TOWARDS ROBUST BILATERAL TELEOPERATION WITH NONLINEAR ENVIRONMENTS VIA MULTIPLIER APPROACH

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Harun Tugal

School of Electrical and Electronic Engineering

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Abbreviations

DOF	Degree of Freedom
FDI	Frequency Dependent Inequality
IMC	Internal Model Control
IQC	Integral Quadratic Constraint
KYP	Kalman Yakubovich Popov
LMI	Linear Matrix Inequality
LTI	Linear Time Invariant
LQG	Linear Quadratic Gaussian
MPC	Model Predictive Controller
P	Position
PD	Position Derivative
P-F	Position-Force
P-FF	Position-Force Force
PD-F	Position Derivative-Force
PD-FF	Position Derivative-Force Force
RNN	Recurrent Neural Network
SISO	Single Input Single Output
QC	Quadratic Constraint
QS	Quadratic Separation
UDP	User Data Protocol

Abstract

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This thesis provides moderate stability conditions for bilateral teleoperation by exploiting the advantages of the Zames-Falb multipliers within the integral quadratic constraint framework, where the environment can be defined as a memoryless, bounded, and monotonic nonlinear operator. Recent advances in multiplier theory for appropriate classes of uncertainties/nonlinearities are applied. Because the classes of multipliers have infinite dimension, parametrization of these multipliers is used to obtain convex searches over a finite number of parameters such that an asymmetric Zames-Falb multiplier search is proposed.

The stability of the system is analysed as a Lur'e system containing time-delay and monotone bounded nonlinearity. As a result, (less) conservative (than typical) delay-dependent stability conditions can be developed. Performance of the results is initially evaluated with case studies based on the numerical examples from the neural networks while using Kalman conjecture as a benchmark. Also, a geometrically intuitive stability analysis approach is provided to show when the Kalman conjecture is true for the time delayed Lur'e interconnections. Thus, one can show that it is possible to find a multiplier for a slope bound equivalent to the Nyquist value without constructing the suitable multiplier by revisiting classical results in clockwise properties of the plants with time delay.

Then, these results are applied to the bilateral teleoperation. Finally, stability conditions are tested with different control architectures and experimentations; in particular, bilateral teleoperation experiments over the internet between Manchester, UK, and Vigo, Spain, are carried out. The advantage of the proposed approach is demonstrated by reaching higher transparency indices for a two-channel position-force and for three-channel bilateral teleoperation architectures while ensuring the absolute stability with nonlinear environments.

Declaration

No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Copyright Statement

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who never cares about anything but us.

Notations and Symbols

$\mathbb{R}$	field of real numbers
$\mathbb{R}_+$	set of non-negative real numbers $\{x \in \mathbb{R} : x \geq 0\}$
$\mathbb{R}^n$	linear space of ordered n-tuples in $\mathbb{R}$
$\mathbb{R}^{n\times m}$	ring of matrices with n rows and m columns with elements in $\mathbb{R}$
$\mathbb{C}$	field of complex numbers
$(a,b]$	interval: $\{x \in \mathbb{R} : a < x \leq b\}$
$C(\Upsilon, \lvert\Upsilon\rvert)$	circle constructed by rational function Υ with radius equal to $\lvert\Upsilon\rvert$
$\in$	belongs to
$\forall$	for all
$\approx$	approximately equal
$\Re\{a\}$	real part of a
$\Im\{a\}$	imaginary part of a
$\lvert a\rvert$	absolute value of a
A^{-1}	inverse of matrix A
$A^\top$	transpose of A
A^*	complex conjugate transpose of A
$A^\perp$	a full rank matrix whose columns span the null-space of A
$\det(A)$	determinant of A
$\mathcal{L}_p^m[0,\infty)$	Hilbert space of integrable and Lebesgue measurable functions $f : [0,\infty) \to \mathbb{R}^m$
$\langle x, y\rangle$	inner product of $x, y \in \mathcal{L}_p$
$\mathcal{H}$	inner product function space $\{f : \mathcal{T} \to \mathcal{V} : \langle f, f\rangle < \infty\}$
f_T	truncation of the function f at T
$\mathbb{P}_T$	the truncation operator
$\mathcal{L}_{pe}^m$	extended $\mathcal{L}_p^m$ space

$\|f\|_p$	$\mathcal{L}_p$-norm of a Lebesgue integrable function $f(\cdot)$
$\|f\|_\infty$	the least upper bound of the absolute value, $\|f\|_\infty = \sup_t \|f(t)\|$
$\hat{f}(j\omega)$	Fourier Transform of the function $f(\cdot)$
$\mathbf{RH}_\infty$	the space of all proper real rational transfer functions such that all their poles have strictly negative real parts
ϕ	static memoryless uncertainty/nonlinearity
$(\star)$	a space holder for the right outer factor of a quadratic form such that $(\star)^* M\Phi G = G^*\Phi^* M\Phi G$.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	state space representation of $G(s) = C(sI - A)^{-1}B + D$

1 Introduction

The technology that we have today is a result of human persistence in following a dream and desire to make real what was simply in the imagination. Unarguably, overcoming our bodies' physical capabilities has been one of those dreams for decades. Today, many technological improvements empower us to extend our physical limitations. Conducting an intercontinental phone call or watching events on the television that are occurring around the world have become standard parts of our daily lives. Electro-mechanical devices have been developed that not only are capable of transmitting auditory and visual information but also allow us to manipulate objects remotely. Commonly, all the names of these devices contain *Tele* as a prefix. *Tele* is a Greek word meaning 'at a distance', and not surprisingly, the gadget used to manipulate objects remotely is called a *teleoperator*.

Teleoperation is the act of remotely carrying out a task by using robotic manipulators that communicate via a communication medium such as the Internet. In a typical teleoperation control system, reference position information generated by a local manipulator, called the master, is transmitted to a distant robot, known as the slave, and the operator conducts the task by relying solely on the visual information delivered from the distant side by a camera. Conversely, in a *bilateral teleoperation* control system, which is the main topic of this thesis, in addition to the visual information position of the distant robot, a slave, or task interaction force is transmitted to the operator side so that, with provided haptic information, the situational awareness of the operator is increased.

Definition 1.1 (Haptic perception [1])**.** *Haptic defines the sense of touch and can also be related to other types of sensation, such as temperature; in this thesis, we will refer to it simply in relation to mechanical interactions such as grasping.*

This additional feedback is crucial in tasks requiring delicate actions such as

telesurgery [2], which require more steps than simply picking and placing, and in tasks where the environment is hazardous, such as space missions, undersea exploration, and nuclear decommissioning [3]. To illustrate the importance of this technology, some of the pioneering application areas are going to be heedfully presented in the subsequent section.

1.1 Teleoperation Systems and Applications

This section briefly introduces the definition, components, and application areas of *teleoperation systems*. *Teleoperation* is a combination of two words: *tele* and *operation*. The remote control systems that are commonly used in the academic research are an example of Teleoperation systems. As a system, the *teleoperator* enables a human to move, sense (if possible), and mechanically manipulate objects from a distance [4]. Any mechanism that enables a human to perform tasks over a distance can be called a teleoperator.

As an electromechanical device, a robot can interact with its surroundings based on the actuators and different types of sensors on the device. A robot can be fully autonomous (acting on its own) or semi-autonomous, with some degree of autonomy. Types of robots vary from humanoid robots with self-intelligence, such as Asimo[1], to industrial robots of different sizes and shapes that conduct work based on predetermined tasks. A manipulator used within the teleoperator might be like a humanoid robot; serial links may be mounted on a robotic body with a camera providing visual information, or an industrial manipulator that looks like a mechanical arm may be used. The level and type of the autonomy, however, differentiate a robot from a teleoperator which has a variety of control algorithms, from remote operation to supervisory control.

However, despite the necessity of using robots for working environments that are hazardous for humans, fully autonomous robots cannot be implemented for all tasks because their capabilities have not yet reached the intelligence and dexterity of humans, and is not expected to in the near future. Additionally, there exist a great number of tasks for which it is not possible nor plausible to pre-plan what needs to be carried out; the task itself might depend on spontaneous decisions which are im-

[1]The Honda's Humanoid Robot

possible to be made with any artificial intelligence that exists currently. Therefore, it can be said that the teleoperation system combines advantages of using electromechanical manipulators and robots with human intelligence within the same architecture. Also, expanding on the human capabilities increases the safety and enhances the task quality. These conveniences paved the way for using teleoperators in many different operation areas; in fact, for some applications they became the key element for success. The following sections highlight some of the main application areas of teleoperation.

1.1.1 Space

Needless to say, environmental conditions in outer space are extremely dangerous and hostile for any living creature on Earth, and particularly for human beings. Conditions are such that an astronaut cannot survive in outer space more than 30 s without a spacesuit, as losing consciousness may occur within less than 15 s due to lack of oxygen. In addition, body fluids and blood start to evaporate and then freeze due to lack of air pressure and heat. See [5] for detailed information regarding the hazardous conditions of a low pressure medium. Furthermore, extreme temperatures occur based on exposure to solar radiation; it can reach 120 °C in sunlight and −100 °C in shade.

Despite these hazardous conditions, space walks are necessary for maintenance of spacecraft such as the International Space Station (ISS) and for exploration missions over extraterrestrial areas. Thus, special suits have been designed for astronauts to protect them during their necessary space walks. These suits must create enough pressure to keep body fluid in a liquid state, have sufficient oxygen for breathing, provide heat isolation from extreme outside temperatures, and contain radiation protection, as cosmic radiation is deadly for any living creature. No matter how functional the suit is, extraterrestrial missions remain hazardous. Despite the necessity of wearing a spacesuit, it reduces the operator's functionality, particularly the dexterity of the human hands that is crucial during maintenance of the spacecraft. In addition to these concerns, when considering the restrictions on control and artificial intelligence that today's fully autonomous robots have, researchers have focused on designing a locally autonomous space robot system that can be con-

trolled by an operator [4].

ROTEX[2] was one of the first teleoperated robots to be designed for space missions. It was a multi-sensory (force-torque sensors, laser range finders, stereo cameras, etc.), six-degree-of-freedom (DOF) serial link manipulator that could be teleoperated with the help of video cameras from a nearby spacecraft. In addition, it could be controlled from the ground control unit on Earth with a special operating mode known as predictive computer graphics (to reduce detrimental effects of time delay) [6,7]. Mainly, the robot had three tasks: connecting-disconnecting mechanical or electrical components from the spacecraft, which is necessary for maintenance; grasping floating objects (might be useful for space junk clean-up); and assembling a simple mechanical structure. Despite the number of the sensors that the robot had, feedback to the human for on-line operation was provided only by a video camera.

ETS-VII[3] was an orbiter that had two robotic arms to be used for space experiments, and which were used particularly for rendezvous docking[4]. As the satellite was unmanned, the robotic arms were controlled (teleoperated) from a control station located on Earth. A communication satellite was used to transfer data between the ground control unit and the satellite. Due to the time delay existing in long-distance communication, predictive computer graphics were used for on-line controlling of the robotic arms. In addition, a test-bed, which was located on the ground and was similar to the robotic system in space, was used to evaluate design of the teleoperation system [8].

ROKVISS was another robotic system designed and used in space missions. It was mounted on the outside of the International Space Station (ISS) to complete and evaluate various experiments. The experiments were conducted using a small robot that had two controlled joints. This robot was different, in terms of design and control perspectives, from the previously used space robots in two aspects: it had a fully autonomous control system (due to the time limit for direct link experiments), and in teleoperation mode it was able to supply force feedback to the operator to achieve a level of telepresence. In teleoperation mode, video images of the robot's joint angles and torque values were fed back to the human operator. A force feedback control

[2]Robot Technology EXperiment
[3]Engineering Test Satellite No.7
[4]An orbital maneuver; connecting two space-crafts

algorithm was used for this experiment, yet the maximum communication time delay was restricted to 0.5 s. Due to this limitation, the teleoperation mode was used only when ISS was flying over the ground station (over-flight situation). In other circumstances, communication satellites were required to be used; in such circumstances the communication delay would exceed the human capability[5] to carry out a task [9]. Ultimately, ROKVISS had two main objectives: testing the usefulness of lightweight robotic arms and verification of the direct teleoperation with force feedback. The latter process was essential for satellite maintenance tasks that required precision.

A number of different robotic systems have been designed, tested, and used in space missions, particularly for maintenance of the ISS. Among them is *Canadarm*, which is a six-DOF robotic arm that has been used on more than 50 space missions. These early designed robots and experiments exposed the importance and usefulness of teleoperation systems in space missions. However, some challenges remain to be resolved in teleoperation of space robots, including communication time delay, limited bandwidth, and limitation of on-board computational systems. In addition, these robots must be highly reliable and able to operate precisely. Consequently, precise analysing, testing, and accurate modelling are essential to validate them on the ground before dispatching them to space.

1.1.2 Undersea

The history of mankind is full of scientific achievements and exploration stories brought about by our intelligence and curiosity. We were able to send spacecraft and fully autonomous high-tech robots deep into space or to distant planets. Massive telescopes such as the Hubble Space Telescope (HST) were built for use in exploration of the cosmos. Despite these strides, the least known places in the universe may be located on our own planet; they are the oceans' depths and deep-sea bottoms. It has been claimed that more is known about the Moon than the deepest part of the oceans [10]. Of course, the reason is not our lack of interest, it is the challenges and conditions that welcome us as we go deeper into the water. For instance, sunlight cannot illuminate and penetrate deep into the water because it is scattered

[5]Consider the level of frustration experienced in conducting such a task using a sluggish computer

while travelling through the water. Light might be detectable at 1,000 m into the ocean, yet it is rare to have significant visibility after 200 m [11]. Second, pressure increases by 1 atmosphere every 10 m toward the bottom of the ocean. The deepest point of the ocean is measured as approximately 10,000 m and, for comparison, the height of Mount Everest is approximately 9,000 m above sea level. Due to the extreme pressure, it is impossible for a human to go to the bottom of the ocean, even with special devices. That is why deep water exploration submarines generally are unmanned and remotely operated vehicles, and are operated from the control unit on a ship at the surface of the sea.

Despite these hazardous environmental conditions, deep locations in the oceans must be reached for scientific and commercial (oil and gas) purposes. In the offshore oil industry, remote undersea vehicles which have two robotic arms are used for maintenance and material handling [12,13]. In some situations, high dexterity is required for undersea material handling. For instance, recovering cockpit voice and flight data recorders, known as 'black-boxes', is a process that must be carried out meticulously for crash investigations. An example of this is the Air France Flight 447 data recovery. Therefore, it is essential to have robust and reliable bilateral teleoperation systems for underwater tasks and explorations. Many submarines which have robotic manipulators were designed and used for these purposes. An example is the Jason undersea vehicle operated by Woods Hole Oceanographic Institution [4]. Research on teleoperated sub-sea vehicles is increasing due to the usefulness of having such robots for marine experiments, geological exploration, and material recovery.

1.1.3 Nuclear Decommissioning and Toxic Material Handling

Discovery of the relationship between mass and energy changed the world in many ways and brought a new concept to our daily lives: nuclear energy. When uranium atoms split, a huge amount of energy releases to the medium. In nuclear power plants, this released energy is used for generating pressurised steam which is used for turning generator shafts and subsequently leads to electricity generation. Today, nuclear energy supplies more than 15% of the world's electricity consumption. This ratio increases dramatically with the rise in energy demand, particularly in countries where this ratio is more severe; France produces over 75%, and U.S derives nearly

20% of their electricity from nuclear energy [14]. In military (no need to include weapons of mass destruction), nuclear energy is used in submarines and gigantic aircraft carriers, which can sail more than 20 years without refuelling.

However, there are high risks in nuclear fission as an energy source, and it is generally described in two ways: radio-toxicity and accessibility. The former describes the damage level of the radiation, which can be exemplified as unseen bullets to the human body. The latter describes the amount of time that the body stayed under the influence of the radiation during transportation, assembling, etc. [15]. The aim of the research in that area is to reduce the accessibility level, because after discharge, the level of radio-toxicity remains high for many years (thousands), so accessibility is crucial for preventing damage from these hazardous materials. For this reason, the hazardous toxic materials generally are reserved in isolated capsules to prevent accessibility. No matter how hazardous the materials are, the problem would be solved as long as it is not accessible to anyone or anything. In some circumstances, the solution might not be that simple, as in the example of the reactor at the Chernobyl nuclear power plant. Reactor number 4 was covered with a 2-meter-thick concrete wall to prevent radiation emission to the environment. But construction workers, fire fighters, military officers, and other people who worked on the region after the disaster were exposed to high levels of radiation that caused the death of many. Therefore, for such situations, a teleoperation system is essential and must be used with precision to separate humans from the hazardous environments. Another example is the more recent disaster in Fukushima. There, remote-controlled robots were sent to the core of the plant to carry out some tasks after the incident because the radiation level was destructive for humans.

Furthermore, each nuclear power plant has a lifetime, and when it completes its life cycle, it must be dissembled so the occupied place can be used for other purposes. The number of power plants that reach their expiration date and require decommissioning has increased gradually as commercial nuclear energy began in the mid 1950s. In the U.K., for instance, decommissioning of 25 nuclear power plants has already begun. It is extremely technical, as during the process all hazardous radioactive materials must be cleaned up completely and precisely. Robot technology is necessary, as radiation is dangerous for any living creatures, yet due to the complexity of the process, fully autonomous robots still are not suitable. However,

teleoperated robots are expected to have a great impact on this application area; thus, a number of teleoperation systems have been designed for decommissioning purposes [16]. Additionally, regulation of nuclear toxic material storage for military use is not as restricted as it is for commercial nuclear power plants. Therefore, for many years radioactive disposals of the military were buried in deserts within capsules. Unfortunately, these capsules were faced with corrosion after many decades and started to leak through the underground water sources, particularly in the U.S. That, of course, is a major problem and might only be solved with a teleoperation system for transportation of massive amounts of heavy toxic materials.

1.1.4 Telesurgery and Telediagnosis

Today, technological improvements and discoveries have made a huge effect on medical sciences. Diagnostic methods for illnesses have improved and accuracy has increased dramatically with the help of electronic devices such as X-rays, magnetic resonance imaging (MRI), and ultrasound imaging. More information has become available and accessible. Despite the adverse effects that some of them have, nobody questions their value, as their usefulness has been shown in different aspects. Another revolution has come about with telesurgery and the use of robotic systems in medicine. These devices increase the performance level of operations beyond the human limitations in terms of dexterity and scale. For instance, a surgeon's hand vibrates, which reduces the quality of the operation. This will not be an issue with robotic surgery, as the vibration is eliminated via filters.

Moreover, minimally invasive surgery decreases the level of trauma drastically and enables the patient to return to the activities of daily life as soon as one day after the operation. The revolution, of course, brought some challenges to doctors or surgeons, as rather than looking directly at the anatomy of the patients, they need to perform their jobs via a computer screen; in addition, they lack the tactile sensation that is vital for special operations [17]. For instance, laparoscopic surgery was the first application for which special types of robotic devices such as the da Vinci robot were designed and used, see Figure 1.1.

With this type of surgery, hand movements of the surgeon are transmitted to the apparatus via electromechanical devices. This paved the way for distance surgery,

in which the patient and surgeon do not need to be in the same room or even in the same country, as in [2]. The impact of this technology might be beyond our estimations. The number of experts that can perform specialized operations is limited, so the ability for the surgeons to conduct different operations, when needed, at opposite sides of the world, without actually being there, will be valuable. However, a number of challenges exist for researchers and designers to overcome; one challenge is latency. Time delay that occurs during communication must be limited due to performance limitations of the mechanical system as well as the human [17]. It has been stated that if the total communication time is more than 1 s, performance of the system decreases drastically, and beyond that the operation is not at all realizable in terms of the human point of view . An acceptable time limit of approximately 350 ms has been established for safety of the overall operation [2].

Figure 1.1: Telesurgery between US (surgeon) and France (patient) (Image is taken from [2])

As robots began to appear in operating rooms, new opportunities and developments arose, and new challenges appeared for engineers and surgeons as well. Many researchers have worked to bring tactile sensations into telesurgery and also to improve robustness against latency.

The reader might ask why, if teleoperation has such wide application areas and resolves many fundamental problems while separating humans from the hazardous mediums, we still sending people to outer space or to nuclear facilities in critical states. The answer will be provided upon solving the indispensable problems that bilateral teleoperation has: modelling dynamics of the human and environment, as they are part of the interconnected system, and providing robustness against com-

munication time delay. In other words, absolute stability, which is discussed rigorously in other sections, must be provided. However, we can state that some of the mentioned technologies already have begun to play a role in the 21st century, and others need more time for improvements. Here, we attempt to show some other methods that can be followed while solving chronical issues in bilateral teleoperation. But before moving on to that, let us briefly discuss some current meritorious efforts in the field of bilateral teleoperation.

1.2 Brief Current Scope of the Related Work

The design aim of a bilateral teleoperation is to include a kinaesthetic feedback channel so that performance and success rate are increased while operational cost and time are decreased. A bilateral teleoperation needs to fulfill two key criteria: absolute stability, which means that functionality must be held for any possible human and environment pairs; and transparency, which means that the environmental interaction forces or the impedances must be transmitted well to the operator side. Unfortunately, a trade-off exists between these two measures; namely, it is impossible to obtain an ideally transparent operation, where the operator feels only task impedance and absolutely stable bilateral teleoperation [18]. With highly variable uncertainties, the overall control design process transforms into a challenging task. Over the years, many efforts have been made to overcome these obstacles that prevent a physically realisable high-performance teleoperation system. Early research on the topic primarily used network theory due to the lack of today's computational power and the simplicity of the theory, and passivity theory became the main tool used by many researchers [19–22].

Passivity is an energy-related phenomenon; a system is called passive if the energy injected into its input ports is higher than the energy extracted from its output ports [23]. Passivity is independent from the model parameters[6], which makes it appropriate for teleoperation, as designs contain highly variable uncertainties; human and environment. Therefore, the stability in a bilateral teleoperation is carried out by transforming the overall system into a network consisting of one port for

[6]The effect of a substantial time delay on the passivity will be discussed in the forthcoming sections.

the passive human-environment pair connected to two ports for the network, containing manipulator and controller dynamics, etc. With the passivity assumption of the operators, if we can find a way to guarantee that the two-port network is also passive, then stability of the interconnection is ensured. Llewellyn's stability criterion proposes certain conditions [24] while ensuring stability of the interconnection, but in addition to the passivity uncertainties, the assumption of linear time invariant (LTI) operators must be made. These conveniences, model avoidance and linearity, come with a price: conservatism of the design, as it needs to be robust against a wide range of uncertainty classes such that stability of the system with a simple PD-controller may not be guaranteed [25, 26].

To reduce conservatism, the definition of uncertainties was restricted such that stability was required for an upper-bounded environment instead of the full class, and reasonable performance specifications were obtained [27, 28]. For instance, defining the environment as a set of models consist of mass, spring, and damper with upper bounded parameters' constants is a frequently applied technique. Those results indicate how descriptions about uncertainties affect the overall performance, yet present stability analysis methods are restricting us to extend or outperform the current definitions.

Polat and Scherer [29] showed that stability analysis of teleoperation can be conducted by using integral quadratic constraints (IQCs). Stability is transformed to convex search for suitable passive multipliers that ensure frequency dependent inequalities by using equivalent linear matrix inequalities (LMI); for more detailed information, we refer the reader to [30]. Those results show that methods such as multipliers for robust control to analyse stability against a class of uncertainty can be applied to bilateral teleoperation as well. Furthermore, possible application of the IQC methodology within the inherently non passive systems (particularly in teleoperation) also have been highlighted in [31]. That implies, while analysing absolute stability of the bilateral teleoperation IQC framework can be practiced alongside with Multiplier theory to extend the definition of the troublesome perturbations such as human and environment without including any additional conservatism within the analyses that current approaches have. In comparison, the Zames-Falb multiplier [32, 33] is attracting significant attention, courtesy of today's computational power. This class of multipliers is used to analyse stability against slope-

restricted monotonic nonlinear uncertainties [34–38] which might exists within the bilateral teleoperation architecture.

Apart from that, in bilateral teleoperation, if there exists a physical distance between two manipulators, the time delay phenomenon, which can hinder the performance and also destabilize the overall system, emerges [39]. A number of efforts have been made to improve performance and recover stability of teleoperation while the communication channel accommodates a constant or time varying delay: wave variables and scattering transformation paved the way for stable, yet conservative, time-delayed teleoperation by making the communication channel passive [40–44]. Less conservative results also were obtained with the controller's passivity property [45, 46]. Techniques that analyse stability against time delay are divided into two categories: delay-independent, where the stability margin is independent from the delay itself, and delay-dependent, where the system is stable against a pre-determined maximum time delay. By considering the possible allowable time delay, more useful results are obtained for the practical applications.

In this thesis, we have tried to show, in brief, that a theoretical framework, which is first implemented for bilateral teleoperation by Polat and Scherer [29], can provide a sufficient solution to the aforementioned challenges and problems in the stability analysis of bilateral teleoperation. IQC framework is not only allowing us to analyse different classes of uncertainties within the same schema but also leading us to less conservative stability condition in bilateral teleoperation by providing combination of the different multiplier sets. Although the proposed methodology has a wide range of application areas, some of which are described here, we will focus our attention on bilateral teleoperation.

1.3 Objectives and Contributions

The objective of this thesis is twofold. The first is to engage the theoretical framework of the IQC approach with a useful tractable computational search for a particular class of multipliers (e.g Zames-Falb multipliers) so that stability analysis of the systems that contain a distinct class of perturbations, such as delay or slope restricted nonlinearity, can be carried out with ease. By virtue of the proposed technique, the second is to carry out stability analysis of the bilateral teleoperation

with slope restricted nonlinear environments, then evaluating the numerical analyses via long- and short-distance bilateral teleoperation experiments. Toward these purposes, the outline of this thesis is highlighted as follows.

1.3.1 Structure of the Thesis

In Chapter 2, we start by providing fundamental information related to the performance measure of the designed bilateral teleoperation: particularly, the one that is based on the transmitted force feedback to the human operator. Then, the main methods which have been used for stability analysis of the teleoperation, such as passivity and network theories, as covered in literature, are discussed and their pros and cons addressed. In addition, we highlight how the time delay problem that naturally occurs during long-distance transmission in the bilateral teleoperation architecture has been handled in literature.

Chapter 3 provides essential information related to methodologies that principally constitute the main scope of this study: IQC framework and Multiplier approach. Differences and common grounds between these two techniques are indicated with a bit test of historical perspective by showing how the Western and Soviet worlds approached the same problem and provided solutions from different perspectives. In the sequel, a number of classes of multipliers and IQC frameworks are provided for the particular class of uncertainties and nonlinearities which naturally occur in the bilateral teleoperation architecture, such as constant or variable time delays. Additionally, these multiplier classes are combined to be able to define an interpretation for the specific classes of structured uncertainties, such as diagonal combination of delay and nonlinearity. Conclusively, prospected multipliers are parametrized to obtain an efficient search within the defined class.

In Chapter 4, we show how the proposed stability analysis framework can be used in the first place for systems containing saturation type nonlinearity along with time delay based on the aforementioned parametrization and search. Compatibility of the proposed method is illustrated with numerical examples that are predominant in literature. In addition, we chose stability of the Retarded Neural Networks as a case study and obtained less conservative stability conditions in a similar fashion, particularly with constant time delay scenarios. Then, recent developments on

Kalman and Aizerman conjectures for time-delayed systems are discussed, as their conclusions are used as benchmarks in the numerical examples. Next, a simple geometrically intuitive sufficient stability condition is derived for the first-, second-, and third-order time-delayed Lur'e[7] systems, where delay and bounded nonlinearity are within the same loop. This understanding is important when choosing an appropriate order of the multiplier.

In Chapter 5, uncertain operators such as the human arm and the environment in bilateral teleoperation are defined as a perturbation, while the overall system is transformed into the Lur'e interconnection. The assumption related to the environment is modified and it is assumed that the slave is interacting with monotonic, slope restricted, and bounded nonlinear type of environment. Zames-Falb multipliers which provide positivity for this class of nonlinearities are used to regain positivity of the admittance matrix (i.e the nominal system) of the teleoperation system. Then, results are extended to time-delayed bilateral teleoperation, and comparison between different types of control architectures is provided numerically and experimentally. Long-distance bilateral teleoperation is evaluated experimentally by setting up two distance laboratories, one in Manchester (UK) and the other in Vigo (Spain). Numerical and experimental results show that the proposed approach in this thesis reduces the conservatism in the main stream designs and allows us to modify freely the assumption related to the perturbation without including any additional complications.

As a final note, Chapter 6 provides some concluding remarks and future works. For completeness, the Appendix provides mathematical details of the basis functions which play a key role within the parametrization of the multipliers. Also, we discuss another technique, the Quadratic Separation approach, which is used for stability analysis of the same type of system. Then, we highlight the similarities between two approaches (IQC versus Quadratic Separation) and when/how these two disassociate from one another.

[7] It has been depicted as Lurye or Lure as well.

1.3.2 Contributions and Publications

The contribution of this thesis is twofold. In the first place, different classes of multipliers are combined to analyse stability of the structured uncertainty consisting of time delay and saturation type nonlinearity. In order to carry out a search for multiplier within the defined class, multipliers are factorized such that asymmetric factorization is provided for the Zames-Falb multiplier. Secondly, multiplier approach, along with IQC theory, is used to analyse absolute stability of the bilateral teleoperation with nonlinear environments. Then, theoretically obtained stability conditions are experimentally evaluated via two distant robotic manipulators. The results are mainly illustrated in Chapters 4 and 5 which are based on the subsequent publications:

- Tugal H., Carrasco J., Heath W., *Absolute stability of clockwise systems with delay and saturation,* Submitted to Conference on Decision and Control, IEEE, 2017.

- Tugal H., Carrasco J., Falcon P., Barreiro A., *Stability Analysis of Bilateral Teleoperation with Bounded and Monotone Environments via Zames-Falb Multipliers,* Transaction of Control System Technology, IEEE, 2016.

  -This study will be also presented in a workshop entitled "Methods for Stability Analysis of Haptic Teleoperation Systems" in the 2017 American Control Conference in Seattle, USA by Dr. Joaquin Carrasco.

- Tugal H., Carrasco J., *Stable Bilateral Teleoperation with Nonlinear Environments: Multiplier Approach*, IFAC-PapersOnLine, 49.30 (2016), pp.308-313.

- Tugal H., Carrasco J., Maya-Gonzalez, M., *Teleoperation with memoryless, monotone, and bounded environments: A Zames-Falb multiplier approach*, European Control Conference 2015 (ECC15), Austria.

2 Brief Literature Survey on Bilateral Teleoperation

As human beings we were only able to reach and manipulate the objects which are within the near distance that our limbs can grasp, as we were limited by the capabilities of our own bodies. But, mankind's history contains many efforts that have tried to eliminate such deficiencies. In accordance with this purpose many inventions/improvements in mechanical devices, now, enable us to reach and manipulate heavy or hazardous materials without any effort or danger. Such tools have became an indispensable part of our daily lives by virtue of their usefulness; a very simple example will be the long metal handles used to retrieve a hot object from the fire. But we were still limited to be in the same place and medium in order to be able to carry on the process. It is hard to tell which one came first; whether the human desire to reach distances or the essential requirement for separating personal from the hazardous environments, yet handling distance objects was became daily practice with the help of the enhanced electromechanical devices. During this continuum the term *Teleoperation* were started to be used in the terminology to define the applications where objects are manipulated remotely via manipulators.

In the early design of teleoperator two manipulators, master and slave, were connected to each other with mechanical gears, cables, and chains. Due to the lack of having electromechanical tools within the design, operating such type of machine was required to apply extensive amount of man power. The first electromechanically coupled teleoperator was designed by Raymond Goertz in 1952 for handling radioactive materials[1] behind a sealed chamber, so that researchers could work more safely without wearing any protection suit. As the design's only ob-

[1]Strictly speaking a bomb making process

jective was minimizing the effect of the harmful radioactivity as much as possible, the feedback information was restricted and operation mainly based on the visual-feedback behind sealed glass. Despite that, as Goertz stated in the first instance, that ensuring stability of coupled manipulators is more complicated than uncoupled one therefore much more effort is required for design of practically feasible stable teleoperation that provides high performance [47].

In addition to the complexity of the coupled dynamics, the problem can be moved to another dimension once we start to concern about how the human and the robot will cooperate. To provide a level of cooperation force feedback is supplied to the human operator by courtesy of the developments in the electronics especially in sensors and transducers. The interaction has been discussed for decades and as always ideal one was aimed and occasionally even without concerning its feasibility. To specify level of the additional feedback (creates differences between lateral and bilateral teleoperations) the term *Telepresence*, which defines an ideal of sensing such that measured information is communicated to the human side sufficiently natural way that human operator feels physically being in the remote side, was introduced.

In this chapter our aim is to analyse and synthesize the prevalent control strategies and analytical methodologies related to the bilateral teleoperation that not only enable us to carry out tasks remotely but also providing fundamental haptic information. But, covering all the relevant literature within this modest chapter is almost impossible, yet we can still refer malcontent reader to exclusive surveys carried out in different time periods such as [22, 48–50]. In the first place we would like to start with details that might be the explanation of the subsequent question: what is the main aim of designing bilateral teleoperation; namely what is desired to be achieved, and how is the achievement measured ?

2.1 Performance Measures

There are some criteria that are used to measure the quality of bilateral teleoperation in terms of performance; position tracking and force reflection. The former simply defines how well the slave manipulator mimics the movements of the master manipulator which is a general aim in most robotic systems. The later will be depicted with different terminologies such as *Transparency* which defines how transparent is

the design in terms of force reflection or environmental impedance transformation. By impedance we imply dynamic that defines the relationship between velocity and force. Within this terminology, here, our main focus related to the performance measure will be based on the human sensation.

Bilateral teleoperation is one of the challenging control problems that engineers/scientists are dealing with because it involves highly uncertain elements, "trouble makers" such as delay, and additionally human sensation as a performance measure. The satisfaction of human operator in terms of perceptual understanding is a demanding task to accomplish. The main point is to provide sufficient situational awareness to the operator via haptic perception. And one of the performance criterion is how well the human 'feels' the environmental interaction force, yet still it is not clear what level of force feedback is enough to categorize it as being 'good', because that might changes from one operator to another. Besides, a human has highly sophisticated neural mechanism that is hard to understand from the engineering point of view (might be only for control engineers), but enables us to identify our surroundings with the limited information obtained via contact. For instance we are capable of identifying an object only by touching it with a simple stick not necessarily with our bare hands [51].

Nonetheless, many researchers have been neglected aforementioned type of performance criterion in a bilateral teleoperation system. They particularly analysed stability margin against time delay, the uncertainties in human and environment dynamics, and non-linearity of the manipulators, yet the human sensation or operator perfective has never been mentioned or stated. It should be noted and clearly indicated that the main aim of a *bilateral* teleoperation is to supply satisfactory (if possible) haptic information to the operator via designed controllers; the design that does not consider human sensation is similar to manufacturing an aircraft that has superb flying characteristic, yet is not able to carry any passenger or cargo supplies which are the main objectives in the first place.

2.1.1 Transparency

It is described as that a physical object is transparent if the light travelling through it has not been scattered. In a similar vein, in bilateral teleoperation transparency de-

fines how well the environmental interaction force or task impedance is transmitted to the human side without being scattered. The level of transparency is measured by describing how well the human operator feels as if he is interacting with the distance task directly.

Definition 2.1 (Ideal Transparency [52]). *A bilateral teleoperation is said to be ideally transparent if the operator is beguiled in a way that interaction with the distant task is taking place directly; without any teleoperator.*

In an ideal condition the operator is able to feel only the impedance[2] of the environment without any interference from the dynamics of the manipulators or their controllers. In order to achieve such level of optimism, the environmental interaction impedance should be transformed to the human side without being diminished and dynamics of the manipulators should be cancelled out with the aid of controllers. Thus, in the optimum conditions transmitted impedance, let us define it as Z_t, should be equivalent to the task impedance Z_e ($Z_t = Z_e$). In a full synchronisation scenario forces (F_i) and positions of the both sides[3] (or velocities (V_i) as in most cases) are required to be equivalent as

$$\begin{bmatrix} F_h \\ -V_e \end{bmatrix} = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \begin{bmatrix} V_h \\ F_e \end{bmatrix},$$

where V_h, V_e, F_h, and F_e represent the master and slave velocities, the hand force on the master, and the slave force on the environment, respectively.

This level of transparency has been aimed to be end-point of the many different controller architectures. And once the determined *optimum transparency* is achieved it can be said that impedance at one side is transformed to the other side precisely. Namely, if this transparency level is achieved the operator feels as he is touching to the task with his bare hands. Now, some critical questions need to be asked or queries need to be raised. A human hand is capable to complete rather complicated tasks, yet it is not useful alone for carrying out some tasks such as driving a nail and there are some useful tools that we use in our daily lives e.q. hammer and screwdriver. While using such devices of course we 'feel' how much force we need to

[2]That is a measure of how much an environment resists motion when interacted with slave manipulator.

[3]Subscripts, $_h$ and $_e$ will be associated with human and environment, respectively.

apply, yet the force we perceive is not the force that is on the interacting point and it should not be, as this is the main idea of using a tool. This picturesque description shows how a performance criterion might confuse us when we pull out human sensations and only focus on the matrix representation of the performance, but we will insist no more on this argument.

On the other hand while measuring the performance of bilateral teleoperation in terms of the human sensation, transparency is not the only characterisation, there are many kinds of it as given delicately in [53]. Perceptual transparency that defines the operator perfection of the task is the one that mentioned above, tried to be obtained in many designs, and the one that we will take into consideration, yet it is worth mentioning some of them as well for the integrity.

2.1.2 Z_{width}

In teleoperation the designed controller should achieve environmental impedance[4] transmission from slave side to master side in order to obtain haptic interaction. And the design should work in a desired way with different types of environments or namely with a wide range of impedances. In a short way, if the controller manages to transmit a wide range of environmental impedances to the operator side then that indicates usefulness of the designed system in wide range of application areas.

