. Since we also wish to include a superpotential, the appropriate contribution will be given by adding to (5.52) the appropriatelly rescaled superpotential W so that the full Lagrangian will be given by

$$\mathcal{L}_{superpotential} = -3 \int d^4\theta E + \left\{ \int d^4\theta \frac{E}{2\mathcal{R}} e^{K/2} W + h.c. \right\}. \tag{5.53}$$

In this new framework, Kähler transformations, generated by holomorphic functions F, are expressed as field dependent transformations gauged by a composite $U_K(1)$ vector B_A. The respective charge now is referred to as "chiral weight" and a superfield $\varPhi$ of chiral weight $w(\varPhi)$ transforms as

$$\varPhi \to \varPhi e^{-\frac{i}{2} w(\varPhi) \text{Im}\mathcal{F}}. \tag{5.54}$$

Gauge covariant superspace derivatives are defined as

$$\mathcal{D}_A \varPhi = E_A^{\ M} \partial_M \varPhi + w(\varPhi) B_A \varPhi \tag{5.55}$$

where the composite connection superfields are

$$\begin{aligned} B_\alpha &= \frac{1}{4} \mathcal{D}_\alpha K\,, \quad \bar{B}^{\dot\alpha} = -\frac{1}{4} \bar{\mathcal{D}}^{\dot\alpha} K \\ B_a &= \frac{1}{4} (\partial_i K) \mathcal{D}_a \Phi^i - \frac{1}{4} (\partial_{\bar j} K) \mathcal{D}_a \bar\Phi^{\bar j} + \frac{3i}{2} \mathcal{G}_a + \frac{i}{8} g_{i\bar j} \bar\sigma^{\dot\alpha\alpha} (\mathcal{D}_\alpha \Phi^i) \bar{\mathcal{D}}_{\dot\alpha} \bar\Phi^{\bar j}. \end{aligned}$$

All component fields are understood to be defined appropriately via projection as usual but now with the use of these Kähler-superspace derivatives. It turns out that the invariant Lagrangian containing both (5.52) and (5.53) depends only on the generalized Kähler potential

$$e^G = e^{K(\Phi,\bar\Phi)} W(\Phi) \bar W(\bar\Phi). \tag{5.56}$$

By taking into account the chiral weights of the gravity sector and performing a Kähler transformation with parameter $\mathcal{F} = \ln W$, we find that the final expression for the most general coupling of matter to supergravity is

$$\mathcal{L} = \int d^4\theta E \left[-3 + \frac{1}{2\mathcal{R}} e^{\frac{G}{2}} + \frac{1}{2\bar{\mathcal{R}}} e^{\frac{G}{2}} \right]. \tag{5.57}$$

It sould be stressed that this form of the action is completely equivalent to the standard $\mathcal{N} = 1$ superspace formulation (5.50) to which is related by appropriate redefinitions of the superspace frames.

5.2.2 Sgoldstino decoupling

We are interested in those classes of models where the sgoldstino may become superheavy and decouples from the spectrum. In this case, it plays no role in the low energy effective theory, and its dynamics can be integrated out by its equations of motion. Essentialy, in order to be able to decouple consistently the sgoldstino degrees of freedom, one has to

- consider the sgoldstino mass as the heavier scale in the problem, and
- find consistent solutions for the equations of motion in that limit.

This is equivalent to taking the limit of infinitely heavy sgolstino and integrate its equations of motion, if possible, in this limit. This work has been done in component form earlier [40] and extended recently [19,73]. We will implement the above procedure in superspace, where as we will see it is quite straightforward.
To study sgoldstino decoupling in supergravity, it is helpful to consider the corresponding decoupling in global supersymmetry.

Sgoldstino decoupling in global supersymmetry

The most general single chiral globally supersymmetric superfield Lagrangian is given by

$$\mathcal{L} = \int d^4\theta\, K(\Phi,\bar{\Phi}) + \left\{ \int d^2\theta\, W(\Phi) + h.c. \right\} \tag{5.58}$$

where, $K(\Phi,\bar{\Phi})$ is the Kähler potential, a hermitian function of the chiral superfield, and $W(\Phi)$ is the superpotential, a holomorphic function of the chiral superfield. From the above action, the superspace equations of motion

$$-\frac{1}{4}\bar{D}\bar{D}K_\Phi + W_\Phi = 0, \tag{5.59}$$

with $K_\Phi = \partial_\Phi K$, $W_\Phi = \partial_\Phi W$ easily follow. For a general, non-renormalizable supersymmetric model where supersymmetry is spontaneously broken, the supertrace mass formula reads [99]

$$\mathrm{Str}M^2 = \sum_J (-1)^{2J}(2J+1)M_J^2 = -2R_{A\bar{A}}f\bar{f} \tag{5.60}$$

where $f = \langle F \rangle$ and $R_{A\bar{A}}$ $(= g^{A\bar{A}}R_{A\bar{A}A\bar{A}})$ is the Ricci tensor of the scalar Kähler manifold evaluated at the vacuum expectation values of the scalars. Eq.(5.60) describes the mass splitting between the components of the supermultiplet. In the case of a single chiral superfield we are discussing, since the goldstino is always massless, the supertrace of the goldstino multiplet is just the square of the sgoldstino mass

$$M^2_{\text{sg}} = -R_{A\bar{A}}f\bar{f} \tag{5.61}$$

We see that necessarily the scalar manifold should be a space of negative curvature in order to have non-tachyonic scalar excitations. In addition, the limit of the infinitely heavy sgoldstino

$$2M^2_{sg} = \mathrm{Str}M^2 \to \infty \quad \text{or} \quad R_{A\bar{A}A\bar{A}} \to -\infty. \tag{5.62}$$

Since

$$R_{A\bar{A}A\bar{A}} = \partial_{\bar{A}}\partial_A\partial_{\bar{A}}\partial_A K - \partial_{\bar{A}}\partial_A\partial_{\bar{A}} K \partial_A\partial_A\partial^A K\,, \tag{5.63}$$

in normal coordinates for the Kähler space in which $g_{A\bar{A}} = \delta_{A\bar{A}}$ and $\partial_i\partial_j\partial_k K = 0$ (for any $i, j = A, \bar{A}$), we have that the infinitely heavy sgoldstino is obtained in the limit

$$-\partial_{\bar{A}}\partial_A\partial_{\bar{A}}\partial_A K \to \infty \tag{5.64}$$

By assuming that the vacuum expectation value of $A = \Phi\big|$ vanish[3], the general form of the Kähler potential

$$K(\Phi, \bar{\Phi}) = \sum_{mn} c_{mn}\Phi^m\bar{\Phi}^n \tag{5.65}$$

will have the following expansion in normal coordinates

$$K(\Phi, \bar{\Phi}) = \Phi\bar{\Phi} + c_{22}\bar{\Phi}^2\Phi^2 + \cdots \tag{5.66}$$

It is easy to see that in fact

$$c_{22} = \frac{1}{4}R_{A\bar{A}A\bar{A}} = \frac{1}{4}R_{A\bar{A}} \tag{5.67}$$

in normal coordinates. By using then (5.60,5.62), we get that the Kähler potential may be expressed in terms of the sgoldstino mass as

$$K(\Phi, \bar{\Phi}) = \Phi\bar{\Phi} - \frac{M_{sg}^2}{4|f|^2}\bar{\Phi}^2\Phi^2 + \cdots \tag{5.68}$$

where the dots stands for M_{sg}-independed terms and $f =< F >$ is the vev of the auxiliary field in the chiral multiplet. From the superspace equations of motion (5.59), one can easily isolate the contribution proportional to M_{sg}^2. Indeed, (5.59) is written as

$$\frac{M_{sg}^2}{4|f|^2}\Phi\bar{D}\bar{D}\bar{\Phi}^2 \;+\; \Big(M_{sg}\text{−independed terms}\Big) = 0. \tag{5.69}$$

Therefore, in the $M_{sg} \to \infty$ limit, the M_{sg}-dependent part of the field equations is turned into the superspace constraint

$$\Phi\bar{D}\bar{D}\bar{\Phi}^2 = 0. \tag{5.70}$$

To explicitly solve (5.70), we note that it leads to three component equations

$$\Phi\bar{D}\bar{D}\bar{\Phi}^2| = 0, \quad D_\alpha(\Phi\bar{D}\bar{D}\bar{\Phi}^2)| = 0, \quad DD(\Phi\bar{D}\bar{D}\bar{\Phi}^2)| = 0. \tag{5.71}$$

The non-trivial solution to the above equations is [139, 174]

$$\Phi_{NL} = \frac{\chi\chi}{2F} + \sqrt{2}\theta\chi + \theta^2 F \tag{5.72}$$

which can be easily checked that it satisfies

$$\Phi_{NL}^2 = 0. \tag{5.73}$$

As a result, the sgoldstino can be safely decoupled in the $M_{sg} \to \infty$ limit as long as Φ satisfies (5.70), or equivalently (5.73).

[3]if not we may shift appropriately A so that $\langle A\rangle = 0$

Sgoldstino decoupling in supergravity

As in the case of global supersymmetry, we are interested in the equations of motion and the mass supertrace. The superfield equations of motion as follow from the action (5.57) are [31]

$$\mathcal{R} = \frac{1}{2} e^{\frac{G}{2}}, \tag{5.74}$$

$$\mathcal{G}_a + \frac{1}{8} G_{\Phi\bar{\Phi}} \bar{\sigma}_a^{\dot{\alpha}\alpha} \mathcal{D}_\alpha \Phi \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Phi} = 0, \tag{5.75}$$

$$(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}) G_\Phi = 0. \tag{5.76}$$

On the other hand, for a general supergravity model with only one chiral multiplet the supertrace is given by [193]

$$\text{Str} M^2 = -2 R_{A\bar{A}} f \bar{f}, \tag{5.77}$$

which means that in the limit of infinite negative Kähler curvature the sgoldstino will become superheavy and can consistently be integrated out. Indeed, (5.77) is explicitly written as

$$M_{sg}^2 = 2 m_{3/2}^2 - R_{A\bar{A}} f \bar{f}. \tag{5.78}$$

Therefore, for finite gravitino mass $m_{3/2}$, the infinite curvature limit

$$R_{A\bar{A}A\bar{A}} \to -\infty \tag{5.79}$$

is equivalent to superheavy sgoldstinos. Again, in normal coordinates

$$R_{A\bar{A}A\bar{A}} = \partial_{\bar{A}} \partial_A \partial_{\bar{A}} \partial_A K = \partial_{\bar{A}} \partial_A \partial_{\bar{A}} \partial_A G \tag{5.80}$$

and therefore with

$$G \supset \frac{2 m_{3/2}^2 - M_{sg}^2}{4|f|^2} \Phi^2 \bar{\Phi}^2 + \cdots \tag{5.81}$$

the decoupling limit we are after is again $M_{sg}^2 \to \infty$. Taking into account that the Kähler curvature $M_{sg}^2/4|f|^2$ will dominate the equations of motion and following the same reasoning as in the global supersymmetric case, we get from (5.76)

$$\Phi (\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}) \bar{\Phi}^2 = 0. \tag{5.82}$$

This constraint is the curved superspace analogue of (5.70). In order to solve it, we take into account that $\Phi(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R})\bar{\Phi}^2$ is a chiral superfield, and we will once again start from its lowest component, namely

$$\Phi (\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R}) \bar{\Phi}^2 | = 0. \tag{5.83}$$

This is written, for

$$\Phi = A + \sqrt{2} \Theta \chi + \Theta\Theta F \,, \quad \mathcal{R}\Big| = -\frac{1}{6} M \tag{5.84}$$

as

$$AM\bar{A}^2 - 24A\bar{A}\bar{F} + 12A\bar{\chi}\bar{\chi} = 0. \tag{5.85}$$

This equation has three solutions

$$A_0 = 0, \quad A_1 = \frac{\chi\chi}{2F}, \quad A_2 = \frac{24F}{M} - \frac{\chi\chi}{2F}. \tag{5.86}$$

The first solution A_0 is the trivial and we will not consider it. The second solution A_1 is the $\Phi^2 = 0$ we already encounter in the global susy case. The third solution A_3 corresponds to $\Phi^2 \neq 0$ and can only be realized as long as the auxiliary field of supergravity M is non vanishing ($M \neq 0$). However, from the equation (5.74) we get

$$\mathcal{R} = \frac{1}{2}e^{\frac{G}{2}} = \frac{1}{2}e^{-\frac{M_{sg}^2}{8|f|^2}\Phi^2\bar{\Phi}^2+\cdots}, \tag{5.87}$$

where only the dominant term was explicitly written in the exponent in the right hand side. Now, in the $M_{sg}^2 \to \infty$ limit, the right hand side goes to zero exponentialy fast so that for $\Phi^2 \neq 0$

$$\mathcal{R} = 0 \quad \text{for} \quad M_{sg}^2 \to \infty \tag{5.88}$$

Therefore also $M = -6\mathcal{R}| = 0$ and the third solution (A_2) cannot consistently be realized. As a result, the only solution to the constraint (5.82) is the $A_1 = \frac{\chi\chi}{2F}$, or in other words the familiar

$$\Phi^2 = 0. \tag{5.89}$$

This constraint leads to

$$e^{\frac{M_{sg}^2}{8|f|^2}\Phi^2\bar{\Phi}^2}|_{\Phi^2=0} = 1 \tag{5.90}$$

and thus, the divergent part of (5.74) completely decouples! Moreover, $\Phi^2 = 0$ also satisfies

$$\mathcal{D}_\alpha\Phi\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\Phi}^2 = 0 \tag{5.91}$$

which is the field equation (5.75) in the $M_{sg}^2 \to \infty$ limit. As a result, we have again arrived to the constraint (5.89) as the only viable and consistent condition for the decoupling of the sgoldstino.

Supercurrent and sgoldstino decoupling

In order to discuss the relation of supersymmetry breaking to conservation laws, let us explore the decoupling limit of the supergravity sector. The supergravity equations of motion (5.74) and (5.75) in superspace, after restoring dimensions with compensating powers of M_P and returning to the Kähler frame where everything is expressed in terms of K and W, are written as

$$\mathcal{R} = \frac{1}{M_P^2}\frac{1}{2}We^{\frac{K}{2M_P^2}}, \tag{5.92}$$

$$\mathcal{G}_a + \frac{1}{M_P^2}\frac{1}{8}g_{i\bar{j}}\bar{\sigma}_a^{\dot{\alpha}\alpha}\mathcal{D}_\alpha\Phi^i\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\Phi}^{\bar{j}} = 0. \tag{5.93}$$

Gravity decouples in the limit $M_P \to \infty$, and from (5.92) and (5.93) we have

$$\mathcal{R} \to 0, \quad \mathcal{G}_a \to 0. \tag{5.94}$$

We note that this is the limit even when $W/M_P =$ finite, which is another possible limit [22] for gauge mediated SUSY breaking scenarios. The fact that these supergravity superfields should vanish can be also understood from the algebra of supergravity when compared to supersymmetry. For example, the global commutation relation (for $w(\Phi^i) = 0$)

$$[\bar{D}_{\dot{\alpha}}, D_a]\Phi^i = 0, \tag{5.95}$$

in supergravity becomes

$$[\bar{\mathcal{D}}_{\dot{\alpha}}, \mathcal{D}_a]\Phi^i = -i\mathcal{R}\sigma_{\alpha\dot{\alpha}}\mathcal{D}^{\alpha}\Phi^i \tag{5.96}$$

thus in order to recover the global supersymmetry algebra the superfield $\mathcal{R}$ should vanish.
Let us now derive the analog of the conservation equation of the Ferrara-Zumino multiplet (5.48) in curved superspace. By using the consistency conditions of the Bianchi identities [30]

$$X_\alpha = M_P^2\mathcal{D}_\alpha\mathcal{R} - M_P^2\bar{\mathcal{D}}^{\dot{\alpha}}G_{\alpha\dot{\alpha}} \tag{5.97}$$

with

$$X_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8\mathcal{R})\mathcal{D}_\alpha K \tag{5.98}$$

and the equations of motion, we find

$$\bar{\mathcal{D}}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = \mathcal{D}_\alpha\mathcal{X} - \frac{16}{3}\mathcal{R}\mathcal{D}_\alpha K + \frac{2}{3}\mathcal{G}_{\alpha\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}}K \tag{5.99}$$

with

$$\mathcal{J}_{\alpha\dot{\alpha}} = 2g_{i\bar{j}}\mathcal{D}_\alpha\Phi^i\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\Phi}^{\bar{j}} - \frac{2}{3}[\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}]K\,, \qquad \mathcal{X} = 4We^{\frac{K}{2M_P^2}} - \frac{1}{3}\bar{\mathcal{D}}\bar{\mathcal{D}}K. \tag{5.100}$$

The extra terms compared to (5.48) arise due to commutation relations like (5.96), and should vanish when supergravity is decoupled.
Now we take the decoupling limit of supergravity ($M_P \to \infty$) with ($\mathcal{R} \to 0,\ \mathcal{G}_a \to 0$) and find exactly the same formula as the global case. As a final comment let us note that now, after the decoupling of supergravity, the superfield X is

$$\mathcal{X} \to X = 4W - \frac{1}{3}\bar{D}\bar{D}K. \tag{5.101}$$

5.3 Summary

The main purpose of this work was to show that in the non-linear MSSM framework, a one Higgs doublet is possible and equally motivating with the two-Higgs scenario. In fact, even when dealing with a two-Higgs MSSM, unavoidably, non-linear goldstino dynamics should be considered as a possibility for the physics beyond MSSM. In this context, higher dimensional operators are introduced in order to study the consequences of the non-linearities of the underlying theory. However, higher dimensional operators is what is needed for a single Higgs MSSM. In this sense, a single Higgs MSSM is quite interesting, as it turns out that it is intrinsically connected to the underlying supergravity theory, as it cannot be constructed without the use of the higher dimensional operators.