Naturally, in free space ($Z_e = 0$) the transmitted impedance (Z_t) should be small enough and indeed in optimum conditions it should be zero. Also maximum impedance occurs while the slave manipulator interacts with a rigid object such as a wall or a stiff object. Minimum and maximum transmitted impedances are generally described with Z_f and Z_c, respectively. These two measures define the extreme situations that the slave manipulator might encounter and the designed system should perform desirably with any other environment that has impedance within the range of Z_f and Z_c [54]. Then, Z_{width} can be described as the area between these two transmittable impedances over a range of frequency [50], subsequently

$$Z_{width} = \frac{1}{\omega_{max} - \omega_{min}} \int_{\omega_{min}}^{\omega_{max}} |logZ_c(j\omega)| - |logZ_f(j\omega)| d\omega.$$

[4]This implies the *mechanical impedance*, yet the term used in electrical engineering terminology will be defined in subsequent section.

Instinctively, increase within the value of Z_{width} indicates that the teleoperation system can be used in many different environments without any danger also that improves sensation of human operator in terms of impedance distinction [55]. Stability, however, restricts the maximum achievable Z_t, due to the substantial trade off the maximum Z_c that can be rendered by relating with the mechanical damping value. And one way to enlarge Z_{width} is to increase damping mechanically or electronically.

2.2 Stability Analysis via Network Theory

Even though early designed teleoperation systems were only concerned about separating the human beings from the hazardous mediums, shortly after that stability became one of the main concerns because interaction with the task itself subsequently changes the dynamics of the overall system and that can imperil the stability [56]. Also, it did not take long to bring in the telepresence terminology within the scope of interacting robotic. As the dynamics became more coupled more systematic analysis of the designs were started to be required. Having limited computational power in the late 1980s and early 1990s paved the way to use well established circuit and network theories within the teleoperation analysis as any interconnected system can be represented like a network interconnection [23]. Somehow, over the past research history this beneficial methodology transformed into a tradition. As a consequence, entire bilateral teleoperation system has frequently been defined and expressed as a two-port network as illustrated in Figure 2.1, where the operator and environment are modelled by the LTI impedances Z_h and Z_e [57]. It should be noted that to simplify the energy calculation of the network and to sustain compatibility with the passivity theory velocity information is used as a port variable instead of more intuitive position information.

Definition 2.2 (m-port network [58])**.** *A physical device that consists of a number of circuit elements or some components with m pairs of terminals to connect to external circuits is called an m(multi)-port network.*

For this case, the literature overall bilateral teleoperation was divided into three main different layers. The first and the third layers are the human operator and

environment. The second layer is the combination of the electromechanical master manipulator, the communication medium which connects master and slave manipulators, and slave manipulator along with its local control properties. In the forward section, we will see that this second layer is also divided amongst themselves to overcome some particular problems such as time delay in the transmission. To increase the comprehensibility we will start with and clarify some of these definitions which are frequently utilized in network theory yet have not been commonly used in the control terminology.

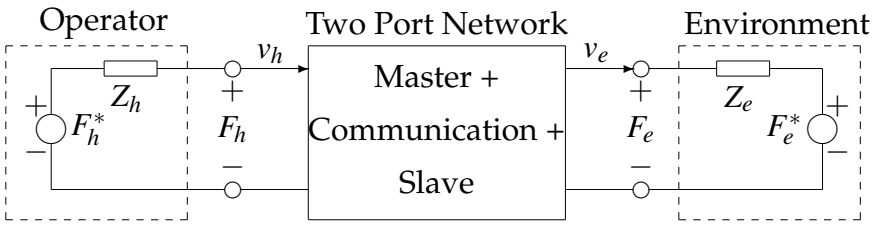

Figure 2.1: Network representation of the bilateral teleoperation, where human and environment are defined as a one port and the rest, including the designed controllers, is depicted as two-port network. Master and slave velocities, operator force on the master, environmental interaction force, and the operator and environment exogenous force inputs are depicted as v_h, v_e, F_h, F_e, F_h^*, F_e^*, respectivelly.

Remark 2.1. *The force applied by the human operator to the master manipulator is resolved into two categories: the voluntarily applied force is depicted as F_h^* which moves master manipulator from one point to another. And, it is assumed that human will not consciously cause any instability while operating the system, hence in general F_h^* is assumed to be bounded (contrary situation might occurs, see [59]). Then, the involuntary force applied to the master is depicted as F_h which is caused by the mass and muscle activities of the human hand/arm and will be utilized while defining the dynamics of the human operator.*

A *port* is simply terminal of the network, namely pinpoint where any element connects to the network. In circuit theory terminology, voltage across the port enables the current transmission between network and the connected circuit element. A model of an electrical socket is a good example for one port network where the entire interconnected system is modelled as one port network that has only two terminals, see Figure 2.2 (a) for the block diagram illustration. A port has two main variables, current and voltage, and each can be defined as an input or output of the system. Based on the determined input or output definition of the network model can be entitled differently. These terms in electrical energy terminology might seem

to be irrelevant to the teleoperation, yet correlation will be clear once forces and velocities in a robotic system are associated with the voltages and currents, respectivelly. If we get back to the topic, the correlation between the different measures in the network is depicted with varied *immitance* matrices.

2.2.1 Immitance Matrices

For an m-port network the matrix that defines linear relationship between the currents and voltages at the ports is generally called immitance matrix; *impedance* and *admittance* matrices are the particular example of it. In order to express admittance matrix the applied voltage, $V(s)$, is defined as an independent and flowing current, $I(s)$, (assuming there is no open circuit circumstance) as a dependent variables. Subsequently, linear relationship between the network's parameters will be designated as $I(s) = Y(s)V(s)$, where $Y(s)$ will be called as *admittance matrix* of the network, N. Besides, if the applied current and measured voltage are defined as independent and dependent variables, respectively, we can specify the impedance matrix as $V(s) = Z(s)I(s)$. Similarly, $Z(s)$ is going to be called as *impedance matrix* of the network. It is clear that these two matrices are the inverse of one another, and mathematically obtaining $Z(s)Y(s) = I$ is distinct (if both exist). Electromechanical devices, i.e. robotic manipulators, are also distinguished based on their dynamics behaviours. An admittance type device, generally, has high inertia and friction and receive force as an input and apply force to its interaction environment. Conversely, impedance type devices are designed to have much lower inertia and friction using the velocity signals as inputs and acts as a velocity source to its environment. As a short note, the mechanical-electrical analogy becomes clear once force and velocity are depicted as voltage and current, respectively. The control paradigm used in bilateral teleoperation becomes dissimilar depending on which type of device is being used to realize high degree of transparency [60].

Then, in some particular circumstances it is infeasible to define all the voltages V and currents I as dependent or independent variables. For instance, impedance representation of an open port circuit or admittance matrix definition of a short circuit is not achievable. Besides, defining all the identical signals ($V(s)$ or $I(s)$) within the same terminology may not be useful for the analysis and synthesis. Therefore,

some voltages or currents variables within the network can be expressed as dependent and the remaining are being defined as independent variables. In this case, *Hybrid matrix*, $H(s)$, can express the overall relations among the interconnection in a straightforward fashion as $R(s) = H(s)U(s)$ while $R(s)$ and $U(s)$ are containing information related to voltages and currents.

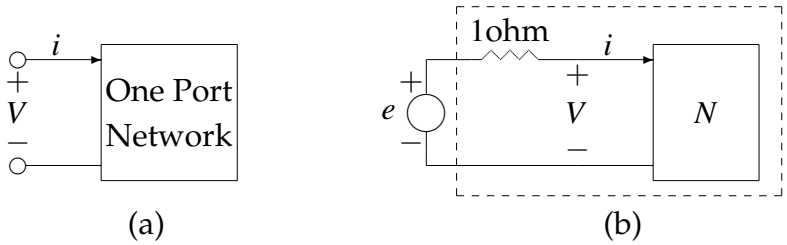

Figure 2.2: Simple one-port network representation (a), and augmented m-port network (b) [58].

Additionally, *Scattering matrix*, $S(s)$, representation is a commonly used expression while analysing the behaviours of a transmission line. We would like to propose more detailed information as applying this matrix into the delayed teleoperation is a frequently used methodology. To express clearly, let us assume that there exists an *augmented m-port network N* with a series of resistance connected at its each ports as illustrated in Figure 2.2 (b). Then, the voltage source can be expressed as $e(t) = v(t) + i(t)$. The only physical response of the network to the voltage source, $e(t)$, is the port current flowing within it. But, it is clear that $e(t) - (v(t) - i(t)) = 2i(t)$, therefore $v(t) - i(t)$ and $v(t) + i(t)$ will be called as response and excitation vectors, respectively. In general, it is common to use $\frac{1}{2}(v(t) + i(t))$ and $\frac{1}{2}(v(t) - i(t))$ as *incident voltage* (v^i) and *reflected voltage* (v^r). As a result the scattering matrix can be defined as $V^r(s) = S(s)V^i(s)$ by using incident and reflected voltages. More detailed discussion will be provided when scattering transformation is used for bilateral teleoperation with time delays.

In this thesis, unless otherwise specified, the term *immitance matrix* will be used to specify all the matrix representations (admittance, impedance, and hybrid) of the network.

2.2.2 Stability with Linear Network Theory

As the network theory dominated the literature of bilateral teleoperation's stability analyses some applicative theories such as Llewellyn stability conditions emerged to

simplify the complex analyses therewithal provide exact stability conditions. Thus, two-port network is investigated by Llewellyn theory whose main motivation was to investigate stability of the linear electricity transmission lines. The method is based on linear behaviour of the network and its ports which can be nonlinear yet required to be linearised at least locally. Many researchers applied the proposed conditions to bilateral teleoperation either by investigating the stability or designing the controllers within the overall system, see [61] and references therein.

It can be said that a linear two-port network is stable if and only if there is no passive sets that can terminate the network and lead unstable behaviours. The following theorem gives exact conditions for stability of the proposed two-port network.

Definition 2.3 (Positive Real System [62]). *A rational transfer function matrix $G(s)$ is called positive real if*

- *poles of all elements of $G(s)$ are in $\Re\{s\} \leq 0$,*
- *for all real ω for which $j\omega$ is not a pole of any element of $G(s)$, the matrix $G(j\omega) + G^*(j\omega)$ is positive semi-definite,*
- *any pure imaginary pole $j\omega$ of any element of $G(s)$ is a simple pole and the residue matrix $lim_{s \to j\omega}(s - j\omega)G(s)$ is positive semi-definite Hermitian.*

For single input single output (SISO) systems second condition reduces to $\Re\{G(j\omega)\} \geq 0, \forall \omega \in \mathbb{R}$.

Theorem 2.1 (Llewellyn's Stability Criteria [24]). *A Linear Time Invariant two-port network, N, described with its immitance matrix*

$$N(j\omega) = \begin{pmatrix} N_{11}(j\omega) & N_{12}(j\omega) \\ N_{21}(j\omega) & N_{22}(j\omega) \end{pmatrix}$$

and interconnected to two LTI passive one-port networks as in Figure 2.1 is stable if and only if the following conditions are satisfied:

- *N_{11} and N_{22} are positive real transfer functions,*
- *The inequality $n(\omega) = -\dfrac{\Re\{N_{12}(j\omega)N_{21}(j\omega)\}}{|N_{12}(j\omega)N_{21}(j\omega)|} + 2\dfrac{\Re\{N_{11}(j\omega)\}\Re\{N_{22}(j\omega)\}}{|N_{12}(j\omega)N_{21}(j\omega)|} \geq 1$, holds for all $\omega \geq 0$,*

where $\Re\{\cdot\}$ denotes real parts of the complex number.

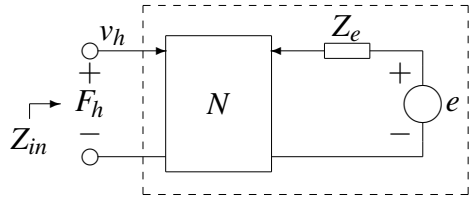

Figure 2.3: A two-port network terminated with an impedance Z_e in port 2 and if input impedance (Z_{in}) of Z_e is also passive, then it can be concluded that network N is passive as well.

The aforementioned theorem is based on the bilinear transformation (Mobius Transformation) along with one of the port's input impedance when the other port terminated with a passive impedance [63]. Let illustrate the correlation between the network representation of the bilateral teleoperation, depicted in Figure 2.1, and the given theorem by assuming that there exists a passive Z_e connected one side of the two port network as in Figure 2.3. The driving point impedance of Z_e from other side of the network can be determined as

$$Z_{in} = N_{11} - \frac{N_{12}N_{21}}{N_{22} + Z_e}$$

where N_{ij} denotes the network quantities for $i, j = 1, 2$. Namely, Z_{in} is basically bilinear transformation of the Z_e subsisting right side of the complex plane (by being passive). As known, in bilinear transformation a straight line, which is the imaginary axis in this case due to the passivity of Z_e, is transformed into a circle centred at C_{in} with radius r_{in}. To define the required impedance transmission of Z_e to Z_{in}. The circle parameters can be defined as follows

$$C_{in} = N_{11} - \frac{N_{12}N_{21}}{2R_{22}}, \quad r_{in} = \frac{|N_{12}N_{21}|}{2R_{22}}. \tag{2.1}$$

This circle needs to lie in the right half side of the complex plain so that one can conclude that the transformation preserves the passivity, in other words one can ensure that the two port network is passive as well. Therefore the following inequality needs to be satisfied

$$\Re\{C_{in}\} - r_{in} \geq 0. \tag{2.2}$$

Substituting (2.1) into (2.2) leads to

$$\frac{2R_{11}R_{22} - \Re\{N_{12}N_{21}\} - |N_{12}N_{21}|}{2R_{22}} \geq 0,$$

which holds when the conditions given in the theorem above are satisfied.

On the contrary, from teleoperation point of view the first condition implies passivity of the master and slave manipulators as positive realness of the transfer function implies passivity of the system, namely first condition is necessary to have positive realness of the network when the ports are not coupled. The second condition[5] implies stability of the interconnected system (effect of the coupling). In other words, this condition shows whether or not the image of a passive operator (human or environment) terminating at one side of two-port network is passive as well from the other side of the network, as illustrated in Figure 2.3.

Remark 2.2. *Within the Network Theory terminology while defining a two-port network the neighbouring port currents are towards the network, yet while representing bilateral teleoperation as a two-port network, as in Figure 2.1, port 2 signal (v_e) is outwards the network as it is assumed that environment is under the influence of slave's velocity, v_e.*

The conditions in Llewellyn's Criteria can be implemented with any kind of immitance matrix, N, such as hybrid, H, or impedance matrices, Z, etc. From analytical point of view, frequency gridding is the most convenient and frequently used methodology while checking the given last condition, but missing the critical frequency is always the hidden danger. It must be noted that, absolute stability is ensured by obtaining the passivity as a passive network will always be stable, but an absolutely stable system is not required to be passive. This is a celebrated phenomenon yet has not been specified intermittently in a way that led to some misconception such that passivity is as if an essential property for a bilateral teleoperation. With this methodology human operator and environment are model as a class of operators depicted with linear mass (M), spring (K), and damper (B), i.e. $F_i(s) = (s^2M_i + sB_i + K_i)X_i(s)$ for $i = h, e$, where F and X denote force and position.

Remark 2.3. *Within the bilateral teleoperation architecture Llewellyn's Stability Theorem implies depicted two-port network is absolutely stable while interconnected with any LTI passive human and environment if the given conditions are satisfied. Stability, here, is based on the passivity which means if the last condition does not hold then the system is said to be critically stable as there might exist a particular passive human-environment pair makes the system unstable.*

[5]The $n(\omega)$ will be called as *stability index*

It is worth mentioning that, proposed Llewellyn criterion only allows us to analyse stability of the bilateral haptic systems with one-DOF manipulators. Yet, an extended version (re-formulation) that presents the criteria for multilateral teleoperation with the m-DOF manipulators is proposed in [64]. As known, stability is not the only concern in a controlled system, so that the subsequent section will be based on the efforts that have been carried out to figure out what control architecture leads to higher transparency if a particular task (for a given environment) is being considered in the bilateral teleoperation system.

2.3 Control Architectures in Bilateral Teleoperation

Challenges in the bilateral teleoperation lead engineers and researchers to design different control architectures where the main aim was ensuring the stability of teleoperation along with improving the haptic information sufficiently. Also, effort to answer some open questions increased varieties of the architectures: with a determined task what type of control architecture gives the better performance measures like fidelity? Sensory information from the robotic manipulators in bilateral teleoperation has been the key element to distinguish difference between each design. Individual data that is being transmitted, in teleoperation, is designated with a transmission channel and the number of these channels determines type of the architecture and eventually its name. All compositions can be gathered in three main branches which are Two-, Three-, and Four-Channel control architectures.

2.3.1 Two-Channel Control Architecture

This is the simplest possible form of the bilateral teleoperation control architecture. Only one measurement is being transmitted from one side to another. For instance, if we are sending position information to the slave side, as a reference signal, and force is sent back to the master manipulator, for obtaining transparency, then the design is going to be called two-channel[6] position-force architecture. Generally, the former name (position in this case) is referred to the signal being transmitted from master to slave and the later one is used to identify the data transmitted from slave to master.

[6]One channel for transmission from master to slave and one for the vice versa

Unless stated otherwise we will follow the same notation. Naturally, within this control architecture different types of design can be constructed depending upon the transmitted data form and that can be listed as follows

- Position-Position (P-P)
- Position-Force (P-F)
- Force-Position (F-P)
- Force-Force (F-F)

We will briefly discuss only the first two of them as they are the most commonly used and implemented architectures within the (impedance type) bilateral teleoperation framework. It is important to highlight that *position* information does not mean explicitly position signal has been particularly used; for instance in [40], and in many others, velocity signal has been used without any indication in order to comply with the passivity analysis. Yet, in our analyses, that will be proposed in the subsequent chapters, we have examined the designs where position information is explicitly being transmitted. Therefore, we have referred to the different architectures with 'Position' instead of 'Velocity', also in this way we retain integrity with the literature.

Strictly speaking Position-Force (P-F) architecture is built on the foundation of the bilateral teleoperation where slave is designed to follow position of the master and a measured force signal, that appears as a result of the environmental interaction, is sent back to the master side to create a sort of situational awareness to the human operator. It provides direct connection between environmental force and human operator and there are only two controllers/parameters required to be tuned, one for position tracking (slave controller C_s[7] aims to reduce position error e_s) and the other one for force scaling (K_f), see Figure 2.4 for the general block diagram representation. This control architecture has been investigated by many researchers in different aspects see [65–67], but still despite its simplicity stability is the main drawback of the design as transmitted force feedback provokes the master position and that might trigger further interaction forces [51]. Thus, to establish the stabil-

[7]Subscripts, $_m$ and $_s$ will be associated with master and slave, respectively.

ity the essential solution is to significantly attenuate the force feedback that is being transmitted to the operator side.

Another common solution for improving stability within P-F architecture is to interpolate additional damping values into the system, yet a sluggish response at free space movements became inevitable when the damping is injected to the master manipulator. Besides the stability might be in danger especially when the slave is in hard contact and human operator is not grasping the master manipulator as it had been concluded that there is a linear relationship between stability and the operator grasp. Inherently a heavier grasp has the same effect as adding extra damping to the master manipulator [65]. Additionally, within this control architecture trade off between two performance measures, transparency and position tracking, needs to be meticulously handled because a high gain in slave controller gives more priority to the position tracking thus leading to worse force fidelity[8] specifically when the slave is in contact with hard mediums. Consequently, that might leads the reader to think the controller design problem in position-force architecture should be handled as an optimization issue with a good objective function such that tracking and robust stability of the system define constraints and fidelity describes objective of the optimization. Yet, as highlighted in [68] such an optimization problem is nonconvex that cannot be solved with tractable methodologies.

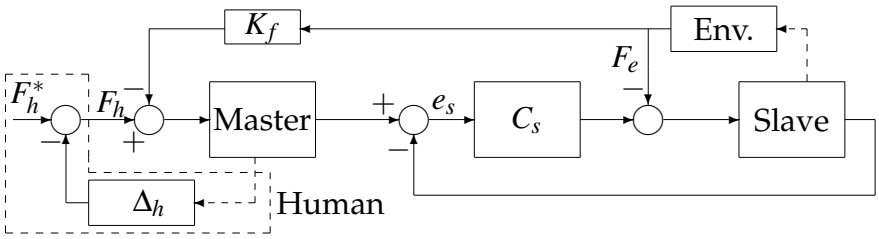

Figure 2.4: Two-channel (delay free) position-force bilateral teleoperation control architecture: the block diagram representation where Δ_h denotes uncertain human-arm dynamics and F_h^* is depicted as the voluntarily bounded force applied by the human operator and it is mostly dismissed in the stability analyses.

On the other hand, being force sensor free system leads to utilisation of the Position-Position (P-P) controller which is prelusively implemented bilateral teleoperation in 1950s [69]. It is also known as position error based control methodology as both sides' control philosophies are based on utterly position mismatches between two manipulators. For instance, a PD (Proportional-Derivative) type con-

[8]Sensitivity of the transmitted impedance to changes in the environmental impedance

troller at both sides simply creates a virtual spring and damper between motors of two manipulators, pulling them together is a kind of imitation of the early mechanical designs. If identical manipulators were used, then both controllers minimize the position difference regardless of which device is falling behind in terms of tracking [70]. Also, it has been stated that highly accurate position controller at the master side (master controller C_m) is not desirable as system response becomes *sluggish* in free space movement. Due to being lack of force sensor, however, eliminating the inertias, frictions, and unmodelled nonlinear dynamics of the slave manipulators that distort the human sensation regarding to the environmental impedance in P-P architecture is impossible.

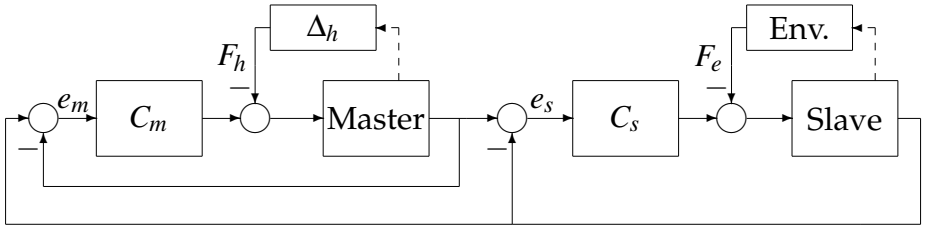

Figure 2.5: Generalized block diagram of two-channel (delay free) position-position bilateral teleoperation control architecture, where e_i defines position error signals for $i = m, s$.

Force feedback attenuation in P-F architecture due to the stability issue and unavoidable sluggish response in P-P architecture pawed the way more complex bilateral teleoperation designs. The complexity generally increased by transmitting additional information between master and slave manipulators.

2.3.2 Three- and Four-Channel Control Architectures

In order to achieve the *optimum* transparency conditions, Lawrence [52,71] claimed that the two-channel control architectures, where only one kind of measured information (force or position) is sent from local side to another, falls short. The reason is that within this control methodology eliminating or cancelling out dynamics of the manipulators and controllers, that are being sensed by the human operator right along with environmental impedance, is infeasible. In four-channel control methodology also known as Lawrence's philosophy, however, both position and force information of two manipulators are being transmitted from one side to another. It was stated that, all four channels illustrated in Figure 2.6, are necessary in order to attain

ideal transparency measure in bilateral teleoperation, see also [72] where similar philosophy has been proposed independently. Parameters in four-channel architecture, as illustrated in Figure 2.6, are chosen such that dynamics of the master and slave (let define Y_m and Y_s as their admittances) and their controllers (C_m and C_s) are cancelled out: namely, $C_1 = (Y_s^{-1} + C_s)$, $C_4 = -(Y_m^{-1} + C_m)$, and optimum transparency is obtained with $C_3 C_2 = I$.

To illustrate the mentioned optimum transparency measure, let us assume that slave manipulator is interacting with a linear passive environment whose impedance is Z_e and the aim is to transmit this impedance to the master side. Inherently, human operator 'feels' the transmitted impedance, Z_t, which is function of the task's impedance, Z_e. Based on the linearised model of manipulators general hybrid matrix of a two-port teleoperation network can be formulated as [70]

$$\begin{bmatrix} F_h \\ -V_e \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} V_h \\ F_e \end{bmatrix} \tag{2.3}$$

where F_i and V_i are forces and velocities of the human and environment for $i = h, e$. As a short note, negative sign is used in the lower part of the matrix at left side of inequality in (2.3) for the sake of keeping uniformity with the Network theory where on the contrary to Figure 2.1 port signal v_e always towards the inwards of the two-port network. If we continue, by adopting the environment's impedance, $F_e = Z_e V_e$, and the equality in (2.3) the relationship between V_e and V_h can be pointed as $-V_e = (I + H_{22} Z_e)^{-1} H_{21} V_h$. Consequently, the relationship between the Z_e and transmitted impedance, $F_h = Z_t V_h$, can be pulled out from

$$(Z_t - H_{11}) V_h = -H_{12} Z_e (I + H_{22} Z_e)^{-1} H_{21} V_h. \tag{2.4}$$

Evidently, the following fundamental features can be depicted from (2.4) which is equivalent to aforementioned ideal hybrid matrix;

- The perfect transparency, which is almost impossible to obtain in practice, is when the Z_t is approximately equal to Z_e, that means H_{22} and H_{11} need to be equal to 0 and off-diagonal elements H_{12} and H_{21} need to be I and $-I$, respectively.

The quantitative measurement of this *feeling* is an open question and has been discussed by many researchers over the years as it is an intuitive measure and de-

pends what the human operator is actually feeling [53,73]. Further, extension version of this design methodology was proposed by introducing local force feedback in a way that performance is improved without increasing level of the feed-forward forces, see [70,74].

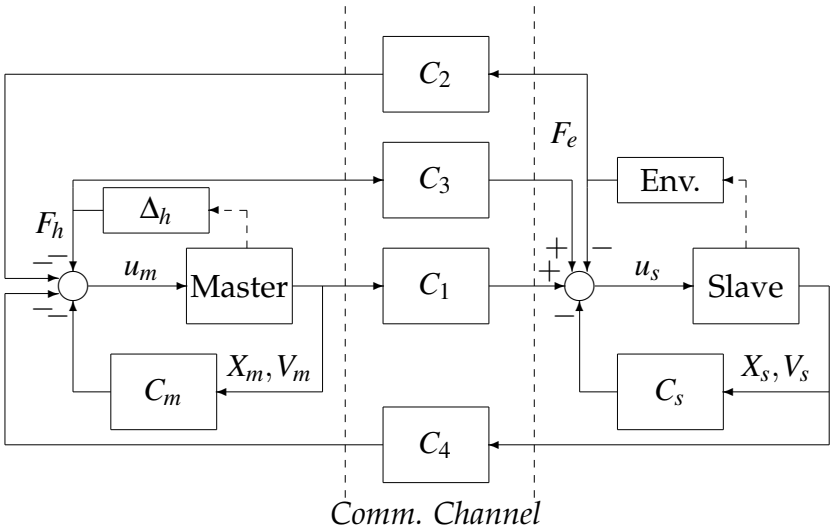

Figure 2.6: General four-channel bilateral teleoperation control architecture: so-called the Lawrence architecture.

Further effort for optimization of transparency indicated that similar performance measures can be achieved with less information transmitted via Three-Channel architecture. Briefly, by cancelling individual communication channel four types of different sub-control methodologies can be designed based on this technique; P-PF, F-PF, PF-P and PF-F [28,75,76], yet we will give more detailed information and comparison with two-channel approach related to the PF-F architecture in chapter 5.

2.3.3 Energy Based Control

Passivity based analysis approach has been implemented more often than not despite providing only sufficient conditions for the stability status of a system. The approach is convenient as it can not only be applied to any linear or nonlinear systems but also highly complex systems by only considering the individual components, and it is simply based on a comprehensible physical concept: energy. In [77], the mentioned approach has been implemented to bilateral teleoperation by observing and adjusting energy flow of each port. Every time instant flowed energy is observed, E_{obsrv}, by Passivity-Observer (PO); that is assumed to be faster than the dynamics of the system. And it is terminated that the system (or port) dissipates the

energy if $E_{obsrv}(t) > 0$ or contrary supplies energy if $E_{obsrv}(t) < 0$. The observation in teleoperation is based on

$$E_{obsrv}(k) = \Delta T \sum_{i=0}^{k} f(t_i)v(t_i),$$

where ΔT is the sampling period. The amount of energy that ruins the dissipativity is resolved/damped with a controller called Passivity-Controller (PC). That is to say that, a time variable element is embedded into the terminals of the ports to dissipate the extra (undesirable) energy. This control methodology can be represented by a resistance connected to the circuit; the energy flow of the overall circuit can be reduced by simply manipulating the value of the resistance itself which eventually regulates the energy of the system. The main convenience of the approach is to be model-insensitive as it purely depends on the measured signals, yet once we consider the worst case scenario where the active energy is higher than the energy that controller cannot dissipate then requirement of the meticulous design is brought to the light. The proposed idea has been implemented to general control systems in [78], to bilateral teleoperation in [19,77] and for time delayed teleoperation in [79] as well.

2.4 Bilateral Teleoperation with Time Delay

An ultimate bilateral telerobotic system should be robust against uncertainties and should give sufficient haptic feedback information to the operator so that complicated tasks can be carried out meticulously. However, it is challenging to have a high performance teleoperation that contains both the uncertainties and additionally latency that naturally occurs in the long-distance communication medium. Time delay is an important phenomenon in bilateral teleoperation, due to the fact that it effects not only performance but also stability of the system [80]. Strictly speaking, passivity can easily be destroyed by the phase lags introduced by communication delays. Also, as known, analysing the time delayed systems is more complicated than the delay free one as with time delay the system transforms from a finite-dimensional to an infinite-dimensional system [81]. In the literature, there are a number of efforts that tried to improve performance and also guarantee the stability of teleoperation system while the communication channel accommodates

constant or time varying delays [29, 40–44, 82, 83]. In this brief section particular methodologies that overcome the problem caused by time delay will be analysed and discussed.

2.4.1 Scattering Transformation

One of the first methods that deal with the delay in bilateral teleoperation is the so-called *Scattering Transformation*. Its fundamental is based on passification of the delayed communication medium. Based on the assumption that all other elements in the bilateral teleoperation system is passive operator, except delayed communication medium that jeopardizes the overall passivity, transforming the delayed communication medium into the passive network leads to obtain overall a passive interconnected system. Thus stability (indeed passivity) of the interconnected system will be ensured.

As mentioned previously, any interconnected system can be defined as an interconnection of one-port and two-port networks. Correspondingly, a teleoperation system will be interpreted as a complete network in such a way that human and environment will be described as one port and master, slave, and communication channel will be described as two-port networks. The main aim in scattering transformation is to transform communication medium into two-port passive network, see Figure 2.7.

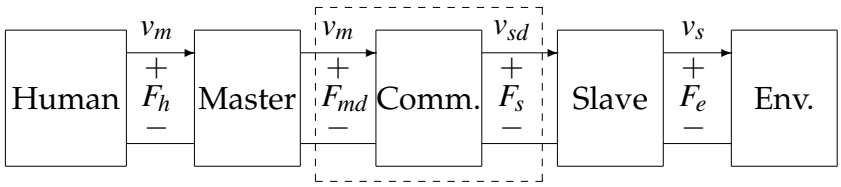

Figure 2.7: Dissociated two-port network block diagram of time delayed bilateral teleoperation where F_{md} and v_{sd} denote desired force and velocity, respectively.

The subsequent information mainly based on [40]. General impedance matrix representation of the each block can be expressed as $F(s) = Z(s)V(s)$. In a similar vein, the scattering operator that maps effort plus flow $(f+v)$ into effort minus flow $(f-v)$ can be defined as scattering matrix $S(s)$

$$F(s) - V(s) = S(s)(F(s) + V(s)).$$

And this scattering matrix can be defined within hybrid matrix description of a two-port network as

$$S(s) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} (H(s) - I)(H(s) + I)^{-1}, \tag{2.5}$$

where $H(s)$ defines hybrid matrix of the network. Passivity of the system can be reviewed from its scattering operator; a system is said to be passive if the norm of its scattering matrix is less than 1. If there exists a delay in the communication channel, the network representation of the transmission will be non-passive. This phenomenon can be shown in the following equations. The hybrid matrix of the communication medium in bilateral teleoperation with delay can be expressed as

$$H(s) = \begin{bmatrix} 0 & e^{-sT_d} \\ -e^{-sT_d} & 0 \end{bmatrix},$$

where time delay depicted as T_d, then its scattering matrix can be calculated as in (2.5):

$$\begin{aligned} S(s) &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & e^{-sT_d} \\ -e^{-sT_d} & -1 \end{bmatrix} \begin{bmatrix} 1 & e^{-sT_d} \\ -e^{-sT_d} & 1 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} -\tanh(sT_d) & \operatorname{sech}(sT_d) \\ \operatorname{sech}(sT_d) & \tanh(sT_d) \end{bmatrix}. \end{aligned}$$

That renders $\|S\|_\infty = \infty$, thus time delayed system is neither bounded nor passive. That implies even a small delay leads to a non-passive communication channel. But, Anderson and Spong [40] justified it so that passivity of the communication channel will be guaranteed independent of the time delay[9]. With their design communication channel mimics the characteristic of an energy transmission line, which is passive and naturally contains time delay.

A linear two-port lossless transmission line element has the following input-output relationship

$$F_{in}(s) = Z_0 \tanh(sl/v_0) v_{in}(s) + \operatorname{sech}(sl/v_0) F_{out}(s), \tag{2.6}$$

$$-v_{out}(s) = -\operatorname{sech}(sl/v_0) v_{in}(s) + (\tanh(sl/v_0)/Z_0) F_{out}(s), \tag{2.7}$$

where F_{in}, v_{in} define input and F_{out}, v_{out} define output forces and velocities, l represents length of the line, $Z_0 = \sqrt{L/C}$, and $v_0 = 1/\sqrt{L/C}$; impedance and capacitance

[9] Yes, even if the duration of delay is equal to one year.

of the transmission line is depicted as L and C, respectively. Similarly, the communication channel, assumed to be two-port network, in teleoperation system can be expressed based on (2.6) and (2.7) by setting $Z_0 = 1$ and $v_0 = l/T_d$ such that;

$$F_{md}(s) = \tanh(sT_d)v_m(s) + \operatorname{sech}(sT_d)F_s(s), \tag{2.8}$$

$$-v_{sd}(s) = -\operatorname{sech}(sT_d)v_m(s) + \tanh(sT_d)F_s(s), \tag{2.9}$$

where F_{md} is depicted as reflected force and v_{sd} is velocity set point based on the two-port representation of the communication channel, with a constant transmission delay T_d; $F_{md} = F_s(t - T_d)$ and $v_{sd} = v_m(t - T_d)$. Let us express (2.8) and (2.9) based on the scattering matrix representation so that the passivity can be investigated conveniently,

$$\begin{bmatrix} F_{md}(s) - v_m(s) \\ F_s(s) + v_{sd}(s) \end{bmatrix} = \begin{bmatrix} 0 & e^{-sT_d} \\ e^{-sT_d} & 0 \end{bmatrix} \begin{bmatrix} F_{md}(s) + v_m(s) \\ F_s(s) - v_{sd}(s) \end{bmatrix}. \tag{2.10}$$

Based on the scattering matrix proposed in (2.10) it can be concluded that $\|S\|_\infty = 1$, hence by using the communication law given the following equations the passivity property of the communication channel in bilateral teleoperation can be recovered.

In an electricity transmission line, in order to reduce energy loses over the lines, a transformer increases the voltage on one end and another one decreases on the other end, with the ratio of n and $1/n$. A similar method is necessary for avoiding implementation problem in the teleoperation when scattering transformation is used as the force and the velocity signals may differ by orders of magnitude.

Scattering transformation is not the only way to passivate the delayed communication medium. Wave variables is another recognized methodology that shares the same philosophy with the aforementioned approach.

2.4.2 Wave Variables

In previous section, it was illustrated that how stability of the time delayed bilateral teleoperation was achieved independent of the delay by using scattering transformation. Niemeyer and Slotine reformulated the scattering transformation and introduced the *wave variables* that can be used for the analysis of time delayed bilateral teleoperation in a similar fashion [41, 82]. In a general manner, the principles

of wave variables are close to the passivity terminology. It basically separates the input-output powers and associates them with the input-output wave variables.

In order to define the framework, assume that there exists a two-port network such that v_1 and F_1 are input variables increasing the power and v_2 and F_2 are output variables that decrease the power flow. Then the total power of the two-port network can be calculated with wave variables as follows

$$P = v_1^T F_1 - v_2^T F_2 = \frac{1}{2} p_1^T p_1 - \frac{1}{2} n_1^T n_1 + \frac{1}{2} p_2^T p_2 - \frac{1}{2} n_2^T n_2 \tag{2.11}$$

where apparently p_i signals increase power of the network, on the other hand n_i signals decreases the total power and they will be referred as input and output wave variables. Thus, transformation from power variables (v, F) to wave variables (p, n) has been introduced implicitly. Distinctly, based on (2.11) this transformation can be defined as follows,

$$p_1 = \frac{1}{\sqrt{2b}}(F_1 + bv_1), \quad p_2 = \frac{1}{\sqrt{2b}}(F_2 - bv_2),$$

$$n_1 = \frac{1}{\sqrt{2b}}(F_1 - bv_1), \quad n_2 = \frac{1}{\sqrt{2b}}(F_2 + bv_2),$$

where b is a positive constant value and going to be referred as *characteristic impedance* or wave impedance. Generally speaking it is desired to have higher and lower values of this impedance when the slave is in contact with environment and in free space, respectively. As the value of the impedance effects behaviour of the system it should be tuned meticulously, we can refer reader to [84] for the method of selection of the b parameter. The wave variables can be transformed from power variables and complementary transformation from wave to power variables is also possible.

A system is said to be passive if the output energy, provided by output waves, is lower than the input energy provided by input waves. Previously, it has been shown that even a small delay might destroy passivity of the communication medium, but with proper transformation this undesired phenomenon can be eliminated. The application of the wave variables to the teleoperation can be illustrated with an example where velocity information is sent from master to slave, and measured force is sent from slave to master via communication channel which accommodates a constant delay such that

$$v_s(t) = v_m(t - T_d), \qquad F_m(t) = F_s(t - T_d),$$

where v_i and F_i denote velocity and force of the master and slave manipulators for $i = m, s$.

In a passive communication medium, power dissipation is always positive, but as mentioned previously once there exists a latency, unfortunately power dissipation becomes negative. In that case, stability of the overall system might be in danger as a combination of some controllers and manipulators might lead to a non-passive interconnection [41]. Conversely, with the properly inserted dissipation elements a time delayed communication channel can be transformed into a passive system regardless of the value of the time delay. For this purpose, instead of power variables, force and speed, wave variables are transmitted via the communication medium, with the following transformation,

$$n_m(t) = p_s(t - T_d) \quad \text{and} \quad n_s(t) = p_m(t - T_d)$$

Based on the proposed wave variables power flow of the architecture illustrated in Figure 2.8 can be calculated as,

$$\begin{aligned} P &= \frac{1}{2}\left(p_m(t)^2 - n_m(t)^2 + p_s(t)^2 - n_s(t)^2\right) \\ &= \frac{1}{2}\left(p_m(t)^2 - n_s(t - T_d)^2 + p_s(t)^2 - n_m(t - T_d)^2\right) \\ &= \frac{1}{2}\frac{d}{dt}\left(\int_{t-T_d}^{t} p_m(\lambda)^2 + p_s(\lambda)^2 d\lambda\right). \end{aligned}$$

As positivity of storage energy is preserved, it can be concluded that the communication channel remains passive regardless of time delay duration. This demonstrates usefulness of the wave variables for obtaining passive delayed communication channel. Additionally passivity is independent of the delay itself and information related to delay duration does not need to be known priorly.

On the other hand, signal reflection is a well-known phenomenon in the power transmission lines; a signal reflects back to its original direction if the impedance of the material, where the signal is being transmitted, is not homogeneous along the material or if there exists a change in the impedance of the transmission medium[10]. The reflected back signal can cause some damages and an unwanted consequences within the system like a deterioration in the original signal. Therefore this phenomenon must be eliminated rigorously. One of the eliminating method is impedance

[10]Similar phenomenon when light reflects from water surface

matching which implies the equivalence of the impedances at each terminal by choosing characteristic impedance b, see [41] for more detailed information. Also, local impedance controller can be implemented into the both sides (master and slave) which receive force information and provide velocity signals; the block diagram representation is illustrated in Figure 2.8. The controllers at both sides are basically a PI (Proportional Integral) controller which consists of a spring and a damper. Energy will be transmitted with spring and will be slightly dissipated over the damper.

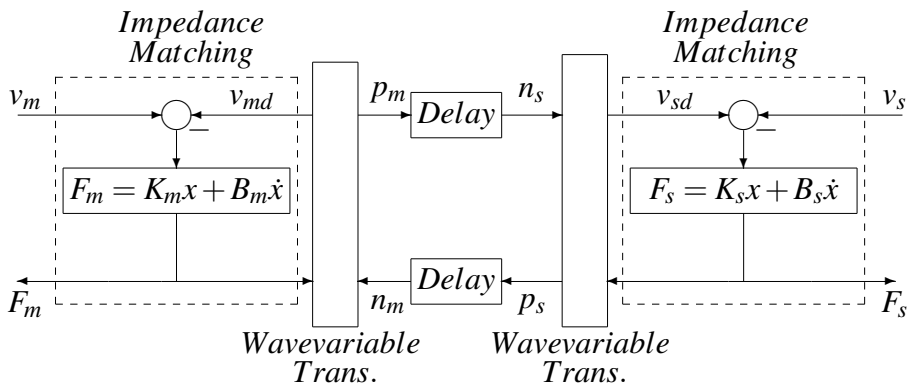

Figure 2.8: Wave transmission and impedance matching in bilateral teleoperation under velocity control at both sides (with a constant transmission delay T_d, $v_{md} = v_s(t - T_d)$), where K_i and B_i need be chosen close to b for $i = m, s$ ($\dot{x} = v$).

To sum up, the main concern of the scattering transformation or wave variables is ensuring stability while there is a time delay in the communication channel. And its aim is to guarantee the stability, whatever the value of delay, we refer reader to [85,86] for more detailed information.

On the contrary, time delay in the communication channel is not always a constant value as discussed previously, it highly depends on the communication method and medium itself. For example, if two manipulators are connected via Internet then latency is going to be a time varying parameter as speed of the transmission depends on various parameters such as network traffic, bandwidth, and distance between the terminals. Strictly speaking, the aforementioned scattering transformation guarantees passivity of the communication channel when the time delay is constant and not varying [87]. Therefore, it is required to reformulate the transformation in order to be used for time varying delays as well. And such a methodology is proposed by Lozano et al. in [88] where passivity is guaranteed not only for constant but also for time varying delays.

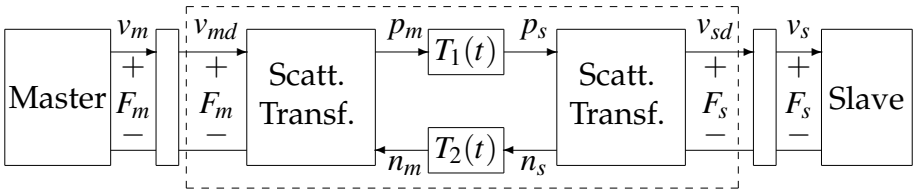

Figure 2.9: Scattering transformation for bilateral teleoperation with variable time delays [88].

As in general, input and output energy of communication medium with variable delay needs to be calculated in order to be able to validate the passivity property. Firstly, total power in the communication channel can be obtained subtracting the output power from the input power

$$P_{in}(t) = v_{md}(t)F_m(t) - v_{sd}(t)F_s(t)$$

In the case of constant network delay we have already showed that the communication channel is going to be passive with wave variable transformation in (2.11), when the network delay is time varying the previous results do not hold the passivity condition as now the transmission equation becomes

$$p_s(t) = p_m(t - T_1(t)), \quad n_m(t) = n_s(t - T_2(t))$$

where, $T_1(t)$ and $T_2(t)$ are time varying delays. The stored energy in the communication channel can be calculated in a similar manner. But in this case, the communication channel is going to be passive only when the delay is decreasing, but otherwise (increasing delay scenario) it shows non-passive behaviours. This phenomenon can be clearly seen in the following equation

$$\int_0^t P_{in}(\tau)d\tau = \frac{1}{2}\Bigg(\int_{t-T_1(t)}^{t} p_m^T(\tau)p_m(\tau) + \int_{t-T_2(t)}^{t} n_s^T(\tau)n_s(\tau)d\tau$$
$$- \int_0^{t-T_1(t)} \frac{\dot{T}_1(\sigma)}{1-\dot{T}_1(\sigma)} p_m^T(\sigma)p_m(\sigma)d\sigma - \int_0^{t-T_2(t)} \frac{\dot{T}_2(\sigma)}{1-\dot{T}_2(\sigma)} n_s^T(\sigma)n_s(\sigma)d\sigma\Bigg),$$

where $\sigma = \tau - T_i(\tau)$. As duration of delay cannot be bigger than the time itself variation of the delay is bounded such that $\dot{T}_i(t) \leq 1$. A time varying gain, k_i, can be inserted after the time varying delay block in Figure 2.9 so that the potential non-passive element of the time varying delay will be eliminated. After the alteration, new transmission equations are

$$p_s(t) = k_1(t)p_m(t - T_1(t)) \quad n_m(t) = k_2(t)n_s(t - T_2(t)).$$

Now the total energy flow can be calculated as

$$\int_0^t P_{in}(\tau)d\tau = \frac{1}{2}\Bigg(\int_{t-T_1(t)}^{t} p_m^T(\tau)p_m(\tau) + \int_{t-T_2(t)}^{t} n_s^T(\tau)n_s(\tau)d\tau + \int_0^{t-T_1(t)} \frac{1-\dot{T}_1(\sigma)-k_1^2}{1-\dot{T}_1(\sigma)} p_m^T(\sigma)p_m(\sigma)d\sigma + \int_0^{t-T_2(t)} \frac{1-\dot{T}_2(\sigma)-k_2^2}{1-\dot{T}_2(\sigma)} n_s^T(\sigma)n_s(\sigma)d\sigma\Bigg).$$

In order to eliminate the second terms, which may destroy passivity, in the above equation the time varying gain k_i is chosen to be as $k_i^2 = 1-\dot{T}_i$. In this way the system is going to be passive and when the time delay is constant, $k_i = 1$, the initial results are going to be obtained. While designing a control law for a bilateral teleoperation, where the Internet medium is used as a communication environment information about time delay such as its variation and maximum delay duration is substantial.

Another method for eliminating difficulties related to the time varying delays is simply using *buffers* at input output ports of the communication medium. A virtual time delay, that is bigger than any delay that might occurs during the transmission, can be designated so that the scattering transformation can be used with this virtual time delay rather than time varying delay and the passivity is going to be ensured conveniently [89]. Complexity and conservativeness of the stability analysis while communication medium accommodating variable time delay will be discussed rigorously with IQC framework as well in the forthcoming stages.

2.4.3 Position Mismatch with Passive Communication

This section highlights the notorious property of the passivity based control methodology used in the time delayed bilateral teleoperation: position mismatch between the manipulators. It is desired that the slave manipulator tracks the position of the master, especially in the free space movement precise tracking is desired. But, when the scattering transformation or wave variables are implemented to eliminate active behaviours of the transmission delay the position error problem starts to rise to surface as such control schemas ensures only the velocity tracking [42, 82, 90–92]. To be able to use the passivity approach and simplify the power calculation velocity has been transmitted and velocity controller is implemented within the system. As a result, the position draft becomes inevitable especially when the initial positions of

two manipulators are different from each others or slave contacts with a stiff environment. Moreover, the delay can be time varying and this aggravates the problem, because the wave variable will be distorted and that deteriorates the position mismatch between master and slave.

In order to eliminate the position mismatch a number of methods have been introduced; for instance transmitting not only the wave variables but also their integrals, which contains position and momentum information, and adding a corrective term that increases the synchronisation between the manipulators is proposed in [82]. Also, a new configuration bilateral teleoperation was designed in [44], where speed and force information are sent via scattering transformation and also delayed position information were used in order to overcome that unwanted mismatch. With the additional position controllers synchronisation between the manipulators is ensured. Lyapunov stability analysis approach were implemented to obtain the range of the position control gains in the both sides. When the controller parameters hold the condition based on the manipulators' damping values such that $K^2 T_d{}^2 < B_m B_s$, it has been proved that the interconnection system is stable and the position error will eventually diminish. The stability property is turned to a delay dependent property from delay independent one as duration of the time delay affects parameters of the controllers.

As a final note, these methods that guarantee stability against any size of the communication delay are analogous with implementing the small gain theorem. Strictly speaking, norm of the delay operator is always equivalent to 1 and by simply ensuring norm of the nominal system is less then the gain of delay operator we would also guarantee the stability independent of the delay without including any other additional transformation within the depicted interconnection, see [23] for more detailed information about the relationship between passivity and small gain theorems.

2.5 Some Other Control Techniques for Teleoperation

Ensuring stability (strictly speaking passivity) of the time delayed teleoperation is a nontrivial problem. Obtaining a passive communication medium that contains latencies is prominent in the literature on bilateral teleoperation. By scattering

and wave transformation the transmission channel remains passive and stability of the teleoperation was guaranteed for any constant and after some restoration for time varying delays as well. In addition, Lee and Spong [93] showed that simple proportional-derivative (PD) controller can be used with time delayed bilateral teleoperation without jeopardizing passivity of the overall system. Namely, master and slave can be directly connected with virtual springs and dampers over time delayed communication channel. Energetic passivity of the closed loop teleoperation is obtained by using passivity concept of the controllers. For this control methodology the delay needs to be symmetric (required to be equal in both directions, from master to slave or vice versa) and it is assumed that time delay was known in advance.

Furthermore, in [45] similar methodology was applied yet in this case only the upper bound of the round trip delay is required to be known. The main advantage of these proposed architectures is that, exact position information is being transmitted therefore tracking and fidelity can be ensured without any additional synthesis. In some aforementioned approaches, such as scattering transformation, the communication channel itself is passivated, yet here with suitably chosen parameters, controllers and communication channel are passivated all together.

The main theoretical premise behind [45, 93] is that stability of the interconnection is ensured for an upper bounded time delay and synchronisation between the manipulators is obtained such that the position mismatch approaches to zero ($x_m(t) - x_s(t) \to 0$). In order to dissipate the energy generated by delayed communication channel and the controller a dissipation gain (K_d) is inserted into the controllers. And also an supplementary damping (P_e) is implemented into the system to ensure master-slave coordination. In order to guarantee the energetic passivity of the system, dissipation gain K_d was introduced as

$$K_d = \frac{T_{total}}{2} K_p,$$

where K_p is the proportional controller gain and T_{total} is round trip communication delay. With the designed PD controllers and with the condition given above the interconnected system is energetic passive besides the position error between master and slave eventually diminish.

On the contrary, [46] states that within the proposed methodology it has been assumed that both human and environment can be described by using stable op-

erators in $\mathcal{L}_\infty$ that mapping from velocity to force but the desired stability and performance criterion with simple PD-like schemas can be obtained under classical assumption of the passive terminals, yet with provided sufficiently large damping injected to both manipulator's subsystems. In that presented system, parameters of the controllers must hold the following inequality in order to obtain required stability and performance criterion

$$B_m B_s > {T_d}^2 K_m K_s,$$

where T_d is the time delay in the communication channel, K_m, K_s are proportional gains for eliminating the position errors, and B_m, B_s are scalar gains for the injected damping. It is worth highlighting that stability analysis covers only master, slave, controllers, and communication channel. In other ways, the affects of human and environment has been ignored, under favour of the assumptions, in order to be able to use Lyapunov equations for the stability analysis.

There seems to be no compelling reason to argue that the proposed framework achieves passable transparency only at low frequencies as no force sensor is used and the mechanical movements occur at the low frequencies [73]. Yet, by using the control law above the human operator feels a force at all the time; phenomenon known as *sluggish* transparency. This type of performance is extensively common when two-channel position error algorithm is being implemented in bilateral teleoperation. Especially within the aforementioned approach due to the time delay, even when the slave is in the free motion the master controller applies relentless force while the operator tries to move the master manipulator. To eliminate such a behaviour, a gain switching control scheme was proposed in [94]. The controller gains were being changed based on the detection of the impedance change at the slave side (whether it is in contact with environment or not). However, it should be noted time delay has not been considered, therefore stability was not an issue compared to the delayed one. It needs to be state that the upper bound of the controller parameters are obtained from only stability point of view, but there is not any specific tuning rules for the designed controller in order to be able to obtain optimum performance criterion particularly transparency point of view.

Despite the fact that passivity theory dominated the research field of the bilateral teleoperation, there are also different algorithms that have been used for analysis

and control of teleoperation system to overcome challenges, such as H_∞, adaptive, sliding mode, and model predictive controllers. Let us start with the H_∞ control that transfers controller design affair to a mathematical optimization problem, so with the optimum controller stability can be ensured along with a desired performance objective. The standard problem in this method is to find a proper controller that minimizes infinity norm of the lower fractional transformation of the system under the constraint of internal stability. An H_∞ controller was designed for delay free bilateral teleoperation in [95]. The dynamic relationship between master and slave manipulators was the main performance criterion. Then, a controller methodology that uses the H_∞ optimum control theory and μ-synthesis was designed for time delayed teleoperation in [96]. Here, depending on the interaction between slave and environment the control algorithm was divided into two different parts. A separate controller was designed with H_∞ technique when there is no environmental interaction and the other one was designed when the slave interacts with the environment.

Additionally, there is sufficient research on predictive controller to draw any firm conclusions about the implementation on bilateral teleoperation. For instance, Sheng and Spong designed a Model Predictive Controller (MPC) for time delayed bilateral teleoperation in [97]. Their main aim is to analyse robustness of MPC when the time delay is unknown, hereby impedance of the environment was assumed to be known. An MPC was designed for teleoperation when the slave is not in an interaction with the environment ($F_e = 0$), and then performance of the MPC was analysed when slave is contacted with environment, and also when there is a mismatch between real forward time delay and the assumed one (predicted one). It was stated that when the mismatch is high, proposed method fails to provide a stable bilateral teleoperation, yet when the time delay is known MPC can provide stable interconnection between master and slave. On the basis of the evidence currently available, it seems fair to claim that information of the time delay is crucial while implementing MPC for bilateral teleoperation.

Likewise, an MPC was designed for time delayed bilateral teleoperation in [98] as well, yet as distinct from the previous approach here the time delay is assumed to be known. Two separate controllers were designed for the free movement and rigid contact. In order to obtain transparency objective an LQG controller/observer pair is used at both master and slave sites. It must be noted that to use this synthesis

methodology, the operator and environment need to be modelled, therefore second order LTI dynamics have been used for the human operator and environment. As the proposed method is model based predictive controller, the models should be sufficiently good in order to obtain robustness even when there exists high latencies. Also, a different control structure was designed in [99] where the slave manipulator was controlled with PI controller and a Smith predictor is inserted in the master side to predict the future behaviours of the slave, thus this predicted signal and master position (displacement) signal is sent to the bilateral predictive controller.

Adaptive controller is used for obtaining high performance when the model used in the system is poorly modelled or time varying. Adaptation mechanism, as the name suggests, modifies the system parameters to adapt the internal or external changes. In teleoperation, adaptive controller is designed to cope with operator, master, slave, environment model uncertainties, and time delays. The adaptive controllers that designed to overcome latency can be divided two main categories; passivity based adaptive controller and Virtual Internal Model (VIM) adaptive controller [100]. There are a number of researches in the literature that use the adaptive method to overcome different problems in teleoperation. It is possible to find all kinds of adaptive controller that have been designed for bilateral teleoperation in a recently published detailed paper by Chan et al. [101].

Apart from these controller methodologies it has been stated that when computers are used as a medium for the controllers discretization needs to be carried out, yet this process might destroy the passivity of the overall interconnection. The findings in [102] lend support to the claim that discretization jeopardizing the passivity and discrete time controller was analysed along with continuous time master, slave, human operator, and environment. The trade off between performance and stability was investigated in the existence of the sampling rate and controller parameters were chosen based on this trade off, see [103] for more detailed information.

3 Preliminaries on Multiplier and IQC Theories

The first task of a control engineer is to obtain a model of the plant that is desired to be controlled. But, a mathematical model will never be able to characterize all the properties of the plant precisely. A meticulously designed controller for the model might produce undesirable or unexpected behaviours due to the sensitivity of the overall design to the model mismatches. At that point, the engineers may use their experience and intuition to slightly modify the controller for achieving a level of desire from the performance of the controlled system. This technique has been practically useful and implemented frequently for simple plants that require to accomplish straightforward tasks. Thus in these type of applications control theory might have little effect. On the other hand, as complexity of the systems boosted with some operators like nonlinearities, intuitional approach becomes difficult to perform and apply. Therefore, the demand for a theory that is able to cope with obscurity of the real plants rose to the surface. And this bring us to the robustness terminology: a design is called robust if it can cope not only with a certain model but also with a set of models which are within the neighbouring subsets of the particular model [104].

In this chapter, in the spirit of [105], we will introduce some particular methodologies in robust control analysis that are mostly based on frequency-domain stability criteria: Multiplier theory and Integral Quadratic Constraints framework.

3.1 Multiplier Approach

An interconnection between a linear system and bounded nonlinearity is said to be *absolutely stable* if the depicted feedback, known as Lur'e system [160], has a globally uniformly asymptotically stable equilibrium point at the origin for all nonlinearity within the given class. Yet, revealing the absolute stability of such an interconnection is a nontrivial task, thus multipliers are the first technique that are being proposed for this purpose. Strictly speaking, multipliers are virtual operators[1] that are being inserted (with its inverse) into a feedback system such a way that overall feedback interconnection remains unchanged, yet stability analysis of the overall system can be carried out more conveniently based on the well established positivity property of the individual loops. Also, complexity of the stability problem in the Lur'e structure is reduced as approach leads to simple solution based on the linear part only. Along with the usefulness, multiplier approach also reduces the conservatism of the stability analysis based on pure small gain and passivity theorems. In the same manner as in [106], let us assume that we would like to investigate stability of the system illustrated in Figure 3.1 (so-called Lur'e structure) where the feedback interconnection is defined by

$$\begin{cases} v = Gw + f, \\ w = \Delta(v) + g; \end{cases} \tag{3.1}$$

where g, w, $\Delta(v) \in \mathcal{L}_{2e}^{m}$ and Gw, v, $f \in \mathcal{L}_{2e}^{l}$. This interconnection is said to be well-posed if the map $(v,w) \mapsto (g,f)$ has a causal inverse on $\mathcal{L}_{2e}^{m+l}[0,\infty)$. The well-posedness condition is for ensuring that there exists a solution for the differential equation of the system, and it can be said that real systems naturally have well-posedness property. Additionally it can also be said that the interconnection is stable if for any $(g,f) \in \mathcal{L}_{2}^{m+l}$, then $(w,v) \in \mathcal{L}_{2}^{m+l}$. Namely, this is the well-known bounded-input bounded-output condition; the system will be called stable if applying external well behaved bounded signals only create well-behaved signals that decay to the zero over the time interval.

In addition to this, positivity property of the each forward channel can be investigated in order to ensure stability of the interconnected system. It leads to so-called

[1] An operator maps from one space domain into another space domain.

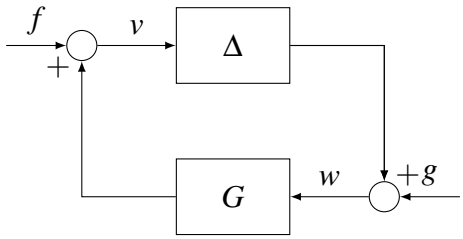

Figure 3.1: Classical feedback interconnection of a nominal plant and perturbation

passivity theory, yet firstly let us propose the following definitions:

Definition 3.1 (Positive Operators [107]). *Let $\mathcal{H}$ be a Hilbert space with inner product denoted by $\langle , \rangle$ and let A be an operator in $\mathcal{H}$; then A is said to be positive if $\Re\{\langle Ax, x\rangle\} \geq 0$ for every $x \in \mathcal{H}$.*

Definition 3.2 (Truncation [108]). *The set $\mathcal{L}_{pe}^n$ consists of all measurable functions $f : [0,\infty) \to \mathbb{R}^n$ such that the truncations*

$$f_T(t) = \begin{cases} f(t), & 0 \leq t \leq T \quad (t \in \mathbb{R}_+) \\ 0, & t > T \end{cases}$$

are in $\mathcal{L}_p^n$ for all finite $T \geq 0$.

Theorem 3.1 (Passivity Theorem [109]). *Let us assume that feedback interconnection in (3.1) is well-posed. The system is stable if the following inequalities*

$$\langle v_T, \Delta v_T \rangle \geq 0,$$
$$\langle v_T, G v_T \rangle \leq -\varepsilon \|v_T\|^2,$$

hold for some $\varepsilon > 0$, for all $v \in \mathcal{L}_{2e}^m[0,\infty)$, and $T \in \mathbb{R}_+$.

Remark 3.1. *Note that positivity is tested on $\mathcal{L}_p$ spaces yet passivity is tested over extended $\mathcal{L}_{pe}$ spaces, which emphasizes that causality condition is required for a positive operator to be passive as well.*

In order to enhance passivity property of the each forward channel a bounded causal operator M can be embedded into the system along with its bounded inverse thereby the entire interconnection transformed from Figure 3.1 to Figure 3.2 (a). The original system's property remained consistent, yet now passivity of the individual feedforward operators can be investigated together with property of the injected

multiplier M. That additional operator can distribute *extra*[2] passivity property of the individual element to the other operators within the interconnection. The main objective with this methodology is to somehow simplify the stability analysis of the interconnection (particularly when there exists a nonlinearity within the loop) in the circumstances where proving or establishing stability of the original system is more complex than the one with multiplier.

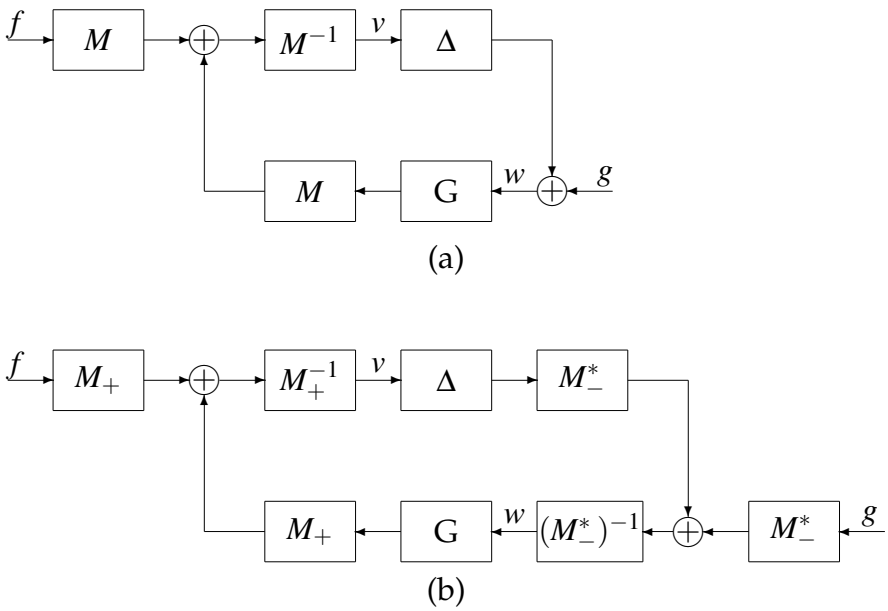

Figure 3.2: Lur'e interconnection with injected bounded multiplier M (a), factorization is implemented to maintain the causality property (b).

Real world physical and realizable systems respect an intuitive and fundamental property called causality. In the nature that we are in, there is always a reaction to every action not other way around: effect cannot create its cause. Consistently, to be able to apply physically intuitive passivity theory while analysing the stability all the operators within the interconnection need to hold the causality property. This is to say both multiplier M and its inverse M^{-1} need to be causal and the passivity theory cannot be applied if any of these are non-causal [110]. In many stability analysis problems as well having or using particularly casual operators is one of the main concerns. In a feedback interconnection being able to obtain two positive operators, which are also causal, is a desired condition as it leads to a nicely stable interconnected system. Therefore, developing causal operators from arbitrary positive operators is crucial. Let us propose the technique that provides causal positive

[2]The intention of this terminology will be clear in the further section once we illustrate passivity of the operator with and without multipliers.

operators from any positive operators:

Theorem 3.2 (Factorization [107]). *Let us assume that there exists a positive operator M on the Hilbert space of functions. And let M holds the factorization such that $M = M_- M_+$, where M_+ is a causal and M_- is invertible bounded operator with causality property of $(M_-^*)^{-1}$. Then, $M_+(M_-^*)^{-1}$ is a positive and causal operator.*

The theorem shows that a causal positive operator can be obtained from a non-causal positive operator and the only condition that is required to satisfy the factorization property. It received a great deal of attention as restriction on using only casual operators while using Multiplier theory can be eliminated as once a positive operator (either causal or non-causal) obtained factorization can be carried out to hold required causality property of the each sub-system within the loop.

Therefore, in such circumstances where causality of the inverse operator became an issue, multiplier M needs to be factorized as $M = M_- M_+$ such that M_-^* and M_+ and their inverses are bounded and causal, as a result the overall loop is transformed from Figure 3.2 (a) to Figure 3.2 (b) without any intervention. Now we are ready to propose the stability theorem based on the multiplier theory:

Theorem 3.3 (Multiplier Theory [106]). *Let $M : j\mathbb{R} \to \mathbb{C}^{m\times l}$ be bounded measurable function. Assume that*

1. *feedback interconnection between $G \in \mathbf{RH}_\infty^{l\times m}$ and $\Delta : \mathcal{L}_{2e}^m[0,\infty) \to \mathcal{L}_{2e}^l[0,\infty)$ is well-posed,*

2. *Δ satisfies,*

$$\int_{-\infty}^{\infty} \begin{bmatrix} \widehat{v}(j\omega) \\ \widehat{\Delta v}(j\omega) \end{bmatrix}^* \begin{bmatrix} 0 & M^*(j\omega) \\ M(j\omega) & 0 \end{bmatrix} \begin{bmatrix} \widehat{v}(j\omega) \\ \widehat{\Delta v}(j\omega) \end{bmatrix} d\omega \geq 0, \quad \forall v \in \mathcal{L}_2^m,$$

3. *M can be factorized into $M = M_- M_+$, where M_+, M_-^*, and their inverses are casual and bounded,*

4. *there exists a positive ε such that*

$$\begin{bmatrix} G(j\omega) \\ I \end{bmatrix}^* \begin{bmatrix} 0 & M^*(j\omega) \\ M(j\omega) & 0 \end{bmatrix} \begin{bmatrix} G(j\omega) \\ I \end{bmatrix} \leq -\varepsilon I, \quad \forall \omega \in \mathbb{R}.$$

Then, the positive feedback interconnection between G and Δ is stable.

On the other hand, a new framework to analyse absolute stability of the systems in the frequency domain was proposed; that is so-called IQC theorem where sufficient stability conditions can be provided for complicated interconnected systems. In other words, by following this aforementioned stability technique multiplier search can be carried out with ease via convex optimization leading to tractable solutions.

3.2 Overview of IQC Theory

Looking for the convenient approaches that simplify stability analysis of the systems containing nonlinearities or uncertainties has always been a great deal of interest. Input-output approach in other words Operator approach has dominated the western literature among the small gain and passivity theorems are the well-recognized, yet it is agreed that these techniques give conservative conditions for the stability tests. To reduce the conservatism gap within the input-output approach, multiplier theory is proposed along with loop transformation and well established especially while analysing systems with the nonlinearities. Then, the focus of the interest among the researches moved to find appropriate multiplier or to illustrate existence of the multiplier that holds the constraints/conditions such that the stability is ensured. Combined with the causality requirement, however, search for the appropriate class of multiplier became the main issue in the input-output stability approach.

On the other hand, within the same period absolute stability technique has dominated the Soviet world where Russian researches used the multipliers in a different sense; instead of defining them as operators they are used simply as mathematical tools. Herein, so-called Integral Quadratic Constraints theory, which was introduced by Megretski and Rantzer in [111, 112] where the operators have been implicitly analysed in frequency domain, was the first bridge between Input-Output and Absolute stability approaches, reader referred to [105] and references therein for detailed analysis and synthesis related to this combination. IQC is simply based on the definition of the nonlinearities or perturbations where they are defined in the format of quadratic constraints frame work.

Definition 3.3 (Integral Quadratic Inequality [112]). *Let $\Pi : j\mathbb{R} \to \mathbb{C}^{(l+m)\times(l+m)}$ be a Hermitian bounded measurable function. Two signals $u \in \mathcal{L}_2^m[0,\infty)$ and $y \in \mathcal{L}_2^l[0,\infty)$ are said to satisfy the IQC defined by Π, if*

$$\int_{-\infty}^{\infty} \begin{bmatrix} \widehat{u}(j\omega) \\ \widehat{y}(j\omega) \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} \widehat{u}(j\omega) \\ \widehat{y}(j\omega) \end{bmatrix} d\omega \geq 0. \tag{3.2}$$

Moreover, a bounded system $\Delta : \mathcal{L}_{2e}^m[0,\infty) \to \mathcal{L}_{2e}^l[0,\infty)$ is said to satisfy the IQC defined by Π if u and Δu satisfy the IQC defined by Π for all $u \in \mathcal{L}_2^m$.

Once the appropriate quadratic constraints expressed for the troublesome operators (e.g. nonlinearities), stability of the system that is depicted as a Lur'e structure can be analysed by using the IQC theorem:

Theorem 3.4 (IQC Theory [112]). *Let $G \in \mathbf{RH}_\infty^{l\times m}$, and Δ be a bounded causal operator. If the following statements hold:*

1. *for every $\tau \in [0,1]$, the interconnection of G and $\tau\Delta$ is well-posed,*

2. *for every $\tau \in [0,1]$, $\tau\Delta$ satisfies the IQC defined by Π,*

3. *there exist $\varepsilon > 0$ such that*

$$\begin{bmatrix} G(j\omega) \\ I \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} G(j\omega) \\ I \end{bmatrix} \leq -\varepsilon I, \quad \forall \omega \in \mathbb{R}.$$

Then, the positive feedback interconnection of G and Δ in Figure 3.1 is stable.

The matrix in the middle, $\Pi(j\omega)$, will be referred as an *IQC multiplier* for sake of preventing possible confusion with the previous multiplier definition given for M. But, the relationship comes to the light as it can be noted that multiplier and IQC theories have similarities such that their equivalence was indicated with the condition that right lower corner of the IQC multiplier Π is negative semi-definite [113, 114]. IQC approach can be seen as a generalization of the multiplier methodology, yet with an additional convenience: there is no need to concern about the causality property as it is not based on the time domain specifications yet the price paid for this convenience is the homotopy argument; well-posedness condition is required for the $G-\tau\Delta$ interconnection for all $\tau \in [0,1]$. Conversely, it can also be stated that in multiplier approach once the positive multiplier is obtained then it can be factorized

as desired (such that having causality property), we refer reader to [110] for more information about in what conditions both approaches are equivalent.

Remark 3.2. *Generally,* $\Pi : j\mathbb{R} \to C^{(l+m)\times(l+m)}$ *can be any Hermitian bounded measurable function in the form as* $\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^* & \Pi_{22} \end{bmatrix}$*, yet throughout this thesis we will restrict to have* $\Pi_{11} \geq 0$ *and* $\Pi_{22} \leq 0$ *so that the second condition in Theorem 3.4 will be satisfied if and only if* Δ *satisfies the IQC defined by* Π*. It implies that the existence of IQC will be equivalent to have a multiplier [113].*

Some well-known stability analysis methods in robust control can be expressed via IQC theory such as passivity and small gain theorems. For instance, let us assume that Δ is a passive LTI operator then the following quadratic constraint holds

$$\begin{bmatrix} \widehat{u}(j\omega) \\ \widehat{\Delta u}(j\omega) \end{bmatrix}^* \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} \widehat{u}(j\omega) \\ \widehat{\Delta u}(j\omega) \end{bmatrix} \geq 0.$$

In a similar fashion, if it is assumed that the perturbation holds the small gain condition then IQC for the Δ operator can be expressed as

$$\begin{bmatrix} \widehat{u}(j\omega) \\ \widehat{\Delta u}(j\omega) \end{bmatrix}^* \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} \widehat{u}(j\omega) \\ \widehat{\Delta u}(j\omega) \end{bmatrix} \geq 0.$$

As shown, frequency independent IQC multipliers can be used to express the passivity and small gain theorems via IQC and as stated previously these methodologies are notorious to give conservative stability conditions [115].

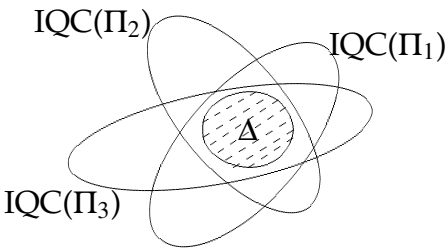

Figure 3.3: Increase in the number of the IQCs defined for an uncertainty set reduces the conservatism gap. IQC(Π) represents the set of systems that satisfy the IQC defined by Π: a graphical illustration.

On the other hand, conservatism due to a particular perturbation can be reduced once the number of IQCs defined for the particular uncertainty class is increased (if feasible). Namely, if some multipliers Π_i, for $i = 1, ..., n$ have already known to

satisfy (3.2), then their conic combination,

$$\Pi(j\omega) = \sum_{i=1}^{n} x_i \Pi_i(j\omega), \quad x_i \geq 0, \quad i = 1, ..., n,$$

also satisfies (3.2) [116]. This can be interpreted as, the set of IQCs for the depicted class of uncertainty is somehow covering the undefined intersection region, which for sure includes the set of perturbation as well, as illustrated in Figure 3.3. For the sake of completeness let us propose the following definition:

Definition 3.4 (Canonical Combination)**.** *Given a number of vectors $x_1, x_2, \ldots, x_n$ in the vector space, the canonical combination of these vectors is*

$$\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n,$$

where α_i is a real number which satisfies $\alpha_i \geq 0$, $i = 1, ..., n$.

Moreover, assume that Δ is a structured uncertainty; diagonal combination of different perturbations such that $\Delta = \mathrm{diag}(\Delta_1, ..., \Delta_n)$ and each subsystem satisfies IQC defined by

$$\Pi_i(j\omega) = \begin{bmatrix} \Pi_{i(11)} & \Pi^*_{i(12)} \\ \Pi_{i(12)} & \Pi_{i(22)} \end{bmatrix}$$

where $i = 1, ..., n$, then overall system satisfies the IQC defined by

$$\Pi = \left[\begin{array}{cccc|cccc} \Pi_{1(11)} & 0 & \cdots & 0 & \Pi^*_{1(12)} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \Pi_{n(11)} & 0 & \cdots & 0 & \Pi^*_{n(12)} \\ \hline \Pi_{1(12)} & 0 & \cdots & 0 & \Pi_{1(22)} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \Pi_{n(12)} & 0 & \cdots & 0 & \Pi_{n(22)} \end{array}\right].$$

After expressing appropriate class for multipliers holding IQC for the set of uncertainties, then robustness of the feedback interconnection between a nominal plant and pre-defined set of uncertainties will be investigated with the last condition in Theorem 3.4. Stability condition can be checked based on the frequency-dependent, infinite dimensional inequality with a frequency gridding, but it can

be transformed into a frequency-independent finite dimensional LMIs by using the Kalman-Yakubovich-Popov (KYP) lemma:

Lemma 3.1 (KYP lemma [117]). *Given $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $M = M^\top \in \mathbb{R}^{(n+m)\times(n+m)}$, with $\det(j\omega I - A) \neq 0$ for all ω, where $[A, B]$ are controllable. The following two statements are equivalent:*

i)
$$\begin{bmatrix} (j\omega I - A)^{-1}B \\ I \end{bmatrix}^* M \begin{bmatrix} (j\omega I - A)^{-1}B \\ I \end{bmatrix} \leq 0,$$

ii) There is a matrix $P \in \mathbb{R}^{n\times n}$ such that $P = P^\top$ and
$$\begin{bmatrix} A^\top P + PA & PB \\ B^\top P & 0 \end{bmatrix} + M \leq 0.$$

Remark 3.3. *One can write the last condition in Theorem 3.4 as a finite dimensional LMI based on the above classical KYP lemma as: Given a minimal transfer function defined with state-space matrices as $G(j\omega) = C(j\omega I - A)^{-1}B + D$, then the following two statements are equivalent:*

i)
$$\begin{bmatrix} G(j\omega) \\ I \end{bmatrix}^* \bar{M} \begin{bmatrix} G(j\omega) \\ I \end{bmatrix} \leq 0,$$

ii) There is a matrix $P \in \mathbb{R}$ such that $P = P^\top$ and
$$\begin{bmatrix} A^\top P + PA & PB \\ B^\top P & 0 \end{bmatrix} + \begin{bmatrix} C & D \end{bmatrix}^\top \bar{M} \begin{bmatrix} C & D \end{bmatrix} \leq 0.$$

Relationship between this multiplier and the one in Lemma 3.1 is distinct as $M = \begin{bmatrix} C & D \end{bmatrix}^\top \bar{M} \begin{bmatrix} C & D \end{bmatrix}$.