In this approach, we have constructed two consistent supersymmetric extensions of the SM where only one scalar field is required to have a non-trivial vacuum expectation value. The energy regime of both models is comparable or above the soft masses. In the first model, the second Higgs superfield is completely missing from the MSSM spectrum and a new leptonic generation has taken its place for anomaly cancellation purposes. This introduction of a new leptonic generation would have significant effects in the Higgs production rates and eventually will change the SM expectations. In the second model, the second Higgs superfield of the MSSM is turned into a spectator. In both cases, mass generation can be implemented by the use of H_u and the constrained superfield X. It should be noted that in both cases the μ problem of the MSSM does not exist, in the first model by construction (as there is no H_d) and in the second case by the employment of a discrete symmetry.

Thus, one can have a non-linear MSSM where there is only one field with the "Higgs" property (i.e., of getting a vacuum expectation value). The constrained superfield framework we used, especially the goldstino, which should be interpreted as the surviving longitudinal low energy component of the gravitino, gives an insight to the connection of the more fundamental supergravity theory with the low energy phenomenology. We stress again that, it is in this sense that the supersymmetric single-Higgs Yukawa couplings are fundamentally connected to the low energy limit of supergravity, rather than being completely unattached to this underlying theory.

We would like to make a final comment in the case of a half quark generation. Electroweak symmetry breaking in the single Higgs non-linear MSSM should happen again radiatively. Quantum corrections drive the initially positive soft mass of the Higgs field to negative values near the electroweak scale and thus triggers symmetry breaking. This happens due to the large Yukawa couplings of the Higgs field to matter, especially the heavy quarks. It will be the new generation heavy quarks that will dominate radiative corrections and will make this effect quite stronger.

In the second section we explored the decoupling limit of sgoldstinos in spontaneously broken SUSY theories. This decoupling was implemented by considering large mass values for the sgoldstino (in fact the infinite mass limit). We used superspace techniques as they allowed for a unified treatment of the spontaneous breaking of SUSY both in local and global supersymmetric cases. The motivation of this study was twofold: first to check if the constraint superfield formalism employed in the global supersummetry still works in supergravity as well and second, to correctly identify in supergavity the chiral superfield that enters in the conservation of the Ferrara-Zumino multiplet and which accomodates the goldstino in global supersymmetry.

The way to approach these targets was to reformulate the goldstino dynamics in global supersymmetry but now in a language appropriate for supergravity. First we have identified the sgoldstino mass in both cases, and found the decoupling limit (supermassive sgoldstino) to be the limit of infinite negative Kähler curvature. Then we impose this limit to the superfield equations of motion and in the case of supersymmetry we found the constraint ($\Phi \bar{D}^2 \bar{\Phi}^2 = 0$) which is solved by $\Phi^2 = 0$ as expected. In the case of supergravity, the super-covariant form of the more general constraint emerges, but again with the same single consistent solution. Thus, the superspace constraint $\Phi^2 = 0$ for the goldstino, when the sgoldstino is supermassive, holds for supergravity as well. However, we should mention a potential problem here. Namely, the expansion of the Kähler potential in (5.68) is written in powers of M_{sg}/f, from where it follows that actually $M_{sg} \sim f/\Lambda$ where Λ is the effective cutoff of the theory. The infinite sgoldstino mass seems therefore to be in conflict with the removal of the cutoff ($\Lambda \to \infty$), which is needed to identify the goldstino superfield with the infrared limit of the superconformal symmetry breaking superfield that enters the Ferrara-Zumino current conservation. This issue is further complicated by the presence of extra light fields. The problem has been pointed out in [18] where conditions for the effective expansion of the supersymmetric Lagrangian in terms of the inverse cuttoff to not be in conflict with a small sgoldstino mass $\sim f/\Lambda$ were given. Note that we have not faced this problem, as we have taken the formal infinite large sgoldstino mass limit.

Chapter 6

Higher Derivative Supergravity and Cosmology

In this chapter we discuss the construction and important technical aspects of highly motivated inflationary and cosmological models in supergravity.
In the $\mathcal{N} = 1$ four-dimensional new-minimal supergravity framework, we supersymmetrise the coupling of the scalar kinetic term to the Einstein tensor. This coupling, although introduces a non-minimal derivative interaction of curvature to matter, it does not introduce harmful higher-derivatives. For this construction, we employ off-shell chiral and real linear multiplets. Physical scalars are accommodated in the chiral multiplet whereas curvature resides in a linear one. We then present consistent supersymmetric theories invariant under the generalization of the Galilean shift symmetry to $\mathcal{N} = 1$ superspace. These theories are constructed via the decoupling limit of certain non-minimally derivative coupled supergravities, thus they correspond to the supersymmetrization of the so-called covariant Galileon. Specifically, these theories are constructed in the linearized $\mathcal{N} = 1$ new-minimal supergravity set-up where the chiral supermultiplet is minimally coupled to gravity via the standard R-current contact term, and, at the same time, non-minimally derivatively coupled to the Einstein superfield.
We then turn to the higher-derivative Starobinsky model of inflation and discuss how it originates from $\mathcal{N} = 1$ supergravity. It is known that, in the old-minimal supergravity description written by employing a chiral compensator in the superconformal framework, the Starobinsky model is equivalent to a no-scale model with F-term potential. We show that the Starobinsky model can also be originated within the so-called new-minimal supergravity, where a linear compensator superfield is employed. In this formulation, the Starobinsky model is equivalent to standard supergravity coupled to a massive vector multiplet whose lowest scalar component plays the role of the inflaton and the vacuum energy is provided by a D-term potential. We also point out that higher-order corrections to the supergravity Lagrangian represent a threat to the Starobinsky model as they can destroy the flatness of the inflaton potential in its scalar field equivalent description.

6.1 Supersymmetric Galileons

It has been recently discovered [118, 158] that there is a set of, non-renormalizable, scalar field self-interactions having the interesting property that their suppression scale do not run at any energies. In addition, these same theories enjoy, in flat space, the following Galilean shift

$$\pi \to \pi + c + b_m x^m \ , \tag{6.1}$$

where c, b_m are respectively a constant and a constant four-vector, x^m are Cartesian coordinates in a Minkowski spacetime and π is the so-called *Galilean* field [166]. The additional requirement to have only up to second order differential equations, in order to avoid possible Ostrogradski instabilities, restrict the Galilean Lagrangians to contain only a product of up to five scalars [166]. The transformation (6.1) may be extended to superspace. In particular, for a chiral superfield (Φ), we propose that the only consistent supersymmetric extension of (6.1) is

$$\Phi \to \Phi + c + b_m y^m \tag{6.2}$$

where

$$y^m = x^m + i\theta\sigma^m\bar{\theta} \ . \tag{6.3}$$

Note that the super-Galilean shift (6.2), when projected to the real space, only shifts the lowest component of Φ as the complex extension of (6.1) while it maintains its superspace chirality.
It has been shown in [140], by brute-force calculation, that the supersymmetrization of a cubic Galileon out of a chiral field [134], is not possible without the appearance of ghosts. The same Authors however, could not exclude the quartic supersymmetric Galilean theories, although their constructions only led to ghost-propagating field theories. Those theories, although invariant under (6.1), were not invariant under the superspace Galilean shift (6.2) introduced here. We believe, that this was the main issue that led the Authors of [140] to conclude that no supersymmetric Galileons can be found without propagating ghosts states.
In this work we indeed show that a ghost-free quartic Galilean theory *does* exist and is invariant under (6.2). In other words, we will construct the supersymmetric version of the so-called "quadratic" and "quartic" Galileons

$$\begin{aligned} \mathcal{L}_2 &= -\frac{1}{2}\partial_m\bar{\pi}\partial^m\pi \ , \\ \mathcal{L}_4 &= -\frac{4}{\Lambda^6}\pi(\partial_{[k}\partial^k\bar{\pi})(\partial_\lambda\partial^\lambda\pi)(\partial_{\zeta]}\partial^\zeta\bar{\pi}) \ , \end{aligned} \tag{6.4}$$

where $\bar{\pi}$ is the complex conjugated of π and Λ is a suppression scale.
The easiest way to find our quartic Galilean theory passes through a decoupling limit of certain supergravities. To appreciate this, let us go back to the non-supersymmetric case.
In Minkowski space, the shift $b_m x^m = b_a \int \xi^a_m dx^m$, where $\xi^a_m = \delta^a_m$ is a set of Killing vectors (the four related to translations and labelled by a) such that $\nabla_m \xi^a_n = 0$ (i.e. integrable), and $b_a = \delta^m_a b_m$. One may then ask the question of whether generalized "Galilean" theories, i.e. with the property that they are invariant under the shift

$$\pi \to \pi + c + b_a \int \xi^a_m dx^m \ , \tag{6.5}$$

exist in non-trivial spacetimes with integrable Killing vectors.
This question has been answered in [102]. In particular, up to quadratic order in π one has

$$\mathcal{A}_2 = -\frac{1}{2}g^{mn}\partial_m\pi\partial_n\bar{\pi} + \frac{1}{2M^2}G^{mn}\partial_m\pi\partial_n\bar{\pi} \ . \tag{6.6}$$

where M is a mass scale and G^{mn} is the Einstein tensor. The sign of the terms in $\mathcal{A}_2$ are chosen in such a way that, whenever energy conditions are satisfied, the effective propagator of π is never ghost-like [103].
Again the theories of [102] enjoy a non-renormalization theorem (up to the Planck scale) of their coupling/suppression constants [102]. Finally the theory $\mathcal{A}_2$ has been dubbed *Slotheonic* theory in [102] (and so π the "*Slotheon*") for its property of a "slow" scalar evolution with respect to the minimal case $M \to \infty$. This property, turned out to be the key issue to produce successful inflationary scenarios even in the case of steep scalar field potentials [103–106].
Thanks to the equivalence principle, locally, any spacetime is approximately flat. Another way to see this is to notice that, in Riemanian coordinates, for any theory where graviton self-interaction is suppressed by the Planck scale,

$$\nabla_\alpha \xi_\beta = \mathcal{O}(\frac{1}{M_p}) \ , \tag{6.7}$$

where $\partial_m \xi_n = 0$ and M_p is the Planck scale. Therefore, there must exist "decoupling limits" involving $M_p \to \infty$, such that (6.6), endowed with the Einstein-Hilbert Lagrangian

$$\mathcal{L}_{\text{grav}} = \frac{1}{2} M_p^2 R \ , \tag{6.8}$$

reproduces (6.4).
These limits have been found in [107] showing an intimate relation between the theories (6.4) and (6.6), i.e. between Galileons and Slotheons. In particular, in [107], it has been shown that the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left[M_p^2 R - \frac{1}{M^2} G^{mn} \partial_m \pi \partial_n \pi \right] \ , \tag{6.9}$$

in the limit $M_p \to \infty$ but $\Lambda = M^2 M_p \to$ finite, reproduces the quartic Galileon $\mathcal{L}_4$. In the non-decoupling limit instead, with the help of the gravity equations, the equation of motion for π are nothing else than the covariant Galileon of [69].

6.1.1 Non-minimally kinetically coupled Supergravity

Following [84], we will work in the $\mathcal{N} = 1$ new-minimal supergravity framework [6, 38, 90, 98, 99, 179, 180]. Apart from [84], higher derivative extensions of new-minimal supergravity have been also studied in [44, 91], whereas consistent higher derivative theories have been discussed in [82, 83, 135]. As we will only be interested in the decoupling limit of gravity, we will only consider Lagrangians at linearized level in the graviton [54]. The non-minimal derivative coupling of a chiral superfield Φ to the linearized new-minimal supergravity, is found by considering the supersymmetric lagrangian [84]

$$\mathcal{L}_0 = \int d^4\theta \frac{2i}{M^2} \Phi E^m \partial_m \bar{\Phi} \tag{6.10}$$

where E^m is the Einstein superfield. We recall that the Einstein superfield is defined in terms of the real superfield ϕ_m as

$$E^m = -\frac{1}{2} \epsilon^{mnrs} \bar{D}_{\dot{\alpha}} \bar{\sigma}_n^{\dot{\alpha}\alpha} D_\alpha \partial_r \phi_s \tag{6.11}$$

where the covariant derivatives with respect to the Grassman co-ordinates of the superspace are defined as usual

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m \ , \quad \bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m. \tag{6.12}$$

The real superfield ϕ_m is invariant under the following gauge symmetry (needed in order to contain the $12+12$ degrees of freedom of new-minimal supergravity)

$$\delta\phi_m = \partial_m V + S_m + \bar S_m \ , \tag{6.13}$$

with V a real superfield and S_m a chiral superfield.
Obviously, the superfield E_m is also invariant under this gauge transformation. In fact, E_m is nothing else than the "field strength" of ϕ_m.
In the appropriate Wess-Zumino (WZ) gauge, ϕ_m contains the graviton h_{mn}, the gravitino ψ_m, a two-form auxiliary B_{mn} and a vector auxiliary A_m. The latter, gauges the continuous R-symmetry in supergravity.
The θ-expansion of ϕ_m is explicitly written as

$$\phi_m|_{WZ} = -\theta\sigma^n\overline\theta(h_{nm}+B_{nm}) + i\theta^2\overline\theta\bar\psi_m - i\overline\theta^2\theta\psi_m + \frac{1}{2}\theta^2\overline\theta^2 A_m \tag{6.14}$$

and it is useful to define the components of ϕ_m in terms of projections as [1]

$$\begin{aligned} -\frac{1}{2}[D_\alpha,\bar D_{\dot\alpha}]\phi_m| &= h_{\alpha\dot\alpha m} + B_{\alpha\dot\alpha m} \qquad (6.15)\\ -\frac{i}{4}\bar D^2 D_\alpha\phi_m| &= \psi_{m\alpha}\\ \frac{i}{4}D^2\bar D_{\dot\alpha}\phi_m| &= \bar\psi_{m\dot\alpha}\\ -\frac{1}{8}D^\alpha\bar D^2 D_\alpha\phi_m| &= A_m. \qquad (6.16)\end{aligned}$$

Using (6.14) in (6.11), we find that E_m can be expanded as

$$\begin{aligned} E_m &= -2H_m - 2i\theta R_m + 2i\overline\theta\bar R_m - \theta\sigma^n\overline\theta(G_{nm} + \partial^\lambda H_{lnm} - {}^*F_{nm})\\ &+ \theta^2\overline\theta\bar\sigma^n\partial_n R_m - \overline\theta^2\theta\sigma^n\partial_n\bar R_m - \frac{1}{2}\theta^2\overline\theta^2\partial^2 H_m \end{aligned} \tag{6.17}$$

where

$$\begin{aligned} H^m &= \frac{1}{3!}\epsilon^{mnrs}H_{nrs} \ ,\\ H_{nrs} &= (\partial_n B_{rs} + \partial_r B_{sn} + \partial_s B_{nr})\\ {}^*F^{mn} &= \frac{1}{2}\epsilon^{mnrs}(\partial_r A_s - \partial_s A_r)\end{aligned}$$

and

$$R_{mr} = -\partial_m\partial_r h^n_n + \partial^n\partial_r h_{nm} + \partial^n\partial_m h_{nr} - \partial^2 h_{mr} \ ,$$