The lemma has various applications in systems theory and control. Also, based on the system that is being analysed, the equivalence between frequency dependent inequality (FDI) and LMI can be proposed for a particular frequency range by using Generalized KYP lemma, see [118] for more detailed information.

3.3 Classes of Multipliers

In this section, definitions of the different classes of multipliers that we shall use while analysing absolute stability of the bilateral teleoperation are given.

3.3.1 Passive and Zames-Falb Multipliers

A SISO LTI system $\Delta \in \mathbf{RH}_\infty$ is said to be passive if $\Delta(j\omega) + \Delta(j\omega)^* \geq 0$ for all $\omega \in \mathbb{R}$. One can define an off-diagonal identity matrix as a multiplier, yet as mentioned that leads to conservative stability conditions. To reduce the conservatism frequency dependent class of multipliers preserving the positivity of this class is defined in [119]:

Definition 3.5. *Let λ be a function, then λ belongs to the class of passive multipliers $\mathcal{P}$ if $\lambda(\omega) = \lambda(\omega)^*$ and $\lambda(\omega) > 0$.*

Lemma 3.2 ([119]). *Given a bounded LTI passive system Δ and $\lambda \in \mathcal{P}$, then Δ satisfies the IQC defined by*

$$\Pi(\omega) = \begin{bmatrix} 0 & \lambda(\omega) \\ \lambda(\omega) & 0 \end{bmatrix}. \tag{3.3}$$

A nonlinearity $\phi : \mathcal{L}_{2e}[0,\infty) \to \mathcal{L}_{2e}[0,\infty)$ is said to be memoryless if there exists $N : \mathbb{R} \to \mathbb{R}$ such $(\phi v)(t) = N(v(t))$ for all $t \in \mathbb{R}$. Henceforth we assume that $N(0) = 0$. A memoryless nonlinearity ϕ is said to be bounded if there exists a positive constant C such that $|N(x)| < C|x|$ for all $x \in \mathbb{R}$. The nonlinearity ϕ is said to be monotone if for any two real numbers x_1 and x_2 we have

$$0 \leq \frac{N(x_1) - N(x_2)}{x_1 - x_2}, \quad \forall x_1 \neq x_2.$$

The class of multiplier that attain positivity of this class of nonlinearity is defined as follows:

Definition 3.6 (Zames-Falb Multiplier [32, 33]). *Let Z be a rational transfer function. Then, Z belongs to the multiplier class of Zames-Falb multipliers $\mathcal{Z}$, if the following three conditions are satisfied:*

- $$Z(j\omega) = z_0 - \int_{-\infty}^{\infty} q(t) e^{-j\omega t} dt, \tag{3.4}$$
- $$\int_{-\infty}^{\infty} |q(t)| dt < z_0, \tag{3.5}$$
- $$q(t) \geq 0, \quad \forall t \in \mathbb{R}. \tag{3.6}$$

Remark 3.4. *A different class of multipliers can be generated by removing the last condition, but further a condition in the nonlinearity (being odd) is required.*

Lemma 3.3 ([33]). *Given a memoryless, monotone and bounded nonlinearity ϕ and any $Z \in \mathcal{Z}$, then the nonlinearity satisfies the IQC defined by*

$$\Pi(j\omega) = \begin{bmatrix} 0 & Z(j\omega)^* \\ Z(j\omega) & 0 \end{bmatrix}. \tag{3.7}$$

A comparison between the rest of the classes for this type of nonlinearities and the class $\mathcal{Z}$ is given in [37], for more detailed information we refer reader to the tutorial paper [120] and its extended version [38].

3.3.2 Multipliers for Time Delay

Delay is an intrinsic behaviour of the most systems hereby over the years it has engaged the attention of many researches [121]. The, literature is dominated by the Lyapunov-Krasovskii functional technique [81, 122], but a considerable effort was made based on IQC framework [123–127] as well.

In this section we will briefly discuss the multipliers for constant and time variable delays that will be used in the robustness analyses. To begin with, in the IQC framework, the uncertainty based on delay operator, $\Delta(j\omega) = e^{-j\omega T_d}$, is encapsulated with a negative unit, i.e. $\Delta_d(j\omega) = e^{-j\omega T_d} - 1$, hence one can think of the block Δ_d as a perturbation of the feedback without delay. Subsequently, a positive feedback interconnection is injected into the nominal system to not make any changes in the original interconnection, see Figure 3.4 for the mentioned "loop shifting" in the block diagram.

The first multiplier that we want to propose for the constant delay is conic combination of the two multipliers; one is based on the magnitude of the frequency response and the other depends on its characterization in the complex plane. On the complex plane, the operator $\Delta(j\omega)$, with a given time delay T_d, characterizes a unit circle whose centre is located at the origin and that can be interpolated as $\Delta_d^*\Delta_d + \Delta_d + \Delta_d^* = 0$. This property can be defined as a quadratic function by using

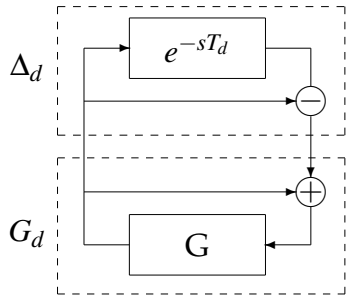

Figure 3.4: Time delayed system with feedback and feedforward transformation, where $\Delta_d = e^{-sT_d} - 1$.

any negative definite function $\Omega : \mathbb{R} \to \mathbb{R}$ as

$$\begin{bmatrix} I \\ \Delta_d(j\omega) \end{bmatrix}^* \begin{bmatrix} 0 & \Omega(\omega) \\ \Omega(\omega) & \Omega(\omega) \end{bmatrix} \begin{bmatrix} I \\ \Delta_d(j\omega) \end{bmatrix} \geq 0, \quad \forall \omega \in \mathbb{R}. \tag{3.8}$$

The negativity condition of Ω can be removed as the quadratic formulation in (3.8) holds for all Ω (negative or positive), yet depicted as a negative definite with proposed positive inequality so that the interior region of the unit circle on the complex plane can be defined while characterizing the uncertainty as a quadratic inequality. Also right lower corner of the IQC multiplier is going to be negative definite. In the contrary case the outer space can be expressed yet that might leads to ill conditions while carrying out the numerical search for the depicted multiplier.

Conservatism that appears when small gain property of the norm bounded delay operator is used can be reduced with a predefined transfer function, $W_d(j\omega)$, in a way that lower amplitudes of the uncertainty in the lower frequencies can be utilized. Amplitude covering property of the W_d can be seen in Figure 3.5, with the following rational transfer function [128];

$$W_d(j\omega) = 2\frac{(j\omega)^2 T_{dmax}^2 + 3.5 j\omega T_{dmax} + 10^{-6}}{(j\omega)^2 T_{dmax}^2 + 4.5 j\omega T_{dmax} + 7.1}.$$

Thus, multiplier eventually the stability condition will be based on the maximum time delay duration and it will be standing on the gain relation depicted as $W_d^*(j\omega)W_d(j\omega) \geq \Delta_d^*(j\omega)\Delta_d(j\omega)$. Quadratic form of the inequality can be reproduced with any positive definite $D : \mathbb{R} \to \mathbb{R}$ as

$$\begin{bmatrix} I \\ \Delta_d(j\omega) \end{bmatrix}^* \begin{bmatrix} W_d(j\omega)^* D(\omega) W_d(j\omega) & 0 \\ 0 & -D(\omega) \end{bmatrix} \begin{bmatrix} I \\ \Delta_d(j\omega) \end{bmatrix} \geq 0, \quad \forall \omega \in \mathbb{R}. \tag{3.9}$$

Let us propose the first multiplier class for the constant time delay by conic combination of the inequalities given in (3.8) and (3.9) as:

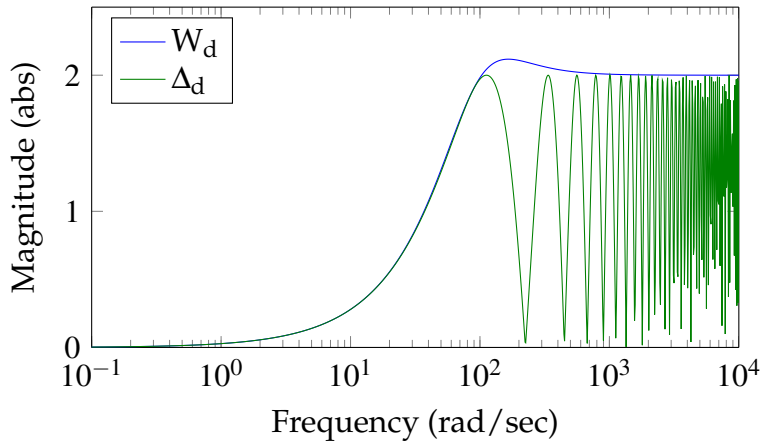

Figure 3.5: Covering gain of the delay with a rational transfer function, $T_{dmax} = 28\,\text{msec}$.

Definition 3.7. *Given a delay T_{dmax}, Π belongs to the class of multipliers $\mathbf{\Pi}_{d1}$ if there exist $D(\omega) = D^*(\omega) \geq 0$, $\Omega(\omega) = \Omega^*(\omega) \leq 0$, and $W_d(j\omega)$ ensures $|W_d(j\omega)| \geq |\Delta_d(j\omega)|$, $\forall T_d \in [0, T_{dmax}], \forall \omega \in \mathbb{R}$, such that*

$$\Pi(j\omega) = \begin{bmatrix} W_d(j\omega)^* D(\omega) W_d(j\omega) & \Omega(\omega) \\ \Omega(\omega) & -D(\omega) + \Omega(\omega) \end{bmatrix}. \tag{3.10}$$

Remark 3.5. *Generally, multiplier classes for time delay are defined with any $\Omega(\omega)$, yet the negativity condition was included to define the interior region of the unit circle and also to ensure that right lower part of the multiplier $(-D(\omega) + \Omega(\omega))$ is negative definite. Yet, it is possible and plausible to relax this condition on Ω with $\Omega(\omega) \leq D(\omega)$ but it will be stated only the circumstances leading to useful results in the further analysis.*

Lemma 3.4. *Let $\Delta_d(j\omega)$ be a time delay operator with constant unknown time delay $T_d \in [0, T_{dmax}]$, it satisfies the IQC defined by $\Pi \in \mathbf{\Pi}_{d1}$.*

Based on the behaviour of the Δ_d on the complex plane another quadratic inequality can be defined with rational approximation of the delay operator. Recently, Pfifer and Seiler [129] illustrated tighter circle constraints constructed by a rational transfer function, Υ_d, that encircles Δ_d and the origin in the complex plane. Namely, the class of uncertainty is defined with a circle centred at the midpoint between origin and Δ_d with a radius equal to the absolute value of this point, see Figure 3.6 for illustration of the circles in the complex plane. An example of Υ_d was defined

in [112] as

$$\Upsilon_d(j\omega) = \frac{-2.19(j\omega)^2 + 9.02(\frac{j\omega}{T_{dmax}}) + \frac{0.089}{T_{dmax}^2}}{(j\omega)^2 - 5.64(\frac{j\omega}{T_{dmax}}) - \frac{17}{T_{dmax}^2}},$$

where T_{dmax} is maximum delay duration that the design is required to be robust. Subsequently, quadratic inequality of the tighter circle description can be depicted with any positive definite $D : \mathbb{R} \to \mathbb{R}$ as

$$\begin{bmatrix} I \\ \Delta_d(j\omega) \end{bmatrix}^* \begin{bmatrix} 0 & \Upsilon_d(j\omega)^* D(\omega) \\ \Upsilon_d(j\omega) D(\omega) & -D(\omega) \end{bmatrix} \begin{bmatrix} I \\ \Delta_d(j\omega) \end{bmatrix} \geq 0, \quad \forall \omega \in \mathbb{R} \tag{3.11}$$

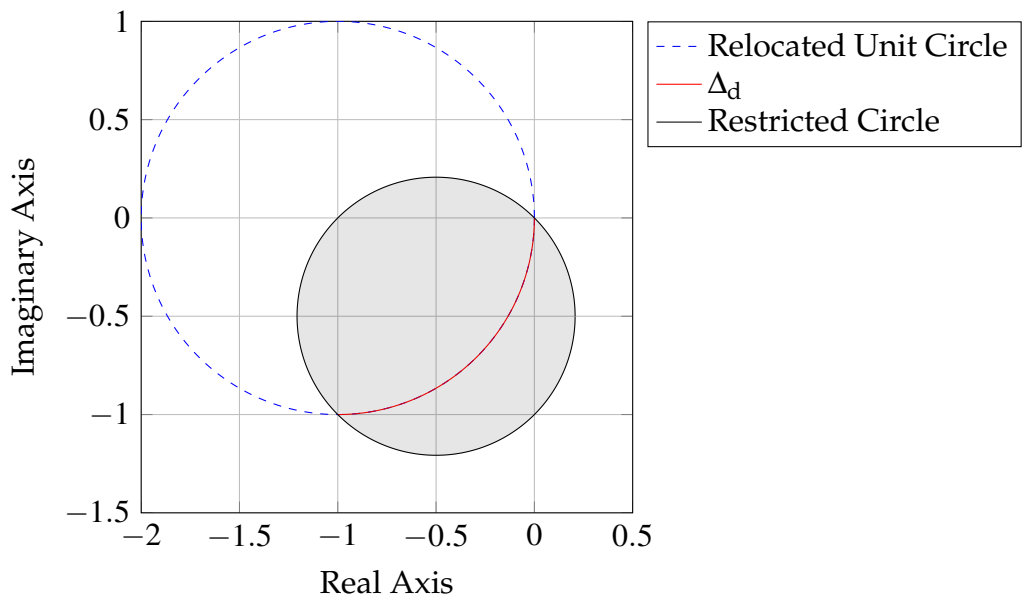

Figure 3.6: Encapsulation of the delay in the complex plane with a restricted circle. See [130] for an analogous interpretation.

A class of multiplier can be expressed based on the restricted circle proposed in (3.8) and (3.11) and their canonical combination as:

Definition 3.8. *Given a delay T_{dmax}, Π belongs to the class of multipliers $\mathbf{\Pi}_{d2}$ if there exist $D(\omega) = D^*(\omega) \geq 0$, $\Omega(\omega) = \Omega^*(\omega) \leq 0$, and $\Upsilon_d(j\omega)$ with $\Delta_d \in C(\Upsilon_d, |\Upsilon_d|)$, $\forall T_d \in [0, T_{dmax}], \forall \omega \in \mathbb{R}$, such that*

$$\Pi(j\omega) = \begin{bmatrix} 0 & \Upsilon_d(j\omega)^* D(\omega) + \Omega(\omega) \\ \Upsilon_d(j\omega) D(\omega) + \Omega(\omega) & -D(\omega) + \Omega(\omega) \end{bmatrix}. \tag{3.12}$$

Lemma 3.5 ([129]). *Let $\Delta_d(j\omega)$ be a time delay operator with constant unknown time delay $T_d \in [0, T_{dmax}]$, then it satisfies the IQC defined by $\Pi \in \mathbf{\Pi}_{d2}$.*

Both multipliers proposed in (3.10) and (3.12) can be used while analysing the uncertainties caused by constant time delays, but unfortunately there is no direct recipe to define which one gives better performance specifications. Also, canonical combination of the proposed multipliers can be implemented to define single perturbation caused by the constant delay. But, we choose not to combine them as combination will eventually increase complexity of the final IQC multiplier, also early synthesis revealed that no profits are gained in the result of the combination.

On the other hand, depending on the architecture of the system, delay may become time variable operator as well, for instance when the Internet is used as a communication medium delay may vary due to the congestion, bandwidth, or the transmission distance. In that case, delay, $T_d(t)$, will be depicted as unknown time-variable parameter that satisfies

$$0 \leq T_d(t) \leq T_{dmax}, \quad 0 < |\dot{T}_d(t)| \leq d, \quad \forall t \geq 0.$$

The subsequent information are particularly based on [127, 131, 132]. Firstly, let us remark that time variable delay is not a bounded operator (in $\mathcal{L}_2$ space) if there is no restriction on the variation of the delay; that can be shown with the example proposed in [127] with the given functions

$$y(t) = \begin{cases} 1, & t \in [0, \varepsilon], \\ 0, & otherwise, \end{cases} \qquad T_d(t) = \begin{cases} t, & t \in [0, T_0], \\ T_0, & otherwise. \end{cases}$$

It can be stated that $y(t - T_d(t))$ is equivalent to 1 for $t \in [0, \varepsilon + T_0]$ and 0 otherwise. The energy of $y(t - T_d(t))$ is equal to $\varepsilon + T_0$, yet energy of $y(t)$ is equal to ε; so that gain of the time variable delay operator becomes unbounded as $\varepsilon \to 0$.

Then, let $\mathcal{V}$ denote a time-delay operator; $\mathcal{V}(v) := v(t - T_d(t))$ also $\Delta_d(v)$ be $\mathcal{V} - I$; $\Delta_d(v) := v(t - T_d(t)) - v(t)$. IQC related to the time variable delay operator can be defined by properties of the $\mathcal{V}$ as it is a bounded operator on $\mathcal{L}_2$ if the variation of the delay is restricted as $d < 1$ and any multiplier depicted for $\mathcal{V}$ leads to a multiplier for Δ_d. Consequently an IQC can be deduced for any symmetric positive definite Ω as

$$\int_0^\infty \begin{bmatrix} v(t) \\ v(t - T_d(t)) \end{bmatrix}^* \begin{bmatrix} \frac{d}{1-d}\Omega & \Omega \\ \Omega & \Omega \end{bmatrix} \begin{bmatrix} v(t) \\ v(t - T_d(t)) \end{bmatrix} dt \geq 0, \quad \forall t \geq 0. \tag{3.13}$$

In a similar fashion as in constant time delay case, Δ_v is a bounded operator on $\mathcal{L}_2$ and this amplitude bound can be covered with a bounded rational transfer function W_v such that $\|W_v\|_2 \geq \|\Delta_v\|_2$ where W_v is any rational transfer function satisfying

$$|W_v(j\omega)| > \begin{cases} 1+\frac{1}{\sqrt{1-d}}, & \text{if } T_{dmax}|\omega| > 1+\frac{1}{\sqrt{1-d}}, \\ T_{dmax}|\omega|, & \text{if } T_{dmax}|\omega| \leq 1+\frac{1}{\sqrt{1-d}}, \end{cases}$$

Figure 3.7 illustrates such an amplitude covering property with different rational transfer functions. Then, this can be rendered with the following multiplier connectively with any positive definite $D : \mathbb{R} \to \mathbb{R}$ as

$$\Pi(j\omega) = \begin{bmatrix} W_v(j\omega)D(\omega)W_v^*(j\omega) & 0 \\ 0 & -D(\omega) \end{bmatrix}. \tag{3.14}$$

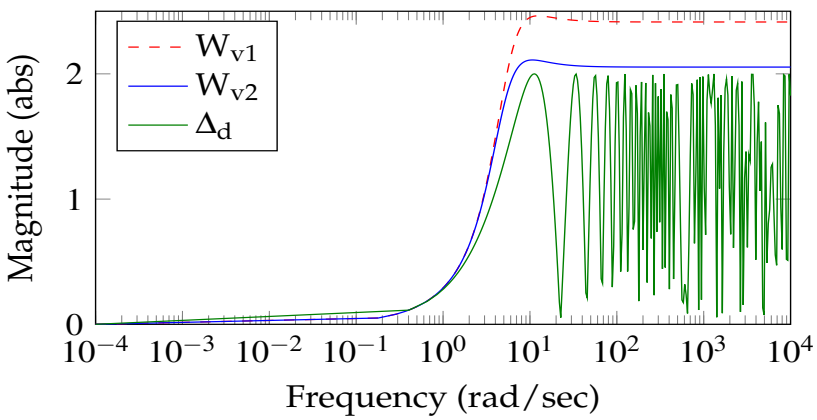

Figure 3.7: Covering gain of the delay with rational transfer functions; W_{v1} is with $\dot{T}_d = 0.5$, W_{v2} is with $\dot{T}_d = 0.1$, $\Delta_d = e^{-sT_{dmax}} - 1$, $T_{dmax} = 28$msec. Sparse frequency grid is the cause of not having legitimate jumps in the gain of Δ_d at the higher frequencies.

Correspondingly, final multiplier for variable delay operator can be depicted with the canonical combination of (3.13) and (3.14) as follows.

Definition 3.9. *Given a variable delay $T_d(t) \in [0, T_{dmax}]$ with $|\dot{T}_d(t)| \leq d < 1, \forall t \geq 0$, Π belongs to the class of multipliers $\mathbf{\Pi}_v$ if there exist $D(\omega) = D^*(\omega) \geq 0$, $\Omega(\omega) = \Omega^*(\omega) \leq 0$, and a rational transfer function $W_v(j\omega)$ satisfying*

$$|W_v(j\omega)| > \begin{cases} 1+\frac{1}{\sqrt{1-d}}, & \textit{if } T_{dmax}|\omega| > 1+\frac{1}{\sqrt{1-d}}, \\ T_{dmax}|\omega|, & \textit{if } T_{dmax}|\omega| \leq 1+\frac{1}{\sqrt{1-d}}, \end{cases}$$

then

$$\Pi(j\omega) = \begin{bmatrix} W_v(j\omega)D(\omega)W_v^*(j\omega) + \frac{d}{1-d}\Omega(\omega) & \Omega(\omega) \\ \Omega(\omega) & -D(\omega)+\Omega(\omega) \end{bmatrix}, \tag{3.15}$$

where, d is variation of the delay, $\dot{T}_d$.

Lemma 3.6 ([127]). *Let Δ_d be a variable time delay operator with delay $T_d(t) \in [0, T_{dmax}]$ and $|\dot{T}_d(t)| \leq d < 1, \forall t \geq 0$, Δ_d satisfies IQC defined by $\Pi \in \mathbf{\Pi}_v$.*

Remark 3.6. *Having a restriction on the variation of the delay as $|\dot{T}_d(t)| \leq d < 1$ is a reasonable assumption as delay cannot grow higher than the time itself. But, contrary situation might occur once there is no package utilization applied when the Internet is used as a communication medium.*

It can be noted that multiplier defined for time variable delays, (3.15), is equal to (3.10) when $\dot{T}_d(t) = 0$ (constant time delay case).

3.4 Parametrization and Combination of the Multipliers

Based on the IQC methodology or multiplier approach absolute stability problem of the classical Lur'e problem can be split into two sub-problems: First, start by defining appropriate class of multipliers in a way that depicted IQCs are hold based on these definitions. Once the proposed IQCs are ensured to hold, then the system is said to be stable if it can be illustrated that there exists a multiplier within the defined class that holds quadratic inequality with the nominal plant as in Theorem 3.4. Therefore, it is required to carry out a search for a multiplier within the defined class, yet one can perform a convex search within the defined class if the depicted multiplier is parametrized in an appropriate fashion. Within this section we will initially discuss the parametrization of the aforementioned class of multipliers and later examine the combination of the different class of multipliers that will be used for the absolute stability analysis of the bilateral teleoperation in the further chapters.

3.4.1 Parametrization of the Class of Multipliers

In the spirit of [29], we propose the following parametrization for the frequency dependent passive multiplier $\lambda(\omega)$. Given $n \geq 2$ and $\alpha > 0$, let define a transfer function vector as

$$^{n}\Lambda_{\alpha}(j\omega) = \begin{bmatrix} 1 & \frac{1}{j\omega+\alpha} & \cdots & \frac{1}{(j\omega+\alpha)^{n-1}} \end{bmatrix}^{\top}, \quad \alpha > 0, \tag{3.16}$$

which will be used to parametrize the multipliers and depicted as *basis function*, see Appendix for the minimal state space representation.

Firstly, we use the same parametrization for frequency dependent passive multiplier $\lambda(\omega)$ as in [29]. Then, a subclass of the set of multipliers $\mathcal{P}$ is given by $\lambda(\omega) = {}^{n}\Lambda_{\alpha}(j\omega)^{*}K_h{}^{n}\Lambda_{\alpha}(j\omega)$, where $K_h = K_h^{\top}$, and $\lambda(\omega) \in \mathbb{R}^{+}$. Henceforth we simplify the notation for passive multipliers by using $\Lambda_h(j\omega) = {}^{n}\Lambda_{\alpha}(j\omega)$ for some n and α.

Secondly, we will parametrize the Zames-Falb multiplier using Szegö's polynomials. Following [133] and [134], $Z \in \mathcal{Z}$ in Definition 3.6 can be approximated as

$$Z(j\omega) = z_0 - \sum_{i=1}^{n-1}\left(\frac{a_i}{(j\omega+\alpha)^i} - \frac{b_i(-1)^{i-1}}{(j\omega-\alpha)^i}\right), \tag{3.17}$$

if n is chosen sufficiently large. Initially we factorize class of Zames-Falb multiplier as $Z(j\omega) = {}^{n}\Lambda_{\alpha}(j\omega)^{*}K_z\,{}^{n}\Lambda_{\alpha}(j\omega)$, where Λ_{α} is a basis function with the same structure as in (3.16), and K_z is a matrix containing free parameters of the Z function (z_0, a_i and b_i) at its first column and row

$$K_z = \begin{bmatrix} z_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \\ -b_1 & 0 & 0 & \cdots & 0 \\ -b_2 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -b_{n-1} & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Condition (3.5) in Definition 3.6 requires the inverse two-sided Laplace transform of the summation in (3.17)[3]. It is given by

$$q(t) = \begin{cases} e^{-\alpha t}(a_1 + ta_2 + \cdots + \frac{t^{n-2}a_{n-1}}{(n-2)!}), & \text{if } t \geq 0; \\ e^{\alpha t}(b_1 - tb_2 + \cdots + (-1)^{n-2}\frac{t^{n-2}b_{n-1}}{(n-2)!}), & \text{if } t < 0; \end{cases} \tag{3.18}$$

and using that $q(t) \geq 0$ for all t, it follows by direct integration such that

$$\int_{-\infty}^{\infty} q(t)dt = (a+b)\bar{\alpha} < z_0, \tag{3.19}$$

where a and b are vectors contain all free parameters as

$$\begin{aligned} a &= \begin{bmatrix} a_1 & a_2 & \cdots & a_{n-1} \end{bmatrix}, \\ b &= \begin{bmatrix} b_1 & b_2 & \cdots & b_{n-1} \end{bmatrix}, \end{aligned}$$

[3]The transfer function in (3.17) is noncausal, and its region of convergence (ROC) includes the imaginary axis. See Table 8.4 in [135].

and $\bar{\alpha}$ is a vector designated as

$$\bar{\alpha} = \begin{bmatrix} 1/\alpha & 1/\alpha^2 & \cdots & 1/\alpha^{n-1} \end{bmatrix}^\top.$$

The final constraint $q(t) \geq 0$ in Definition 3.6 is implemented as an LMI as follows. Firstly, two transfer functions are generated:

$$\begin{aligned} H_1(s) &= \frac{\sum_{i=1}^{n-1} a_i(-1)^{i-1} s^{2(i-1)}(i-1)!^{-1}}{(-s+1)^{n-1}(s+1)^{n-1}}, \\ H_2(s) &= \frac{\sum_{i=1}^{n-1} b_i(-1)^{i-1} s^{2(i-1)}(i-1)!^{-1}}{(-s+1)^{n-1}(s+1)^{n-1}}; \end{aligned} \tag{3.20}$$

then $q(t) \geq 0$ for all $t \in \mathbb{R}$ is equivalent to the conditions $H_j(j\omega) \geq 0$ for $j = 1,2$ and all $\omega \in \mathbb{R}$; finally these two conditions are expressed as LMIs via KYP Lemma, see [134] for the minimal state space representations of these transfer functions.

Remark 3.7. *Note that the positivity of $q(t)$ given in (3.18) is equivalent to the positivity of the polynomials*

$$\left(a_1 + ta_2 + \frac{t^2 a_3}{2!} + \cdots + \frac{t^{n-2} a_{n-1}}{(n-2)!}\right),$$

and

$$\left(b_1 + tb_2 + \frac{t^2 b_3}{2!} + \cdots + \frac{t^{n-2} b_{n-1}}{(n-2)!}\right),$$

for all $t \geq 0$; since $e^{\pm\alpha t} > 0$ for all t. Then positivity of $q(t)$ is independent on the value of α. By default, we check the positivity of $q(t)$ assuming $\alpha = 1$. See the proof in [133] for further details. Similarly, the same conclusion can be reached for the anticausal component of the multipliers.

Although this parametrization has been shown to be a complete parametrization when $N \to \infty$ [136], this limit is not feasible due to numerical issues. To increase the flexibility without increasing the order of the multiplier, we propose a second method similar to the parametrization proposed in [137], $Z(j\omega) = \Lambda_{\alpha,\beta}(j\omega)^* K_e \Lambda_{\alpha,\beta}(j\omega)$, so that flexibility of the asymmetric poles in a multiplier can be used (see example in [32,38]). Let $\Lambda_{\alpha,\beta}(j\omega)$ be a second basis function given by

$$\Lambda_{\alpha,\beta}(j\omega) = \begin{bmatrix} {}^n\Lambda_\alpha(j\omega)^\top & {}^n\Lambda_\beta(j\omega)^\top \end{bmatrix}^\top, \tag{3.21}$$

with $\beta, \alpha > 0$, and K_e containing free parameters of Z function expressed as

$$K_e = \begin{bmatrix} z_1 & -a_1 & -a_2 & \cdots & -a_{n-1} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & z_2 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & -b_1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & -b_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -b_{n-1} & \cdots & 0 \end{bmatrix} \tag{3.22}$$

and condition proposed in (3.19) is redefined as

$$a\bar{\alpha} + b\bar{\beta} < z_1 + z_2, \tag{3.23}$$

where $\bar{\beta}$ is defined similarly to $\bar{\alpha}$. Equivalently, conditions on H_1 and H_2 still ensure that the new $q(t)$ is non-negative for all $t \in \mathbb{R}$, as has been mentioned in Remark 3.7. Henceforward, we will use Λ_e as the basis associated with the Zames-Falb multiplier, regardless of the selection on the poles and order. If one pole is used, it means that the first option has been used whereas the second option has been used if two poles are given.

Furthermore, parametrization of the multipliers for uncertainties caused by time delay can be performed in a similar fashion as in passivity multiplier. For constant time delay case, infinite dimensional frequency dependent functions $\Omega(\omega)$ and positive definite $D(\omega)$ can be parametrized as

$$\begin{aligned} D(\omega) &= {}^n\Lambda_\alpha(j\omega)^* K_d \, {}^n\Lambda_\alpha(j\omega), \\ \Omega(\omega) &= {}^n\Lambda_\alpha(j\omega)^* K_r \, {}^n\Lambda_\alpha(j\omega), \end{aligned} \tag{3.24}$$

where $K_{d,r} = K_{d,r}^\top$ and basis functions Λ_α are designed as in (3.16) with different poles and orders. Henceforward we will use the notation Λ_d and Λ_r to represent some selection for the poles and orders of these two multipliers.

3.4.2 Combination of the Class of Multipliers

IQC is a powerful framework that enables us to analyse different types of uncertainties in one characterization. By dint of this property previously proposed IQCs will

be combined in a regular base to overcome stability issues caused by specific obstacles such as saturation in actuator or latency in the feedback. Here, we are interested in three combinations of multipliers. Firstly, we will combine Zames-Falb and time delay multipliers in order to analyse stability of the systems with saturation and delayed states. Secondly, combination of passive and Zames-Falb multipliers will be proposed as that is going to be used in the stability analysis of delay free bilateral teleoperation while assuming human is linear and environment is nonlinear operators. Thirdly, together with these two, multipliers for time delay also will be combined to analyse stability of the time delayed bilateral teleoperation.

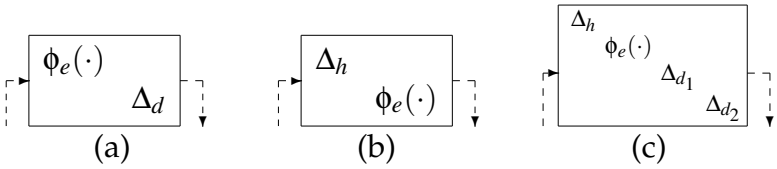

Figure 3.8: Structured uncertainties: (a) is nonlinearity, ϕ_e, and time delay, Δ_d, (b) is passive, Δ_h, plus nonlinear, and (c) is combination of the passive, nonlinear and time delays.

Firstly, let us define the class of multipliers for the structure in Figure 3.8 (a) by combining the class of Zames-Falb multipliers (3.7) and multipliers proposed for time delay (3.12).

Definition 3.10. *Given some selection of Λ_e, Λ_d, and Λ_r, let us define the class of multipliers $\mathbf{\Pi}_a$ as the set of multipliers with the structure given by*

$$\Pi = \begin{bmatrix} \Psi_a(1,1) \\ \Psi_a(2,1) \end{bmatrix}^* \underbrace{\left[\begin{array}{cccc|cccc} 0 & 0 & K_e^\top & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_d & 0 & 0 & 0 & 0 \\ K_e & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_d & 0 & -K_d & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_r \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_r & 0 & K_r \end{array}\right]}_{K_a} \underbrace{\begin{bmatrix} \Psi_a(1,1) \\ \Psi_a(2,1) \end{bmatrix}}_{\Psi_a}, \tag{3.25}$$

where $\Psi_a(1,1) = diag(\Lambda_e, \Lambda_d \Upsilon_d, \Lambda_e, \Lambda_d)$, $\Psi_a(2,1) = diag(0, \Lambda_r, 0, \Lambda_r)$, and K_e, K_d, K_r are defined by (3.22), and (3.24), respectively; and satisfying the following constraint and LMIs:

- *The parameters in the matrix K_e satisfies*

$$a\bar{\alpha} + b\bar{\beta} < z_1 + z_2, \tag{3.26}$$

- *there exists $X_j = X_j^\top$ such that,*

$$\begin{bmatrix} A_j^\top X_j + X_j A_j & C_j^\top - X_j B_j \\ C_j - B_j^\top X_j & -(D_j + D_j^\top) \end{bmatrix} \leq 0, \tag{3.27}$$

where A_j, B_j, C_j, D_j are state space parameters of $H_j(s)$ in (3.20), for $j = 1, 2$,

- *there exist symmetric matrices, P_d, K_d such that,*

$$\begin{bmatrix} A_d^\top P_d + P_d A_d & P_d B_d \\ B_d^\top P_d & 0 \end{bmatrix} - \begin{bmatrix} C_d & D_d \end{bmatrix}^\top K_d \begin{bmatrix} C_d & D_d \end{bmatrix} \leq 0, \tag{3.28}$$

- *there exist symmetric matrices P_r, K_r such that,*

$$\begin{bmatrix} A_r^\top P_r + P_r A_r & P_r B_r \\ B_r^\top P_r & 0 \end{bmatrix} + \begin{bmatrix} C_r & D_r \end{bmatrix}^\top K_r \begin{bmatrix} C_r & D_r \end{bmatrix} \leq 0, \tag{3.29}$$

where A_j, B_j, C_j, D_j are state space parameters of the basis functions $\Lambda_j(j\omega)$, for $j = d, r$.

Lemma 3.7. *Given an uncertainty $\Delta = \ diag(\phi_e, \Delta_d)$ as in Figure 3.8 (a) and $\Pi \in \mathbf{\Pi}_a$, then Δ satisfies the IQC defined by $\Pi(j\omega)$.*

Secondly, we develop the class of multipliers that we will use to analyse the absolute stability of the bilateral teleoperated system when the human block is modelled as an LTI passive system and the environment as a bounded and monotone nonlinearity and defined as a structured uncertainty as in Figure 3.8 (b). We combine frequency dependent multipliers for LTI passive systems, (3.3), and Zames-Falb multipliers, (3.7) as follows.

Definition 3.11. *Given some selection of Λ_h and Λ_e, let us define the class of multipliers $\mathbf{\Pi}_b$ as the set of multipliers with the structure given by*

$$\Pi(j\omega) = \begin{bmatrix} \Psi_b(j\omega) & 0 \\ 0 & \Psi_b(j\omega) \end{bmatrix}^* \underbrace{\left[\begin{array}{cc|cc} 0 & 0 & K_h & 0 \\ 0 & 0 & 0 & K_e^\top \\ \hline K_h & 0 & 0 & 0 \\ 0 & K_e & 0 & 0 \end{array}\right]}_{K_b} \begin{bmatrix} \Psi_b(j\omega) & 0 \\ 0 & \Psi_b(j\omega) \end{bmatrix}, \tag{3.30}$$

where K_h is any symmetric matrix, K_e is defined in (3.22), and $\Psi_b(j\omega) = \begin{bmatrix} \Lambda_h(j\omega) & 0 \\ 0 & \Lambda_e(j\omega) \end{bmatrix}$; and satisfying the following conditions:

- *there exist symmetric matrices P_h and K_h such that,*

$$\begin{bmatrix} A_h^\top P_h + P_h A_h & P_h B_h \\ B_h^\top P_h & 0 \end{bmatrix} - \begin{bmatrix} C_h & D_h \end{bmatrix}^\top K_h \begin{bmatrix} C_h & D_h \end{bmatrix} \leq 0, \tag{3.31}$$

  where A_h, B_h, C_h, and D_h are the state space representation of $\Lambda_h(j\omega)$,

- *constraint (3.26) and LMI (3.27) are satisfied.*

Lemma 3.8. *Given a structured uncertainty block $\Delta = \ diag(\Delta_h, \phi_e)$ as in Figure 3.8 (b), then Δ satisfies the IQC defined by $\Pi(j\omega) \in \mathbf{\Pi}_b$.*

Lastly, let us define the class of multipliers for the structured uncertainty illustrated in Figure 3.8 (c); namely we combine passive, Zames-Falb, and time delay multipliers as follows;

Definition 3.12. *Given some selection of Λ_h, Λ_e, Λ_{d_1}, Λ_{d_2}, Λ_{r_1}, Λ_{r_2} and rational transfer functions Υ_{d_i}, $i = 1, 2$; let us define the class of multipliers $\mathbf{\Pi}_c$ as the set of multipliers with the structure given by*

$$\Pi = \begin{bmatrix} \Psi_c(1,1) \\ \Psi_c(2,1) \end{bmatrix}^* \underbrace{\left[\begin{array}{cc|cc} 0 & \bar{K}_1^\top & 0 & 0 \\ \bar{K}_1 & \bar{K}_2 & 0 & 0 \\ \hline 0 & 0 & 0 & \bar{K}_3 \\ 0 & 0 & \bar{K}_3 & \bar{K}_3 \end{array}\right]}_{\bar{K}} \begin{bmatrix} \Psi_c(1,1) \\ \Psi_c(2,1) \end{bmatrix}, \tag{3.32}$$

where $\bar{K}_1 = diag(K_h, K_e, K_{d_1}, K_{d_2})$, $\bar{K}_2 = diag(0, 0, -K_{d_1}, -K_{d_2})$, $\bar{K}_3 = diag(0, 0, K_{r_1}, K_{r_2})$, and
$\Psi_c(1,1) = diag(\Lambda_h, \Lambda_e, \Lambda_{d_1}\Upsilon_{d_1}, \Lambda_{d_2}\Upsilon_{d_2}, \Lambda_h, \Lambda_e, \Lambda_{d_1}, \Lambda_{d_2})$,
$\Psi_c(2,1) = diag(0, 0, \Lambda_{r_1}, \Lambda_{r_2}, 0, 0, \Lambda_{r_1}, \Lambda_{r_2})$; and satisfying the constraint (3.26) and LMIs (3.27), (3.28), (3.29), and (3.31) [4].

Lemma 3.9. *A constructed uncertainty block $\Delta = \ diag(\Delta_h, \phi_e, \Delta_{d_1}, \Delta_{d_2})$, as in Figure 3.8 (c), satisfies the IQC defined by $\Pi(j\omega) \in \mathbf{\Pi}_c$.*

[4] LMIs for $D_{1,2}(\omega)$ and $\Omega_{1,2}(\omega)$ need to be defined for each delay

Once the proper multipliers are combined to define the structured nonlinearities existence of the inequality given in Theorem 3.4 with a multiplier within the defined subclasses (if any) will be sufficient to conclude that the proposed nominal system-uncertainty interconnections are absolutely stable. In the subsequent chapters LMI conditions for absolute stability of three different interconnections are going to be introduced.

4 Stability of the Systems Subject to Saturation and Time Delay

IQC is a framework that provides us a powerful tool to combine different classes of multipliers for the stability analysis. For this reason, particular operators in bilateral teleoperation that lead to stability loses or performance degredation will be systematically analysed through the thesis via IQC framework. But, before moving on to the results related to teleoperation, we examine absolute stability of the systems with saturation and time delay. Thus, the usefulness of the framework and power of the Zames-Falb multipliers come to light while reducing the conservatism of a well-known problem, also this will be a basis to the results based on the bilateral teleoperation.

Therefore, this chapter reviews methodologies applied to the robustness analysis of systems containing time delay and saturation type nonlinearities. Also, so-called Kalman and Aizerman conjectures will be investigated for the time delayed systems as results based on these conjectures will be used as a benchmark while validating the results obtained via IQCs. As a last effort on this topic, based on the clockwise properties and graphical frameworks we will show that the Kalman conjecture is true for a wide class of time delayed Lur'e systems.

4.1 Robustness Against Saturation and Time Delay via Multipliers

In this section we analyse absolute stability of the systems containing both nonlinearity and constant time delay, which is within an interval $T_d \in [0, T_{dmax}]$, via multipliers. If time-domain techniques as Lyapunov-Krasovskii functional is a versatile

tool for time-delay systems [138, 139]; when delay is combined with other nonlinearities such as saturation, the modularity of IQC framework may provide some advantages. For examples, if the nonlinearity is slope restricted or sector bounded then Zames-Falb multipliers outperform Lyapunov techniques [37, 38].

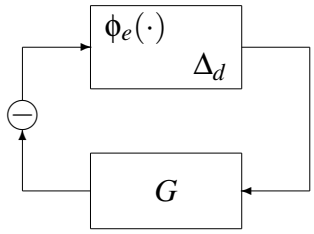

Figure 4.1: Time delayed Lur'e system, where Δ is structural uncertainty containing bounded nonlinearity, ϕ_e, and time delay, Δ_d.

Consequently, assume that there exists a system with slope restricted monotonic nonlinearity plus delayed states. After making certain interconnections the system is transformed into the feedback connection between nominal plant G and structured uncertainty as illustrated in Figure 4.1. Then, based on Theorem 3.4 on page 71 absolute stability of the whole system is transformed to a search for appropriate multipliers within the defined class such that $\Pi \in \mathbf{\Pi}_a$, where $\mathbf{\Pi}_a$ has been defined in Definition 3.10 on page 86.

Corollary 4.1. *Consider the negative feedback interconnection between G and a structured uncertainty block in Figure 4.1. Let us define a minimal state space representation as*

$$\Psi_a(j\omega)\begin{bmatrix} -G(j\omega) \\ I \end{bmatrix} \sim \left[\begin{array}{c|c} A_a & B_a \\ \hline C_a & D_a \end{array}\right],$$

where $\Psi_a(j\omega)$ is defined in (3.25). Assume that interconnection between G and $\tau\Delta$ is well-posed for $\tau \in [0,1]$. Then, the interconnection is absolutely stable if there exist symmetric matrices P_a and K_a, where K_a has the form as in (3.25), such that there exists $\Pi = (\star)^* K_a \begin{bmatrix} \Psi_a(j\omega) & 0 \\ 0 & \Psi_a(j\omega) \end{bmatrix} \in \mathbf{\Pi}_a$ and following LMI holds

$$\begin{bmatrix} A_a^\top P_a + P_a A_a & P_a B_a \\ B_a^\top P_a & 0 \end{bmatrix} + \begin{bmatrix} C_a & D_a \end{bmatrix}^\top K_a \begin{bmatrix} C_a & D_a \end{bmatrix} \leq 0. \tag{4.1}$$

Proof. If LMIs (3.27), (3.28), and (3.29) hold with defined conditions, then uncertainty block (a) in Figure 3.8 on page 86 satisfies IQC defined by $\Pi(j\omega) \in \mathbf{\Pi}_a$, see Lemma 3.7 on page 87.

Well-posedness condition in IQC theorem was assumed to be satisfied and as the lower right corner of $\Pi(j\omega) \in \mathbf{\Pi}_a$ is negative semi-definite, $\tau\Delta$ satisfies the IQC defined by for $\tau \in [0,1]$ if Δ satisfies, so second condition in the theorem is streamlined. Then, based on the KYP Lemma, satisfying the LMI (4.1) implies that

$$\begin{bmatrix} -G(j\omega) \\ I \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} -G(j\omega) \\ I \end{bmatrix} \leq -\varepsilon I, \quad \forall \omega \in \mathbb{R},$$

also holds for some $\varepsilon > 0$ and based on Theorem 3.4 on page 71 it can be concluded that the proposed interconnection is stable. □

Infinite dimensional inequality is converted into a finite dimensional LMI which is a convex cone and search can be carried out with semi definite programs. We have used Yalmip with *sdpt3* solver [140] to test the LMI conditions search for suitable class of multipliers.

4.1.1 Numerical Examples

Initially, particular examples from the literature are chosen to verify conservatism of the proposed IQC methodology on the systems containing time delays only. Stability analysis are based on delay dependent conditions, so that the results are found based on the bisection search for the maximum delay duration while using semi-definite programming for the LMIs. A numerical example which is frequently encountered in the literature and contains only time delay as a perturbation is going to be analysed prelusively.

Example 1.

Consider an uncertain LTI system with single time delay in the state as

$$\dot{x}(t) = Ax(t) + A_d x(t - T_d), \tag{4.2}$$

where $T_d \in [0, T_{dmax}]$, and $A + A_d \in \mathbb{R}^{n \times n}$ is Hurwitz that implies stability of the system is obvious when there is no delay. Delay, T_d, is assumed to be constant but unknown. The given system in (4.2) is transformed into positive feedback interconnection of G and $\Delta_d = e^{-sT_d} - 1$ as in Figure 4.2. And the aim is to find the maximum allowable

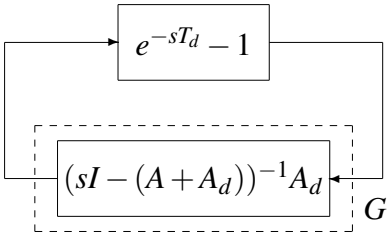

Figure 4.2: Uncertain LTI system with single time delay as nominal system-uncertainty feedback interconnection.

time delay, T_{dmax}, such that robustness against delay is guaranteed by using multipliers proposed in (3.10) and (3.12). Consider the following state-space parameters for the system in (4.2)

$$A = \begin{bmatrix} -2 & 0 \\ 0 & 0.25 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 0 \\ -0.1 & -0.85 \end{bmatrix}. \tag{4.3}$$

To demonstrate the reduced conservatism of the proposed method and for comparison with the methods in the literature Table 4.1 is proposed so that maximum possible achievable time delays for the system with parameters given in (4.3) can be observed.

Method	Maximum delay: T_{dmax} s
Fu et al. [123]	0.6417
Theorem 7 in [123]	0.9848
Jun and Safonov [124, 125]	0.9999
Jun and Safonov [126]	1.489
Using $\mathbf{\Pi}_{d2}$	**1.5390**
Using $\mathbf{\Pi}_{d1}$	**1.54**
Optimal value [141]	1.54

Table 4.1: Maximum achievable time delays for **Example** 1 with different methodologies in the literature and with the ones proposed previously.

As seen from the Table 4.1 proposed method with both class of multipliers, $\mathbf{\Pi}_{d1}$ and $\mathbf{\Pi}_{d2}$ which are respectively proposed in Definition 3.7 on page 78 and in Definition 3.8 on page 79, give delay durations which are the closest ones to the optimal value given in [141]. Over and above it is possible to obtain the optimum value with $\Pi \in \mathbf{\Pi}_{d1}$. Figure 4.3 shows the last condition in Theorem 3.4 on page 71 is satisfied with maximum available time delay. The order and pole location of the multipliers are given in Table 4.3, it must be noted that similar results can also be obtained with multipliers with less orders yet we have used the proposed ones for the consistency

within the thesis. Lastly, it can be concluded that besides Lyapunov-Krasovskii, IQC is also proposing efficient methodology to analyse stability of the time delayed system.

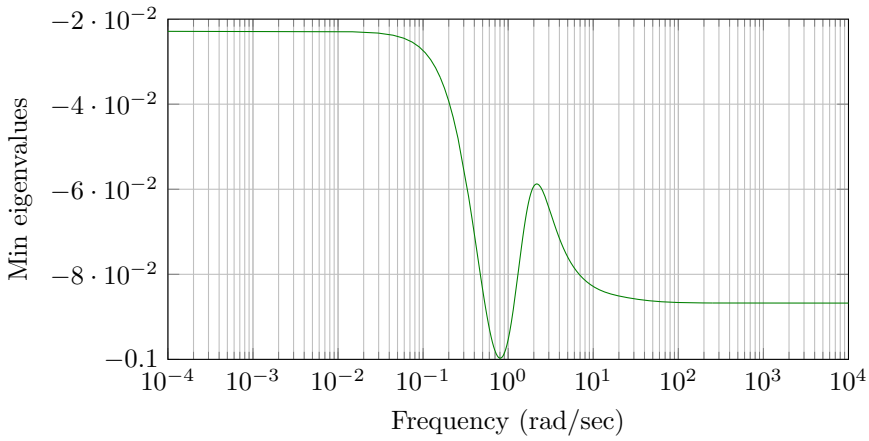

Figure 4.3: Minimum eigenvalues of $[G;I]^*\Pi[G;I]$ with multiplier $\Pi \in \mathbf{\Pi}_{d1}$ when maximum time delay is obtained as 1.54 s.

Henceforth, we can move on to investigate the conservatism of the Corollary 4.1 which can be carried out with the following system parameters containing time delay and saturation type nonlinearity simultaneously; this numerical example has been frequently used in the literature [142–145].

Example 2.

Let us assume that there exists a system with a class of nonlinear uncertainty and time-delay as illustrated in Figure 4.1 and described as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bx(t-T_d) + Dw(t), \\ m(t) &= Mx(t) + Nx(t-T_d), \\ w(t) &= -\phi(t, m(t)), \end{aligned} \tag{4.4}$$

where, $\phi(t, m(t))$ belongs to the sector $[K_1, K_2]$, $x(t) \in \mathbb{R}^n$, $w(t) \in \mathbb{R}^k$, $m(t) \in \mathbb{R}^k$ are the state, input, and output of the system, respectively. Let $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$, $D \in \mathbb{R}^{n\times k}$,

$M \in \mathbb{R}^{k\times n}$, and $N \in \mathbb{R}^{k\times n}$ be known constant matrices with the following values

$$\begin{aligned} A &= \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} \\ D &= \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix}, \quad M = \begin{bmatrix} 0.6 & 0.8 \end{bmatrix} \\ N &= \begin{bmatrix} 0 & 0 \end{bmatrix}, \quad K_1 = 0.2, K_2 = 0.5 \end{aligned} \tag{4.5}$$

Stability of the interconnection with parameters given in (4.5) was transformed to a convex search for multiplier class in Definition 3.10 on page 86. For comparison and evaluation of the method results are given in the Table 4.2.

Method	Maximum delay: T_{dmax} s
Han [142]	2.4859
Wu et al. [143] (n = 2)	3.0080
Wu et al. [143] (n = 3)	3.1110
Kazemy and Farrokhi [144]	3.0216
Zeng et al. [145] ($k = 4$)	3.1730
$\mathbf{\Pi}_a$ (i.e., using $\mathbf{\Pi}_{d2}$)	**3.2191**
Counterpart of $\mathbf{\Pi}_a$ using $\mathbf{\Pi}_{d1}$	**3.2378**
Nyquist value ($\phi(u) = ku$)	3.2520

Table 4.2: Maximum allowable time delays for **Example** 2: system with delay and slope restricted nonlinearity.

Meanwhile, similar results were obtained while using asymmetric poles in multiplier with (3.21) as in both cases positivity of the nominal system was recovered, see Figure 4.4. Results were obtained with the basis function parameters given in Table 4.3 unless otherwise stated parameters for Λ_d and Λ_r[1] will remain same for the future analysis. Generally speaking, dimensions of the basis functions and poles locations have a significant effect on the result that is being obtained. One can choose non-dynamic multiplier with minimal order yet that might leads to conservative conditions. Here, we have chosen the lengths based on a linear search such that increasing the parameters beyond the given values (in Table 4.3) provides nothing but complexity in the analysis. Namely, no reduction in the conservatism (increase in the maximum delay duration) is observed as the length of the basis function increased beyond the given values, in order to require less computational power we

[1]Number of the delay operators based on the system's dimension.

have used these given parameters. Similar search was carried out for the pole location. However, it can be stated that for this particular example pole locations of the basis functions for time delay multipliers are not sensitive, yet they are chosen to be different from one another (also from nominal system's poles) to prevent any possible numerical issues while carrying out the search. Further information related to this issue will be provided in the future sections.

	Λ_e	Λ_{d_1}	Λ_{r_1}	Λ_{d_2}	Λ_{r_2}
Order (n)	5	4	4	4	4
Pole ($-\alpha$)	-10	-21	-14	-13	-8

Table 4.3: Basis functions' parameters for retarded system with/without nonlinearity.

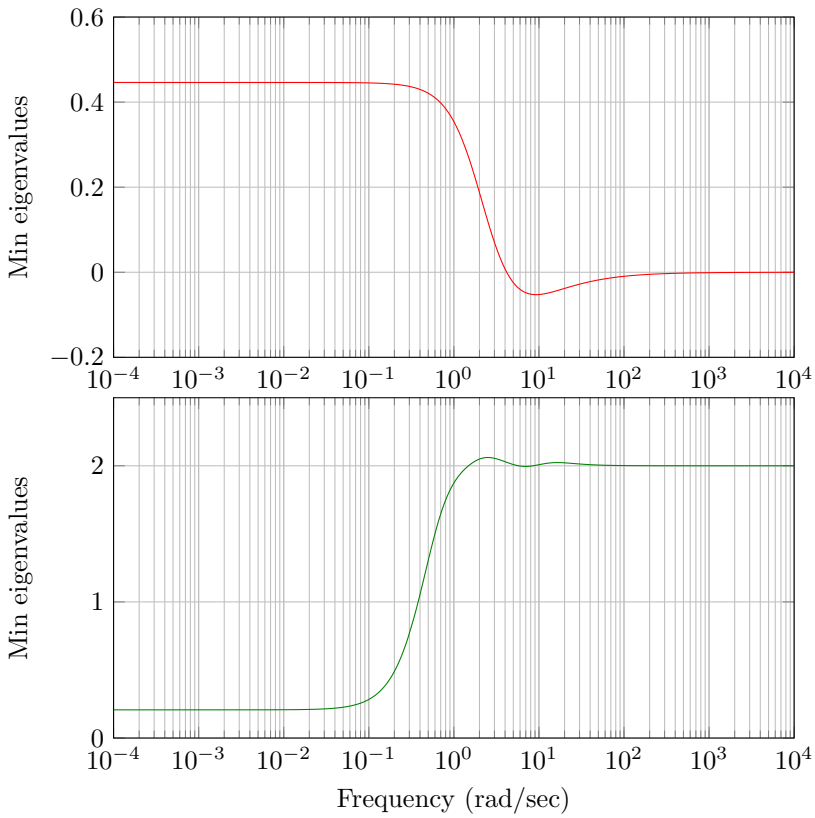

Figure 4.4: Minimum eigenvalues of $G + G^*$ (Top): Nominal system is not positive definite without multiplier. Minimum eigenvalues with multiplier that ensures stability against $T_{dmax} \approxeq 3.2191$ sec (bottom).

4.1.2 Stability of the Retarded Neural Networks: Case Study

Neural networks have found a wide range of application areas in the recent years such as signal processing, feedback control, parallel computing, etc. Thus leads to

huge amount of interest to analyse and synthesis behaviours of the neural networks both from theoretical and practical points of view. Here, there are two main reasons why neural networks have been considered as case study: the first one is the time delays which are frequently encountered phenomenon due to the communication among neurons or finite speed of the information processing. And the second one is the activation function inside a neural which usually satisfies slope restriction criteria. Therefore, overall network can be depicted as a time delayed Lur'e structure and asymptotic stability analysis can be carried out with aforementioned methodologies in the robust control. Literature is dominated with Lyapunov-Krasovskii functional theorem incorporating with LMI technique while analysing the stability of the network, yet apart from that there exist recent researches that use different techniques such as Quadratic Separation (QS) principles [146], as well. The QC approach might be associated with the IQC theorem, so we would like to investigate the usefulness of these techniques (particularly IQC) with neural networks.

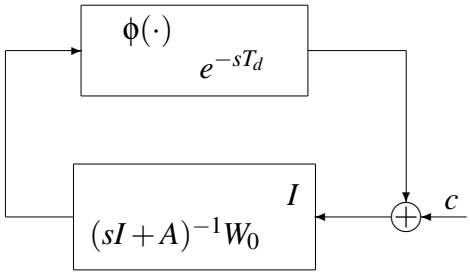

Figure 4.5: Hopfield Neural Network in (4.6) as a feedback interconnection of nominal system and perturbation.

Let start to analyse absolute stability of an Hopfield Neural Network (HNN) [147] which is a type of Recurrent Neural Network (RNN) in which all connections are symmetric. Dynamical equation of a time delayed HNN can be proposed as follows [148]:

$$\dot{x}_i(t) = -a_i x_i(t) + \sum_{j=1}^{n} w_{ij} f_j(x_j(t-T_d)) + c_i$$

and equivalently as,

$$\dot{x}(t) = -Ax(t) + W_0 f(x(t-T_d)) + c \tag{4.6}$$

where, $A = \text{diag}(a_1, a_2, \ldots, a_n)$ is positive matrix, delayed weight coefficients are represented with W_0 matrix; and $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^\top$ is neural state vector,

$c = [c_1, c_2, ..., c_n]^\top$ is time independent external input,

$$f(x(t-T_d)) = [f_1(x_1(t-T_d)), f_2(x_2(t-T_d)), ..., f_n(x_n(t-T_d))]^\top$$

denotes activation function which is expressed as a slope restricted nonlinearity with maximum slope K, and T_d is constant transmission delay between neurons. It is generally assumed that activation functions f_i are bounded and satisfy following inequality with $f_i(0) = 0$

$$0 \leq \frac{f_i(x_1) - f_i(x_2)}{x_1 - x_2} \leq k_i, \quad \forall x_1 \neq x_2,$$

where k_i is positive constant and will be used in the maximum slope matrix K. For clarity, we have used the terminology in the neural network while defining the dynamics, yet in the analysis and synthesis activation function will be depicted as ϕ to have consistency throughout the thesis.

As mentioned, once the proposed network is transformed into a nominal plant and structured uncertainty interconnection as in Figure 4.5 then well established methodologies, such as IQCs, can be utilized to investigate stability of the system. The operators, time delay and activation function, will be inserted diagonally into a structured uncertainty block then bisection algorithm can be used to gain maximum allowable time delay within the system while searching for appropriate multipliers attain positivity of the perturbation block. The examples are chosen from the literature such that they have been introduced frequently (see the references in the given tables) for comparison of the proposed methodologies, in this way one can evaluate the given methodology and compare with the state of the art results in the literature.

Example 3.

Consider the following 3rd order delayed HNN:

$$A = \text{diag}\{4.1989, 0.7160, 1.9985\},$$

$$W_0 = \begin{bmatrix} -0.1052 & -0.5069 & -0.1121 \\ -0.0257 & -0.2808 & 0.0212 \\ 0.1205 & -0.2153 & 0.1315 \end{bmatrix},$$

$$K = \text{diag}\{0.4219, 3.8998, 1.0160\},$$

$$c_i = 0, \quad i = 1, 2, 3.$$

Maximum delay durations that stability of the proposed neural network is guaranteed based on the different methodologies are given in Table 4.4.

Method	Maximum delay: T_{dmax} s
Lou et al. [149]	1.7644
Mou. et al. [148]	2.597
Z. Wang [146]	2.5992
Multiplier approach; $\mathbf{\Pi}_a$	**2.6781**
Nyquist value ($f(u) = Ku$)	2.7563

Table 4.4: Computational results of **Example** 3: upper bound of the delays such that the depicted system is stable.

As seen from Table 4.4 multiplier approach gives less conservative stability conditions with systems which contain both nonlinearity and constant time delays. Strictly speaking, in this particular example usefulness of the proposed approach is mainly based on the multiplier defined for the time delay because the nominal system ($G_{21}(s) = (sI + A)^{-1}W_0$) that associated with Zames-Falb multiplier for regaining the positive realness is already positive real. In other words, there is no deficiency of positivity in the linear part that multiplier can regain so there is not much to benefit from the multiplier for the nonlinearity.

On the other hand, time delays in the neural networks are not always time independent; the switching in the communication medium between the neurons or implementing the network by digital circuits might cause to have time variable delays. As the network require to have a unique and global stable equilibrium point once it has even variable delays, there has been a great interest to ensure the stability of the RNN with time variable delay as well. Generally, a RNN with variable time delay is defined as

$$\dot{x}(t) = -Ax(t) + W_0 f(x(t)) + W_1 f(x(t - T_d(t))) + c \tag{4.7}$$

where $0 \leq T_d(t) \leq T_{dmax}$ variable delay with $\dot{T}_d(t) \leq d \leq 1$. Other parameters have been defined in the constant time delay case, yet it can be noted that in this particular example not all weighting coefficients have delayed activation function, yet that does not make any significant changes in the analysis.

Similarly, the time variable delayed system will be depicted as an interconnection between a nominal system and a structured uncertainty. An illustrative example is proposed to verify effectiveness of the given criteria.

Example 4.

Consider the given neural network in (4.7) with subsequent parameters

$$A = \text{diag}\{1.2769, 0.6231, 0.9230, 0.4480\},$$

$$W_0 = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -1.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix},$$

$$K = \text{diag}\{0.1137, 0.1279, 0.7994, 0.2368\},$$

$$c = 0.$$

It can be stated with the achieved maximum admissible delay durations given in Table 4.5 that IQC approach for time variable delay is compatible with frequently used methodologies in the literature while variation of the delay has lover values. But as the variation increases, Lyapunov technique becomes less restrictive and conservative. This is caused by the multiplier used to define the variable time delay; in small variations uncertainty gap is restricted, yet opposite phenomena occurs when the variation increases. Figure 3.7 on page 81 shows how the robustness gap increases with respect to variation in the time delay based on the proposed multiplier. In addition to that, multiplier for nonlinearity could not provide additional benefit due to fundamentally having positive real nominal system in most numerical examples proposed for the RNN in the literature as a result conservative results become inevitable.

To illustrate the main drawbacks of the multiplier that we have used to define variable time delay another numerical example which is less complex than the previous one was examined as well, yet similar conservative conditions were obtained.

Example 5.

Method	$\dot{T}_d = 0.1$	$\dot{T}_d = 0.5$	$\dot{T}_d = 0.9$
Zhang et al. [150]	3.5989	2.4530	1.8593
Zhang et al. [151] ($\rho = 0.6$)	3.3574	2.5915	2.1306
Wang et al. [152]	3.4886	2.6056	2.2522
Shen et al. [153]	3.5546	2.6438	2.1349
Zeng et al. [154]	3.7665	2.6814	2.2274
Ge et al. [155]	3.8428	2.7081	2.2485
Lee et al. [156]	4.0067	2.9242	2.5165
Zeng et al. [157]	4.1903	3.0779	2.8268
Counterpart of $\mathbf{\Pi}_a$, i.e., using $\mathbf{\Pi}_v$	**3.5089**	**1.7998**	**1.3549**

Table 4.5: Computational results for **Example** 4: upper bound of T_{dmax} with various $\dot{T}_d$, ($T_{dmax} = 4.6548\,\mathrm{s}$ (const.)).

Consider Neural Network (4.7) with parameters below,

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \qquad W = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix},$$
$$W_1 = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix}, \quad K = \mathrm{diag}\{0.4, 0.8\}.$$

Results for this particular example are illustrated in Table 4.6.

Method	$\dot{T}d = 0.8$	$\dot{T}d = 0.9$
Hua et al. [158]	1.2281	0.8636
He et al. [159]	2.3534	1.6050
Ge et al. [155]	2.8980	1.9562
Lee et al. [156]	4.1993	2.5979
Zeng et al. [157]	4.8167	3.4245
Counterpart of $\mathbf{\Pi}_a$, i.e using $\mathbf{\Pi}_v$	**1.8487**	**1.2783**

Table 4.6: Achievable upper bound of T_{dmax} with various $\dot{T}_d$ for **Example** 5.

These results establish the usefulness of the Zames-Falb and delay multipliers while interconnected system contains both monotonic nonlinearity and latency such as delayed neural networks. It needs to be stated that, we have used the Nyquist criteria as a benchmark result which implicitly states that it was assumed that Aizerman and/or Kalman Conjectures are true for the proposed time delayed systems subject to saturation type nonlinearity. Yet, this is still an unsupported assumption for some higher order systems particularly with latencies.

4.2 Kalman Conjecture for Plant with Delay: A Graphical Criterion

The usefulness of the absolute stability concept is indisputable: not only it provides stability conditions for arbitrary forms of nonlinearities with only certain properties mainly based on linear part of the system but also many practical control systems can be represented as feedback interconnection of a linear part and a class of nonlinearity. Thus, there has been many theoretical and practical contributions to this methodology that enable us to analyse the system with classes of nonlinearities [161], like Popov frequency domain approach [162]. Despite its advantages this methodology still only provides sufficient conditions related to stability of the interconnection. Therefore, there have been many efforts to find *necessary and sufficient* conditions for absolute stability and the first study was proposed by Aizerman for the systems containing nonlinearities with sector restrictions via subsequent conjecture.

Definition 4.1 (Nyquist value, k_N)**.** *The Nyquist value of a stable transfer function $G(s)$ is*

$$k_N = \sup_k \{k > 0 : (1 + \tau k G(s))^{-1} \text{ is stable } \forall \tau \in [0,1]\} \tag{4.8}$$

Conjecture 4.1 (Aizerman Conjecture [163])**.** *The following system*

$$\dot{x} = Ax - B\phi(Cx) \tag{4.9}$$

with A, B, and C are being constant matrices and sector bounded nonlinearity $\phi \in [0, K]$ is absolutely stable if and only if the subsequent linear system

$$\dot{x} = Ax - kBCx \tag{4.10}$$

is asymptotically stable for all $k \in [0, K]$

Namely, the nonlinearity within the system (4.9) is replaced with a linear gain such that the overall interconnection is transformed into a linear system (4.10), see Figure 4.6 with $G(s) = C(Is - A)^{-1}B$. In brief, the Aizerman conjecture states that *Hurwitz*, k_N, and nonlinearity, $\phi \in [0, K]$, sectors coincide, whereas in general $k_N > K$.

A similar conjecture is stated by Kalman for system with slope restricted nonlinearity in [164] as

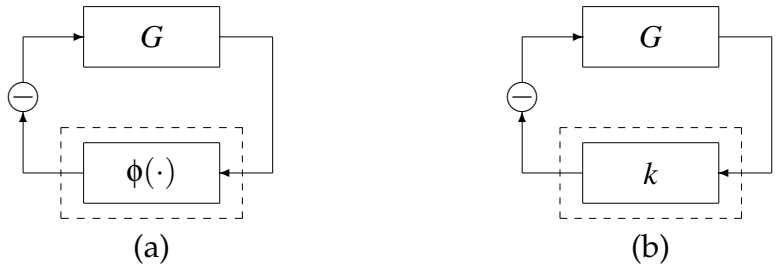

Figure 4.6: Conjecture in absolute stability: if the system (b) is stable for any feedback gain in the interval $[0,k]$ then it is concluded that system (a) is stable for any $\phi(\cdot) \in [0,k]$.

> *If nonlinearity in the feedback interconnection is replaced by a constant k corresponding to all possible values of* $\frac{d\phi(u(t))}{du(t)} \leq K$*, and it is found that the closed-loop system is stable for all* $k \in [0,K]$*, then it is intuitively clear that the system must be monostable; i.e., all transient solutions will converge to a unique, stable critical point.*

That is to say;

Conjecture 4.2 (Kalman Conjecture [164]). *The following system*

$$\dot{x} = Ax - B\phi(Cx) \tag{4.11}$$

with A, B, and C are being constant matrices and slope restricted nonlinearity $\frac{d\phi(u(t))}{du(t)} \in [0,K]$ *is absolutely stable if and only if the subsequent linear system*

$$\dot{x} = Ax - kBCx \tag{4.12}$$

is asymptotically stable for all $k \in [0,K]$.

These conjectures play an indispensable role in absolute stability: for instance system in (4.11) can correspond to a control architecture containing a class of nonlinearity and by virtue of the Kalman conjecture stability of the nonlinear system is depicted with simple Nyquist criterion based on the linear system description in (4.12). Thus, a complex problem of showing global stability was solved with ease. But these conjectures have been shown to be false in general [165]. Aizerman conjecture has been shown to be false for 3rd order system [166] and Kalman conjecture has been shown to be false for 4th order system [167], see Table 4.7 [168].

On the contrary, the problem of finding necessary and sufficient stability conditions for nonlinear systems with time delay draw little attention except Aizerman

Conjecture	Maximum order of validity n	Proof of sufficiency	Counter example for $n+1$
Kalman	3	[167]	[165]
Aizerman	2	[161]	[165]

Table 4.7: Validity of the conjectures in continuous time.

(or Kalman) conjecture was shown to be true for 1st and 2nd order systems with single time delay [169, 170] for the interconnection illustrated in Figure 4.7.

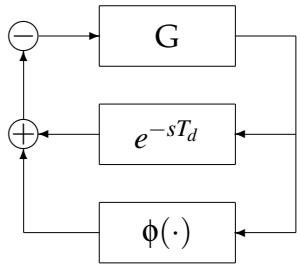

Figure 4.7: Feedback interconnection between nominal system (G), delay (e^{-sT_d}), and bounded nonlinearity ($\phi(\cdot)$).

Here, our main concern is the exactness of the Kalman conjecture for time delayed systems where delay and nonlinearity are within the same feedback loop. Initially we will discuss previous attempts where there have been efforts to show whether aforementioned conjectures are true or false when there exists delay within the nonlinear system. Then, we simply illustrate that there exists a positive answer to the subsequent question when the nominal system within the depicted interconnection contains some strict properties.

> *Given an exponential stability conditions for linear time delayed system, are they still valid if linear gain replaced with sector or slope bounded nonlinearity?*

In contrast with [169, 170], our approach is to study convexity properties of the Nyquist plot [171–173]. Instead of providing a multiplier, we will ensure its existence by using these properties. For stability of linear systems with delay, these properties have been used to analyse stability of internet congestion control [174, 175].

4.2.1 Previous Works

To begin with, assume that we have time delayed system as

$$\dot{x} = A_0 x(t) + A_d(x - T_d) \tag{4.13}$$

where A_0, A_d are constant matrices, and T_d is corresponding time delay. The problem is to find the conditions related to stability of the delayed system is so-called Routh-Hurwitz problem for quasipolynomials as exponential stability is based on location of the characteristic equation $\det(\lambda I - A_0 - A_d e^{-\lambda T_d}) = 0$. Stability conditions for $\dot{x} + a_0 x(t) + a_1 x(t - T_d) = 0$ which is a kind of the scalar version of the system in (4.13) for *small delays* are given by following inequalities

$$a_1 > |a_0|, \quad a_1 + a_0 > 0, \quad 0 \leq T_d < \frac{\arccos(-\frac{a_0}{a_1})}{\sqrt{a_1^2 - a_0^2}}. \tag{4.14}$$

Rasvan in [169] replaced the system (4.13) with

$$\dot{x} = -a_1 x(t - T_d) - \phi(x(t))$$

where $\phi(\tau)$, $\tau > 0$ is sector bounded nonlinearity. Once we can analyse the stability of the linear system by replacing ϕ with a linear gain k and it can be concluded that (given in (4.14) as well) interconnection is stable with all positive delay values if $|a_1| > k$. That is to say that for delay independent stability condition lower sector bound required to be greater than $|a_1|$. With a loop transformation as $\hat{\phi}(\tau) = \phi(\tau) - |a_1|\tau$ the system can be transformed to feedback interconnection between sector bound nonlinearity, $\hat{\phi}(\tau) \in [0, \infty)$, and the subsequent transfer function

$$G(s) = \frac{1}{s + |a_1| + a_1 e^{-sT_d}}.$$

It can be said that feedback interconnection between nominal system (G) and bounded nonlinearity is stable once there exists a multiplier, M, such that $\Re\{M(j\omega)(G(j\omega) + \frac{1}{k})\} \geq 0$. Therefore, in [169] Popov multiplier, $M(j\omega) = 1 + \beta j\omega$, was searched to ensure that $\Re\{M(j\omega)G(j\omega)\} \geq 0, \forall \omega \geq 0$, so that frequency domain inequality can be depicted as

$$\beta\omega - (\beta a_1 sin(\omega T_d))\omega + |a_1| + a_1 cos(\omega T_d) \geq 0,$$

and it can be concluded that there exists a Popov multiplier with the subsequent parameter range which satisfies positivity of the nominal transfer function within

the forward loop

$$0 < \beta|a_1| < 2.$$

That concludes delay-independent Aizerman condition is true for the given example above.

The results are extended to 2nd order retarded system by Altshuller in [170] described by following delay differential equation

$$\ddot{x}(t) + a_1\dot{x}(t) + \phi(x) + b_1\dot{x}(t - T_d) + bx(t - T_d) = 0,$$

where it is assumed that $\phi(\cdot)$ is a sector bounded nonlinearity such that $0 < \phi(x)x \leq x^2k$. One can define the second order retarded system as interconnection of bounded nonlinearity and the nominal system

$$G(s) = \frac{1}{s^2 + a_1 s + (b_1 s + b)e^{-sT_d}}.$$

Then, absolute stability of the depicted interconnection can be investigated with the frequency domain Popov criterion and it can be said that system is stable if the following inequality holds

$$\Re\{(1 + j\omega\beta)G(j\omega)\} + \frac{1}{k} > 0. \tag{4.15}$$

In addition to that the following linear equation is under investigation,

$$\ddot{x}(t) + a_1\dot{x}(t) + a + b_1\dot{x}(t - T_d) + bx(t - T_d) = 0. \tag{4.16}$$

The problem is to check whether k in the stability condition (4.15) coincides with a value such that (4.16) is globally asymptotically stable. Aizerman conjecture states that these two values are same and by courtesy of the Popov multiplier it has been shown that Aizerman conjecture is true for the 2nd order retarded system [170,176].

In the other respect, extending these aforementioned results to 3rd order systems with symbolic solution for the appearance of a multiplier is not a straight forward task. But, our main concern is neither providing a symbolic multiplier solution for a given nominal system nor extending these solutions for the interconnection where time delay and nonlinearity are in the different loops. Instead, our main concern is to obtain a graphical condition for the Kalman conjecture while considering the interconnection where nonlinearity and delay are within the same loop as many practical

control systems can be depicted in this way, see Figure 4.8 (a). Thus, absolute stability of nonlinear systems with delays can be carried out with ease once we can show that conjectures are true. Also, the obtained results based on this approach will be a benchmark to the solutions achieved by less intuitive and cumbersome LMIs [177].