[1] As standard we use the notation "|" to mean $\theta = \bar\theta = 0$. For our superspace conventions see [193].

$$G_{mn} = R_{mn} - \frac{1}{2}\eta_{mn}R \tag{6.18}$$

are the linearized Ricci and Einstein tensors respectively.
The components of E^m can be found using the definitions (6.16) and the supersymmetry algebra

$$\begin{aligned} -\frac{1}{2}E^m| &= H^m = \frac{1}{3!}\epsilon^{mnrs}H_{nrs} \ , \\ \frac{i}{2}D_\alpha E^m| &= R^m_\alpha = -\frac{1}{2}\epsilon^{mnrs}\sigma_{n\alpha\dot{\alpha}}\partial_r\bar{\psi}^{\dot{\alpha}}_s \ , \\ -\frac{1}{2}[D_\alpha, \bar{D}_{\dot{\alpha}}]E_m| &= \sigma^n_{\alpha\dot{\alpha}}(E_{nm} + \partial^l H_{lnm} - \ {}^*F_{nm}). \end{aligned} \tag{6.19}$$

Note that E^m is a real linear superfield as it satisfies the conditions

$$\bar{E}^m = E^m \, , \quad D^2 E^m = 0 \tag{6.20}$$

as well as the superspace Bianchi identity

$$\partial_m E^m = 0. \tag{6.21}$$

The components of the chiral superfield are defined as usual

$$\begin{aligned} \Phi| &= \pi \\ \frac{1}{\sqrt{2}}D_\alpha\Phi| &= \chi_\alpha \\ -\frac{1}{4}D^2\Phi| &= F. \end{aligned} \tag{6.22}$$

Taking into account the standard coupling of the Einstein superfield E^m with the graviton multiplet ϕ_m, we can write the leading terms of a chiral superfield Φ coupled to the new-minimal linearized supergravity as

$$\mathcal{L}_1 = \int d^4\theta \left(M_P^2 E^m\phi_m + \bar{\Phi}\Phi + \phi^m R_m - \frac{2i}{M^2}\Phi E^m \partial_m \bar{\Phi} \right) + \mathcal{O}\left(\frac{1}{M_P}\right) \ . \tag{6.23}$$

In addition, R_m is the supersymmetric R-current (see for example [143, 147]) which is defined as

$$R_m = -\bar{\sigma}^{\dot{\alpha}\alpha}_m D_\alpha \Phi \bar{D}_{\dot{\alpha}}\bar{\Phi} \tag{6.24}$$

and satisfies (on-shell)

$$\bar{D}^{\dot{\alpha}} R_{\alpha\dot{\alpha}} = \chi_\alpha \tag{6.25}$$

with

$$\chi_\alpha = \bar{D}^2 D_\alpha(\bar{\Phi}\Phi). \tag{6.26}$$

Note that, in the spirit of the already mentioned decoupling limit, we have silently assumed that M is not proportional to M_P (in which case we could have omitted the term (6.10) from (6.23)) but rather, as we will see later, proportional to $1/M_P^{1/2}$.
Concerning dimensions, we have assigned mass dimension zero to the graviton but the graviton multiplet has $[\phi_m] = -1$. For the chiral superfield $[\Phi] = 1$ and for the superspace derivatives $([D_\alpha][\bar{D}_{\dot{\alpha}}]) \sim [\partial_m] = 1$.

6.1.2 Linearized Lagrangian for the non-minimal derivative coupling

The linearized Lagrangian invariant under global supersymmetry reads

$$\begin{aligned} L &= \frac{1}{2}h^{mn}R_{mn} + \frac{1}{4}\bar{\psi}^m_{\dot{\alpha}}\epsilon_{mnrs}\bar{\sigma}^{n\dot{\alpha}\alpha}\partial^r\psi^s_\alpha - (2A^n + H^n)H_n \\ &+ A\partial^2\bar{A} + i\partial_n\bar{\chi}_{\dot{\alpha}}\bar{\sigma}^{n\dot{\alpha}\alpha}\chi_\alpha + F\bar{F} \\ &+ w^2[-2i\partial^2\bar{A}H^m\partial_m A + i\sqrt{2}\sigma^n_{\gamma\dot{\gamma}}\partial_n\bar{\chi}^{\dot{\gamma}}\bar{R}^{m\gamma}\partial_m A + 2\sigma^n_{\gamma\dot{\gamma}}\partial_n\bar{\chi}_{\dot{\gamma}}H^m\partial_m\chi^\gamma \\ &\quad -\sqrt{2}\bar{F}\bar{R}^{m\alpha}\partial_m\chi_\alpha + \sqrt{2}\bar{\chi}_{\dot{\alpha}}R^{m\dot{\alpha}}\partial_m F - 2i\bar{F}H^m\partial_m F \\ &\quad +(E^{mn} - 2i\partial^n H^m + \partial_r H^{rnm} + {}^*F^{mn})\partial_m A\partial_n\bar{A} \\ &\quad +i\sqrt{2}\bar{\chi}_{\dot{\alpha}}\bar{\sigma}^{n\dot{\alpha}\alpha}\partial_n\bar{R}^m_\alpha\partial_m A - i\sqrt{2}\bar{\sigma}^{n\dot{\alpha}\alpha}\partial_n\bar{A}R^m_{\dot{\alpha}}\partial_m\chi_\alpha \\ &\quad +\frac{i}{2}\bar{\chi}_{\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}_n(E^{mn} - 2i\partial^n H^m + \partial_r H^{rnm} + {}^*F^{mn})\partial_m\chi_\alpha + h.c.] \end{aligned} \tag{6.27}$$

where

$$E_{mn} = R_{mn} - \frac{1}{2}\eta_{mn}(\eta^{kl}R_{kl}) \tag{6.28}$$

$$R_{mn} = R_{mrnl}\eta^{rl} \tag{6.29}$$

$$R_{mnrl} = -\partial_m\partial_r h_{nl} + \partial_n\partial_r h_{ml} + \partial_m\partial_l h_{nr} - \partial_n\partial_l h_{mr} \tag{6.30}$$

$$\bar{R}^m_\alpha = -\frac{1}{4}\epsilon^{mnrs}\sigma_{n\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}_{rs} \tag{6.31}$$

$$\bar{\psi}^{\dot{\alpha}}_{rs} = \partial_r\bar{\psi}^{\dot{\alpha}}_s - \partial_s\bar{\psi}^{\dot{\alpha}}_r \tag{6.32}$$

$$H^m = \frac{1}{3!}\epsilon^{mnrs}(\partial_n B_{rs} + \partial_r B_{sn} + \partial_s B_{nr}) \tag{6.33}$$

$$H_{nrs} = \partial_n B_{rs} + \partial_r B_{sn} + \partial_s B_{nr} \tag{6.34}$$

$$^*F^{mn} = \frac{1}{2}\epsilon^{mnrs}(\partial_r A_s - \partial_s A_r) \tag{6.35}$$

the field h_{mn} is the graviton, B_{mn} is a two form, η_{mn} is mostly plus.
Let us now do the redefinition

$$2A^n + H^n = 2U^n \tag{6.36}$$

then the Lagrangian (6.27) will schematically take the form

$$L = L_{\text{reduced}} + H^{mnr}K_{mnr} + U^n J_n \tag{6.37}$$

due to the fact that the redefined auxiliary fields now appear only linearly coupled. It is clear then that the equations of motion will be

$$\begin{aligned} K_{mnr} &= 0 \\ J_m &= 0 \end{aligned} \tag{6.38}$$

and the higher derivatives disappear from the Lagrangian. What is very important to note is that the higher derivatives will survive inside the supersymmetry transformations, this is because equations (6.38) will be solved in terms of U^m and H^n.

Our Lagrangian then becomes

$$\begin{aligned}
L_{\text{reduced}} &= \frac{1}{2}h^{mn}R_{mn} + \frac{1}{4}\bar{\psi}^m_{\dot\alpha}\epsilon_{mnrs}\bar\sigma^{n\dot\alpha\alpha}\partial^r\psi^s_\alpha \\
&+ A\partial^2\bar{A} + i\partial_n\bar\chi_{\dot\alpha}\bar\sigma^{n\dot\alpha\alpha}\chi_\alpha + F\bar{F} \\
&+ w^2[+i\sqrt{2}\sigma^n_{\gamma\dot\gamma}\partial_n\bar\chi^{\dot\gamma}\bar{R}^{m\gamma}\partial_m A - i\sqrt{2}\sigma^n_{\gamma\dot\gamma}R^{m\dot\gamma}\partial_n\chi^\gamma\partial_m A \\
&\qquad -2\sqrt{2}\bar{F}\bar{R}^{m\alpha}\partial_m\chi_\alpha - 2\sqrt{2}\partial_m\bar\chi_{\dot\alpha}R^{m\dot\alpha}F \\
&\qquad +2E^{mn}\partial_m A\partial_n\bar{A} + i\bar\chi_{\dot\alpha}\bar\sigma^{\dot\alpha\alpha}_n E^{mn}\partial_m\chi_\alpha \\
&\qquad +i\sqrt{2}\bar\chi_{\dot\alpha}\bar\sigma^{n\dot\alpha\alpha}\partial_n\bar{R}^m_\alpha\partial_m A - i\sqrt{2}\partial_n R^m_{\dot\alpha}\bar\sigma^{n\dot\alpha\alpha}\chi_\alpha\partial_m A \\
&\qquad -i\sqrt{2}\bar\sigma^{n\dot\alpha\alpha}\partial_n\bar{A}R^m_{\dot\alpha}\partial_m\chi_\alpha + i\sqrt{2}\bar\sigma^{n\dot\alpha\alpha}\partial_n\bar{A}\partial_m\bar\chi_{\dot\alpha}\bar{R}^m_\alpha],
\end{aligned} \tag{6.39}$$

and after integrating out the auxiliary F we have

$$\begin{aligned}
L_{\text{reduced}} &= \frac{1}{2}h^{mn}R_{mn} + \frac{1}{4}\bar{\psi}^m_{\dot\alpha}\epsilon_{mnrs}\bar\sigma^{n\dot\alpha\alpha}\partial^r\psi^s_\alpha + A\partial^2\bar{A} + i\partial_n\bar\chi_{\dot\alpha}\bar\sigma^{n\dot\alpha\alpha}\chi_\alpha \\
&+w^2[2E^{mn}\partial_m A\partial_n\bar{A} + i\bar\chi_{\dot\alpha}\bar\sigma^{\dot\alpha\alpha}_n E^{mn}\partial_m\chi_\alpha - 8\partial_m\bar\chi_{\dot\alpha}R^{m\dot\alpha}\bar{R}^{n\alpha}\partial_n\chi_\alpha \\
&\quad +i\sqrt{2}\sigma^n_{\gamma\dot\gamma}\partial_n\bar\chi^{\dot\gamma}\bar{R}^{m\gamma}\partial_m A - i\sqrt{2}\sigma^n_{\gamma\dot\gamma}R^{m\dot\gamma}\partial_n\chi^\gamma\partial_m A \\
&\quad +i\sqrt{2}\bar\chi_{\dot\alpha}\bar\sigma^{n\dot\alpha\alpha}\partial_n\bar{R}^m_\alpha\partial_m A - i\sqrt{2}\partial_n R^m_{\dot\alpha}\bar\sigma^{n\dot\alpha\alpha}\chi_\alpha\partial_m A \\
&\quad -i\sqrt{2}\bar\sigma^{n\dot\alpha\alpha}\partial_n\bar{A}R^m_{\dot\alpha}\partial_m\chi_\alpha + i\sqrt{2}\bar\sigma^{n\dot\alpha\alpha}\partial_n\bar{A}\partial_m\bar\chi_{\dot\alpha}\bar{R}^m_\alpha].
\end{aligned} \tag{6.40}$$

6.1.3 Decoupling limit

We now proceed to the decoupling of gravity as in the previous discussions and [107]. The (gravity) equations of motion for ϕ_m are

$$E^s + \frac{1}{2M_P^2}R^s + \frac{i}{2M_P^2M^2}\bar{D}_{\dot\alpha}\partial_m\bar\Phi\bar\sigma^{\dot\alpha\alpha}_n D_\alpha\partial_r\Phi\epsilon^{mnrs} = 0 \tag{6.41}$$

and for Φ we have

$$\bar{D}^2(\bar\Phi - \bar\sigma^{\dot\alpha\alpha}_m D_\alpha(\phi^m\bar{D}_{\dot\alpha}\bar\Phi) - 2i\frac{1}{M^2}E^m\partial_m\bar\Phi) = 0. \tag{6.42}$$

Solving for E^m in (6.41) and plugging into (6.42) we find

$$\begin{aligned}
&\bar{D}^2(\bar\Phi - \bar\sigma^{\dot\alpha\alpha}_m D_\alpha(\phi^m\bar{D}_{\dot\alpha}\bar\Phi) - \frac{i}{M^2M_P^2}R^m\partial_m\bar\Phi \\
&\qquad - \frac{1}{M_P^2M^4}(\bar{D}_{\dot\alpha}\partial_m\bar\Phi\bar\sigma^{\dot\alpha\alpha}_n D_\alpha\partial_r\Phi)\epsilon^{mnrs}\partial_s\bar\Phi) = 0.
\end{aligned} \tag{6.43}$$

Now, in the limit

$$M_P \to \infty, \quad \text{such that} \quad M^2M_P = \Lambda^3 \quad \text{is finite} \tag{6.44}$$

gravity decouples

$$E_m = 0 \quad \to \quad \phi_m = \text{pure gauge} \tag{6.45}$$

and

$$\bar{D}^2(\bar{\Phi} - \frac{1}{\Lambda^6}(\bar{D}_{\dot{\alpha}}\partial_m\bar{\Phi}\bar{\sigma}_n^{\dot{\alpha}\alpha}D_\alpha\partial_r\Phi)\epsilon^{mnrs}\partial_s\bar{\Phi}) = 0 \tag{6.46}$$

where in (6.46), using the fact that ϕ_m is a pure gauge, we have set it to zero.
The component form of (6.46) (ignoring all fermions and auxiliary fields) are

$$\partial^2\bar{\pi} - \frac{4}{\Lambda^6}(\partial_{[k}\partial^k\bar{\pi})(\partial_l\partial^l\pi)(\partial_{\varsigma]}\partial^\varsigma\bar{\pi}) = 0 \ , \tag{6.47}$$

which is just the complex Galilean equation of motion coming from the variation of the action (6.4), as anticipated.