It can be noted that deriving a conclusion for the depicted interconnection (Figure 4.8 (a)) based on the delay independent stability condition is straight forward because once the feedback channel depicted with constant gain k, stability of the linear interconnection states that close loop system's Nyquist curve remains in the right side of the line passing through the point $(-\frac{1}{k},0)$ in the complex plane as illustrated in Figure 4.8 (b). Then, when the absolute stability is investigated with the nominal system obtained based on the loop transformation, $G_k = G + \frac{1}{k}$, it can be concluded that G_k is always passive with k attained depending on the linear stability conditions. In other words, conjectures are true as passivity will be guaranteed even without any multiplier[2], yet as known delay independent stability condition are conservative and leads to smaller slope bounds.

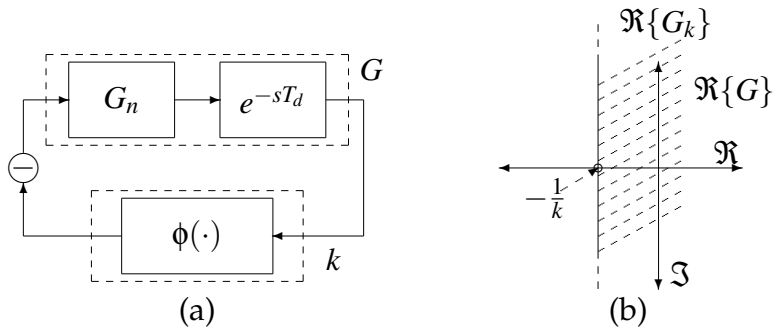

Figure 4.8: (a) The Lur'e interconnection with its linear counterpart and (b) the Nyquist region of the nominal systems in the complex plane, i.e. illustration of classical circle criterion.

4.2.2 Clockwise Property and Graphical Framework

The results here are based on the properties of RC/RL multipliers which can preserve positivity of the monotone slope restricted nonlinearities [178] and geometrical interpretation of the Off-Axis circle criterion [179]. RC/RL multipliers are a particular class of Zames-Falb multipliers [33, 180].

Loosely speaking, existence of a Zames-Falb multiplier, that concludes stability of the nonlinear system, can be depicted via phase property of the $1 + kG(j\omega)$ as it

[2]There is no need to search for a multiplier

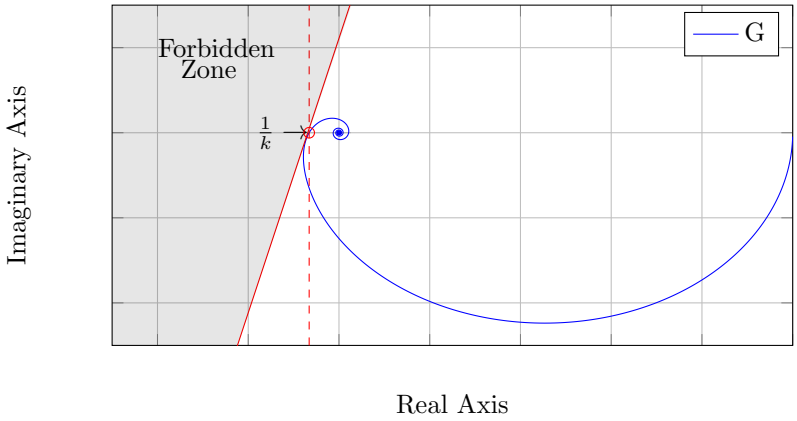

Figure 4.9: Graphical interpretation of Off-Axis circle criterion with a nominal system G.

needs to be between $(90^\circ - \theta_0, -90^\circ - \theta_0)$ with $\theta_0 \in [-90^\circ, 90^\circ]$, see [180]. In particular, when the maximum difference in the phase of $1 + kG(j\omega)$ at any two frequencies is smaller than 180°, then the Off-Axis circle criterion provides stability.

Theorem 4.1 (Off-Axis circle criterion [179]). *Let $G \in \mathbf{RH}_\infty$ be a nominal system in a feedback interconnection with a slope restricted nonlinearity $\phi \in [0, k]$ as in Figure 4.6 (a). If the Nyquist curve of the nominal system $G(j\omega)$ lies entirely to the right of a straight line with non-zero slope passing through the point $(-\frac{1}{k} + \varepsilon, 0)$ with $\varepsilon > 0$ (as in Figure 4.9) and ϕ is monotonically increasing, then the proposed feedback interconnection is $\mathcal{L}_2-$stable.*

It is well known that this result can ensure that a set of plant with *convex* properties in their frequency response satisfy the Kalman conjecture. Convexity of frequency response arcs has been analysed by Hamann and Barmish [172], and it is closely related with clockwise properties of the Nyquist plot [173]. Systems with clockwise properties of the Nyquist plot and decreasing magnitude for all frequency ensure that they satisfies the Kalman conjecture.

Definition 4.2 (Clockwise property [171]). *Let Γ be the Nyquist curve of a transfer function G in the complex plane defined by two parametric equations*

$$X = \Re\{G(j\omega)\},$$
$$Y = \Im\{G(j\omega)\},$$

with $\omega \in [\omega_1, \omega_2]$. Then, the curvature $\mathcal{C}(\omega)$ of Γ is defined as

$$\mathcal{C}(\omega) = \frac{X_\omega Y_{\omega\omega} - X_{\omega\omega} Y_\omega}{(X_\omega^2 + Y_\omega^2)^{\frac{3}{2}}} = \frac{\Im\{G_\omega^* G_{\omega\omega}\}}{|G_\omega|^3}, \tag{4.17}$$

where subscripts denote the argument with respect to which derivatives are taken. Once we assume that the curvature exists within the interval, then the curve said to satisfy the clockwise property in ω_0 *if*

$$\mathcal{C}(\omega_0) < 0$$

holds $\forall \omega_0 \in [\omega_1, \omega_2]$.

Lemma 4.1 ([171]). *A system G satisfies the Kalman conjecture if it satisfies the following two properties:*

i) The magnitude of the Bode plot of its transfer function is decreasing at all frequencies, i.e.,

$$M(\omega) = \frac{d}{d\omega}|G(j\omega)| < 0, \quad \forall \omega. \tag{4.18}$$

ii) Its Nyquist plot is clockwise.

Remark 4.1. *The first condition in the given lemma above implies that* $G(j\omega)$ *is an all pole transfer function (as a zero in the left hand side of the complex plane contributes to the magnitude of the transfer function). Therefore, the results that we will derive only contain systems have no zeros.*

4.2.3 Results For Time Delay Systems

The outcome follows from the direct application of the Off-Axis circle criterion. Thus, the implications of the results in [171] regarding to absolute stability can be extended to time delay systems as follows:

Corollary 4.2. *A first order time delayed system* $G(s) = \frac{e^{-sT_d}}{s+a}$ *with* $a > 0$ *and* $T_d \geq 0$ *satisfies the Kalman conjecture.*

Proof. The clockwise property of the Nyquist curve of the depicted first order system $G(s) = \frac{e^{-sT_d}}{s+a}$ is investigated with the given inequality in (4.17) such that

$$\Im\{G_\omega^* G_{\omega\omega}\} = -\frac{T_d^3 a^4 + 2T_d^3 a^2\omega^2 + T_d^3\omega^4 + 3T_d^2 a^3 + 3T_d^2 a\omega^2 + 4T_d a^2 + 2a}{(a^2+\omega^2)^3} < 0,$$

hence $\mathcal{C}(\omega) < 0$ for all $\omega \geq 0$ and $T_d \geq 0$. Therefore, one can say that the clockwise property condition is satisfied and the proof is concluded based on Lemma 4.1. □

Subsequently, results can be derived for the second order systems as well.

Corollary 4.3. *A second order time delayed system* $G(s) = \frac{e^{-sT_d}}{(s+a)(s+b)}$ *with* $a > 0$, $b > 0$, *and* $T_d \geq 0$ *satisfies the Kalman conjecture.*

Proof. In a similar manner, the clockwise property can be analysed via curvature of the second order time delayed nominal system's Nyquist curve and that can be derived with

$$G_\omega = \frac{d}{d\omega}G(j\omega) = -\frac{jT_d e^{-j\omega T_d}}{(j\omega+a)(j\omega+b)} - \frac{je^{-j\omega T_d}}{(j\omega+a)^2(j\omega+b)} - \frac{je^{-j\omega T_d}}{(j\omega+a)(j\omega+b)^2},$$

$$\begin{aligned} G_{\omega\omega} = \frac{d}{d\omega}G_\omega(j\omega) = &-\frac{T_d^2 e^{-j\omega T_d}}{(j\omega+a)(j\omega+b)} - \frac{2T_d e^{-j\omega T_d}}{(j\omega+a)^2(j\omega+b)} - \frac{2T_d e^{-j\omega T_d}}{(j\omega+a)(j\omega+b)^2} \\ &- \frac{2e^{-j\omega T_d}}{(j\omega+a)^3(j\omega+b)} - \frac{2e^{-j\omega T_d}}{(j\omega+a)^2(j\omega+b)^2} - \frac{2e^{-j\omega T_d}}{(j\omega+a)(j\omega+b)^3}, \end{aligned}$$

then sign of $\mathcal{C}(\omega)$ is given by the sign of

$$\Im\{G_\omega^* G_{\omega\omega}\} = -\frac{1}{(a^2+\omega^2)^3(b^2+\omega^2)^3}\left(\omega^8 p_8 + \omega^6 p_6 + \omega^4 p_4 + \omega^2 p_2 + p_0\right),$$

where

$$\begin{aligned} p_8 &= T_d^3, \\ p_6 &= 2T_d + (3a+3b)T_d^2 + (2a^2+2b^2)T_d^3, \\ p_4 &= 6a+6b+(6a^2+6ab+6b^2)T_d + (3a^3+6a^2b+6ab^2+3b^3)T_d^2 + (a^4+4a^2b^2+b^4)T_d^3, \\ p_2 &= 12a^2b+12ab^2+(6a^3b+18a^2b^2+6ab^3)T_d + (3a^4b+6a^3b^2+6a^2b^3+3ab^4)T_d^2 \\ &\quad + (2a^4b^2+2a^2b^4)T_d^3, \\ p_0 &= 2a^4b+4a^3b^2+4a^2b^3+2ab^4+(4a^4b^2+6a^3b^3+4a^2b^4)T_d + (3a^4b^3+3a^3b^4)T_d^2 \\ &\quad + a^4b^4T_d^3. \end{aligned}$$

All above polynomial are non-negative, hence we conclude that $\mathcal{C}(\omega)$ is negative for all $\omega \geq 0$. As a result Kalman conjecture is guaranteed based on Lemma 4.1. □

So far, we have proposed some results for the nominal systems with real poles, yet conclusion for second order systems with complex conjugate poles can be derived with additional conditions on the parameters:

Corollary 4.4. *A second order time delayed system* $G(s) = \frac{e^{-sT_d}}{s^2+2\xi s+1}$ *with* $\xi > \frac{1}{\sqrt{2}}$ *and* $T_d \geq 0$ *satisfies the Kalman conjecture.*

Proof. Restriction on the damping factor will be clear with the subsequent curvature definition for the depicted system:

$$G_\omega = \frac{d}{d\omega}G(j\omega) = -\frac{jT_d e^{-j\omega T_d}}{(-\omega^2+2j\xi\omega+1)} - \frac{e^{-j\omega T_d}(-2\omega+2j\xi)}{(-\omega^2+2j\xi\omega+1)^2},$$

$$\begin{aligned} G_{\omega\omega} = \frac{d}{d\omega}G_\omega(j\omega) = &-\frac{T_d^2 e^{-j\omega T_d}}{(-\omega^2+2j\xi\omega+1)} + \frac{2jT_d e^{-j\omega T_d}(-2\omega+2j\xi)}{(-\omega^2+2j\xi\omega+1)^2} \\ &+ \frac{2e^{-j\omega T_d}(-2\omega+2j\xi)^2}{(-\omega^2+2j\xi\omega+1)^3} + \frac{2e^{-j\omega T_d}}{(-\omega^2+2j\xi\omega+1)^2}, \end{aligned}$$

then sign of $\mathcal{C}(\omega)$ can be depicted with

$$\Im\{G_\omega^* G_{\omega\omega}\} = -\frac{1}{(\omega^4+2\omega^2(2\xi^2-1)+1)^3}\left(\omega^8 p_8 + \omega^6 p_6 + \omega^4 p_4 + \omega^2 p_2 + p_0\right),$$

where

$$\begin{aligned} p_8 &= T_d^3, \\ p_6 &= 2T_d + 6T_d^2\xi + 4T_d^3(2\xi^2-1), \\ p_4 &= 12\xi + 6T_d(4\xi^2-1) + 6\xi T_d^2(4\xi^2-1) + T_d^3(4(2\xi^2-1)^2+2), \\ p_2 &= 24\xi + 4T_d^3(2\xi^2-1) + 6T_d(4\xi^2+1) + 6\xi T_d^2(4\xi^2-1), \\ p_0 &= T_d^3 + T_d^2 6\xi + 2T_d(8\xi^2-1) + 4\xi(4\xi^2-1). \end{aligned}$$

All polynomials will be positive if damping factor is chosen such that $\xi > \frac{1}{\sqrt{2}}$ as sign of the numerator and de-numerator remain consistent (positive) for any given frequency and time delay. Similarly, that concludes the proof with respect to Lemma 4.1.

□

Remark 4.2. *Note that the condition $\xi > \frac{1}{\sqrt{2}}$ is required to satisfy (4.18), so the delay does not affect the result.*

Corollary 4.5. *A third order time delayed system $G(s) = \frac{e^{-sT_d}}{(s^2+2\xi\omega_n s+\omega_n^2)(s+c)}$ with $\xi > \frac{1}{\sqrt{2}}$, $T_d \geq 0$, $c > 0$, and $\omega_n > 0$ satisfies the Kalman conjecture.*

Proof. The proof is similar to the previous result, but with more tedious algebra. The condition to avoid resonant poles, i.e. $\xi > \frac{1}{\sqrt{2}}$, is enough to show that the negative sign of the curvature is preserved when a delay is introduced. □

4.2.4 Discussion

With motivation of the aforementioned results it can be noted that the derived conditions for the Kalman conjecture are independent of the value of the time delay. In fact, delay contributes positively into the convexity property of the curve: i.e., as delay increases curvature in (4.17) becomes more negative for the given examples above. As time-delay does not affect the magnitude, i.e.

$$\frac{d}{d\omega}|G(j\omega)| = \frac{d}{d\omega}|G_n(j\omega)|, \tag{4.19}$$

then it seems natural that the clockwise properties of transfer functions with delay ($G_n(j\omega)e^{-sT_d}$) are inherited from the transfer function without delay (G_n). If this property is satisfied, then it would be possible to find a very large class of transfer functions with delay that satisfies the Kalman conjecture. We pose the following conjecture:

Conjecture 4.3. *A transfer function $G_n e^{-sT_d}$ where G_n is an all pole transfer function with damping factor greater than $\frac{1}{\sqrt{2}}$ and $T_d > 0$ satisfies the Kalman conjecture.*

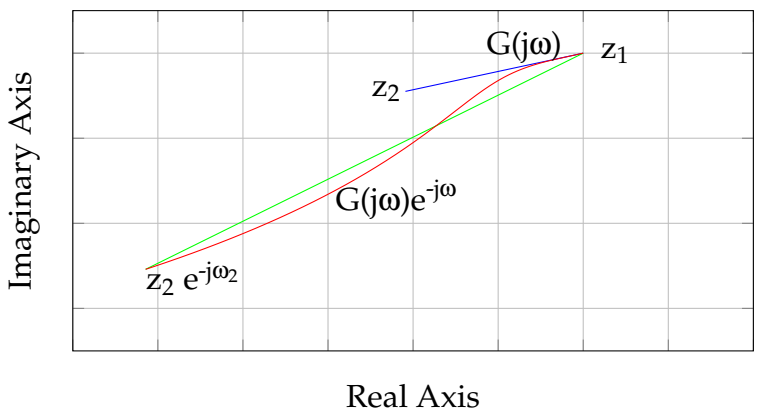

Figure 4.10: A geometrically intuitive counter example for the proposed generalization of Kalman conjecture for the time delayed nonlinear systems (Depicted $G(j\omega)$ is not a rational transfer function).

The conjecture has been confirmed to be true for third order systems, see corollary 4.5. Such "natural" assumptions on clockwise properties have previously been shown to be false, see [181, 182], thus a formal proof of the above result deserves some attention. For a general curve in the complex plane which is transformed under delay, it is easy to find counterexamples when a curve without curvature is

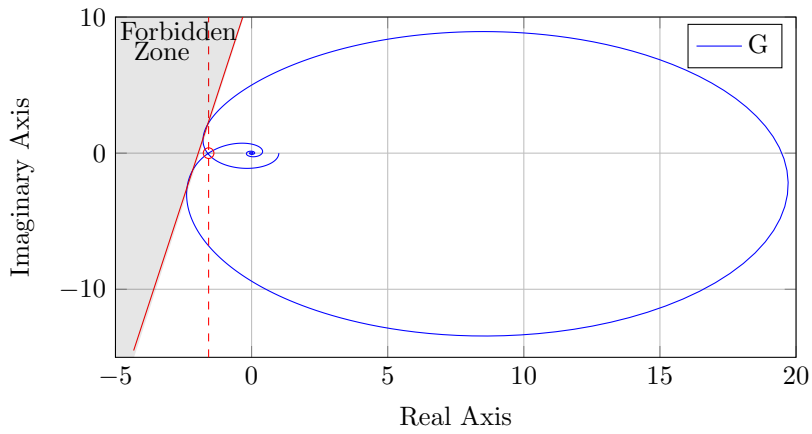

Figure 4.11: The Off-Axis criterion does not reach the Nyquist value ($k_N \simeq 0.618$).

transformed into a curve with a change in the sign of its curvature as shown in Figure 4.10.

It is then clear that geometrical properties are not enough to prove Conjecture 4.3, but it could be shown to be true by using the *Bode's phase-gain* relation in the transfer functions [183].

Finally we discuss second order plants when the plant exhibits a resonant peak in the magnitude, then the argument that used by [171] can no longer be used as it is not possible to ensure that the first crossing with the negative axis of the Nyquist plot is the only information that we need to ensure stability. So far, we have not found any second order plant with delay that does not satisfy the Kalman conjecture. However, more complicated multipliers are required, they must be designed for each time delay, so the complexity of the problem grows exponentially as we do not have an analytical value of the Nyquist gain.

As an example, let us consider

$$G(s) = \frac{e^{-5s}}{s^2 + 0.05s + 1}. \tag{4.20}$$

The Nyquist plot of this plant shows that the Off-Axis circle criterion provides a result that does not correspond with the Nyquist gain. In particular, $k_{\text{OACC}} = 0.502$ and $k_N \simeq 0.618$. Other simple multipliers such as the Popov multipliers (used in [170]) are not useful either. However, it is possible to find a Zames-Falb multiplier to show that the Nyquist gain can be reached. In this example, let us consider the Zames-Falb multiplier given by

$$M(j\omega) = 1 - 0.802e^{3.77j\omega} \tag{4.21}$$

then the real part of $(1+0.618G(j\omega))M(j\omega)$ is positive (see Figure 4.12).

As the plant has a delay, it is natural to use the class of irrational Zames-Falb multipliers [33, 180]. We have carried out a manual search, but more sophisticated searches over this class of multipliers have been proposed [184, 185].

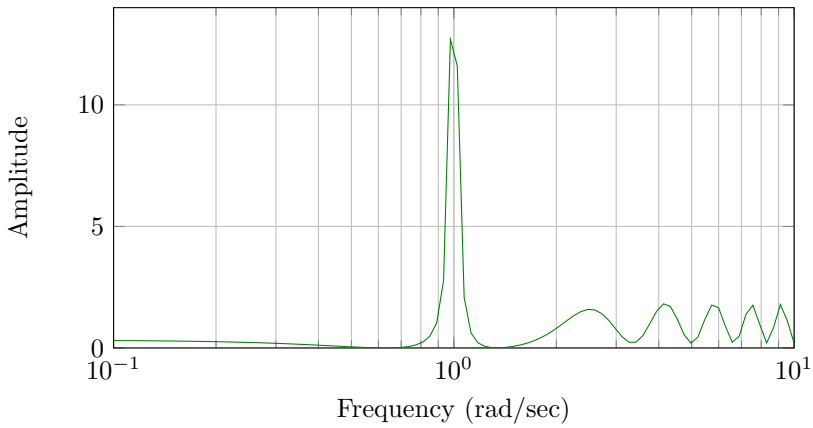

Figure 4.12: Real part of $(1+k_nG(j\omega))M(j\omega)$ with $M(j\omega) = 1-0.802e^{3.77j\omega}$.

This analysis clearly shows the advantages of multiplier theory over Lyapunov-Krasovskii functional for input/output delays. None of the current techniques based on Lyapunov-Krasovskii functionals can exploit the properties of slope-restricted nonlinearities as Zames-Falb multipliers as the analysis can fully be carried out in the frequency domain.

To sum up, a simple geometrically intuitive sufficient stability condition can be derived for the time delayed Lur'e system where delay and bounded nonlinearity are within the same loop. Strictly speaking, this extends the results in [171], where it has been revealed that clockwise transfer function satisfy the Kalman conjecture, for time-delayed plants. We show that clockwise transfer functions with any time-delay also satisfy the Kalman conjecture for particular nominal systems. As a result, first, a large class of second, and third order systems with no zeros and no resonant poles satisfies the Kalman conjecture. Generalizing this result is required more efforts, yet we conjecture that to any clockwise plant G as clockwise properties are increased when delay is introduced and we have not been able to find any system where the Kalman conjecture is not satisfied. However, the search of a suitable multiplier becomes challenging since irrational multipliers seem to be required. We illustrate these difficulties with an example. Referring to the previous arguments, understanding of this problem has significant relevance in the context of teleoperation or

neural network.

5 Multipliers for Bilateral Teleoperation

A great deal of effort has been made while analysing bilateral teleoperation in order to ensure the stability as any unpredicted/unexpected behaviour in the designed system is not well-appreciated at all, due to the critical applications which these devices are used in. That is why in most design a considerable performance measure, particularly related to the tactical feedbacks, has been sacrificed to guarantee absolute stability of the bilateral teleoperation where the design is robust against a class of (mostly passive and LTI) human and environment pair. Therefore, it can be noted that if one desires to reduce the conservatism within the bilateral teleoperation architecture, additional structural information about the human and environment operators should be included.

Here, multiplier approach will be used while analysing stability of the bilateral teleoperation where environment is assumed to be within the class of bounded monotonic nonlinear operators. Zames-Falb multiplier is used; in this way robustness of the design was increased along side with the performance criterion. It will be shown that the key element leading to high performance designs is the methodology that is being used for the analysis and assumptions related to the perturbation within the designed interconnection.

This chapter is mainly based on the previous publications given in [186–188]. Let us start with defining operators that will be depicted as an uncertainty within the bilateral teleoperation architecture.

5.1 Perturbations in Teleoperation

To begin with, in a teleoperation system, the human is the main source of the energy as they apply force to stir the master manipulator. Despite some circumstances where the operator unintentionally fights against the system's dynamics and leads to the overall system into instability[1] [59], there is a general acceptance that the applied force is bounded and does not cause any instability or undesired behaviours [189]. In other words, except some circumstances where human operator acts as a high gain controller leading to the system into instability, it is assumed that, in general, (particularly in bilateral teleoperation applications) human consciously does not apply a force causing the overall system to be unstable. Moreover, it is also assumed that the constant grip of the human arm is equivalent to an injected mass, damper, and springs into the master manipulator [190], see for instance [191] and references therein where it has been shown how human hand impedance fits a mass-damper-spring model, which can be depicted as an LTI passive operator, during a manual welding process. Thus, despite being the main source of the energy it has been accepted that a human arm in the teleoperation system has similar dynamics with a passive operator [192, 193] such that Adams and Hannaford stated in [189] as:

> *Treating energetic interaction between the human arm and a mechanical device as passive appears to be a reasonable assumption.*

Inherently, it needs to be taken into account that there exist counter statements about the passivity of the human arm, for instance see [194]. Yet for comparison we have also assumed that human is an LTI passive operator (like the great majority of the literature) in the bilateral teleoperation. Based on this assumption, the multiplier for this type of uncertainty set given in Lemma 3.2 on page 75 will be utilized for the stability analysis.

On the other hand, as stated previously, the assumptions about the environment are revisited and it is assumed that the slave manipulator is interacting with a class of memoryless, monotonic, nonlinear environment. In other words, it is assumed that the environmental interaction force is a monotonic bounded nonlinearity with

[1] The phenomenon known as Pilot Induced Oscillation (PIO)

respect to slave's speed and that can be rendered as slave is interacting with a viscosity type environment. Recent interest in this sort of nonlinear force feedback has been also proposed in [39]. For instance, that is a particular case scenario for the teleoperation systems used for sub-sea explorations. Also, a mechanical damper with nonlinear constant can be characterized such type of environment. The class of multiplier that satisfies positivity of this type of nonlinear operators is given in Lemma 3.3 on page 76 and they will be used in the stability analysis of bilateral teleoperation along with their parametrizations. As a short note, extra conditions (assumptions) can be embedded into definition of the environment such as having LTI mass and spring along with nonlinear damping constant. Subsequently, structured uncertainty definition can be extended, yet as an initial study we stick only to the monotonic nonlinearity assumption.

5.2 Teleoperation with Monotonic Nonlinear Environments

While analysing stability of the bilateral teleoperation we have assumed that human operator and environment are modelled as LTI passive operator and monotone and bounded nonlinearity, respectively. Then, we have used parametrization of both multipliers, $\lambda(\omega)$ and $Z(j\omega)$ in order to be able to carry out a convex search for the proper class of multipliers such that existence of them provides information about the stability condition of the interconnection depicted in Figure 5.1, where nominal system Y denotes admittance matrix representation of the bilateral teleoperation containing dynamics of the LTI one-DOF manipulators.

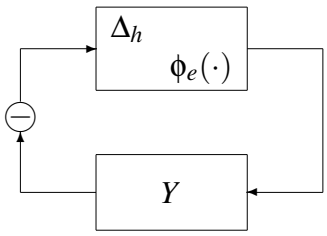

Figure 5.1: Delay free bilateral teleoperation as a classical nominal plant-uncertainty interconnection, where Δ_h and ϕ_e depict uncertain human and environment, respectively.

Remark 5.1. *One should also note that, any given delay free bilateral teleoperation architecture can be defined as a Lure structure given in Figure 5.1 or as a 2-port network given in Figure 2.1 on page 39, see [195]. Namely, the nominal system in the Lure structure is simply corresponding the immitance matrix of the network.*

Corollary 5.1. *Consider the negative feedback interconnection between admittance matrix Y and structured uncertainty block (b) in Figure 3.8 on page 86, i.e. Figure 5.1. Let us consider the following minimal state space representation:*

$$\begin{bmatrix} \Psi_b(j\omega) & 0 \\ 0 & \Psi_b(j\omega) \end{bmatrix} \begin{bmatrix} -Y(j\omega) \\ I \end{bmatrix} \sim \left[\begin{array}{c|c} A_b & B_b \\ \hline C_b & D_b \end{array}\right].$$

where Ψ_b has been defined in (3.30). Assume that the interconnection between Y and $\tau\Delta$ is well posed for $\tau \in [0,1]$. Then, interconnection in Figure 5.1 is absolutely stable if there exist symmetric matrices P_b and K_b, where K_b has the form as in (3.30), such that $(\star)^* K_b \begin{bmatrix} \Psi_b(j\omega) & 0 \\ 0 & \Psi_b(j\omega) \end{bmatrix} \in \mathbf{\Pi}_b$, and following LMI holds

$$\begin{bmatrix} A_b^\top P_b + P_b A_b & P_b B_b \\ B_b^\top P_b & 0 \end{bmatrix} + \begin{bmatrix} C_b & D_b \end{bmatrix}^\top K_b \begin{bmatrix} C_b & D_b \end{bmatrix} \leq 0. \tag{5.1}$$

Proof. If constraint (3.23) is satisfied with LMI (3.27), and LMI (3.31) hold, then there exists a multiplier class such that structured uncertainty satisfies IQC defined by $\Pi(j\omega) \in \mathbf{\Pi}_b$, see Lemma 3.8 on page 88. Finally, with KYP Lemma LMI (5.1) implies that,

$$\begin{bmatrix} -Y(j\omega) \\ I \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} -Y(j\omega) \\ I \end{bmatrix} \leq -\varepsilon I, \quad \forall \omega \in \mathbb{R}, \tag{5.2}$$

is satisfied for some $\varepsilon > 0$, where $Y(j\omega)$ is admittance transfer function matrix of the designed teleoperation system. □

Remark 5.2. *In [29] it has been stated that nominal system, Y, needs to be perturbed because the inequalities (5.2) cannot be satisfied when $\omega \to \infty$, as the plant is strictly proper. Here we use the same approach as [29]; using $Y + \zeta I$, with $\zeta = 10^{-4}$, instead of Y. This constant is interpreted as having uncertainties within the sector (0, ζ^{-1}).*

Additionally, it can be noted that the aforementioned corollary based on the stability of the undelayed bilateral teleoperation, yet delay is an inevitable phe-

nomenon if master and slave manipulators are placed at different locations. Subsequently, to analyse absolute stability of the time delayed bilateral teleoperation, multipliers for passive, Zames-Falb, and time delay are required. Therefore, the existence of any multipliers within the suitable class as in Definition 3.12 on page 88 needs to be searched. Overall, the system is transformed into the interconnection of $\bar{Y}$ and uncertainty block (c) in Figure 3.8 on page 86, where $\bar{Y}(j\omega)$ is a transformation of the admittance matrix, correspondingly whole time delayed bilateral teleoperation is reconstructed as system illustrated in Figure 5.2.

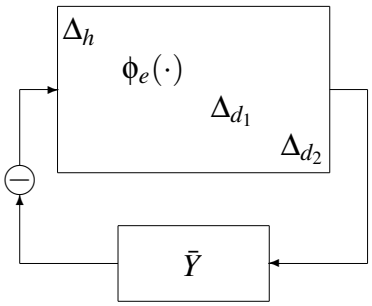

Figure 5.2: Time delayed bilateral teleoperation as $\bar{Y}$ - Δ interconnection, where human and environment are defined with Δ_h and ϕ_e, respectively, and latencies between manipulators are characterized as $\Delta_{di} = e^{-sT_{di}} - 1$, $i = (1,2)$.

Corollary 5.2. *Consider feedback interconnection between $\bar{Y}$ and uncertainty block (c) in Figure 3.8 on page 86. Let us consider the following minimal state space representation:*

$$\Psi_c(j\omega)\begin{bmatrix}\bar{Y}(j\omega)\\ I\end{bmatrix} \sim \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & D_c \end{array}\right],$$

where Ψ_c has been defined in (3.32). Assume that feedback interconnection between $\bar{Y}$ and $\tau\Delta$ is well posed for $\tau \in [0,1]$. Then, the feedback between $\bar{Y}$ and $diag(\Delta_h, \phi_e, \Delta_{d1}, \Delta_{d2})$ in Figure 5.2 is stable if there exist symmetric matrices P_c and $\bar{K}$; where $\bar{K}$ is given in (3.32), such that $\Psi_c^*(j\omega)\bar{K}\Psi_c(j\omega) \in \mathbf{\Pi}_c$ and subsequent LMI holds

$$\begin{bmatrix} A_c^\top P_c + P_c A_c & P_c B_c \\ B_c^\top P_c & 0 \end{bmatrix} + \begin{bmatrix} C_c & D_c \end{bmatrix}^\top \bar{K} \begin{bmatrix} C_c & D_c \end{bmatrix} \leq 0 \tag{5.3}$$

Proof. If constraint (3.23) is satisfied with LMI (3.27), and LMIs (3.28), (3.29), (3.31) hold, then there exists a multiplier class such that structured uncertainty satisfies

IQC defined by $\Pi(j\omega) \in \mathbf{\Pi}_c$, see Lemma 3.9 on page 88. In the end, based on the KYP Lemma, LMI (5.3) implies that

$$\begin{bmatrix} \bar{Y}(j\omega) \\ I \end{bmatrix}^* \Psi_c^*(j\omega)\bar{K}\Psi_c(j\omega) \begin{bmatrix} \bar{Y}(j\omega) \\ I \end{bmatrix} \leq -\varepsilon I, \quad \forall \omega \in \mathbb{R},$$

is satisfied for some $\varepsilon > 0$ and with IQC theorem it can be concluded that depicted interconnection in Figure 5.2 is stable. □

5.3 Numerical and Experimental Case Studies

In this section, aforementioned methodologies are numerically and experimentally evaluated and absolute stabilities of the systems are tested against predefined class of uncertainties. Experimental evaluations are carried out with one-DOF Omni manipulators. For delay free and constant time delay scenarios two Omnis are used in the same laboratory, yet for variable time delay case the test carried out between two distant laboratories located in the Universities of Manchester (UK) and Vigo (Spain).

The analyses of bilateral teleoperation firstly are based on the two-channel position-force control architecture where master and slave are one-DOF rigid robotic manipulators, yet later on the design is extended to three-channel architecture. For the initial analyses, where the two-channel architecture is considered as in Figure 5.3, the system's equation of motion is

$$v_m = Y_m(F_h + \tau_m), \quad v_s = Y_s(\tau_s - F_e),$$

where $Y_m(s) = (sM_m + B_m)^{-1}$ and $Y_s(s) = (sM_s + B_s)^{-1}$ are admittances of the manipulators with simple mass (M_m, M_s) and damper (B_m, B_s), then τ_m and τ_s are forces generated by the controllers, F_h and F_e are applied human and environmental contact forces, respectively. Due to the rigid body, it is assumed that velocities of the manipulators are equal to velocity of the operator and environment such that $v_m = v_h$ and $v_s = v_e$. So that controllers' forces are given by

$$\tau_m = -K_f F_e, \tag{5.4}$$

$$\tau_s = C_s(\mu x_m - x_s), \tag{5.5}$$

where C_s is controller at the slave side and it is used for motion tracking, K_f and μ are environmental interaction force and position scaling factors, respectively, x_m and x_s are positions of the manipulators, see Figure 5.3 for a graphical illustration.

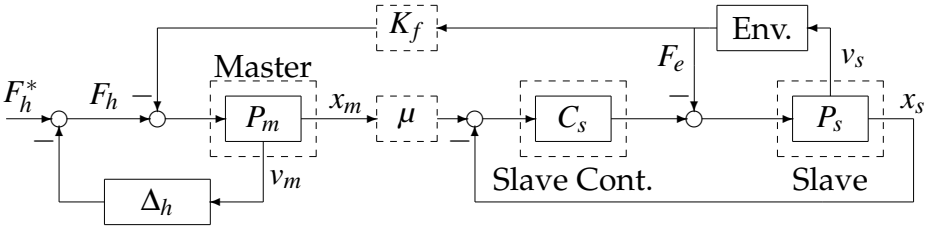

Figure 5.3: Delay free two-channel position-force bilateral teleoperation control architecture, $P_i(s) = Y_i(s)/s$ for $i = m, s$.

Based on the architecture illustrated in Figure 5.3, different types of slave controllers (C_s) can be implemented, yet it is generally designed as a Proportional-Derivative (PD) type controller such that $C_s(s) = K_p + K_v s$ [45,46]. Also, P controller has been implemented, yet extra damping parameter was inserted to certify the stability

$$\tau_s = K_p(\mu x_m - x_s) - K_v v_s. \tag{5.6}$$

The values of the system's parameters, which are going to be used in the analyses, are given in Table 5.1 [196]. The parameters appertain to linearised one-DOF Phantom Omni haptic manipulator, which was used in the experiments, and parameters of the controller are acquired with Internal Model Control (IMC) principles [197]; see Figure 5.4 for outputs comparison between the given model and the Phantom Omni robot based on a predetermined trajectory.

Model		Controller
$M_m = 0.001\,\mathrm{kg}$	$M_s = 0.001\,\mathrm{kg}$	$K_p = 10\,\mathrm{N/m}$
$B_m = 0.02\,\mathrm{N\,s/m}$	$B_s = 0.02\,\mathrm{N\,s/m}$	$K_v = 0.18\,\mathrm{N\,s/m}$

Table 5.1: Values of the system's parameters.

Network representation of the system can be any of the immitance matrices; impedance, admittance or hybrid [58], in our analyses admittance matrix is going to be used. In two-channel architecture initially two types of control algorithms are going to be tested with delayed and delay free bilateral teleoperation systems.

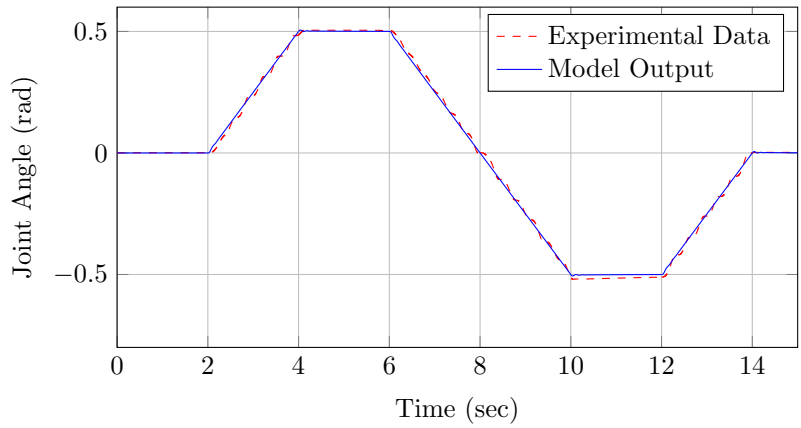

Figure 5.4: Time response comparison between model and the Omni manipulator on a predefined trajectory; model fit is 96.71%.

5.3.1 Delay Free Case

In some circumstances, master and slave manipulators are located close to one another, for instance a robotic surgery where patient and surgeon are in the same room. Based on that, initially we assumed that there is no latency in the communication medium. Then, let Y_m and Y_s be admittance of the manipulators, the admittance matrix representation of damping injected, controlled with (5.4) and (5.6), P-F controller (illustrated in Figure 5.3) can be derived with the subsequent equalities

$$V_m(s) = Y_m(F_h - K_f F_e), \tag{5.7}$$

$$V_s(s) = \frac{Y_s C_s Y_m \mu}{s + Y_s C_s} F_h - \frac{Y_s C_s \mu Y_m K_f + s Y_s}{s + Y_s C_s} F_e, \tag{5.8}$$

where $C_s = K_p$, V_m and V_s are laplace transform of the velocity signals of the master and slave manipulators, respectively. After substituting the slave controller as a proportional contoller gain K_p and admittance of the slave as $Y_s = (sM_s + B_s + Kv)^{-1}$ (damping injected), the admittance matrix representation of the depicted P-F controller can be defined as follows

$$\begin{bmatrix} V_m(s) \\ -V_s(s) \end{bmatrix} = \underbrace{\begin{bmatrix} Y_m & -K_f Y_m \\ -K_p \mu Y_m P_p & (K_p \mu Y_m K_f + s) P_p \end{bmatrix}}_{Y_P(s)} \begin{bmatrix} F_h \\ F_e \end{bmatrix}, \tag{5.9}$$

where $P_p(s) = (s^2 M_s + s(B_s + K_v) + K_p)^{-1}$. The system controlled with (5.4) and (5.5) will be called as PD-F architecture, in a similar manner admittance matrix representaion of the PD-F architecture can be derived by defining the slave controller in (5.8)

as $C_s(s) = K_p + sK_v$. Then, its admittance matrix representation becomes slightly different such that

$$Y_{PD}(s) = \begin{bmatrix} Y_m & -K_f Y_m \\ -\mu(sK_v + K_p)Y_m P_p & Z_{22} Y_m P_p \end{bmatrix}, \tag{5.10}$$

where $Z_{22}(s) = s^2 M_m + s(B_m + \mu K_f K_v) + \mu K_f K_p$. These admittance matrices are used within FDI, such as in (5.2). As previously mentioned, stability is necessary but not the only criterion that needs to be considered while designing the bilateral teleoperation system.

	Controller	
	P-F Architecture	PD-F Architecture
Passivity	0.399	–
With symmetric poles	**0.645**	**0.418**
With asymmetric poles	**0.809**	**0.622**

Table 5.2: Maximum obtainable transparency indexes ($K_f\mu$) with different techniques/multipliers and controllers at slave side.

Firstly, the absolute stability of the P-F control architecture was analysed. In the first stage Llewellyn's stability criterion, which is given in Theorem 2.1 on page 43, was used under LTI passive operators assumption for both human and environment with the parameters given in Table 5.1, and the admittance matrix given in (5.9). For the analysis, we choose the frequency range to be as 0 rad/s to 1×10^6 rad/s where the maximum achievable transparency index is searched without destroying passivity of the two-port network via bisection algorithm. It was concluded that with this hypothesis maximum achievable $K_f\mu$ value is approximately 0.399 so that admittance matrix is on the boundary of the positive realness and stability index remains positive, see Figure 5.5. If the environment is also monotone and bounded, less conservative results were obtained with Zames-Falb multipliers by using Corollary 5.1; results are given in Table 5.2. If we use symmetric poles, we obtain 0.645, whereas the result reaches 0.809 when asymmetric poles are considered.

Secondly, we have analysed stability of the two-channel bilateral teleoperation with PD-F controller architecture whose admittance matrix is defined in (5.10). As it is highlighted in [196] it is not possible to fulfil Llewellyn's stability criteria with this controller unless $M_s = 0$. Namely the admittance matrix is not positive real, so it cannot be ensured whether the design is absolutely stable or not with the parameters given in Table 5.1, when both human and environment are assumed to

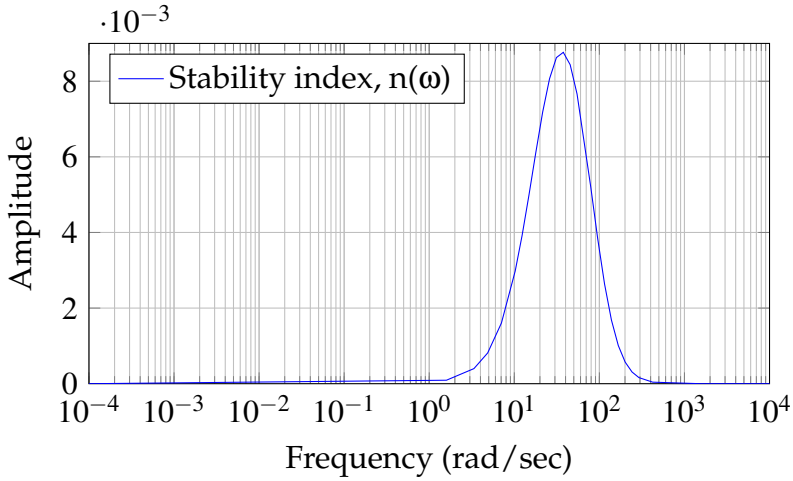

Figure 5.5: Llewellyn's stability index $n(\omega)$ remains positive over a range of frequencies when maximum obtainable $K_f\mu = 0.399$ within the P-F architecture.

be passive LTI systems. However, with the novel assumption on the environment, the use of Zames-Falb multipliers in Corollary 5.1 allows us to conclude that PD-F controller architecture is absolutely stable and maximum achievable transparency index is 0.418. Similarly, the transparency index can be improved by using asymmetric poles, reaching 0.622. These results are obtained with parameters given in Table 5.3.

	P-F Architecture			PD-F Architecture		
	Λ_h	Λ_e		Λ_h	Λ_e	
Order (n)	5	6	6	5	6	6
Pole ($-\alpha$)	-110	-75	-500 & -0.1	-110	-380	-400 & -0.01

Table 5.3: Parameters of the basis functions for stability analyses of bilateral teleoperation.

5.3.2 Time Delayed Case

Admittance matrix representations of architectures are need to be redefined if there exists time delay in the communication medium. For instance, with a constant delay dynamics of the P-F architecture proposed in (5.7) and (5.8) are modified (the transmission delay is embedded) as follows

$$V_m(s) = Y_m(F_h - K_f e^{-sT_d} F_e), \tag{5.11}$$

$$V_s(s) = \frac{Y_s C_s Y_m e^{-sT_d}\mu}{s + Y_s C_s} F_h - \frac{Y_s C_s \mu Y_m e^{-2sT_d} K_f + sY_s}{s + Y_s C_s} F_e, \tag{5.12}$$

where T_d defines one way communication time delay between the master and slave manipulators. For simplicity, initially it was assumed that delay duration, T_d is equal in both direction. In a similar manner, one can substitute the slave controller as $C_s = K_p$ and admittance of the slave as $Y_s = (sM_s + B_s + Kv)^{-1}$ (damping injected), then the overall architecture's admittance matrix can be defined as follows

$$\begin{bmatrix} V_m(s) \\ -V_s(s) \end{bmatrix} = \underbrace{\begin{bmatrix} Y_m & -K_f Y_m e^{-sT_d} \\ -K_p \mu Y_m P_p e^{-sT_d} & (K_p \mu Y_m K_f e^{-2sT_d} + s) P_p \end{bmatrix}}_{Y_{Pd}(s)} \begin{bmatrix} F_h \\ F_e \end{bmatrix},$$

where $P_p(s) = (s^2 M_s + s(B_s + K_v) + K_p)^{-1}$. Then, based on the given equations in (5.11) and (5.12) the communication delay can be pulled out from the admittance matrix to treat as a perturbation

$$\begin{bmatrix} V_m(s) \\ -V_s(s) \\ y_1(s) \\ y_2(s) \end{bmatrix} = \underbrace{\begin{bmatrix} Y_m & 0 & -Y_m & 0 \\ 0 & \frac{sY_s}{s+Y_sC_s} & 0 & \frac{Y_sC_s}{s+Y_sC_s} \\ 0 & K_f & 0 & 0 \\ -\mu Y_m & 0 & \mu Y_m & 0 \end{bmatrix}}_{Y_d(s)} \begin{bmatrix} F_h \\ F_e \\ u_1 \\ u_2 \end{bmatrix},$$

where $y_1 = K_f Fe$, $y_2 = -\mu Y_m F_h + \mu Y_m K_f e^{-sT_d} Fe$, $u_i = e^{-sT_d} y_i$ for $i = 1,2$, and $Y_d(s)$ is interconnected with structured uncertainty block, $\Delta = \mathrm{diag}(\Delta_h, \phi_e, e^{-sT_d}, e^{-sT_d})$. In order to get the interconnection as illustrated in Figure 5.2, due to the feed-forward inclusion in the delay channels, initially, positive feedbacks are included to the last two ports of the transfer function matrix, then last two channels of Y_d need to be multiplied with negative sign, in that way Y_d is transformed to $\bar{Y}$, see Figure 5.6 for a graphical illustration of the mentioned loop transformation.

Initial studies have shown that having light and fast manipulators does largely degrade the stability margin when there exists time delay in the communication medium. With the mass and damping values given in Table 5.1, performance index leads to small values so the scaled force signal might not be perceivable by human being. In order to obtain reasonable performance indexes in delayed teleoperation, additional damping has virtually been injected into the master manipulator. Thus $B_m = 0.2\,\mathrm{N\,s/m}$ is used in the following analyses parameters. Test with several delay duration, $T_{dmax} \in [0.01, 0.1]s$, was carried out with bisection algorithm and maximum performance indexes for both types of controllers used for two-channel control de-

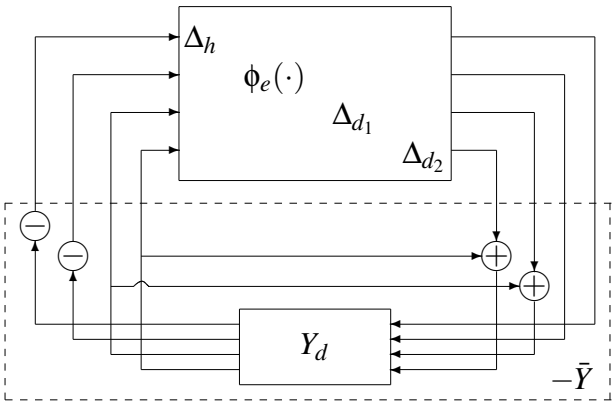

Figure 5.6: Delay encapsulation in time delayed bilateral teleoperation architecture depicted as Lure interconnection.

sign are shown in Figure 5.7. Basis function parameters in Table 5.3 were used with Corollary 5.2. For Zames-Falb multiplier the selection of the poles is very important, but for the delay we can reach same results with different poles.

For variable time delay, on the other hand, we have analysed characteristic of the Internet communication medium between two laboratories[2]. While transmitting the User Datagram Protocol (UDP) packets, each packet was composed of time stamps such that the delay could be measured in terms of the round trip communication time delay. Then, it was also observed that one way maximum transmission delay is approximately 28 msec and variation of the delay is less than 0.45, $\dot{T}_d \leq 0.45$ (with a transmission rate of 1000 packets per second), see [198], where the same network was used for a different experiment, as well. Based on these features bisection algorithm was carried out for searching maximum performance indexes and it was stated that with P-F controller $\mu K_f = 0.0744$ and with PD-F architecture $\mu K_f = 0.0261$, when $n = 5$ in both controllers while using multipliers given in (3.3), (3.7), and (3.15) for uncertain operators human, environment, and time delay, respectively. As a result, these performance indexes are going to be used in the experimental evaluations.

5.3.3 Experimental Evaluation with Two-Channel Architecture

Experimental evaluation of the numerical results has been carried out with Phantom Omni haptic manipulators which have six-DOF (3-actuated and 3-non actu-

[2]The communication network used in the experiment belongs to the GÉANT pan-European research and education network.

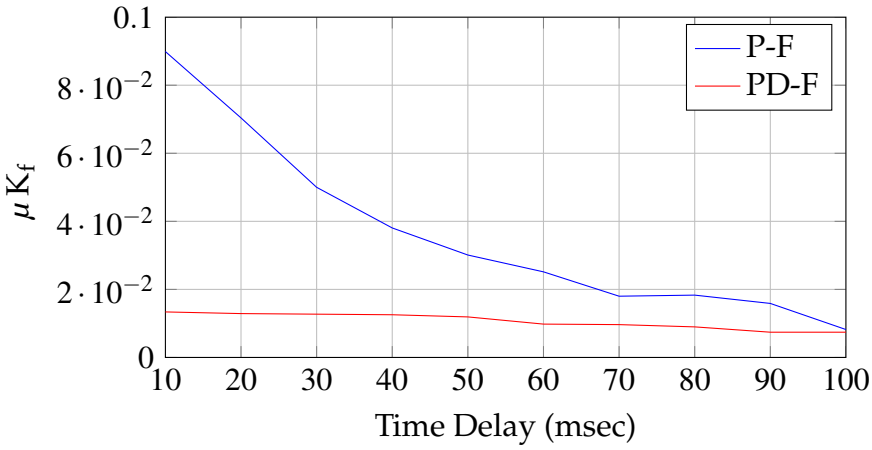

Figure 5.7: Transparency index reduction against maximum delay duration in the communication medium.

ated joints), see Figure 5.8. As analyses are not based on multi-DOF, only one-DOF has been used while moving first joint and blocking/immobilizing the remaining. Omni manipulators have only position sensors so master's and slave's positions are sensed in radians with sensor located at the first joints, yet human and environmental forces are simulated with pre-designed passive and nonlinear operators, respectively.

Figure 5.8: Experimental setup with Phantom Omni manipulators (Image taken from [199])

Delay-free and constant time delay scenarios were carried out in the same laboratory while Omni manipulators are connected to a computer, where controllers are embedded into the Matlab/Simulink environment[3], through IEEE 1394 FireWire cables and constant time delays are virtually injected to the communication

[3]Applied at 1.000 Hz; the maximum value supported by the Omni manipulator

medium. Variable time delay case, however, was carried out in two different laboratories; master and slave were located in the universities of Manchester (UK) and Vigo (Spain), respectively. Internet, inevitably, was used as a communication medium and the data (position and force) were carried out with UDP[4] packets. To be consistent in all cases, end effector of the master was aimed to follow conical shape trajectory (the same as in simulations), meanwhile monotonic nonlinear type environmental force; $F_e(v_s) = k\arctan(v_s), k > 0$, was acting on the slave. Velocity signals can be numerically determined from position signals via a simple first order compensator $H(s) = \frac{Ns}{s+N}$ with a positive filter coefficient, $N > 0$. Yet due to the sensor noises within the position signals using such filter causes high spikes in the determined velocities. One can use suchlike signals throughout the architecture, but this causes high frequency vibrations within the manipulators. Thus, we have derived velocity signals by using second order low pass filters with cut-off frequency and damping ratio are equivalent to 250 rad/s and 4, respectively. Reference signal for the first joint and master-slave manipulators' behaviours under the designed controllers can be seen in Figure 5.9.

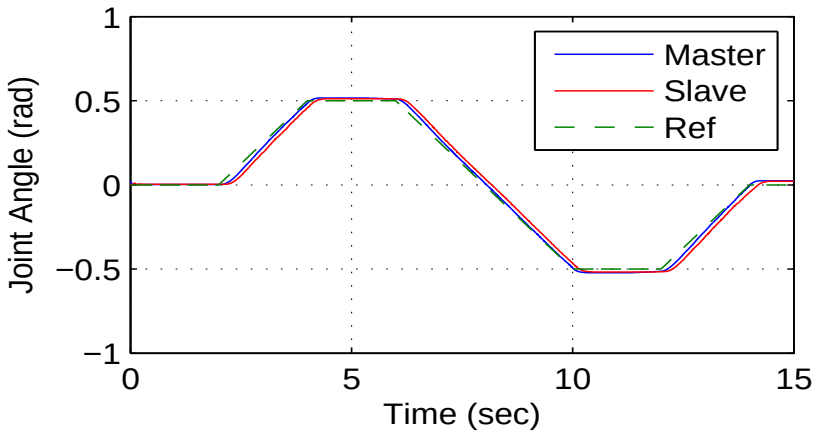

Figure 5.9: First joints' positions and reference signal; experiment between Manchester and Vigo.

Slave needs to mimic the behaviour of the master manipulator in exchange for being able to complete challenging tasks in an high quality manner. In order to evaluate this we have tested the designed two control architectures, P-F and PD-F, with the maximum achievable transparency indexes. It is remarked that P-F architecture's position error is approximately 1.5 times higher than the PD-F architec-

[4]User Datagram Protocol (UDP) is one of the main internet communication protocols

ture's in all scenarios, see Figure 5.10. That concludes more complex controllers give better performance specifications; less transparency index yet higher position tracking features. Present stability analyses, however, are restricting us to design systems controlled by these controllers. Therefore, more powerful and less conservative stability analyses methods need to be used to analyse bilateral teleoperation and evaluate control algorithms that can be implemented.

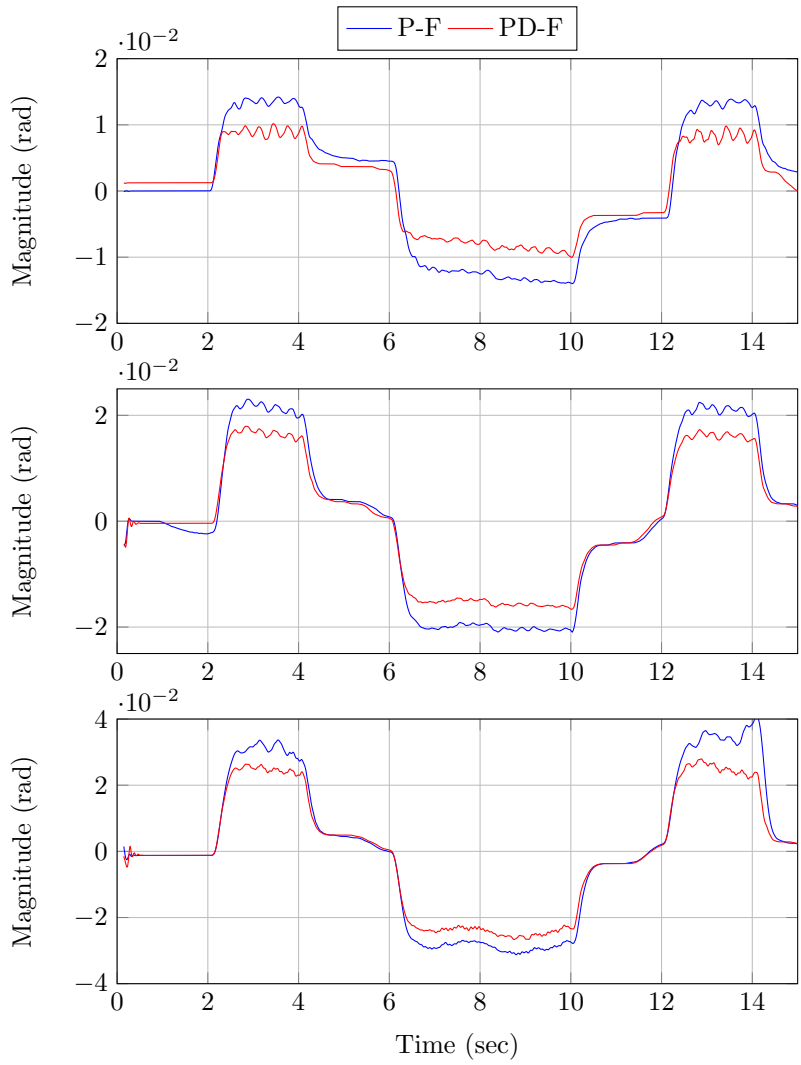

Figure 5.10: Position mismatches between master and slave: Top is delay free case, middle is simulated constant delay ($T_{dmax} = 28msec$) between manipulators, and bottom is the experiment taken between Manchester and Vigo.

Figure 5.11 shows that the improvement in the position tracking is not due to a reduction in the transparency, as PD-F architecture provides similar force mismatches as the P-F architecture under delay in the communication. As a result, Figures 5.10 and 5.11 demonstrate the benefit of the current analysis within the two-channel control architecture.

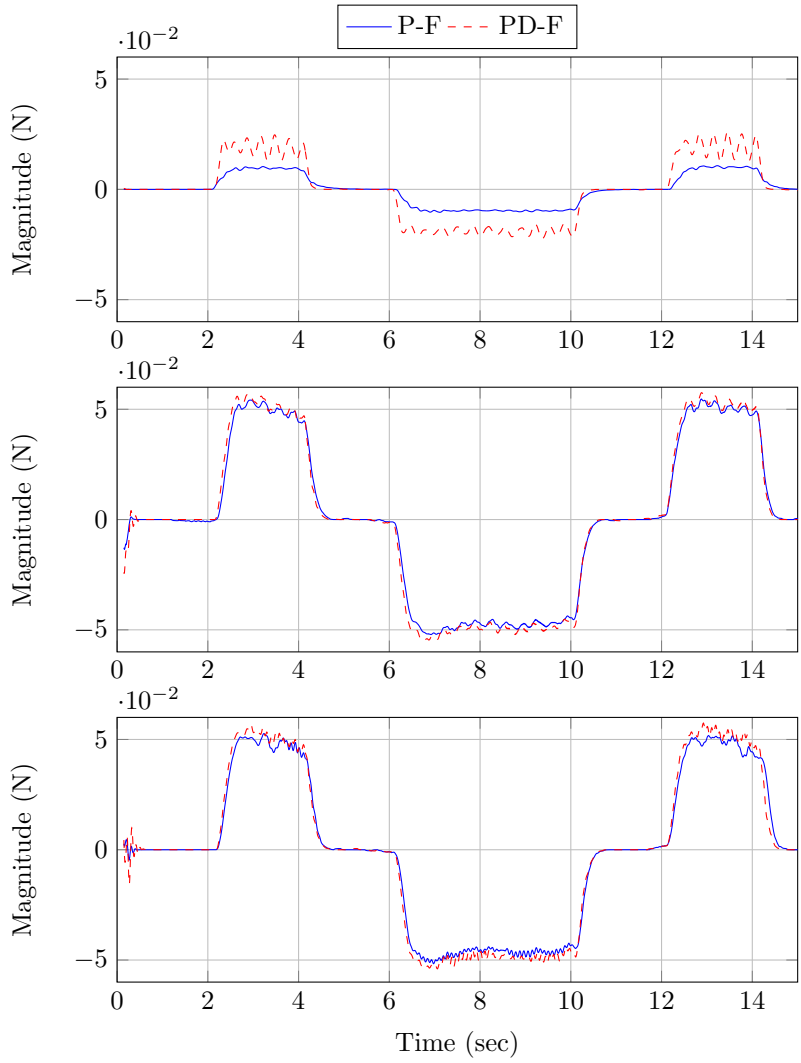

Figure 5.11: Difference between the actual and transmitted (to master side) environmental interaction forces: Top is delay free case, middle is simulated constant delay (in one way $T_{dmax} = 28msec$) between manipulators, and bottom is the experiment taken between Manchester and Vigo.

5.3.4 Enhanced to Three-Channel Architecture

In two-channel position-force architecture we have shown that having PD controller at the slave side increases the performance specifications of the design once Zames-Falb multiplier is used for nonlinear environment. In this section, we extend our previous studies to the three-channel (3C) architecture. Further analyses were carried out to compare 3C Position-Force Force (P-FF) with 2C P-F architecture while both designs have PD-controllers at slave sides. Then, Phantom Omni manipulators were used for the experimental validations. It is observed that performance specifications of the proposed 3C is superior to 2C architecture when master and slave

manipulators have similar dynamics [200].

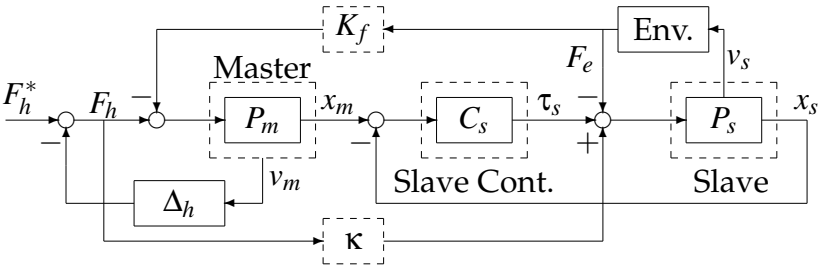

Figure 5.12: Three-channel PD-FF bilateral teleoperation control architecture. Interconnection will be transformed to a two-channel PD-F architecture by substituting $\kappa = 0$ ($P_i(s) = Y_i(s)/s$ for $i = m, s$.).

In 3C case system's equation of motion is slightly different from the two-channel case as

$$v_m = Y_m(F_h + \tau_m), \quad v_s = Y_s(\tau_s - F_e + \kappa F_h), \tag{5.13}$$

where κ which is either 0 or 1 that specifies the control structure: $\kappa = 1$ implies the existence of the extra channel that transmit F_h to the slave side, see Figure 5.12 for block diagram representation of the proposed control architecture. Controller force at human side is given in (5.4) and the one at the slave side is a slight modification of (5.5) where it is assumed that $\mu = 1$ such that

$$\tau_s = C_s(x_m - x_s). \tag{5.14}$$

As stated previously, different types of controller (C_s) can be implemented at the slave side within the same architecture. Eligibility of the PD over single P controller has been stated previously, so herein a PD-controller such $C_s(s) = K_p + K_v s$ is going to be designed for both architectures. In this case, we assume that there is no latency in the communication channel. And let $Y_m(s) = (M_m s + Bm)^{-1}$, $Y_s(s) = (sM_s + (B_s + K_v))^{-1}$ (damping injected), and $\kappa = 0$ in (5.13), equation of the motion of the PD-F architecture, which is controlled with (5.4) and (5.5), can be defined subsequently

$$\begin{aligned} V_m(s) &= Y_m(F_h - K_f F_e), \\ V_s(s) &= \frac{Y_s C_s Y_m}{s + Y_s C_s} F_h - \frac{Y_s C_s Y_m K_f + s Y_s}{s + Y_s C_s} F_e. \end{aligned}$$

Then, admittance matrix representation of the depicted PD-F architecture is given as follows

$$\begin{bmatrix} V_m(s) \\ -V_s(s) \end{bmatrix} = \underbrace{\begin{bmatrix} Y_m & -K_f Y_m \\ -\frac{Y_s C_s Y_m}{s+Y_s C_s} & \frac{sY_s+Y_s C_s K_f Y_m}{s+Y_s C_s} \end{bmatrix}}_{Y_{PDF}(s)} \begin{bmatrix} F_h \\ F_e \end{bmatrix}.$$

The system with $\kappa = 1$ in (5.13) will be called as PD-FF architecture, and its admittance matrix representation is slightly different such that

$$Y_{PDFF}(s) = \begin{bmatrix} Y_m & -K_f Y_m \\ -\frac{sY_s+Y_s C_s Y_m}{s+Y_s C_s} & \frac{sY_s+Y_s C_s K_f Y_m}{s+Y_s C_s} \end{bmatrix}. \tag{5.15}$$

These admittance matrices were used in FDI, (5.2), and it was converted to an equivalent LMI, (5.1), hereby stability analysis was transformed to a convex optimization search for suitable multipliers.

In a similar fashion bisection algorithm was used while searching existence of the suitable multipliers with maximum K_f value; the maximum achievable values are proposed in Table 5.4.

Controller	
PD-F Architecture	PD-FF Architecture
0.5265	**0.8242**

Table 5.4: Maximum obtainable K_f values for different control architectures.

Based on the obtained maximum K_f values, validation of the architectures were carried out; initially with simulations by using parameters given in Tables 5.1 and 5.4 and later with experiments. As a result, it was observed that position error between master and slave is minimized with PD-FF architecture, see Figure 5.13. Synchronisation between two manipulators is enhanced with the additional feedforward force channel because having similar manipulators' dynamics lead to identical parameters in the first column of admittance matrix $Y_{PDFF}(s)$ in (5.15).

Experimental results also support the same argument with the simulations, namely position error obtained with PD-FF is half of the error that we get with PD-F architecture, the results are illustrated in Figure 5.14. In the same manner, difference between the actual environmental and transmitted (to human operator) forces is reduced with PD-FF architecture compare with PD-F, see Figure 5.15.

To sum up, this methodology enables us to use controllers such as simple PD, that disturb passivity of the network representation yet improve position tracking

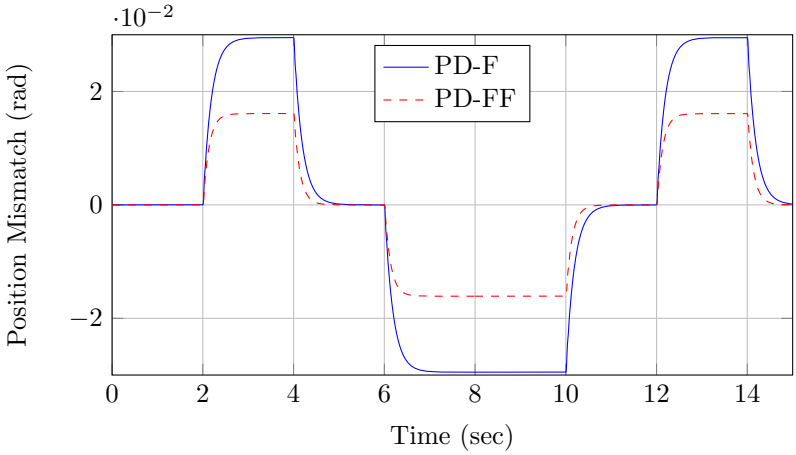

Figure 5.13: Position error between master and slave manipulators in simulation.

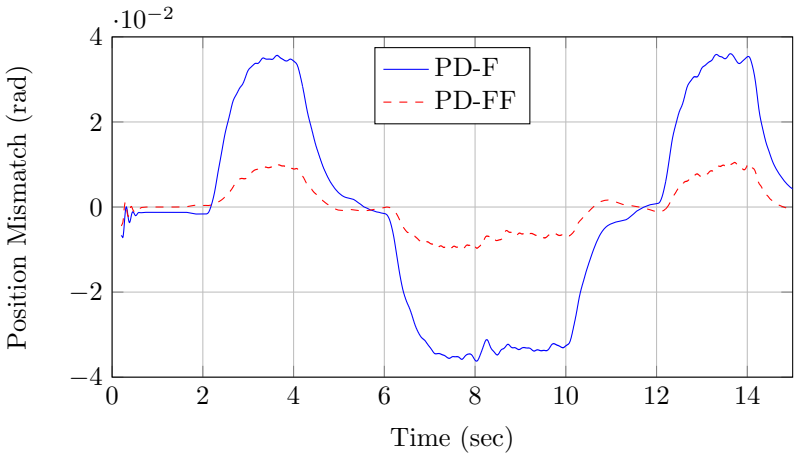

Figure 5.14: Position error between master and slave, experiment with 2 Omni manipulators within the same lab.

significantly. All theory based arguments were validated with experiments using similar manipulators that show superior performance of 3C over 2C. It should be noted that, we do not claim that the proposed methodology transcends general passivity approach in all aspects, indeed there are still some open questions to answer. For instance, in the aforementioned experimental analyses designed controllers are embedded into digital computers so that continuous data is discretized without referring to the detrimental effect of the discretization error, yet needless to say that more rigorous effort is required which is out of the scope of this study.

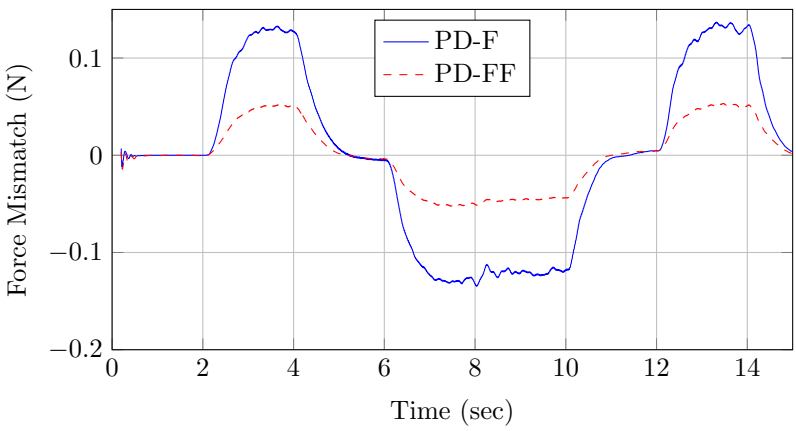

Figure 5.15: Mismatch between actual environmental interaction force and the force transmitted to human operator side.

6 Conclusions and Future Work

In this modest study, our aim was to illustrate usefulness of a theoretical framework that could be used for the analysis of systems consisting a class of nonlinearity, yet absolute stability of the bilateral teleoperation has been our main concern. The path that we have followed bring us to define classes of multipliers and their parametrizations such that stability of the systems, that varies from teleoperation to the neural networks, containing particular class of uncertainties/nonlinearities is simply transformed to a tractable search for a multiplier within the proposed classes.

For this reason, in order to have a solid comparison initially stability of the systems that contain saturation type nonlinearity along with time delay was analysed based on the IQC framework and multiplier approach, also validity of the proposed method was compared with prevalent techniques. Among the other things, it was observed that particular neural networks can be expressed as delayed Lur'e interconnection so that their stability analysis can be carried out with a similar fashion. Based on the achieved numerical results, it can be stated that multiplier approach is compatible with the widely used methodologies such as Lyapunov technique while analysing stability of the systems containing time delay and saturation, including neural networks, in fact it outperforms in particular circumstances such as when there exists constant time delay or when asymmetric Zames-Falb multiplier is searched.

For completeness Quadratic Separation (QS) approach, that has been mainly used for reducing conservatism in the analysis of time delayed systems, was discussed in the appendix and the relationship (equivalence) between IQC technique and QS approach was illuminated.

On the contrary, while carrying out the analyses and comparing the obtained results with current state of the literature we have used the well-known Nyquist crite-

ria as a benchmark[1] that inherently leaded us to Kalman and Aizerman conjectures. We have shown that there exists little interest and still there are more works need to be carried out parallel to these conjectures when latency occurs along with bounded nonlinearity within the prospective interconnection. Nevertheless, we provide a graphically intuitive solution such that absolute stability of the nonlinear systems with time delay can be determined only with linear stability conditions based on a particular class of delay free nominal system within the interconnection. Thus, one can show that there exists a class of multiplier for a slope bound equivalent to the Nyquist value without constructing the suitable multiplier.

If we return to stability of the teleoperation, in the first instance absolute stability of the delay free bilateral teleoperation was analysed by using IQC framework and Zames-Falb multipliers via transforming the overall interconnection into the Lur'e structure. For comparison, definition about the uncertainty caused by human arm was not changed: human arm was assumed as an LTI passive operator. On the other hand, definition about environment's linearity was being questioned and redefined because majority of the objects' physical contact forces show non-linear phenomena. Therefore, in our analyses we considered the environment as memoryless, monotone, bounded, and nonlinear operator. In order to obtain an analytical advantage from this assumption, we have searched for Zames-Falb multipliers which are the widest available class of multipliers. Also, Zames-Falb multiplier is able to 're-gain' passivity of the network so that there is no need to pre-design controllers in a way that network is critically passive.

Furthermore, stability of the two-channel position-force bilateral teleoperation with PD controller (at slave side) was validated without any additional restrictions. Then, analyses were broadened to the bilateral teleoperation architectures that accommodate constant or time variable latencies within the communication medium. Afterwards, an experimental set-up was constructed between Manchester (UK) and Vigo (Spain) Universities so that proposed analyses were experimentally evaluated with two Phantom Omni manipulators located at two remote laboratories. As a final effort, analyses are extended to the three-channel architecture; preliminary experimental results show predominance of the extended design when two manipulators have similar dynamics.

[1] By replacing the slope restricted nonlinearity with a linear gain

Based on the aforementioned numerical and experimental results we can clearly conclude that the more 'realistically' we define the unknown elements within the bilateral teleoperation interconnection the better performance criteria are going to be. Yet, slightly avoiding from straightforward assumptions requires more complex tools for the analyses and naturally that demands more computational power. Despite these requirements, reductions on the conservatism particularly in bilateral teleoperation promote the methodology that we have proposed and adopted. Still more effort must be expended in order to answer some currently open questions leading to physically realizable and also high performance bilateral teleoperations. One of them might be finding a way to analyse stability of the bilateral teleoperation consisting nonlinear high-DOF manipulators based on this methodology. And another one could be extending definition of the human operator or including their perception into the synthesises. Despite its complexity and difficulty assumption on the human operator needs to be relaxed even though that might be beyond the scope of control engineering, it is a challenge that researchers in the relevant field needs to face. The gap in that section comes to the light with controversia of the passivity assumption. Also, we have not proposed a complete solution for the problem caused by the time delay including the human operator. Strictly speaking, that problem can be divided into two different categories; operator point of view and stability point of view which we have considered the most in this study. Yet, human sensory system is not capable of sensing every single delay in a mechanism, that is why a cell phone talk seems to be continuous for us even though the transmission is not. Also, carrying a distance task via teleoperation with a certain transmission delay is not practically feasible from the operator point of view. But, a clear majority of the previous studies (including the one proposed here) have been only focused on the stability point of view as delay might jeopardize the absolute stability.

Therefore, future research direction can be diverted in a way that we can answer or might solve these highlighted problems also can insert human sensation into the design stage of the bilateral teleoperation.

A Appendix

A.1 Minimal State Space Realization of Basis Function

In the spirit of [133,134,137] minimal state space representation of the basis function $\Lambda_\alpha(j\omega)$ is defined as

$$\Lambda_\alpha \sim \left[\begin{array}{c|c} A_{\Lambda_\alpha} & B_{\Lambda_\alpha} \\ \hline C_{\Lambda_\alpha} & D_{\Lambda_\alpha} \end{array}\right] = \left[\begin{array}{ccccc|c} -\alpha & 0 & \cdots & \cdots & 0 & 1 \\ 1 & \ddots & \ddots & \ddots & \vdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 1 & -\alpha & 0 \\ \hline 0 & \cdots & \cdots & \cdots & 0 & 1 \\ 1 & 0 & \cdots & \cdots & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & \cdots & 0 & 1 & 0 \end{array}\right],$$

and other basis functions', $\Lambda(j\omega), D(j\omega)$, etc., state space parameters can be defined similarly.