6.1.4 Supersymmetric Galileon

Now that we learned the structure of the quartic supersymmetric Galileon as a decoupling limit of the new non-minimally coupled $\mathcal{N} = 1$ supergravity of [84], we can infer the superspace Lagrangian that gives rise to the superspace equations of motion (6.46).
After a straightforward calculation one then finds that the Lagrangian describing the super-galileon is given by

$$\mathcal{L} = \int d^4\theta(\bar{\Phi}\Phi - \frac{1}{\Lambda^6}\Phi(\bar{D}_{\dot{\alpha}}\partial_m\bar{\Phi}\bar{\sigma}_n^{\dot{\alpha}\alpha}D_\alpha\partial_r\Phi)\epsilon^{mnrs}\partial_s\bar{\Phi}). \tag{6.48}$$

The Lagrangian (6.48), on top of the standard supersymmetries, enjoys the galilean symmetry extended to superspace, i.e.

$$\Phi \rightarrow \Phi + c + b_m y^m \tag{6.49}$$

where c is a complex constant, b_m is a complex constant vector and

$$y^m = x^m + i\theta\sigma^m\bar{\theta}. \tag{6.50}$$

The latter satisfies the relations

$$\bar{D}_{\dot{\alpha}}y_n = 0, \quad D_\alpha y_n = 2i\sigma_{n\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}, \quad D^2 y_n = 0, \quad \partial_m y_n = \eta_{mn}. \tag{6.51}$$

The super-galilean symmetry (6.49) is defined in a way such that:

- it preserves the chirality of the superfield Φ ($\bar{D}_{\dot{\alpha}}\Phi = 0$)
- it induces the following galileon transformations for the scalar (π), its fermionic super-partner (χ_α) and the auxiliary field (F)

$$\begin{array}{rcl} \pi & \rightarrow & \pi + c + b_m x^m \ , \\ \chi_\alpha & \rightarrow & \chi_\alpha \ , \\ F & \rightarrow & F \ . \end{array}$$

The component form of (6.48) is

$$
\begin{aligned}
\mathcal{L} &= \pi\partial^2\bar{\pi} + i\partial_n\bar{\chi}\bar{\sigma}^n\chi + F\bar{F} \\
&- \frac{1}{\Lambda^6}(4\pi(\partial_{[k}\partial^k\bar{\pi})(\partial_l\partial^l\pi)(\partial_{\zeta]}\partial^\zeta\bar{\pi}) - 8F\partial_m\bar{F}\partial_s\bar{\chi}\bar{\sigma}_n\partial_r\chi\epsilon^{mnrs} \\
&\quad -4i\partial_m\chi\sigma^\tau\partial_\tau\bar{\chi}\partial_s\bar{\chi}\bar{\sigma}_n\partial_r\chi\epsilon^{mnrs} - 2i\partial_m\bar{\chi}\bar{\sigma}_n\sigma^k\partial_s\bar{\chi}\epsilon^{mnrs}\partial_k\chi\partial_r\chi \\
&\quad +4i\partial_s\chi\sigma^k\bar{\sigma}_n\partial_r\chi\epsilon^{mnrs}\partial_m\bar{F}\partial_k\bar{\pi} + 4i\partial_r F\partial_m\bar{\chi}\bar{\sigma}_n\sigma^k\partial_s\bar{\chi}\epsilon^{mnrs}\partial_k\pi \\
&\quad +4\partial_m\pi\partial^2\bar{\pi}\epsilon^{mnrs}\partial_s\bar{\chi}\bar{\sigma}_n\partial_r\chi + 2\partial_m\bar{\chi}\bar{\sigma}_n\sigma^l\bar{\sigma}^k\partial_l\chi\epsilon^{mnrs}\partial_r\pi\partial_k\partial_s\bar{\pi} \\
&\quad +4\partial_\tau\bar{\chi}\bar{\sigma}^\tau\sigma^k\bar{\sigma}^n\partial_r\chi\epsilon^{mnrs}\partial_m\pi\partial_k\partial_s\bar{\pi} + 2\chi\sigma^k\bar{\sigma}_n\sigma^l\partial_s\bar{\chi}\epsilon^{mnrs}\partial_k\partial_m\bar{\pi}\partial_l\partial_r\pi \\
&\quad +2\partial_m\bar{\chi}\bar{\sigma}_n\sigma^k\bar{\sigma}^l\chi\epsilon^{mnrs}\partial_l\partial_s\bar{\pi}\partial_k\partial_r\pi - 2\partial_l\chi\sigma^k\bar{\sigma}_n\sigma^l\partial_s\bar{\chi}\epsilon^{mnrs}\partial_m\partial_k\bar{\pi}\partial_r\pi). \qquad (6.52)
\end{aligned}
$$

In order to find the final Lagrangian, one should integrate out the auxiliary field F in (6.52). The way to do that closely resemble the case studied in [138].
Variation of (6.52) with respect to $\bar{F}$ gives schematically an equation of type

$$F + \frac{\alpha^m}{\Lambda^6}\partial_m F + \frac{\beta}{\Lambda^6} = 0\ , \qquad (6.53)$$

where α^m and β are functions of the scalar field π but most importantly of the fermionic field χ. Finally, the scale Λ has been explicitly extracted. To solve (6.53) one can use an iterative procedure. The first step is to invert this equation as

$$F = -\frac{\alpha^m}{\Lambda^6}\partial_m F - \frac{\beta}{\Lambda^6}\ . \qquad (6.54)$$

The second step would be to substitute again the inversion, i.e.,

$$F = -\frac{\alpha^m}{\Lambda^6}\partial_m\left(-\frac{\alpha^n}{\Lambda^6}\partial_n F - \frac{\beta}{\Lambda^6}\right) - \frac{\beta}{\Lambda^6}\ , \qquad (6.55)$$

and so on. Thanks to the Grassmanian properties of the fermions χ this recursion eventually ends as soon as more than two equal fermions are multiplied (this is typical in supersymmetric theories, see for example [139]). The final Lagrangian is very involved and not very enlightening, for this reason we leave the interested reader to do the full inversion. Nevertheless, as the cut-off of the theory is Λ, which also corresponds to the suppression scale of the pure Galilean term, it is interesting to consider the supersymmetric action (6.52) up to $\mathcal{O}(\Lambda^{-12})$. Equation (6.53) is solved for

$$F = \mathcal{O}(\Lambda^{-6})\ . \qquad (6.56)$$

Therefore the supersymmetric Galilean action to leading order in the cut-off scale Λ reads

$$\mathcal{L}^{(\Lambda)}_{\rm gal} = \mathcal{L}_{\rm WZ} + \frac{1}{\Lambda^6}\left[\mathcal{L}_{\pi\pi} + \mathcal{L}^{(0)}_{\pi\chi}\right] \qquad (6.57)$$

where

$$\mathcal{L}_{\rm WZ} = \pi\partial^2\bar{\pi} + i\partial_n\bar{\chi}\bar{\sigma}^n\chi \qquad (6.58)$$

is the Wess-Zumino action,

$$\mathcal{L}_{\pi\pi} = -4\pi(\partial_{[k}\partial^k\bar{\pi})(\partial_l\partial^l\pi)(\partial_{\zeta]}\partial^\zeta\bar{\pi}) \tag{6.59}$$

is the scalar Galilean self-interaction, and finally the mix fermion-scalar interaction Lagrangian is

$$\begin{aligned}\mathcal{L}^{(0)}_{\pi\chi} =& - 4i\partial_m\chi\sigma^\tau\partial_\tau\bar{\chi}\partial_\sigma\bar{\chi}\bar{s}_n\partial_r\chi\epsilon^{mnrs} - 2i\partial_m\bar{\chi}\bar{\sigma}_n\sigma^k\partial_s\bar{\chi}\epsilon^{mnrs}\partial_k\chi\partial_r\chi\\ &+ 4\partial_m\pi\partial^2\bar{\pi}\epsilon^{mnrs}\partial_s\bar{\chi}\bar{\sigma}_n\partial_r\chi + 2\partial_m\bar{\chi}\bar{\sigma}_n\sigma^l\bar{\sigma}^k\partial_l\chi\epsilon^{mnrs}\partial_r\pi\partial_k\partial_s\bar{\pi}\\ &+ 4\partial_\tau\bar{\chi}\bar{\sigma}^\tau\sigma^k\bar{\sigma}^n\partial_r\chi\epsilon^{mnrs}\partial_m\pi\partial_k\partial_s\bar{\pi} + 2\chi\sigma^k\bar{\sigma}_n\sigma^l\partial_s\bar{\chi}\epsilon^{mnrs}\partial_k\partial_m\bar{\pi}\partial_l\partial_r\pi\\ &+ 2\partial_m\bar{\chi}\bar{\sigma}_n\sigma^k\bar{\sigma}^l\chi\epsilon^{mnrs}\partial_l\partial_s\bar{\pi}\partial_k\partial_r\pi - 2\partial_l\chi\sigma^k\bar{\sigma}_n\sigma^l\partial_s\bar{\chi}\epsilon^{mnrs}\partial_m\partial_k\bar{\pi}\partial_r\pi\ .\end{aligned} \tag{6.60}$$

Note that, from (6.53), the full $\mathcal{L}_{\rm gal}$, i.e. at all orders in Λ, would only involve extra π, χ interaction terms suppressed by higher powers of the cut-off scale Λ. In other words, the full Galilean action would only modify eq. (6.60) by additional $\mathcal{O}(\Lambda^{-6})$ terms. Explicitly

$$\mathcal{L}_{\rm gal} = \mathcal{L}_{\rm WZ} + \frac{1}{\Lambda^6}\left[\mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi\chi}\right]$$

where

$$\mathcal{L}_{\pi\chi} = \mathcal{L}^{(0)}_{\pi\chi} + \mathcal{O}\left(\frac{1}{\Lambda^6}\right)\ .$$

6.2 Non-minimal Derivative Coupling in New Minimal Supergravity

The most generic theory propagating a massless spin-2 and a scalar degree of freedom is *not* General Relativity minimally coupled to a scalar field (GRM). Indeed, Horndeski [121] proved that tensor-scalar theories with only second order differential equations are not restricted to GRM. Up to quadratic terms in matter fields and in four-dimensions, Horndeski showed that the most generic theories propagating a massless spin-2 and a spin-0 are

$$\mathcal{L} = \mathcal{L}_{\rm GRM} \pm \frac{1}{M_I^2}\mathcal{L}_I\ \pm \frac{1}{M_{II}^2}\mathcal{L}_{II} + \xi\mathcal{L}_{III}\ , \tag{6.61}$$

where

$$\begin{aligned}\mathcal{L}_{\rm GRM} &= \frac{1}{2}\left[M_P^2 R - \partial_a\phi\partial^a\phi\right], & (6.62)\\ \mathcal{L}_I &= \left(M_\phi^I\phi + \phi^2\right)R^2_{GB}\,, & (6.63)\\ \mathcal{L}_{II} &= G^{mn}\partial_m\phi\partial_n\phi\,, & (6.64)\\ \mathcal{L}_{III} &= \left(M_\phi^{III}\phi + \phi^2\right)R\,, & (6.65)\end{aligned}$$

and

$$G_{mn} = R_{mn} - \frac{1}{2}g_{mn}R\,,\quad R^2_{GB} = R_{mn\gamma\delta}R^{mn\gamma\delta} - 4R_{mn}R^{mn} + R^2 \tag{6.66}$$

are the Einstein and Gauss-Bonnet tensors, respectively, $M_{(I,II)}$, $M_\phi^{I,II}$ are mass scales, ξ a constant and finally M_P is the Planck constant. That $\mathcal{L}_I$ leads to second order evolution equation follows easily from the fact that the Gauss-Bonnet combination is a total derivative in four-dimensions and it is linear in second order derivatives. Instead, $\mathcal{L}_{II}$ leads to second order equations as, in Hamiltonian ADM formalism [163], G_{tt} and G_{it} contain only first time derivatives, since G_{tt} and G_{ti} are the Hamiltonian and momentum constraints.

While the supersymmetrization of $\mathcal{L}_I$ has been worked out in [44,45] and $\mathcal{L}_{III}$ for the $\mathcal{N}=1$ case in an arbitrary Jordan frame in [93], to our knowledge, the supersymmetric theory containing $\mathcal{L}_{II}$ was never found. It is the purpose of this work to construct the supersymmetric version of $\mathcal{L}_{II}$.

Apart from the obvious interest of studying the most generic supersymmetric theories avoiding Ostrogradski (higher derivatives) instabilities [167, 195], we note that the interaction (6.64) effectively describe part of the cubic graviton-dilaton-dilaton vertex in heterotic superstring theory and therefore appear in the low-energy 10D heterotic string effective action [115].[2] Moreover, it has also been shown in [162], that there exists a field redefinition up to α' corrections, such as to generate the terms $\mathcal{L}_I, \mathcal{L}_{II}$ out of a stringy effective action.

From a more phenomenological point of view, the theory $\mathcal{L}_{II}$ plays a fundamental role in the so called "Gravitationally Enhanced Friction" (GEF) mechanism developed in [103–106, 108]. There, thanks to the GEF, any steep (or not) scalar potential, can in principle produce a cosmic inflation for (relatively) small mass scale M_{II}. This is due to an enhanced friction produced by the Universe expansion acting on the (slow) rolling scalar field. Obviously then, the supersymmetrization of the GEF may notably enlarge the possibilities to find inflationary scenarios in supergravity and/or string theory. An additional motivation for studying supergravities with higher derivative terms, is related to the well known fact that they appear in the effective field theory action for the massless states of the superstring theory, after integrating out all superstring massive states.

All efforts to build higher-derivative supergravities in 4D are based on off-shell formulations. The latter are drastically different from the on-shell ones and, most importantly, they are not unique. This also happens in global supersymmetry where there are more than one off-shell formulations of an on-shell theory. We may recall for example the $\mathcal{N}=1$ 4D theory where a scalar and a pseudoscalar may be completed off-shell by an auxiliary scalar field resulting in a chiral multiplet. Replacing the pseudoscalar by an antisymmetric two-form, a linear multiplet arises. In this case, there is no need of extra auxiliary fields as the off-shell degrees of freedom of an antisymmetric form field are more than those of a scalar. These degree of freedom are the exact number needed to complete the off-shell content of the linear multiplet. On-shell, of course, the two multiplets are the same.

This feature persists also in local supersymmetry where at least for the minimal $\mathcal{N}=1$ 4D supergravity we are interested in, many off-shell formulations exist. The reason is that $\mathcal{N}=1$ superfields carry highly reducible supersymmetry multiplets and additional constraints should be implemented for their truncation. Then the constraints together with the torsion and Bianchi identities are used to solve for the independent fields. As there are various ways implementing this procedure, there are also various off-shell formulations. Known examples are the off-shell supergravity formulation based on the $12+12$ multiplet [95, 183] and the new minimal $12+12$ multiplet [6, 32, 98, 179, 180]. There are also other non-minimal formulations like the one based on the non-minimally $20+20$ [33, 34, 177] or $16+16$ [111, 178] multiplets. Nevertheless, these formulations may be considered reducible in the sense that they can be mapped to the minimal $\mathcal{N}=1$ supergravities coupled with extra multiplets. What is important to know though, is that it

[2] However, it should also be noted that this term has not been found in the heterotic quartic effective supergravity action constructed in [28].

has been proven [92] that when no higher derivative terms are present, the off-shell formulations of minimal supergravities are equivalent. For example old-minimal and new minimal supergravities at the two-derivative level are connected by a duality transformation, where the chiral compensator of the former is mapped to a linear compensator of the latter. When higher derivatives are present, the duality transformation does not work any more due to derivatives of the compensator and the two formulations are *not equivalent* [54, 92].
In this work we will construct the supersymmetrization of $\mathcal{L}_{II}$ in the new-minimal supergravity framework of [6, 98, 179, 180]. Our attempts in the old minimal supergravity setup have so far failed to reproduce $\mathcal{L}_{II}$. In particular, consideration of corresponding higher-derivative supergravity terms, like the ones we employ here, in old minimal formulation does not seem to give rise to such a term [25]. Whether or not one might nevertheless find a way of obtaining $\mathcal{L}_{II}$ in the old minimal supergravity formalism is an interesting open question that will not be discussed here but postponed for future research.

New Minimal $\mathcal{N} = 1$ 4D Supergravity

The simplest example of $\mathcal{N} = 1$ four-dimensional Poincaré supergravity is based on 12 bosonic and 12 fermionic off-shell degrees of freedom. These can be arranged into a multiplet in two ways. In the first one, the gravitational multiplet consists of

$$e_m^a\,, \quad \psi_m\,, \quad b_m\,, \quad M \tag{6.67}$$

and describes the dynamics of the so-called old minimal (standard) supergravity. Here, e_m^a is the vierbein, ψ_m is the gravitino, b_m is a vector, and M a scalar. As usual the vierbein should be used to convert tangent space indices $(a, b, ...)$ to world space indices $(m, n, ..)$ and throughout this work the tangent space metric is mostly plus (more on conventions can be found in the introduction).
In the new minimal supergravity instead, the multiplet consists of the vierbein ${e^a}_m$ and its supersymmetric partner, the gravitino ψ_m^α. In order to implement supersymmetry off-shell and the propagation of the physical degrees of freedom only, one has to also add auxiliary fields, as in the old minimal supergravity. However, in this case, the auxiliary fields are no longer a vector and a scalar but a 2-form B_{mn} with gauge invariance (B-gauge)

$$\delta B_{mn} = \partial_m \xi_n - \partial_n \xi_m, \tag{6.68}$$

and a gauge vector A_m with associated R gauge invariance

$$\delta A_m = -\partial_m \phi \ . \tag{6.69}$$

Thus, to wrap it up, the off-shell new minimal supergravity is based on the gravitational multiplet

$$e_m^a\,, \quad \psi_m\,, \quad A_m\,, \quad B_{mn} \ . \tag{6.70}$$

For more specific details on the structure of this theory the reader should consult [90].
It has been argued that the natural superspace geometry for four-dimensional $\mathcal{N} = 1$ heterotic superstring corresponds to the new minimal formulation of the $\mathcal{N} = 1$ supergravity [53, 152, 168]. This R symmetry is however anomalous (actually it is a mixed superconformal-Weyl-$U(1)$ anomaly [100]). Nevertheless, by using the Green-Schwarz mechanism, the symmetry is restored at one loop thanks to the introduction of a matter linear multiplet together with supersymmetric Lorentz and Chern-Simons terms [26, 157]. Note that this R symmetry has interesting implications on the gravitino over-abundance problem [63, 64].

Supersymmetric Actions

Chiral multiplets with chiral weight $n = 1$ can be used to form invariant actions by the F-density formula [180]

$$[\Sigma]_F = e\left\{F + \frac{i}{2}\chi\sigma\cdot\bar{\psi} + \frac{i}{2}A\,\bar{\psi}^a\bar{\sigma}_{ab}\bar{\psi}^b\right\}. \tag{6.71}$$

In superfield notation this can be written as

$$[\Sigma]_F = \int d^2\theta\,\mathcal{E}\Sigma, \tag{6.72}$$

with

$$\mathcal{E} = e\left\{1 - i\theta\sigma\cdot\bar{\psi} + \frac{i}{2}\theta^2\bar{\psi}^a\bar{\sigma}_{ab}\bar{\psi}^b\right\}. \tag{6.73}$$

The restriction $n = 1$ follows as $d\theta$ has $n = -\frac{1}{2}$ ($d\bar{\theta}$ has $n = \frac{1}{2}$). Furthermore, one can also build invariant actions from a multiplet with chiral weight zero, using the D-density formula

$$[V]_D = e\left\{D - \frac{1}{2}\bar{\psi}\cdot\gamma\gamma_5\,\lambda + \left(V_m + \frac{i}{2}\bar{\psi}_m\gamma_5\chi\right)\varepsilon^{mnrl}\partial_n B_{rl}\right\} + \text{surface terms.} \tag{6.74}$$

We mention here that the F and D density formulas are related by $[V]_D = 2[\Pi(V)]_F$+surface terms.

Non-Minimal Derivative Couplings

In order to construct non-minimal derivative couplings, we will introduce a chiral superfield Φ with chiral weight $n = 0$. Since the kinetic term of a general chiral superfield is given by the F-term density formula (6.72), we will have in our case

$$\mathcal{L}^{(0)}_{kin} = \int d^2\theta\,\mathcal{E}\Phi\left[-\frac{1}{4}\bar{\nabla}^2\Phi^\dagger\right] + \text{h.c.}\ , \tag{6.75}$$

where $-\frac{1}{4}\bar{\nabla}^2$ is the chiral projection operator for the new minimal supergravity. In component form, and recalling that Φ has a zero chiral weight $n = 0$, the bosonic part of the Lagrangian (6.75) is found to be

$$\mathcal{L}^{(0)}_{kin} = 2e\,A\Box A^* + 2eFF^* - 2ieH^c\left(A\,\partial_c A^* - A^*\partial_c A\right)\ . \tag{6.76}$$

We should couple now the chiral multiplet Φ to some curvature multiplet in order to get the the desired non-minimal derivative coupling (6.64). As both Φ and E_a have zero chiral weight, the term $\Phi^\dagger E^a\nabla^-_a\Phi$ is a general superfield with zero chiral weight as well. Therefore $\bar{\nabla}^2\left[\Phi^\dagger E^a\nabla^-_a\Phi\right]$ is a chiral superfield with chiral weight $n = 1$ and thus the superspace Lagrangian

$$\mathcal{L}^{(0)}_{int} = \int d^2\theta\,\mathcal{E}\left\{-\frac{i}{4}\bar{\nabla}^2\left[\Phi^\dagger E^a\nabla^-_a\Phi\right]\right\} + \text{h.c.}\ . \tag{6.77}$$

is supersymmetric. Now, (6.77) can be expanded as

$$\mathcal{L}^{(0)}_{int} = \frac{i}{16}e\nabla^2\bar{\nabla}^2\left[\Phi^\dagger E^a\nabla^-_a\Phi\right]\Big| + \text{h.c.} = A + B + C, \tag{6.78}$$

where

$$
\begin{aligned}
A &= \frac{i}{16} e \left[\left(\nabla^2 \bar{\nabla}^2 \Phi^\dagger \right) E^a \nabla_a^- \Phi \right] \Big| + \text{h.c.}, \\
B &= \frac{i}{16} e \left[\left(\bar{\nabla}^2 \Phi^\dagger \right) E^a \left(\nabla^2 \nabla_a^- \Phi \right) \right] \Big| + \text{h.c.}, \\
C &= \frac{i}{16} e \left[4 \left(\nabla_\gamma \bar{\nabla}_{\dot{\gamma}} \Phi^\dagger \right) \left(\nabla^\gamma \bar{\nabla}^{\dot{\gamma}} E^a \right) \left(\nabla_a^- \Phi \right) \right] \Big| + \text{h.c.} \ . \qquad (6.79)
\end{aligned}
$$

Keeping only bosonic fields, after a straightforward calculation we find

$$
\begin{aligned}
A &= 2eH^b \mathcal{D}_b A^* \, H^a \mathcal{D}_a A + ie \Box A^* \, H^a \mathcal{D}_a A + \text{h.c.} \\
B &= -\frac{i}{4} e \, F^* \, H^a \left(8iFH_a - 4\mathcal{D}_a^- F \right) + \text{h.c.} \\
C &= \frac{1}{2} e \, \partial^d A^* \, \partial^c A \left(G_{dc} - \eta_{dc} H^a H_a - 2H_d H_c \right) + ie \, \partial_b A^* \, \partial_c A \, \mathcal{D}^b H^c + \text{h.c.} \ . \qquad (6.80)
\end{aligned}
$$

In the above formulas we used that $\mathcal{D}_a^- F = \partial_a F - iA_a^- F$ with $A_a^- = A_a - 3H_a$, since F has a chiral weight $n_F = -1$. Additionally, in the above derivation one should use the helpful splitting $\nabla^\gamma \bar{\nabla}^{\dot{\gamma}} E^a = \frac{1}{2} \left[\nabla^\gamma, \bar{\nabla}^{\dot{\gamma}} \right] E^a + \frac{1}{2} \left\{ \nabla^\gamma, \bar{\nabla}^{\dot{\gamma}} \right\} E^a$.
We see that the desired nonminimal derivative coupling with the Einstein tensor indeed appears in C. Thus, the bosonic part of the interaction reads

$$
\begin{aligned}
\mathcal{L}_{int}^{(0)} &= e \, G^{ab} \partial_a A \, \partial_b A^* + 2eFF^* H^a A_a - 2eFF^* H^a H_a + ieH^a \left(F^* \partial_a F - F \partial_a F^* \right) \\
&- e \partial_b A \, \partial^b A^* H_a H^a + 2eH^a \partial_a A \, H^b \partial_b A^* - ieH_c \left(\partial_b A^* \, \mathcal{D}^c \partial^b A - \partial_b A \, \mathcal{D}^c \partial^b A^* \right) \ . \qquad (6.81)
\end{aligned}
$$

In summary, assembling the Lagrangians (3.167,6.75,6.77) we find that the bosonic sector of the theory is

$$
\begin{aligned}
\mathcal{L}_0 &= \frac{1}{\kappa^2} \mathcal{L}_{sugra} + \frac{1}{2} \mathcal{L}_{kin}^{(0)} + w^2 \mathcal{L}_{int}^{(0)} \\
&= \frac{1}{\kappa^2} \left[\frac{1}{2} e\mathcal{R} + 2eH^a A_a - 3eH^a H_a \right] \\
&\quad + eA \Box A^* + eFF^* - ieH^c \left(A \, \partial_c A^* - A^* \partial_c A \right) \\
&\quad + w^2 \Big[e \, G^{ab} \partial_b A^* \, \partial_a A + 2eFF^* H^a A_a - 2eFF^* H^a H_a \\
&\qquad + ieH^a \left(F^* \partial_a F - F \partial_a F^* \right) - e \partial_b A \, \partial^b A^* H_a H^a \\
&\qquad + 2e \, H^a \partial_a A \, H^b \partial_b A^* - ieH_c \left(\partial_b A^* \, \mathcal{D}^c \partial^b A - \partial_b A \, \mathcal{D}^c \partial^b A^* \right) \Big] , \qquad (6.82)
\end{aligned}
$$

where we have introduced the dimensionful parameter $w^2 = \pm M_{II}^{-2}$ and $\kappa^2 = M_P^{-2}$.
We may now integrate out the auxiliary fields to find the on-shell action. For $w^2 > 0$ we may define

$$
\begin{aligned}
V^a &= A^a \Big(1 + \kappa^2 w^2 FF^* \Big) + \frac{\kappa^2}{2} \Big(iA^* \partial^a A \\
&\quad - iA \partial^a A^* - iw^2 F \partial^a F^* + iw^2 F^* \partial^a F \\
&\quad - iw^2 \partial_b A^* \mathcal{D}^a \partial^b A + iw^2 \partial_b A \, \mathcal{D}^a \partial^b A^* \Big), \qquad (6.83)
\end{aligned}
$$

in terms of which (6.82) is written as

$$
e^{-1} \mathcal{L}_0 = \frac{1}{\kappa^2} \left[\frac{1}{2} \mathcal{R} + 2V^a \, H_a - 3H^a H_a \right] + A \Box A^* + FF^*
$$

$$+w^2 \left[G^{ab}\partial_b A^* \, \partial_a A - 2FF^* H^a H_a \right.$$
$$\left. -\partial_b A \, \partial^b A^* \, H_a H^a + 2H^a \partial_a A \, H^b \partial_b A^* \right] . \qquad (6.84)$$

It is important to notice here that since A, F have chiral weights $n = 0, -1$, respectively, V_m transforms under the $U(1)$ symmetry as it should, i.e.,

$$\delta V_m = \partial_m \phi \qquad (6.85)$$

and thus it is physically equivalent to A_m.
To find the on-shell action, we should eliminate the auxiliary fields V_m, B_{mn}, F. This can be done exactly in the same way as in the pure supergravity case (3.167) where we find $V_m = H_m = 0$. Similarly, the elimination of the auxiliary F of the chiral superfield is straightforward and the bosonic part of the supersymmetric Lagrangian (6.77) turns out to be

$$e^{-1}\mathcal{L}_0 \;=\; \frac{1}{2\kappa^2}\mathcal{R} + A\Box A^* + w^2 \, G^{ab} \, \partial_a A^* \, \partial_b A \; . \qquad (6.86)$$

There is a difference when $w^2 < 0$. Variation with respect to A_a gives the following equation

$$\left(\frac{1}{\kappa^2} + w^2 FF^* \right) H_a = 0 \; . \qquad (6.87)$$

For $w^2 > 0$ the only solution is $H_a = 0$ and we may define V^a in (6.83) as described above. However, for $w^2 < 0$, there are two solutions: i) a supersymmetric solution $H_a = 0$ and ii) a non-supersymmetric one $FF^* = \frac{1}{\kappa^2 w^2}$. For the supersymmetric solution, we arrive at the bosonic part (6.86) of our supersymmetric theory. On the other hand for the no-supersymmetric solution, A^a cannot anymore be traded for V^a. Moreover, it generates a cosmological constant as expected, introducing at the same time higher derivatives. Indeed, in this case, the last term of (6.82) would not vanish leading to harmful higher-derivative interactions.
The properties of the theory (6.86) have been studied in [102,175]. In particular, in [102] the scalar A has been dubbed as the *Slotheon* for the reason that, generically, for a given kinetic energy, its time derivative is smaller than the same calculated for a canonical scalar field. This again proves the usefulness of this theory for Inflation, where, in order to get an accelerated expansion of the primordial Universe, the scalar field should have a very small time derivative. In [102] it has also been proven that spherically symmetric Black Holes cannot have slotheonic hairs and, finally, it has been conjectured that this theory does not violate the no-hair theorem generically.
We should note that the Lagrangian (6.77) can easily be generalized to describe more general non-minimal couplings of the form $V(A, A^*)G^{mn}\partial_m A \partial_n A^*$. Indeed, we may employ a holomorphic function $W(\Phi)$ as follows

$$\mathcal{L}^{(W)}_{int} = \int d^2\theta \, \mathcal{E} \left\{ -\frac{i}{4} \bar{\nabla}^2 \left[\bar{W}(\Phi^\dagger) E^a \nabla^-_a W(\Phi) \right] \right\} + \text{h.c.} \; . \qquad (6.88)$$

The computation of (6.88) goes straightforward as in the previous case and the result, after combining with (3.167,6.75,6.77) and by doing an appropriate shifting of the $U(1)$ vector, turns out to be

$$e^{-1}\mathcal{L}^{(W)} \;=\; \frac{1}{\kappa^2}\left[\frac{1}{2}\mathcal{R} + 2V^a \, H_a - 3H^a H_a \right] + A\Box A^* + FF^*$$

$$+w^2\left|\frac{\partial\mathcal{W}}{\partial A}\right|^2\Big(G^{ab}\partial_b A^*\partial_a A - 2FF^*H^aH_a$$
$$-\partial_b A\partial^b A^* H_a H^a + 2H^aH^b\partial_a A\partial_b A^*\Big)\ , \qquad (6.89)$$

where $\mathcal{W}$ is the lowest component of W.
Again, field equations for V^m and H^m force the latter to vanish and the former to be a pure gauge. With this in mind, the bosonic part of the Lagrangian, after elimination of the auxiliary fields is

$$e^{-1}\mathcal{L}^{(W)} = \frac{1}{2\kappa^2}\mathcal{R} + A\Box A^* + w^2\left|\frac{\partial\mathcal{W}}{\partial A}\right|^2 G^{mn}\,\partial_m A^*\,\partial_n A\ . \qquad (6.90)$$

An obvious question concerns possible potential terms. Due to the requirement of R-invariance, one cannot use the F-density formula (6.72) to write general Lagrangians, unless the F-density has a total chiral weight of $n = 1$. For the neutral chiral multiplet we have used to construct our theory, it is not possible to write an R-symmetric potential term, unless new chiral fields are introduced. However, one can introduce explicit soft supersymmetry breaking terms of the form m^2AA^*, as potential for the neutral scalar.
A second question is why the neutral $n = 0$ prescription in (6.77) is fundamental to avoid higher-derivatives. An R-charged multiplet with $n \neq 0$ would give charge to the scalar A. In this case, A would be minimally coupled to the $U(1)$ gauge field A_m inducing quadratic terms for the gauge field. Moreover, kinetic terms for both A_m and H_m will appear. In this case, A_m and H_m could not be eliminated algebraically anymore. Specifically, the equation for A_m would read $H_m \sim \partial_m A + \ldots$. It is then clear that the elimination of H_m would produce quartic derivatives of the scalar A and consequently a higher derivative theory from, for example, the last term of (6.82). Therefore, only for a neutral $n = 0$ chiral field a theory with no harmful higher derivatives can be obtained. However, for completeness, we present later the bosonic sector of a general R-charged chiral multiplet of chiral weight n.
Finally, we note that in the fermionic sector of the theory, among the various fermionic interactions that arise, the term

$$\mathcal{L}_\chi = -w^2\, e\frac{i}{4}\hat{G}^{ab}\chi\sigma_b\hat{D}^-_a\bar{\chi} - iw^2 e\mathcal{D}_d A^*\mathcal{D}_a\chi\sigma^d\bar{r}^a\ , \qquad (6.91)$$

is the direct supersymmetric partner of the Einstein coupling in (6.86) needed to cancel scalar supersymmetry variations of $\mathcal{L}_{II}$. The first term in (6.91) was for first time introduced in non supersymmetric models in [109]. In [109] it has been shown that each time couplings of the form (6.91) or (6.86) are introduced, dependently upon the scale w, fields get dynamically localized around domain walls.

Lagrangian for non-zero chiral weight

The bosonic part of the Lagrangian for a general chiral weight n reads:

$$e^{-1}\mathcal{L}_n = \frac{1}{\kappa^2}\left[\frac{1}{2}\mathcal{R} + 2H^aA_a - 3H^aH_a\right]$$
$$+A\Box^- A^* + FF^* - \frac{1}{2}n\,AA^\star\left(\mathcal{R} + 6H^aH_a\right) - iH^c\left(A\,\mathcal{D}^-_c A^* - A^*\mathcal{D}^-_c A\right)$$
$$+w^2\Big\{iH^b\left[\Box^- A^*\mathcal{D}^-_b A - \Box^- A\mathcal{D}^-_b A^*\right] + \frac{i}{2}nH^b\left(\mathcal{R} + 6H^aH_a\right)\left(A\mathcal{D}^-_b A^* - A^*\mathcal{D}^-_b A\right)$$

$$
\begin{aligned}
&+4H^c\mathcal{D}_c^- A^* H^b \mathcal{D}_b^- A + i\mathcal{D}_d^- A^* \mathcal{D}_a^- A \left(\mathcal{D}^d H^a - \mathcal{D}^a H^d\right) \\
&+\mathcal{D}_d^- A^* \mathcal{D}_a^- A \left[G^{da} - g^{da} H^b H_b - 2H^d H^a\right] \\
&+iH^a \left(F^* \mathcal{D}_a^- F - F\mathcal{D}_a^- F^*\right) + 4FF^* H^a H_a \\
&+\frac{i}{2} n H^d \mathcal{R} \left(A\mathcal{D}_d^- A^* - A^* \mathcal{D}_d^- A\right) + 3in\, H^a H_a H^d (A\mathcal{D}_d^- A^* - A^* \mathcal{D}_d^- A) \\
&+in\ {}^*F^{da} H_a (A\mathcal{D}_d^- A^* - A^* \mathcal{D}_d^- A) + in\, H_a (\mathcal{D}_l H_b) \varepsilon^{blda} (A\mathcal{D}_d^- A^* - A^* \mathcal{D}_d^- A) \\
&-\frac{1}{2} n H_a\ {}^*F_{lb}\, \varepsilon^{blda} \left(A\mathcal{D}_d^- A^* + A^* \mathcal{D}_d^- A\right) - nH^l (\mathcal{D}_l H^d) \left(A\mathcal{D}_d^- A^* + A^* \mathcal{D}_d^- A\right) \\
&+nH^b (\mathcal{D}^d H_b)(A\mathcal{D}_d^- A^* + A^* \mathcal{D}_d^- A) \\
&-\frac{1}{4} n AA^* \left(\mathcal{R} + 6H^a H_a\right)^2 - \frac{1}{2} n AA^*\ {}^*F_{dc}\ {}^*F^{dc} - nAA^* \varepsilon^{cdka}\ {}^*F_{ka} \mathcal{D}_d H_c \\
&+nAA^* (\mathcal{D}_d H_c) \left(\mathcal{D}^d H^c - \mathcal{D}^c H^d\right)\Big\}\ . \qquad (6.92)
\end{aligned}
$$

It is clear that, for $n \neq 0$, the vector A_a of field strength F^{ab} becomes dynamical and therefore, as discussed in the text, cannot be removed by a gauge transformation.

6.3 Starobinski Model of Inflation in Supergravity

If the perturbations during inflation [160] are originated by the same field driving inflation, the inflaton, then the recent Planck data on the cosmic microwave background radiation anisotropies have severely constrained the models of single-field inflation [4]. Indeed, successful models have to predict a significant red tilt in the two-point correlator of the scalar curvature perturbation, measured by the spectral index $n_s = 0.960 \pm 0.007$, and a low enough amount of tensor perturbations quantified by the current bound on the tensor-to-scalar ratio, $r < 0.08$. One of the models which better passes these constraints is the higher-derivative R^2 Starobinsky model [164, 181, 182]. It is described by the Lagrangian (we set from now on the reduced Planckian mass to unity)

$$\mathcal{L}_{\rm star} = \sqrt{-g}\left(R + \lambda_0 R^2\right), \quad \lambda_0 > 0 \qquad (6.93)$$

and it contains, besides the graviton, one additional degree of freedom. The coupling constant λ_0 is positive in order to avoid instabilities. Indeed, one can rewrite the Lagrangian (6.93) as [194]

$$\mathcal{L}_{\rm star} = \sqrt{-g}\left(R + \lambda_0 R\psi - \frac{1}{4}\lambda_0 \psi^2\right) \qquad (6.94)$$

and, upon integrating out ψ, one gets back the original theory (6.93). Note that this is a classical equivalence. After writing the expression (6.94) in the Einstein frame by means of the conformal transformation

$$g_{mn} \to e^{-2\phi} g_{mn} = \left(1 + \lambda_0 \psi\right)^{-1} g_{mn}, \qquad (6.95)$$

we get the equivalent scalar field version of the Starobinsky model

$$\mathcal{L}_{\rm star} = \sqrt{-g}\left[R - 6\partial_m \phi \partial^m \phi - \frac{1}{4\lambda_0}(1 - e^{-2\phi})^2\right]. \qquad (6.96)$$

and the positivity of λ_0 is now obvious. Inflation takes place when the scalar field is slowly-rolling along its potential plateau obtained for $\phi \gg 1$ and in order to achieve a sufficient number of e-folds the plateau must be at least as wide as $\mathcal{O}(5)$ in Planckian units.
In this work we investigate the possibility of embedding the Starobinsky model into superconformal theory and $\mathcal{N}=1$ supergravity. This extension is not unique. The reason is that there are two ways for the graviton, sitting along with the gravitino, to fill a supergravity multiplet and one needs a set of auxiliary fields to define the off-shell supergravity multiplet. The minimal case should contain only the gravitino as fermionic content. This means that we need a total of 12 bosonic degrees of freedom to match the 12 degrees of freedom of the gravitino. This is the so-called (12+12) supergravity theories and there are two of them [39, 99]: the old-minimal supergravity and the new-minimal one. The auxiliary fields of the old-minimal supergravity are a complex scalar and a vector, whereas the new-minimal supergravity has a gauge one-form (gauging an R-symmetry) and a gauge two-form field. In particular, at the two derivative level, these two minimal supergravities are the same as they are related by some duality transformation of their auxiliary sectors. However, when higher derivatives appear, this duality does not work and the two theories are different. Earlier [47, 129–131] and recent [36, 78, 127] embeddings of the Starobinsky model have been all based on the old-minimal formulation of $\mathcal{N}=1$ supergravity. There have also been attempts to interpret gravitino condensate as the inflaton [77, 132].
An appropriate framework to discuss minimal supergravities is the superconformal calculus [35, 39, 99, 144, 145] which we employ here. To go to the desired Poincaré supergravity one fixes the appropriate compensator field and breaks the conformal symmetry. This framework offers a connection between the different auxiliary field structure of the minimal Poincaré supergravities [92]. Depending on the compensator, after gauge fixing the superconformal symmetry, one recovers either old or new-minimal supergravity: with a chiral compensator, the old-minimal supergravity is obtained whereas with a real linear compensator superfield the new-minimal one is recovered.
The goal of this section is two-fold [88]: on one side, we wish to show that the Starobinsky model can be derived also from the new-minimal formulation of supergravity in such a way that the vacuum energy driving inflation can be identified with a D-term; on the other hand we want to point out that the embedding of the Starobinsky model both in old- and new-minimal supergravity suffers of a potential problem deriving from the presence of higher-order corrections which may spoil the plateau of the potential of the scalar field driving inflation. This is reminiscent of the so-called η-problem [160] which arises when a model of inflation is embedded in supersymmetry and the flatness of the potential is usually spoiled by supergravity corrections [160].
This section is organized as follows. We describe the embedding of the Starobinsky model within the old-minimal supergravity formulation in section 2 and within the new-minimal supergravity formulation in section 3. In section 4 we describe the potential danger represented by higher-order corrections in both formulations. Finally, we conclude in section 5.

6.3.1 Starobinsky model in the old-minimal supergravity

We start by writing the Lagrangian that is appropriate to reproduce the supergravity version of the Starobinsky model in the old-minimal framework

$$\mathcal{L} = -3[S_0\overline{S}_0]_D + 3\lambda_1[\mathcal{R}\overline{\mathcal{R}}]_D, \tag{6.97}$$

with

$$\mathcal{R} = -\frac{1}{4} S_0^{-1} \overline{\nabla}^2 \overline{S}_0. \tag{6.98}$$

Here S_0 is the compensator chiral superfield, with scaling weight equal to 1 and chiral weight 2/3, the curvature chiral superfield $\mathcal{R}$ has scaling weight equal to 1 and chiral weight 2/3 as well, and $[\mathcal{O}]_D$ is the standard D-term density formula of conformal supergravity [35], where $\mathcal{O}$ is a real superfield with scaling weight 2 and vanishing chiral weight. After gauge fixing the superconformal symmetry and choosing

$$S_0 = 1, \tag{6.99}$$

the superspace geometry is described by the old-minimal formulation, see for example Ref. [193]. Then Eq. (6.97) becomes

$$\mathcal{L} = -3 \int \mathrm{d}^2\Theta \, 2\,\mathcal{E} \left\{ \mathcal{R} + \frac{\lambda_1}{8} \left(\overline{\mathcal{D}\mathcal{D}} - 8\mathcal{R} \right) \left(\mathcal{R}\overline{\mathcal{R}} \right) \right\} + \text{h.c.} \tag{6.100}$$

It is easy to verify that the bosonic part of Eq. (6.100) contains the Lagrangian (6.93)

$$\mathcal{L} \supset R + \lambda_1 R^2 \tag{6.101}$$

and therefore is a good candidate for the supergravity theory we are after [154, 185, 190]. The next step is to write the expression (6.97) as standard supergravity with additional degrees of freedom in the same way we have traded the R^2 term in non-supersymmetric case (6.93) by a scalar field coupled to Einstein gravity in (6.94). This can be implemented by using appropriate Lagrange multipliers. Hence, we introduce a chiral superfield J with scaling weight 1 (chiral weight 2/3) and a chiral Lagrange multiplier Λ with scaling weight 2 (chiral weight 4/3) and the equivalent Lagrangian to (6.97) is [47]

$$\mathcal{L} = -3[S_0\overline{S}_0]_D + 3\lambda_1[J\overline{J}]_D + 3([\Lambda(J - \mathcal{R})]_F + \text{h.c.}) \tag{6.102}$$

where $[\mathcal{O}]_F$ is the standard F-term density formula of conformal supergravity [35], with $\mathcal{O}$ a chiral superfield having scaling weight 3 and chiral weight 2. Indeed, integrating out the Lagrange multiplier chiral superfield Λ from Eq. (6.102) we get

$$J = \mathcal{R} \tag{6.103}$$

and Eq. (6.97) is reproduced by Eq. (6.102). By using the identity

$$[\Lambda\mathcal{R}]_F = [\Lambda\overline{S_0}S_0^{-1}]_D, \tag{6.104}$$

Eq. (6.102) can be recast in the form

$$\mathcal{L} = -3[S_0\overline{S}_0]_D + 3\lambda_1[J\overline{J}]_D - 3[\Lambda\overline{S_0}S_0^{-1}]_D - 3[\overline{\Lambda}S_0\overline{S}_0^{-1}]_D + 3([\Lambda J]_F + \text{h.c.}), \tag{6.105}$$

and will lead to standard Poincaré supergravity. By defining new chiral superfields $\mathcal{C}$ and $\mathcal{T}$ defined in terms of our original J and Λ as

$$\mathcal{C} = \sqrt{\lambda_1} S_0^{-1} J \,, \qquad \mathcal{T} = \frac{1}{2} + S_0^{-2}\Lambda, \tag{6.106}$$

Eq. (6.105) turns out to be

$$\mathcal{L} = -3\Big[S_0\overline{S}_0(\mathcal{T}+\overline{\mathcal{T}}-\mathcal{C}\overline{\mathcal{C}})\Big]_D + 3\lambda_1^{-1/2}\left(\left[\mathcal{C}\left(\mathcal{T}-\frac{1}{2}\right)S_0^3\right]_F + \text{h.c.}\right). \tag{6.107}$$

We recognize in Eq. (6.107) the characteristic form of a no-scale model [47, 151]. I n particular, the fields $\mathcal{C}$ and $\mathcal{T}$ parametrize the scalar manifold $SU(2,1)/U(2)$. Note that the theory is not gauged and the potential is due to an F-term, the second term in (6.107). We now gauge fix the superconformal symmetry in order for the superspace to be described by the old-minimal formulation. Then Eq. (6.107) turns out to be the standard old-minimal supergravity Lagrangian coupled to chiral superfields

$$\mathcal{L} = \int \mathrm{d}^2\Theta\, 2\,\mathcal{E}\left\{\frac{3}{8}\Big(\overline{\mathcal{D}}\overline{\mathcal{D}}-8\mathcal{R}\Big)e^{-K/3} + W\right\} + \text{h.c.} \tag{6.108}$$

with Kähler potential

$$K = -3\,\ln\Big(\mathcal{T}+\overline{\mathcal{T}}-\mathcal{C}\overline{\mathcal{C}}\Big) \tag{6.109}$$

and superpotential

$$W = \frac{3}{\sqrt{\lambda_1}}\mathcal{C}\left(\mathcal{T}-\frac{1}{2}\right). \tag{6.110}$$

The bosonic sector of the final Lagrangian is

$$e^{-1}\mathcal{L} = \frac{1}{2}R - K_{i\bar{j}}\partial_m z^i\partial^m\bar{z}^{\bar{j}} - V_F \tag{6.111}$$

with

$$V_F = e^K\left[K^{i\bar{j}}(D_iW)(D_{\bar{j}}\overline{W}) - 3W\overline{W}\right], \quad i,j=1,2, \tag{6.112}$$

where $z^1 = T$ and $z^2 = C$ the scalar lowest components of the chiral superfields $\mathcal{T}$ and $\mathcal{C}$. We have used the standard notation

$$K_i = \frac{\partial K}{\partial z^i}, \quad K_{i\bar{j}} = \frac{\partial^2 K(z,\bar{z})}{\partial z^i\partial\bar{z}^{\bar{j}}}, \quad D_iW = W_i + K_iW, \quad W_i = \frac{\partial W}{\partial z^i}. \tag{6.113}$$

The superpotential (6.110) belongs to a specific class of supergravity theories studied in [124–126], where together with a Kähler potential invariant under $C \to -C$, a local extremum at $C = 0$ appears. This also happens in our case as there is a local extremum at $C = \mathrm{Im}\,T = 0$ where we have the inflationary potential. Indeed, by parametrizing the complex scalar T by two real scalar ϕ, b, as

$$T = \frac{1}{2}e^{\frac{2}{\sqrt{3}}\phi} + ib, \tag{6.114}$$

we find that there is an extremum at $C = b = 0$, where the effective bosonic theory turns out to be

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \partial_m\phi\partial^m\phi - \frac{3}{2\lambda_1}\left(1 - e^{-\frac{2}{\sqrt{3}}\phi}\right)^2. \tag{6.115}$$

This is just the Starobinsky theory formulated in terms of the extra scalar degree of freedom. However, there is a possibility of a tachyonic instability for excitations along the inflationary trajectory $C = 0$. Indeed, the mass of such excitations are [124–126]

$$m_{\pm}^2 = -\left(K_{CC\bar{C}\bar{C}} \pm \left|K_{CC\bar{C}\bar{C}} - K_{CC}\right|\right)|f|^2 + |\partial_T f|^2 \tag{6.116}$$

for a general superpotential $W = Cf(T)$ and a Kähler potential invariant under $C \to -C$. It is easy to check that in our case we have in fact a tachyonic instability during the inflationary phase. A remedy can be modifying the Kähler potential appropriately [127]. We may consider, for example, instead of (6.97) the theory

$$\mathcal{L} = -3[S_0\overline{S}_0]_D + 3\lambda_1[\mathcal{R}\overline{\mathcal{R}}]_D + 3\zeta\,[\mathcal{R}\overline{\mathcal{R}}\;\mathcal{F}\left(\mathcal{R}\overline{\mathcal{R}}(S_0\overline{S}_0)^{-1}\right)]_D. \tag{6.117}$$

After writing this theory as standard supergravity as we have done for (6.97) and gauge fixing $S_0 = 1$, the new term does not change the superpotential and changes only the corresponding Kähler potential, which turns out to be

$$K = -3\ln\left(\mathcal{T} + \overline{\mathcal{T}} - \mathcal{C}\overline{\mathcal{C}}[1 + \zeta\,\lambda_1^{-1}\,\mathcal{F}(\mathcal{C}\overline{\mathcal{C}}\,\lambda_1^{-1})]\right). \tag{6.118}$$

As suggested in Ref. [127], the choice

$$\mathcal{F} = -\lambda_1\mathcal{C}\overline{\mathcal{C}} + \cdots \tag{6.119}$$

will stabilize the inflationary trajectory and give rise to a consistent theory for appropriate values of ζ. Theories similar to this have been discussed also in Ref. [129–131].
Let us now turn to the alternative derivation of the same Lagrangian in the new-minimal supergravity formulation.

6.3.2 Starobinsky model in new-minimal supergravity

In this section we want to show that there is another way to write a supergravity which contains the Lagrangian (6.93) in its bosonic sector. The appropriate compensator for the new-minimal supergravity gauging is a real linear multiplet (L_0) with scaling weight 2 and vanishing weight under chiral rotations. We employ now the following Lagrangian

$$\mathcal{L} = [L_0 V_R]_D + \lambda_2([W^\alpha(V_R)W_\alpha(V_R)]_F + \text{h.c.}), \tag{6.120}$$

where

$$V_R = \ln\left(\frac{L_0}{Y_0\overline{Y}_0}\right), \tag{6.121}$$

$$W_\alpha(V_R) = -\frac{1}{4}\overline{\nabla}^2\,\nabla_\alpha(V_R) \tag{6.122}$$

and Y_0 a chiral superfield with scaling weight 1. After gauge fixing the superconformal symmetry and choosing

$$L_0 = 1 \tag{6.123}$$

the superspace geometry is described by the new-minimal formulation, see for example Refs. [90, 180]. Indeed, fixing the superconformal symmetry by $L_0 = 1$, we get that the graviton multiplet contains four fields, the physical graviton e_m^a, the gravitino ψ_m and two auxiliary gauge fields A_m, and B_{mn} with corresponding gauge invariances

$$\delta A_m = \partial_m \phi \,, \qquad \delta b_{mn} = \partial_m b_n - \partial_n b_m . \tag{6.124}$$

In fact, A_m gauges the $U(1)_R$ R-symmetry of the superconformal algebra, which survives after the gauge fixing (6.123). Then, the desired theory is described by the following new-minimal Poincaré superspace density

$$\mathcal{L} = \int \mathrm{d}^2\Theta \, 2\,\mathcal{E} \left\{ -\frac{1}{8}\overline{\nabla\nabla} V_R + \lambda_2 \, W^\alpha(V_R) W_\alpha(V_R) \right\} + \text{h.c.}, \tag{6.125}$$

where now V_R turns out to be the gauge multiplet of the supersymmetry algebra, namely

$$V_R = \left(-H_m + \frac{1}{3}A_m, -\frac{1}{3}\gamma_5\gamma^n r_n, -\frac{1}{6}\hat{\mathcal{R}} - H_m H^m \right), \tag{6.126}$$

where r_n is the supercovariant gravitino field strength, $\hat{R}$ is the (supercovariant) Ricci scalar and H^m the Hodge dual of the (supercovariant) field strength for the auxiliary two-form [90]. The first terms in Eq. (6.125) is easily recognized as the Fayet-Iliopoulos term for the gauge multiplet, whereas the second is its standard kinetic term. Since the highest component D_R of the gauge multiplet contains the Ricci scalar ($D_R \sim R$), clearly we will get the desired $D_R + \lambda_2 D_R^2 \sim R + \lambda_2 R^2$ from the terms in Eq. (6.125). See Ref. [54] for a thorough discussion.
As a first step to write Eq. (6.120) as standard Poincaré supergravity, we consider L_0 as an unconstrained real superfield (note that by employing the equation of motion for Y_0 we can make L_0 real linear again). Then one can check that the following Lagrangian

$$\mathcal{L} = [L_0 V_R]_D + \lambda_2([W^\alpha(V) W_\alpha(V)]_F + \text{h.c.}) + [L'(V - V_R)]_D \tag{6.127}$$

reproduces Eq. (6.120) when we integrate out the real linear superfield L' to find

$$V = \ln\left(\frac{L_0}{Y_0 \overline{Y}_0}\right) - \ln\Phi - \ln\overline{\Phi} + c \tag{6.128}$$

and plug it back into Eq. (6.127). Now, in order to write the theory as standard supergravity, we go in the opposite direction. We again perform a variation with respect to L', but now we interpret the equation of motion as

$$\ln\left(\frac{L_0}{Y_0 \overline{Y}_0}\right) = V + \ln\Phi + \ln\overline{\Phi} + c, \tag{6.129}$$

which can be solved for L_0 by

$$\frac{L_0}{Y_0 \overline{Y}_0} = \overline{\Phi} e^{V+c} \Phi. \tag{6.130}$$

The final step is to plug back Eq. (6.130) (or (6.129)) into Eq. (6.127) to get

$$\mathcal{L} = [Y_0 \overline{Y}_0 (\overline{\Phi} e^{V+c}\Phi \, \ln(\overline{\Phi} e^{V+c}\Phi))]_D + \lambda_2([W^\alpha(V) W_\alpha(V)]_F + \text{h.c.}). \tag{6.131}$$

The action (6.131) is the dual action to (6.120) [46, 54]. Since c is just an integration constant we can take $c = 0$. Note that our theory here is gauged and that the potential is thus due to the standard D-term, in contrast to the expression (6.107). Again gauge fixing superconformal invariance and setting

$$Y_0 = 1,$$

we recover the following standard $\mathcal{N} = 1$ supergravity

$$\mathcal{L} = \int \mathrm{d}^2\Theta\, 2\,\mathcal{E} \left\{ \frac{3}{8}\Big(\overline{\mathcal{D}\mathcal{D}} - 8\mathcal{R}\Big) e^{-K/3} + \lambda_2\, W^\alpha W_\alpha \right\} + \text{h.c.}, \tag{6.132}$$

with the Kähler potential

$$K = -3\ln\left[-\frac{1}{3}\overline{\Phi} e^V \Phi \ln(\overline{\Phi} e^V \Phi)\right]. \tag{6.133}$$

In component form the expression (6.132) reads

$$\begin{aligned} e^{-1}\mathcal{L} &= \frac{1}{2}R - K_{A\overline{A}} D_m A \overline{D^m A} + \frac{1}{2}\left(K_A A + K_{\overline{A}}\overline{A}\right) D \\ &- 2\lambda_2 \left(\frac{1}{2}F^{mn}F_{mn} - \frac{i}{4}\epsilon^{mnrs}F_{mn}F_{rs} - D^2\right) \end{aligned} \tag{6.134}$$

with

$$D_m A = \partial_m A + i A_m A, \tag{6.135}$$

and after integrating out the auxiliary D we get

$$e^{-1}\mathcal{L} = \frac{1}{2}R - 2\lambda_2\left(\frac{1}{2}F^{mn}F_{mn} - \frac{i}{4}\epsilon^{mnrs}F_{mn}F_{rs}\right) - \frac{3}{A\overline{A}\left[\ln(A\overline{A})\right]^2} D_m A\, D^m\overline{A} - \frac{9}{8\lambda_2}\left[1 + \frac{1}{\ln(A\overline{A})}\right]^2. \tag{6.136}$$

With the redefinition

$$\ln A = -\frac{1}{2} e^{\frac{2}{\sqrt{3}}\phi} + i a, \tag{6.137}$$

the expression (6.136) is finally written as (with $\lambda_2 = 1/4g^2$)

$$\begin{aligned} e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{4g^2}F^{mn}F_{mn} + \frac{i}{8g^2}\epsilon^{mnrs}F_{mn}F_{rs} - 3e^{-\frac{4}{\sqrt{3}}\phi}\big(\partial_m a + A_m\big)^2 \\ - \partial_m\phi\partial^m\phi - \frac{9}{2}g^2\left(1 - e^{-\frac{2}{\sqrt{3}}\phi}\right)^2. \end{aligned} \tag{6.138}$$

This describes a massive vector with mass

$$m_A = \sqrt{6}\, e^{-2\phi/\sqrt{3}} \tag{6.139}$$

in Planck units, and a singlet scalar ϕ. The latter can be considered as the inflaton field with a D-term potential

$$V_D = \frac{9}{2}g^2\left(1 - e^{-\frac{2}{\sqrt{3}}\phi}\right)^2. \tag{6.140}$$

Therefore, the R^2 new-minimal supergravity is described by standard supergravity coupled to a massive vector superfield. The latter contains a real scalar in its lowest component (the ϕ field here) and a massive $U(1)$ vector in its bosonic sector. Thus, the Starobinsky model stems from the new-minimal supergravity constructed by means of a massless vector multiplet and a chiral multiplet. The vector eats one of the scalars of the chiral multiplet and becomes massive, whereas the other scalar of the chiral acquires a D-term potential. All together, the massless vector and the two scalars of the chiral, rearrange themselves such that to form standard supergravity coupled to a massive vector multiplet. Note that the scalar ϕ is what is usually fixed to zero by imposing the Wess-Zumino gauge in exact gauge invariance.

6.3.3 The issue of higher-order corrections

Before concluding, we would like to discuss a relevant issue that might represent a potential danger to the embedding of the the Starobinsky model into supergravity: higher order corrections. As we shall see, both in the old- and new-minimal supergravity formulation of the Starobinsky model, one can add non-renormalizable higher-order corrections which are admitted by the symmetries and might spoil the plateau of the inflaton potential necessary to drive inflation.

Corrections in new-minimal supergravity

Let us first discuss the possible corrections to the inflaton potential (6.140) obtained in the new-minimal version. These corrections are generated as corrections to the superconformal action (6.120). However, all possible non-renormalizable terms are restricted by gauge invariance. Possible corrections could arise from higher-order D-terms of the supersymmetric field strength W_α. In conformal superspace we may consider

$$\begin{aligned}\mathcal{L} &= [L_0 V_R]_D + \lambda_2([W^\alpha(V_R)W_\alpha(V_R)]_F + \text{h.c.}) \\ &+ \xi[(W^\alpha(V_R)W_\alpha(V_R)\overline{W}_{\dot\alpha}(V_R)\overline{W}^{\dot\alpha}(V_R))(L_0)^{-2}]_D.\end{aligned} \tag{6.141}$$

In the $L_0 = 1$ gauge, this theory will contain in it bosonic sector terms of the form

$$\mathcal{L} \supset R + \lambda_2 R^2 + \xi R^4, \tag{6.142}$$

which represent corrections to Starobinsky theory in the new-minimal supergravity framework. To recover the dual theory, we write Eq. (6.141) as

$$\begin{aligned}\mathcal{L} &= [L_0 V_R]_D + \lambda_2([W^\alpha(V)W_\alpha(V)]_F + \text{h.c.}) \\ &+ \xi[(W^\alpha(V)W_\alpha(V)\overline{W}_{\dot\alpha}(V)\overline{W}^{\dot\alpha}(V))(L_0)^{-2}]_D + [L'(V - V_R)]_D.\end{aligned} \tag{6.143}$$

Again we perform a variation with respect to L', and we interpret the equation of motion as

$$\ln\left(\frac{L_0}{Y_0\overline{Y}_0}\right) = V + \ln\Phi + \ln\overline{\Phi} + c, \tag{6.144}$$

which can be solved for L_0

$$\frac{L_0}{Y_0\overline{Y}_0} = \overline{\Phi}e^V\Phi. \tag{6.145}$$

We have also set $c = 0$ here. The final step is to plug back (6.145) (or (6.144)) into (6.143), which gives

$$\begin{aligned}\mathcal{L} &= [Y_0\overline{Y}_0(\overline{\Phi}e^V\Phi\,\ln(\overline{\Phi}e^V\Phi))]_D + \lambda_2([W^\alpha(V)W_\alpha(V)]_F + \text{h.c.}) \\ &+ \xi[(W^\alpha(V)W_\alpha(V)\overline{W}_{\dot\alpha}(V)\overline{W}^{\dot\alpha}(V))(Y_0\overline{Y}_0)^{-2}(\overline{\Phi}e^V\Phi)^{-2}]_D. \end{aligned} \tag{6.146}$$

We now gauge fix the conformal symmetry and set $Y_0 = 1$ to recover the standard supergravity theory that corresponds to (6.146)

$$\begin{aligned}\mathcal{L} &= \int \mathrm{d}^2\Theta\, 2\,\mathcal{E}\left\{\frac{3}{8}\Big(\overline{\mathcal{D}\mathcal{D}} - 8\mathcal{R}\Big)e^{-K/3} + \lambda_2\, W^\alpha W_\alpha \right.\\ &- \left.\frac{1}{4}\Big(\overline{\mathcal{D}\mathcal{D}} - 8\mathcal{R}\Big)\left[\frac{\xi}{2(\overline{\Phi}e^V\Phi)^2}W^\alpha W_\alpha \overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha}\right]\right\} + \text{h.c.},\end{aligned} \tag{6.147}$$

with

$$K = -3\ln\left[-\frac{1}{3}\overline{\Phi}e^V\Phi\,\ln(\overline{\Phi}e^V\Phi)\right]. \tag{6.148}$$

Importantly, the Kähler potential is the same as in Eq. (6.133) and thus there are no corrections to the Kähler potential. In addition, the theory (6.147) has been studied in Refs. [43,83], where now the general functions in the higher derivative gauge sector are fixed by the form of the integrated out L_0. The component form reads

$$\begin{aligned} e^{-1}\mathcal{L} &= \frac{1}{2}R - K_{A\overline{A}}D_mA\overline{D^mA} + \frac{1}{2}\left(K_AA + K_{\overline{A}}\overline{A}\right)D \\ &- 2\lambda_2\left(\frac{1}{2}F^{mn}F_{mn} - \frac{i}{4}\epsilon^{mnrs}F_{mn}F_{rs} - D^2\right) \\ &+ \frac{\xi}{(\overline{A}A)^2}e^{\frac{-2K}{3}}\left[\frac{1}{4}(F^{mn}F_{mn})^2 - F^{mn}F_{mn}D^2 + \frac{1}{16}(\epsilon^{mnrs}F_{mn}F_{rs})^2 + D^4\right]. \end{aligned} \tag{6.149}$$

To find the scalar potential we have to integrate over D. Since we are interested only in the scalar potential in what follows we ignore all other contributions to D, but those from A. For a more complete discussion, one may consult [43,83]. By defining the functions

$$a = -\frac{1}{2}[K_AA + K_{\overline{A}}\overline{A}], \tag{6.150}$$

$$b = 2\lambda_2, \tag{6.151}$$

$$c = \frac{\xi e^{\frac{-2K}{3}}}{(\overline{A}A)^2}, \tag{6.152}$$

the equation of motion for D turns out to be

$$0 = a + 2bD + 4cD^3. \tag{6.153}$$

The solution to Eq. (6.153) was found in Ref. [43] and is given by

$$D = \sqrt{\frac{2b}{3c}}\sinh n, \tag{6.154}$$

with

$$n = \frac{1}{3}\,\text{arcsinh}\left(-\frac{3a}{4b}\sqrt{\frac{6c}{b}}\right). \tag{6.155}$$

The scalar potential reads

$$V_D = \frac{4\lambda_2 b}{3c}\cosh(2n)(\sinh n)^2. \tag{6.156}$$

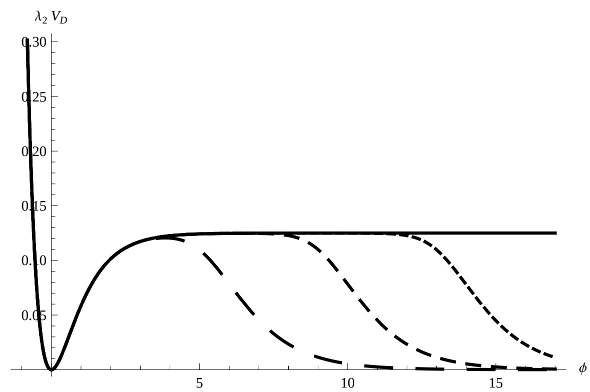

Figure 6.1: The scalar potential for three different values of $s = \frac{\sqrt{\xi}}{4}\left(\frac{3}{\lambda_2}\right)^{3/2}$: *i*) $s = 10^{-2}$ (continuous line), *ii*) $s = 10^{-4}$ (long dashed line) and *iii*) $s = 10^{-6}$ (short dashed line). The horizontal line corresponds to $\xi = 0$.

To find the corrections to the inflaton potential (6.140), we rewrite n as

$$n = \frac{1}{3}\,\text{arcsinh}\,\omega, \tag{6.157}$$

where

$$\omega = \frac{\sqrt{\xi}}{8}\left(\frac{3}{\lambda_2}\right)^{\frac{3}{2}}\left(1 - e^{\frac{2}{\sqrt{3}}\phi}\right). \tag{6.158}$$

For $\omega \ll 1$, the potential is written as

$$V_D \approx \frac{9}{8\lambda_2}\left(1 - e^{-\frac{2}{\sqrt{3}}\phi}\right)^2 - \frac{9\xi}{256\lambda_2^4}e^{-\frac{4}{\sqrt{3}}\phi}\left(1 - e^{\frac{2}{\sqrt{3}}\phi}\right)^4, \qquad \omega \ll 1. \tag{6.159}$$

On the other side, if $|\omega| \gg 1$ ($\phi \gg 1$), the potential is approximately given by

$$V_D \approx \frac{\lambda_2^2}{3\xi}e^{-\frac{4}{3\sqrt{3}}\phi}, \qquad \phi \gg 1. \tag{6.160}$$

We have plotted the potential in Fig. 1 for various values of the parameter $s = \frac{\sqrt{\xi}}{4}\left(\frac{3}{\lambda_2}\right)^{3/2}$. If $s = 0$ (i.e., $\xi = 0$), the potential has a plateau for large positive values of ϕ and one recovers the nice feature of the Starobinsky model formulated in terms of the extra scalar degree of freedom. However, for non-zero values of s, the plateau is restricted to smaller regions and it disappears for larger values of s with a fall-off $V_D \sim e^{-\frac{4}{3\sqrt{3}}\phi}$ after the plateau. Therefore, the higher-order corrections pose a problem to the Starobinsky model: we know that successful inflation is achieved when the number of e-folds is about 60. This requires the field plateau to be as large as $\mathcal{O}(5)$ in Planckian units. This imposes the parameter s to be smaller than about 10^{-4}. Even so, one should explain why the initial value field is positioned on the plateau, instead of being on the fall-off region.

Corrections in old-minimal supergravity

Higher-order corrections are also expected in the old-minimal supergravity case. It is straightforward to verify that the following superspace Lagrangian

$$\mathcal{L} = -3[S_0\overline{S}_0]_D + 3\lambda_1[\mathcal{R}\overline{\mathcal{R}}]_D + \xi[(S_0\overline{S}_0)^{-2}\nabla^\alpha\mathcal{R}\nabla_\alpha\mathcal{R}\overline{\nabla}_{\dot\alpha}\overline{\mathcal{R}}\overline{\nabla}^{\dot\alpha}\overline{\mathcal{R}}]_D \tag{6.161}$$

reproduces (6.142) after gauge fixing $S_0 = 1$. We can rewrite (6.161) as

$$\mathcal{L} = -3[S_0\overline{S}_0]_D + 3\lambda_1[J\overline{J}]_D + \xi[(S_0\overline{S}_0)^{-2}\nabla^\alpha J\nabla_\alpha J\overline{\nabla}_{\dot\alpha}\overline{J}\overline{\nabla}^{\dot\alpha}\overline{J}]_D + 3([\Lambda(J-\mathcal{R})]_F + \text{h.c.}). \tag{6.162}$$

Now, making the redefinitions (6.106), the theory (6.162) becomes

$$\begin{aligned}\mathcal{L} \;&=\; -3[S_0\overline{S}_0(\mathcal{T}+\overline{\mathcal{T}}-\mathcal{C}\overline{\mathcal{C}})]_D + 3(\sqrt{\lambda_1})^{-1}([\mathcal{C}\,(\mathcal{T}-\frac{1}{2})S_0^3]_F + \text{h.c.})\\ &+\; \frac{\xi}{\lambda_1^2}[(S_0\overline{S}_0)^{-2}\nabla^\alpha(S_0\mathcal{C})\nabla_\alpha(S_0\mathcal{C})\overline{\nabla}_{\dot\alpha}(\overline{S}_0\overline{\mathcal{C}})\overline{\nabla}^{\dot\alpha}(\overline{S}_0\overline{\mathcal{C}})]_D.\end{aligned} \tag{6.163}$$

Again, by gauge fixing $S_0 = 1$ we go to the old-minimal supergravity, and the Lagrangian (6.163) is written as

$$\begin{aligned}\mathcal{L} \;&=\; \int \mathrm{d}^2\Theta\, 2\,\mathcal{E}\left\{\frac{3}{8}\Big(\overline{\mathcal{D}\mathcal{D}}-8\mathcal{R}\Big)e^{-K/3} + W\right\} + \text{h.c.}\\ &+\; \frac{\xi}{\lambda_1^2}\int \mathrm{d}^2\Theta\, 2\,\mathcal{E}\left\{\left(-\frac{1}{8}\right)\Big(\overline{\mathcal{D}\mathcal{D}}-8\mathcal{R}\Big)\mathcal{D}^\alpha\mathcal{C}\mathcal{D}_\alpha\mathcal{C}\overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{C}}\;\overline{\mathcal{D}}^{\dot\alpha}\overline{\mathcal{C}}\right\} + \text{h.c.},\end{aligned} \tag{6.164}$$

with Kähler potential

$$K = -3\ln\left(\mathcal{T}+\overline{\mathcal{T}}-\mathcal{C}\overline{\mathcal{C}}\right) \tag{6.165}$$

and superpotential

$$W = \frac{3}{\sqrt{\lambda_1}}\mathcal{C}\left(\mathcal{T}-\frac{1}{2}\right). \tag{6.166}$$

Theories of the form (6.164) have been discussed in Ref. [43] and more recently extensivelly studied in [82,83,133,135,136,176]. After integrating out the auxiliary fields (except F_c, the auxiliary field of the $\mathcal{C}$ superfield), and performing the rescalings, the Lagrangian becomes

$$e^{-1}\mathcal{L} \;=\; \frac{1}{2}R - K_{i\bar{j}}\partial_m z^i\partial^m\overline{z}^{\bar{j}} + \frac{16\xi}{\lambda_1^2}\partial_m C\partial^m C\partial_n\overline{C}\partial^n\overline{C} - V_T + \mathcal{L}_{F_c}, \tag{6.167}$$

with

$$\mathcal{L}_{F_c} = \mathcal{A}F_c + \overline{\mathcal{A}}\,\overline{F}_c + \mathcal{B}F_c\overline{F}_c + \mathcal{S}(F_c\overline{F}_c)^2, \tag{6.168}$$

$$V_T = e^K \frac{1}{K_{T\overline{T}}} D_T W D_{\overline{T}}\overline{W} - 3e^K W\overline{W} \tag{6.169}$$

and

$$\mathcal{A} = e^{2K/3}\frac{K_{C\overline{T}}}{K_{T\overline{T}}} D_T W - e^{2K/3} D_C W, \tag{6.170}$$

$$\mathcal{B} = e^{K/3}K_{C\overline{C}} - e^{K/3}\frac{K_{T\overline{C}}K_{C\overline{T}}}{K_{T\overline{T}}} - \frac{32\xi}{\lambda_1^2}e^{K/3}\partial^m C\partial_m\overline{C}, \tag{6.171}$$

$$\mathcal{S} = \frac{16\xi}{\lambda_1^2}e^{2K/3}. \tag{6.172}$$

The equations of motion for F_c are

$$0 = \mathcal{A} + \mathcal{B}\overline{F}_c + 2\mathcal{S}F_c\overline{F}_c^2 \quad , \quad 0 = \overline{\mathcal{A}} + \mathcal{B}F_c + 2\mathcal{S}\overline{F}_c F_c^2, \tag{6.173}$$

which can be combined into the single equation

$$\alpha = X(1+\beta X)^2, \tag{6.174}$$

where

$$\alpha = \frac{\mathcal{A}\overline{\mathcal{A}}}{\mathcal{B}^2}, \tag{6.175}$$

$$\beta = \frac{2\mathcal{S}}{\mathcal{B}}, \tag{6.176}$$

$$X = F_c\overline{F}_c. \tag{6.177}$$

The solution to the above equation is then easily found to be

$$X = \frac{2}{3\beta}\big(\cosh m - 1\big), \tag{6.178}$$

with

$$m = \frac{1}{3}\,\mathrm{arccosh}\left(\frac{27}{2}\alpha\beta + 1\right). \tag{6.179}$$

The final scalar potential will have the following compact form

$$V_F = \mathcal{B}X + 3\mathcal{S}X^2 + V_T. \tag{6.180}$$

To study the implications of the corrections on the inflaton potential we look again at the minimum $C = \bar{C} = b = 0$ with the redefinition (6.114). The inflaton field ϕ will now have a potential

$$V_F = \frac{3e^{-4\phi/\sqrt{3}}}{8s}\cosh\frac{u}{3}\left(\sinh\frac{u}{6}\right)^2, \tag{6.181}$$

with

$$u = \text{arccosh}\left\{1 + 36\Big(-1 + e^{2\phi/\sqrt{3}}\Big)^2 s\right\}, \quad s = \frac{\xi}{\lambda_1^3}. \tag{6.182}$$

The potential (6.181) has been plotted in Fig. 2 for various values of the parameter s. If $s = 0$ (i.e, $\xi = 0$), the potential has a plateau for large positive values of ϕ. For non-zero values of s, the plateau is restricted to smaller regions of the scalar field and, like for the new-minimal version, it disappears for larger values of s with a fall-off $V_F \sim e^{-4\phi/\sqrt{3}}$ after the plateau.

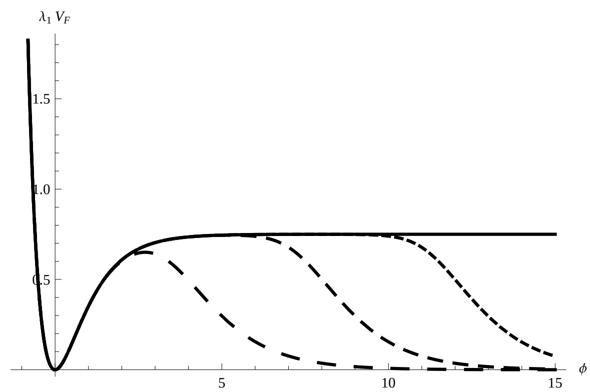

Figure 6.2: The scalar potential for three different values of $s = \frac{\xi}{\lambda_1^3}$: *i*) $s = 10^{-4}$ (long dashed line), *ii*) $s = 10^{-8}$ (continuous line) and *iii*) $s = 10^{-12}$ (short dashed line). The horizontal line corresponds to $\xi = 0$.

6.4 Summary

Galilean invariant theories have attracted a huge attention lately. One of the most striking properties is that their suppression scales do not (quantum mechanically) run with energy. Using the superspace formalism, one would already guess that, if the projected theory onto the real space should be Galilean invariant, in superspace, this symmetry must be incorporated into a larger symmetry. Indeed, we showed that a Galilean theory must be embedded into a super-space Lagrangian (i.e. before projection to real space) enjoying the super-space Galilean shift symmetry

$$\Phi \to \Phi + c + b_m y^m \tag{6.183}$$

where

$$y^m = x^m + i\theta\sigma^m\bar{\theta}\ , \tag{6.184}$$

and where Φ is the Galilean chiral superfield. Note that the super-Galilean shift (6.183), in components, only shifts the scalar π.

The way we found our supersymmetric Galilean Lagrangian was however somehow indirect.

Inspired by the result of [107] showing that the complex Galilean Lagrangians may be found as a decoupling limit ($M_p \to \infty$ but $\Lambda = M^2 M_p$ finite) of

$$\mathcal{L}_{\text{slotheon}} = \frac{1}{2}\left[M_p^2 R - \frac{G^{mn}}{2M^2}\partial_m\pi\partial_n\bar{\pi}\right]\ , \tag{6.185}$$

we used the supergravity extension of the theory (6.185) found in [84] to obtain our supersymmetric Galilean Lagrangian (6.52). Thus, the theory [84] is the supergravity extension of Galilean theories, i.e. the covariant super-Galilean theory.

We would like to conclude by noticing that the theory [84] could only be found in the new-minimal supergravity formalism which requires a conservation of R-charge. In particular it turned out that the chiral superfield could only have vanishing R-charge. In the decoupling limit this is perfectly consistent with the Galilean shift (6.183). In fact, the super Galilean shift has vanishing R-charge and therefore it can only be applied to superfields with vanishing R-charge as well. Thus, in order to have a consistent R-invariant theory, the super-Galileon must have vanishing R-charge, as required by the supergravity extension.

This observation may also be related to the statement of [140] that cubic super-Galilean theories cannot be constructed out of chiral superfields. It seems, as already stated, that super-Galilean theories should be R-symmetry invariant. If the chiral superfield has non-zero charge under the R-symmetry, then, the cubic (and the quintic) Galilean theory cannot exist [196]. Therefore, the only possibility for the existence of a cubic and quintic super-Galilean theory out of a chiral superfield, is that the chiral superfield has vanishing chiral weight. In [107], it has been shown that the cubic Galilean theory can be obtained as a decoupling limit of a theory containing both the "Slotheonic door" $G^{\alpha\beta}\partial_\alpha\pi\partial_\beta\pi$ and the conformal coupling πR. However, it turns out that the two terms cannot coexist in the new-minimal supergravity formalism, as, the first would requires a vanishing R-charge contrary to the second. Thus, the cubic super-Galilean cannot be obtained as a decoupling limit of a supergravity theory coupled to chiral superfields.

The quintic Galileon is instead more mysterious. In [107] no consistent decoupling limit has been found such to lead to the quintic Galileon. Although this is not a proof it is tempting to conjecture that no super-Galilean theories exist for odd number of chiral superfields in the Lagrangian. However, we leave the proof of this conjecture for the future.

We then turn to the full theory of supergravity. The supergravity extension of the non-derivatively coupled theories such as $\mathcal{L}_{III}$ has been already constructed in the literature [93]. However, non-minimal derivative coupled supergravities to matter fields, without extra propagating modes, are restricted to the Gauss-Bonnet interactions $\mathcal{L}_I$. Here we focused on the supersymmetrization of the non-minimal derivative coupled Lagrangian, $\mathcal{L}_{II}$. This was achieved in the framework of new-minimal supergravity by employing a chiral multiplet and the linear curvature multiplet.

A theory described by (6.86) or, more generically, (6.90), may have many phenomenological interesting properties. The first one is that, each time a domain wall is present in the theory, dependently upon the scale w, the scalar field gets dynamically localized around the domain wall itself [109]. In fact, one may consider $\mathcal{L}_{II}$ as a field theoretical realization of the quasi-localization mechanism of [75]. A second, perhaps more important, phenomenological aspect is related to Inflation. Whenever the background Einstein tensor is larger than the mass scale w^{-2}, no matter what potential is driving A, Inflation is naturally produced without exceeding the perturbative cut-off scale of the theory, which is below the Planck scale as it should be for a ghost-free theory [76]. This is due to an enhanced gravitational friction acting on the evolving scalar field and sourced by the Universe expansion itself [103–106, 108]. We therefore believe that the supersymmetrization of the $\mathcal{L}_{II}$ might open new possibilities for exploring inflation in supergravity/string theory. For more applications of the non-minimal derivative coupling in cosmology, black hole physics and condenced matter physics see for example [7, 56, 57, 141, 142, 150].

In the final section we have discussed the embedding of the Starobinsky model of inflation within $\mathcal{N} = 1$ supergravity. We have shown that the Starobinsky model can be derived from the new-minimal supergravity, where a linear compensator superfield is employed. The Starobinsky model becomes equivalent to standard supergravity coupled to a massive vector multiplet whose lowest scalar component plays the role of the inflaton and the vacuum energy is provided by a D-term potential. We have subsequently investigated the robustness of the model against higher-order corrections allowed by the symmetries and concluded that they may represent a threat to the success of the model as they may destroy the flatness of the potential. This is true both in the old- and in the new-minimal formulation. In this sense, the Starobinsky model suffers from the same difficulty one encounters when trying to embed a model of inflation in supersymmetry where the flatness of the potential is easily destroyed by supergravity corrections [160].

Chapter 7

Concluding Remarks

It has been 40 years since the discovery of supersymmetry. Nevertheless, no experimental evidence has been found until now. But let us be patient, even the Higgs scalar, the fundamental constituent of the standard model, predicted some 50 years ago, was just discovered at the LHC. It is the next round of LHC data that will shed further light in the quest for finding the supersymmetric partners of the standard model particles. One has to be alerted though: If indeed supergravity is the low energy limit of the superstring, then there is no reason found until now (from the string point of view) to have a low SUSY breaking scale. Research on supersymmetry and supergravity has to be carried out, the major discoveries have been made, but there is still many open questions waiting to be answered. Indeed, new observational data from the PLANCK satellite bring inflationary cosmology to the forefront.

In this dissertation we have presented progress on the most important subjects in supersymmetry and supergravity: Supersymmetry breaking, supersymmetry and particle physics, supergravity and cosmology. These subjects where treated rather technically, but stimulating phenomenological input was given when needed. We have first presented new methods for supersymmetry breaking which rely on higher dimensional operators, contrary to the common known models of SUSY breaking used as a hidden sector. We discussed the possibility of a single-Higgs MSSM, where we found an emergent hierarchy for the heavy fermion masses and moreover an intriguing modification to the Higgs potential allowing better fitting to the observed Higgs particle mass. We have worked out consistent models of higher derivative supersymmetry and supergravity, and pointed out their relevance to inflationary cosmology. Of course many questions are left open which we hope to address in the future.

Finally, whatever the experimental evidence, the theoretical elegance of supergravity is definitely a reason behind the fact that theorists are not willing yet to give up hope of its relevance to nature; or in the words of P. van Nieuwenhuizen on Supergravity: ”It is the most beautiful gauge theory known, so beautiful, in fact, that Nature should be aware of it!”

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