A.2 Quadratic Separation Approach

Quadratic Separation is another methodology that is being implemented while analysing stability of the time delayed systems [201–205]. Additionally, it has also been considered for analysing systems subject to bounded nonlinearities and time delays [146]. Therefore, in this section initially detailed analysis, synthesis, and discussion related to the Quadratic Separation technique will be proposed. Then,

similarities between QS and IQC approaches are going to be investigated so that the relationship (if exists) between these two methodologies will be brought to the light.

Shortly, in Quadratic Separation approach the main objective is to combine fundamental operators such as time delay and integral within a framework as (possibly) a non-structured block, which depicts complex-valued uncertain matrix gain, and redefine the overall system like as an interconnection of the constructed block and rational matrices, see the block diagram proposed in Figure A.1. Once the system is expressed with two non-square matrices ($\mathcal{E}$ and $\mathscr{A}$) and proposed as interconnection in Figure A.1 then stability can be investigated with the following theorem.

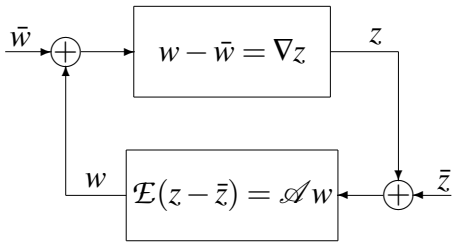

Figure A.1: General feedback interconnection depicted in quadratic separation approach.

Definition A.1. *Let $A \in \mathbb{R}^{n\times n}$ be a matrix, then $A^{\perp}$ is set of column vectors $x \in \mathbb{R}^{n\times 1}$ which satisfies $Ax = 0$.*

Theorem A.1 (Quadratic Separation [206])**.** *The uncertain feedback system proposed in Figure A.1 is well-posed if and only if there exist a Hermitian matrix $\Theta = \Theta^*$ satisfies the following inequalities;*

$$\begin{bmatrix} \mathcal{E} & -\mathscr{A} \end{bmatrix}^{\perp *} \Theta \begin{bmatrix} \mathcal{E} & -\mathscr{A} \end{bmatrix}^{\perp} > 0, \tag{A.1}$$

$$\begin{bmatrix} I \\ \nabla \end{bmatrix}^{*} \Theta \begin{bmatrix} I \\ \nabla \end{bmatrix} \leq 0, \quad \forall \nabla \in \boldsymbol{\nabla} \tag{A.2}$$

where $\mathscr{A}^{\perp}$ is a full rank matrix whose columns span the null space of $\mathscr{A}$.

The term *well-posed*, here, is different from the terminology that has been used in the absolute stability such that it implies boundedness.

Definition A.2 (Well-posedness in QS [201])**.** *The feedback system illustrated in Figure A.1 is said to be well-posed if for all uncertainties and all bounded input vectors, the internal vectors characterizing the system are unique and bounded.*

Note that the inequalities given in (A.1) and (A.2) can be re-expressed as a special case of the IQC theorem where the uncertainty holds only the pointwise inequality;

$$\begin{bmatrix} \mathscr{A} \\ I \end{bmatrix}^* \Pi \begin{bmatrix} \mathscr{A} \\ I \end{bmatrix} < 0,$$
$$\begin{bmatrix} I \\ \nabla \end{bmatrix}^* \Pi \begin{bmatrix} I \\ \nabla \end{bmatrix} \geq 0, \quad \forall \nabla \in \boldsymbol{\nabla}$$

if $\mathcal{E} = I$ and $\Pi = -\Theta$.

Kalman-Yakubovich-Popov (KYP) Lemma plays a crucial role to transform infinite dimensional frequency dependent inequality into finite dimensional linear matrix inequality (LMI) which tender tractable solution with semi-definite programs. But, QS does not require such a transformation as separating dynamics in the first place leads to a direct final LMI condition as in (A.1).

Generally, in both methodologies a number of inequalities are proposed to define the overall dynamic elements or the uncertainties. For instance, in QS the integral term $1/s$ is expressed with a separator based on the passivity property of the integral term. For any $P > 0$,

$$\begin{bmatrix} I \\ \frac{1}{s} \end{bmatrix}^* \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} I \\ \frac{1}{s} \end{bmatrix} \geq 0, \tag{A.3}$$

holds $\forall s \in \mathbb{C}_+$, the closed right half-plane. This particular separator allows us to connect first bridge between QS, Graph theory, and Lyapunov stability analysis. Let us assume that there exists a descriptor system[1] $\dot{x} = Ax$ that required to be analysed from the stability point of view. Based on the Lyapunov theory it can be concluded that the system is stable if there exists any $P > 0$ such that $A^\top P + PA < 0$ holds. Some conclusion can be derived once the system is transformed into the feedback interconnection illustrated in Figure A.2. Then, it is sufficient to check that the following inequality ensures the stability;

$$\begin{bmatrix} A \\ I \end{bmatrix}^* \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} A \\ I \end{bmatrix} < 0,$$

which is the same condition obtained with Lyapunov technique. Furthermore, based on graph separation [209,210] the following two inequalities can be used to validate

[1] A system modelled by differential and algebraic equations, see [207,208].

the stability;

$$\begin{aligned}
&\int_0^T \begin{bmatrix} (w+\bar{w}) \\ z \end{bmatrix}^* \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} (w+\bar{w}) \\ z \end{bmatrix} dt \geq 0, \\
&\int_0^T \begin{bmatrix} A(z+\bar{z}) \\ (z+\bar{z}) \end{bmatrix}^* \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} A(z+\bar{z}) \\ (z+\bar{z}) \end{bmatrix} dt < -\varepsilon I.
\end{aligned} \tag{A.4}$$

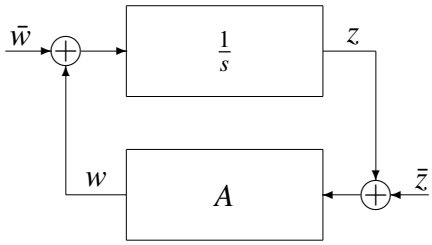

Figure A.2: Basic interconnection of a simple descriptor system as QS architecture.

The only condition that satisfies both inequalities in (A.4) simultaneously is the when internal signals are 0. Therefore, the system is stable if there exist any $P > 0$ such that conditions in (A.4) hold as the graphs of two blocks are separated. We must indicate that parameters in (A.4), such as $\bar{w}$ and $\bar{z}$, were assumed to be time dependent signals, yet in QS approach these parameters are assumed to be vectors in the frequency domain.

Further connections can be illustrated while analysing stability of the time delayed system as follows:

$$\dot{x}(t) = Ax(t) + A_d x(t - T_d), \tag{A.5}$$

where T_d is constant time delay which is unknown yet belong to the an interval $T_d \in [T_{dmin}, T_{dmax}]$. Initially, the proposed delayed system in (A.5) can be defined with two different block diagrams as illustrated in Figure A.3.

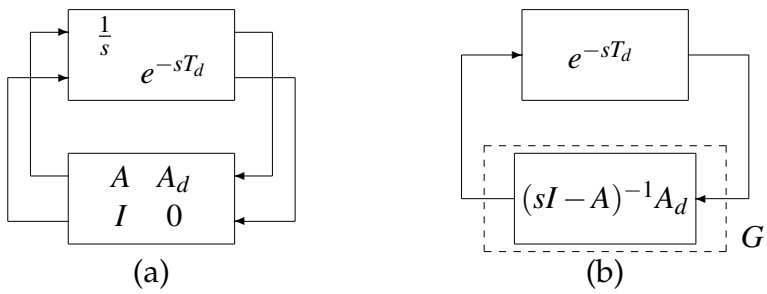

Figure A.3: Basic interconnection of a system with constant time delay; (a) as a QS architecture, (b) as a Lure system.

Separator/multiplier for the delay can basically be expressed with small gain conditions, as a result final inequalities present stability for the QS and IQC can be expressed respectively as follows:

$$\begin{bmatrix} A & A_d \\ I & 0 \\ I & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} 0 & 0 & P & 0 \\ 0 & Q & 0 & 0 \\ P & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q \end{bmatrix} \begin{bmatrix} A & A_d \\ I & 0 \\ I & 0 \\ 0 & I \end{bmatrix} < 0, \tag{A.6}$$

and

$$\begin{bmatrix} (sI-A)^{-1}A_d \\ I \end{bmatrix}^* \begin{bmatrix} Q & 0 \\ 0 & -Q \end{bmatrix} \begin{bmatrix} (sI-A)^{-1}A_d \\ I \end{bmatrix} < 0. \tag{A.7}$$

Based on the KYP lemma inequality in (A.7) can be converted into an LMI as

$$\begin{bmatrix} A^\top P + PA & PA_d \\ A_d^\top P & 0 \end{bmatrix} + \begin{bmatrix} Q & 0 \\ 0 & -Q \end{bmatrix} < 0. \tag{A.8}$$

These results provide confirmatory evidence that final LMIs obtained with two approaches; (A.6) and (A.8) are identical. The same conclusion can be derived if delay is time variable:

$$\dot{x}(t) = Ax(t) + A_d x(t - T_d(t)), \tag{A.9}$$

where $T_d(t)$ is variable time delay such that $T_d(t) \in [0, T_{dmax}]$ and $|\dot{T}_d(t)| \leq d < 1$. But, the proposed Theorem A.1 needs to be modified due to time variable delay operator such that ∇ will be depicted as a linear operator from $\mathcal{L}_{2e}$ to $\mathcal{L}_{2e}$ instead of complex-valued matrix gain. Thus, condition given in (A.2) is altered as

$$\left\langle \begin{bmatrix} I \\ \mathbb{P}_T \nabla \end{bmatrix} u_T, \Theta \begin{bmatrix} I \\ \mathbb{P}_T \nabla \end{bmatrix} u_T \right\rangle \leq 0, \quad \forall u \in \mathcal{L}_{2e}, \forall T > 0, \tag{A.10}$$

where inner product $\langle , \rangle$ can be defined as $\langle f, g \rangle = \int_0^\infty f^*(t) g(t) dt$ and $\mathbb{P}_T$ is the truncation operator. Then, final LMIs for QS and IQC approaches can be expressed as below,

$$\begin{bmatrix} A & A_d \\ I & 0 \\ I & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} 0 & 0 & P & 0 \\ 0 & Q & 0 & 0 \\ P & 0 & 0 & 0 \\ 0 & 0 & 0 & (d-1)Q \end{bmatrix} \begin{bmatrix} A & A_d \\ I & 0 \\ I & 0 \\ 0 & I \end{bmatrix} < 0, \tag{A.11}$$

$$\begin{bmatrix} (sI-A)^{-1}A_d \\ I \end{bmatrix}^* \begin{bmatrix} Q & 0 \\ 0 & (d-1)Q \end{bmatrix} \begin{bmatrix} (sI-A)^{-1}A_d \\ I \end{bmatrix} < 0.$$

Based on the KYP lemma the last inequality can be converted into an LMI as

$$\begin{bmatrix} A^\top P + PA & PA_d \\ A_d^\top P & 0 \end{bmatrix} + \begin{bmatrix} Q & 0 \\ 0 & (d-1)Q \end{bmatrix} < 0, \tag{A.12}$$

Once the same separator/multiplier is used for defining the uncertainties, as expected, both methodologies lead to the same final LMI conditions. But, stability conditions proposed for the time delayed systems are delay range independent which leads to conservative results. To reduce the conservatism, in QS approach ∇ block is extended either diagonally or non-diagonally with operators containing information related to the delay range. In a similar vein in IQC theory, more IQCs are defined for the perturbation with predefined transfer function that covers the delay gain over the full frequency range. This is the point where the two approaches are starting to dissociate.

Let us re-analyse the stability condition given for constant time delay case based on the mentioned extensions. So, to reduce the conservatism gab caused by the delay Ariba et al. in [211] proposed an additional operator along with a multiplier as follows:

$$\delta_0(s) = \int_{-T_d}^{0} e^{s\theta} d\theta = \frac{1-e^{-sT_d}}{s}, \quad \Pi_0 = \begin{bmatrix} T_d^2 & 0 \\ 0 & -1 \end{bmatrix}, \tag{A.13}$$

which is the similar operator used by Jun and Safonov in [125]: $\Delta = \frac{e^{-sT_d}-1}{sT_d}$. Then, system in (A.5) is depicted as the following interconnection.

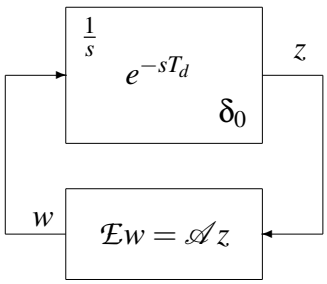

Figure A.4: Extended interconnection of constant time delayed system.

In other words, Figure A.3 (a) is extended with the proposed operator. And subsequent equation makes the connection between the interconnection in Figure A.4

and equation in (A.5);

$$\underbrace{\begin{bmatrix} I \\ 0 \end{bmatrix}}_{\mathcal{E}} \begin{bmatrix} \dot{x}(t) \\ x(t) \\ \dot{x}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A & A_d & 0 \\ I & 0 & 0 \\ A & A_d & 0 \\ I & -1 & -1 \end{bmatrix}}_{\mathscr{A}} \begin{bmatrix} x(t) \\ x_d(t) \\ x(t) - x_d(t) \end{bmatrix}, \tag{A.14}$$

where $x_d(t) = x(t - T_d)$. As a result, stability of the delayed system can be depicted based on the Theorem A.1 by using multipliers given in (A.3), (A.7), and (A.13) and given $\mathscr{A}$ and $\mathcal{E}$ matrices. In a similar fashion more operators can be defined and included into the expressed interconnection based on the so-called Bessel inequality. But, we have to answer the subsequent questions: How does adding extra operator (related to the uncertainty which is delay in this particular case) reduce the conservatism gap? Also, what is the function of the last row in $\mathscr{A}$ matrix given in (A.14)? For that purpose, let us explicitly write and analyse right outer factor of the quadratic term in (A.1) by using full expressions of $\mathcal{E}$ and $\mathscr{A}$ in (A.14);

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \mathcal{E} & -\mathscr{A} \end{bmatrix}^{\perp} \rightarrow \begin{bmatrix} I & 0 & 0 & -A & -A_d & 0 \\ 0 & I & 0 & -I & 0 & 0 \\ 0 & 0 & I & -A & -A_d & 0 \\ \hline 0 & 0 & 0 & -I & I & I \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = 0, \tag{A.15}$$

where null space vector, X, is equivalent to the combination of the inputs and outputs of the structured uncertainty proposed in Figure A.4, so

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \dot{x}(t) \\ x(t) \\ \dot{x}(t) \\ x(t) \\ x_d(t) \\ x(t) - x_d(t) \end{bmatrix}. \tag{A.16}$$

It can be concluded that, without considering the last row in $\mathscr{A}$; it is equivalent to say that the system is stable if there exists $P \leq 0$ such that $X^\top P X \leq 0$, holds $\forall X$ which

implies no constraints on the states and the delay itself. By considering the last row, however, namely adding property of $\delta_0(s)$: it can be depicted that the system is stable if there exists $P \leq 0$ and $X^\top PX \leq 0$ holds $\forall X : -x_4 + x_5 + x_6 = 0$. Including additional constraint on the states of the system simply reduces the aforementioned conservatism as restriction solely can be hold with a particular delay term: $-x(t) + x(t-T_d) + x(t) - x(t-T_d) = 0$.

$$\begin{bmatrix} x \\ x_d \end{bmatrix}^\top \begin{bmatrix} A^\top P + PA + A^\top T_d^2 A + Q - I & A^\top T_d^2 A_d + PA_d + I \\ A_d^\top P + A_d^\top T_d^2 A + I & A_d^\top T_d^2 A_d - Q - I \end{bmatrix} \begin{bmatrix} x \\ x_d \end{bmatrix} \leq 0. \tag{A.17}$$

Namely, what we imply; stability of the system in (A.5) depicted in Figure A.4 can be ensured if the given quadratic inequality in (A.17) holds. Note that multipliers given in (A.3), (A.7), and (A.13) and equalities in (A.15) and (A.16) are being used to derive the final condition in (A.17). As a final comment, Jun and Safonov in [125] concluded that their final LMI condition which was determined for a particular delay value ($\hat{T}_d$) holds for all delay $T_d \leq \hat{T}_d$ as proposed middle matrix in the quadratic inequality does not change its sign, i.e. always remains negative semi-defined. Such a remark can be indicated here as well: middle matrix in (A.17) can be expanded as $M_1 + \hat{T}_d^2 M_2^\top M_2 \leq 0$, where once the upper bound of the delay holds the inequality sufficient condition can be proposed that scaling down the delay value does not affect the sign of the inequality. Thus, once a stability condition is derived for a particular known delay based on the QS, then one can depict the derived result as a delay range stability condition for any delay $T_d \leq \hat{T}_d$.

In this effort, QS approach has been briefly proposed, also we have tried to bring into the light the relationship between QS and IQC methodologies by using non-complex systems and straightforward multipliers/separators. The consensus view seems to be listed as;

- Once the similar multipliers/separators are used both QS and IQC approach lead to the same final LMI condition.
- Relation between QS, Graph theory, Lyapunov analysis, and IQC can be exemplified with uncomplicated systems such as $\dot{x} = Ax$.

It needs to be mentioned that the big picture, however, is still missing and it might be illuminated with extra effort.

Bibliography

[1] S. J. Lederman and R. L. Klatzky, "Haptic perception: A tutorial," *Attention, perception & psychophysics*, vol. 71, no. 7, pp. 1439–1459, 2009. (Cited on page 18.)

[2] J. Marescaux, J. Leroy, M. Gagner, F. Rubino, D. Mutter, M. Vix, S. E. Butner, and M. K. Smith, "Transatlantic robot-assisted telesurgery," *Nature*, vol. 413, no. 6854, pp. 379–380, 2001. (Cited on pages 19 and 26.)

[3] J. De Geeter, M. Decréton, and E. Colon, "The challenges of telerobotics in a nuclear environment," *Robotics and Autonomous Systems*, vol. 28, no. 1, pp. 5–17, 1999. (Cited on page 19.)

[4] T. B. Sheridan, "Teleoperation, telerobotics and telepresence: A progress report," *Control Engineering Practice*, vol. 3, no. 2, pp. 205–214, 1995. (Cited on pages 19, 21, and 23.)

[5] A. A. Pilmanis and W. J. Sears, "Physiological hazards of flight at high altitude," *The Lancet*, vol. 362, 2003. (Cited on page 20.)

[6] G. Hirzinger, B. Brunner, J. Dietrich, and J. Heindl, "Sensor-based space robotics-ROTEX and its telerobotic features," *IEEE Transactions on Robotics and Automation*, vol. 9, no. 5, pp. 649–663, 1993. (Cited on page 21.)

[7] ——, "ROTEX-the first remotely controlled robot in space," in *IEEE International Conference on Robotics and Automation*, 1994, pp. 2604–2611. (Cited on page 21.)

[8] M. Oda, K. Kibe, and F. Yamagata, "ETS-VII, space robot in-orbit experiment satellite," in *IEEE International Conference on Robotics and Automation*, vol. 1, 1996, pp. 739–744. (Cited on page 21.)

[9] K. Landzettel, C. Preusche, A. Albu-Schaffer, D. Reintsema, D. Rebele, and G. Hirzinger, "Robotic on-orbit servicing - DLR's experience and perspective," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2006, pp. 4587–4594. (Cited on page 22.)

[10] R. D. Ballard and W. Hively, *The eternal darkness: a personal history of deep-sea exploration*. Princeton University Press, 2002. (Cited on page 22.)

[11] S. Q. Duntley, "Light in the sea," *JOSA*, vol. 53, no. 2, pp. 214–233, 1963. (Cited on page 23.)

[12] J. Yuh, "Design and control of autonomous underwater robots: A survey," *Autonomous Robots*, vol. 8, no. 1, pp. 7–24, 2000. (Cited on page 23.)

[13] T. Hirabayashi, J. Akizono, T. Yamamoto, H. Sakai, and H. Yano, "Teleoperation of construction machines with haptic information for underwater applications," *Automation in Construction*, vol. 15, no. 5, pp. 563–570, 2006. (Cited on page 23.)

[14] "World Nuclear Association," Date of access 2016-06-10. [Online]. Available: http://www.world-nuclear.org (Cited on page 24.)

[15] A. Hedin, "Spent nuclear fuel-how dangerous is it?" Swedish Nuclear Fuel and Waste Management Co., Stockholm, Tech. Rep., 1997. (Cited on page 24.)

[16] L. Cragg and H. Hu, "Application of mobile agents to robust teleoperation of internet robots in nuclear decommissioning," in *IEEE International Conference on Industrial Technology*, vol. 2, 2003, pp. 1214–1219. (Cited on page 25.)

[17] R. M. Satava, "Emerging technologies for surgery in the 21st century," *Archives of Surgery*, vol. 134, no. 11, pp. 1197–1202, 1999. (Cited on pages 25 and 26.)

[18] B. Hannaford, "Stability and performance tradeoffs in bi-lateral telemanipulation," in *IEEE International Conference on Robotics and Automation*, 1989, pp. 1764–1767. (Cited on page 27.)

[19] J.-H. Ryu, D.-S. Kwon, and B. Hannaford, "Stable teleoperation with time-domain passivity control," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 2, pp. 365–373, apr 2004. (Cited on pages 27 and 51.)

[20] E. Nuño, L. Basañez, and R. Ortega, "Passivity-based control for bilateral teleoperation: A tutorial," *Automatica*, vol. 47, no. 3, pp. 485–495, 2011. (Cited on page 27.)

[21] A. Villaverde, A. Blas, J. Carrasco, and A. Torrico, "Reset control for passive bilateral teleoperation," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 7, pp. 3037–3045, 2011. (Cited on page 27.)

[22] P. F. Hokayem and M. W. Spong, "Bilateral teleoperation: An historical survey," *Automatica*, vol. 42, no. 12, pp. 2035–2057, 2006. (Cited on pages 27 and 34.)

[23] C. A. Desoer and M. Vidyasagar, *Feedback systems: input-output properties*. SIAM, 1975. (Cited on pages 27, 38, and 60.)

[24] F. B. Llewellyn, "Some fundamental properties of transmission systems," in *IEEE Proceedings of the IRE*, vol. 40, no. 3, 1952, pp. 271–283. (Cited on pages 28 and 42.)

[25] M. Tavakoli, A. Aziminejad, R. V. Patel, and M. Moallem, "Enhanced transparency in haptics-based master-slave systems," in *American Control Conference, ACC '07*, 2007, pp. 1455–1460. (Cited on page 28.)

[26] A. Jafari and J.-H. Ryu, "Input-to-state stable approach to release the conservatism of passivity-based stable haptic interaction," in *IEEE International Conference on Robotics and Automation (ICRA)*, 2015, pp. 285–290. (Cited on page 28.)

[27] A. Haddadi and K. Hashtrudi-Zaad, "Bounded-impedance absolute stability of bilateral teleoperation control systems," *IEEE Transactions on Haptics*, vol. 3, no. 1, pp. 15–27, 2010. (Cited on page 28.)

[28] B. Willaert, B. Corteville, D. Reynaerts, H. Van Brussel, and E. B. V. Poorten, "Transparency trade-offs for a 3-channel controller revealed by the bounded environment passivity method," in *Third International Conference on Advances in Computer-Human Interactions ACHI '10.*, 2010, pp. 66–72. (Cited on pages 28 and 50.)

[29] I. Polat and C. W. Scherer, "Stability analysis for bilateral teleoperation: An IQC formulation," *IEEE Transactions on Robotics*, vol. 28, no. 6, pp. 1294–1308, 2012. (Cited on pages 28, 29, 52, 82, 83, and 119.)

[30] I. Polat, "Robustness analysis and controller synthesis for bilateral teleoperation systems via IQCs," Phd dissertation, TU Eindhoven, 2014. (Cited on page 28.)

[31] J. T. Wen, "Review of Passivity-based control and estimation in networked robotics, Takeshi Hatanaka, Nikhil Chopra, Masayuki Fujita, Mark W. Spong," *Automatica*, vol. 72, pp. 251–252, 2016. (Cited on page 28.)

[32] R. O'Shea, "An improved frequency time domain stability criterion for autonomous continuous systems," *IEEE Transactions on Automatic Control*, vol. 12, no. 6, pp. 725–731, 1967. (Cited on pages 28, 75, and 84.)

[33] G. Zames and P. L. Falb, "Stability conditions for systems with monotone and slope-restricted nonlinearities," *SIAM Journal on Control*, vol. 6, no. 1, pp. 89–108, 1968. (Cited on pages 28, 75, 76, 107, and 114.)

[34] M. Turner, M. Kerr, and I. Postlethwaite, "On the existence of multipliers for MIMO systems with repeated slope-restricted nonlinearities," in *ICCAS-SICE*. IEEE, 2009, pp. 1052 – 1057. (Cited on page 29.)

[35] D. Materassi and M. V. Salapaka, "A Generalized Zames-Falb Multiplier," *IEEE Transactions on Automatic Control*, vol. 56, no. 6, pp. 1432–1436, 2011. (Cited on page 29.)

[36] J. Carrasco, W. P. Heath, and A. Lanzon, "Equivalence between classes of multipliers for slope-restricted nonlinearities," *Automatica*, vol. 49, no. 6, pp. 1732–1740, 2013. (Cited on page 29.)

[37] J. Carrasco, M. Maya-Gonzalez, A. Lanzon, and W. P. Heath, "LMI searches for anticausal and noncausal rational Zames-Falb multipliers," *Systems & Control Letters*, vol. 70, pp. 17–22, 2014. (Cited on pages 29, 76, and 91.)

[38] J. Carrasco, M. C. Turner, and W. P. Heath, "Zames-Falb multipliers for absolute stability: From O'Shea's contribution to convex searches," *European Journal of Control*, vol. 28, pp. 1–19, 2016. (Cited on pages 29, 76, 84, and 91.)

[39] H. Morimitsu and S. Katsura, "Circle theorem-based realization of nonlinear force control for teleoperation under time delay," in *IEEE International Conference on Mechatronics (ICM)*, Nagoya, 2015, pp. 562–567. (Cited on pages 29 and 118.)

[40] R. Anderson and M. Spong, "Bilateral control of teleoperators with time delay," *IEEE Transactions on Automatic Control*, vol. 34, no. 5, pp. 494–501, 1989. (Cited on pages 29, 46, 52, and 53.)

[41] G. Niemeyer and J.-J. E. Slotine, "Stable adaptive teleoperation," *IEEE Journal of Oceanic Engineering*, vol. 16, no. 1, pp. 152–162, 1991. (Cited on pages 29, 52, 54, 56, and 57.)

[42] ——, "Using wave variables for system analysis and robot control," in *IEEE International Conference on Robotics and Automation*, vol. 2, 1997, pp. 1619–1625. (Cited on pages 29, 52, and 59.)

[43] A. Eusebi and C. Melchiorri, "Force reflecting telemanipulators with time-delay: Stability analysis and control design," *IEEE Transactions on Robotics and Automation*, vol. 14, no. 4, pp. 635–640, 1998. (Cited on pages 29 and 52.)

[44] N. Chopra, M. W. Spong, R. Ortega, and N. E. Barabanov, "On tracking performance in bilateral teleoperation," *IEEE Transactions on Robotics*, vol. 22, no. 4, pp. 861–866, aug 2006. (Cited on pages 29, 52, and 60.)

[45] D. Lee and M. W. Spong, "Passive bilateral teleoperation with constant time delay," *IEEE Transactions on Robotics*, vol. 22, no. 2, pp. 269–281, 2006. (Cited on pages 29, 61, and 122.)

[46] E. Nuño, R. Ortega, N. Barabanov, and L. Basañez, "A globally stable PD controller for bilateral teleoperators," *IEEE Transactions on Robotics*, vol. 24, no. 3, pp. 753–758, 2008. (Cited on pages 29, 61, and 122.)

[47] M. G. Spooner and C. H. Weaver, "An analysis and analogue-computer study of a force-reflecting positional servomechanism," *Transactions of the American Institute of Electrical Engineers, Part II: Applications and Industry*, vol. 74, no. 6, pp. 384–387, 1956. (Cited on page 34.)

[48] T. B. Sheridan, "Telerobotics," *Automatica*, vol. 25, no. 4, pp. 487–507, 1989. (Cited on page 34.)

[49] ——, "Space teleoperation through time delay: review and prognosis," *IEEE Transactions on Robotics and Automation*, vol. 9, no. 5, pp. 592–606, 1993. (Cited on page 34.)

[50] C. Passenberg, A. Peer, and M. Buss, "A survey of environment-, operator-, and task-adapted controllers for teleoperation systems," *Mechatronics*, vol. 20, no. 7, pp. 787–801, 2010. (Cited on pages 34 and 37.)

[51] R. W. Daniel and P. R. McAree, "Fundamental limits of performance for force reflecting teleoperation," *The International Journal of Robotics Research*, vol. 17, no. 8, pp. 811–830, 1998. (Cited on pages 35 and 46.)

[52] D. A. Lawrence, "Stability and transparency in bilateral teleoperation," *IEEE Transactions on Robotics and Automation*, vol. 9, no. 5, pp. 624–637, 1993. (Cited on pages 36 and 48.)

[53] I. Nisky, F. A. Mussa-Ivaldi, and A. Karniel, "Analytical study of perceptual and motor transparency in bilateral teleoperation," *IEEE Transactions on Human-Machine Systems*, vol. 43, no. 6, pp. 570–582, 2013. (Cited on pages 37 and 50.)

[54] J. E. Colgate and J. M. Brown, "Factors affecting the Z-Width of a haptic display," in *IEEE International Conference on Robotics and Automation*, 1994, pp. 3205–3210. (Cited on page 37.)

[55] D. W. Weir, J. E. Colgate, and M. A. Peshkin, "Measuring and increasing z-width with active electrical damping," in *IEEE Symposium on haptic interfaces for virtual environment and teleoperator systems*, 2008, pp. 169–175. (Cited on page 38.)

[56] N. Hogan, "On the stability of manipulators performing contact tasks," *IEEE Journal of Robotics and Automation*, vol. 4, no. 6, pp. 677–686, 1988. (Cited on page 38.)

[57] R. J. Adams and B. Hannaford, "Stable haptic interaction with virtual environments," *IEEE Transactions on Robotics and Automation*, vol. 15, no. 3, pp. 465–474, 1999. (Cited on page 38.)

[58] B. Anderson and S. Vongpanitlerd, *Network analysis and synthesis: a modern systems theory approach*. Prentice-Hall Englewood Cliffs, NJ, 1973. (Cited on pages 38, 41, and 122.)

[59] D. T. McRuer, "Pilot-induced oscillations and human dynamic behavior," National Aeronautics and Space Administration, Tech. Rep., 1995. (Cited on pages 39 and 117.)

[60] A. Peer, "Design and control of admittance-type telemanipulation systems," PhD dissertation, Technical University of Munich, 2008. (Cited on page 40.)

[61] C. Hatzfeld and A. T. Kern, *Engineering Haptic Devices A Beginner's Guide*. Springer, 2009. (Cited on page 42.)

[62] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Pearson Education, 2001. (Cited on page 42.)

[63] W. H. Ku, "Stability of linear active nonreciprocal N-ports," *Journal of the Franklin Institute*, vol. 276, no. 3, pp. 207–224, 1963. (Cited on page 43.)

[64] J. Li, M. Tavakoli, and Q. Huang, "Absolute stability of multi-dof multilateral haptic systems," *IEEE Transactions on Control Systems Technology*, vol. 22, no. 6, pp. 2319–2328, 2014. (Cited on page 45.)

[65] B. Hannaford and R. Anderson, "Experimental and simulation studies of hard contact in force reflecting teleoperation," in *IEEE International Conference on Robotics and Automation*, 1988, pp. 584–589. (Cited on pages 46 and 47.)

[66] K. B. Fite, L. Shao, and M. Goldfarb, "Loop shaping for transparency and stability robustness in bilateral telemanipulation," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 3, pp. 620–624, 2004. (Cited on page 46.)

[67] B. Willaert, B. Corteville, D. Reynaerts, H. Van Brussel, and E. B. Vander Poorten, "Bounded environment passivity of the classical position-force teleoperation controller," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2009, pp. 4622–4628. (Cited on page 46.)

[68] G. De Gersem, H. Van Brussel, and F. Tendick, "Reliable and enhanced stiffness perception in soft-tissue telemanipulation," *International Journal of Robotics Research*, vol. 24, no. 10, pp. 805–822, 2005. (Cited on page 47.)

[69] R. C. Goertz, R. A. Blomgren, J. H. Grimson, G. A. Forster, W. M. Thompson, and W. H. Kline, "The ANL model 3 master-slave electric manipulator–Its design and use in a cave," *Trans. Am. Nuclear Soc.*, vol. 4, no. 2, 1961. (Cited on page 47.)

[70] K. Hashtrudi-Zaad and S. E. Salcudean, "Analysis of control architectures for teleoperation systems with impedance/admittance master and slave manipulators," *The International Journal of Robotics Research*, vol. 20, no. 6, pp. 419–445, 2001. (Cited on pages 48, 49, and 50.)

[71] D. A. Lawrence, "Designing teleoperator architectures for transparency," in *IEEE International Conference on Robotics and Automation*, 1992, pp. 1406–1411. (Cited on page 48.)

[72] Y. Yokokohji and T. Yoshikawa, "Bilateral control of master-slave manupulators for ideal kinesthetic coupling," *IEEE Transactions on Robotics and Automation*, vol. 10, no. 5, pp. 605–620, 1994. (Cited on page 49.)

[73] K. J. Kuchenbecker, "Characterizing and controlling the high-frequency dynamics of haptic interfaces," Phd dissertation, Stanford University, 2006. (Cited on pages 50 and 62.)

[74] K. Hashtrudi-Zaad and S. E. Salcudean, "Transparency in time-delayed systems and the effect of local force feedback for transparent teleoperation," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 1, pp. 108–114, 2002. (Cited on page 50.)

[75] A. Sherman, M. C. Cavusoglu, and F. Tendick, "Comparison of teleoperator control architecture for palpation task," in *Proceedings of the ASME Dynamic Systems and Control Division, part of the ASME Internatioal Mechanical Engineering Congress and Exposition (IMECE 2000)*, 2000, pp. 1261–1268. (Cited on page 50.)

[76] A. Albakri, C. Liu, and P. Poignet, "Stability and performance analysis of three-channel teleoperation control architecture for medical application," in *IEEE International Conference on Intelligent Robots and Systems (IROS)*, Tokyo, 2013, pp. 456–462. (Cited on page 50.)

[77] B. Hannaford and J.-H. Ryu, "Time-domain passivity control of haptic interfaces," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 1, pp. 1–10, 2002. (Cited on pages 50 and 51.)

[78] J. H. Ryu, D. S. Kwon, and B. Hannaford, "Stability guaranteed control: Time domain passivity approach," *IEEE Transactions on Control Systems Technology*, vol. 12, no. 6, pp. 860–868, 2004. (Cited on page 51.)

[79] J. H. Ryu, J. Artigas, and C. Preusche, "A passive bilateral control scheme for a teleoperator with time-varying communication delay," *Mechatronics*, vol. 20, no. 7, pp. 812–823, 2010. (Cited on page 51.)

[80] W. R. Ferrell, "Delayed force feedback," *Human Factors: The Journal of the Human Factors and Ergonomics Society*, vol. 8, no. 5, pp. 449–455, 1966. (Cited on page 51.)

[81] J.-P. Richard, "Time-delay systems: an overview of some recent advances and open problems," *Automatica*, vol. 39, no. 10, pp. 1667–1694, 2003. (Cited on pages 51 and 76.)

[82] G. Niemeyer and J.-J. Slotine, "Designing force reflecting teleoperators with large time delays to appear as virtual tools," in *IEEE International Conference on Robotics and Automation*, vol. 3, 1997, pp. 2212–2218. (Cited on pages 52, 54, 59, and 60.)

[83] Y. Yokokohji, T. Imaida, and T. Yoshikawa, "Bilateral teleoperation under time-varying communication delay," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, vol. 3, 1999, pp. 1854–1859. (Cited on page 52.)

[84] N. J. D'Amore, "Wave impedance selection for passivity-based bilateral teleoperation," PhD dissertation, University of Maryland, 2016. (Cited on page 55.)

[85] G. Niemeyer, "Using wave variables in teleoperation time delayed force reflecting teleoperation," PhD dissertation, Massachusetts Institute of Technology, 1996. (Cited on page 57.)

[86] D. Sun, F. Naghdy, and H. Du, "Application of wave-variable control to bilateral teleoperation systems: A survey," *Annual Reviews in Control*, vol. 38, no. 1, pp. 12–31, 2014. (Cited on page 57.)

[87] Y.-C. Liu and N. Chopra, "Control of robotic manipulators under input/output communication delays: Theory and experiments," *IEEE Transactions on Robotics*, vol. 28, no. 3, pp. 742–751, 2012. (Cited on page 57.)

[88] R. Lozano, N. Chopra, and M. W. Spong, "Passivation of force reflecting bilateral teleoperators with time varying delay," in *Proceedings of the 8th Mechatronics Forum*, 2002, pp. 954–962. (Cited on pages 57 and 58.)

[89] K. Kosuge, H. Murayama, and K. Takeo, "Bilateral feedback control of telemanipulators via computer network," in *Proceedings of the 1996 IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS 96*, vol. 3, 1996, pp. 1380–1385. (Cited on page 59.)

[90] N. Chopra, M. W. Spong, R. Ortega, and N. E. Barabanov, "On position tracking in bilateral teleoperation," in *IEEE Proceedings of the American Control Conference*, vol. 6, 2004, pp. 5244–5249. (Cited on page 59.)

[91] T. Namerikawa and H. Kawada, "Symmetric impedance matched teleoperation with position tracking," in *45th IEEE Conference on Decision and Control*, 2006, pp. 4496–4501. (Cited on page 59.)

[92] N. Chopra, M. W. Spong, and R. Lozano, "Synchronization of bilateral teleoperators with time delay," *Automatica*, vol. 44, no. 8, pp. 2142–2148, 2008. (Cited on page 59.)

[93] D. Lee and M. W. Spong, "Passive bilateral control of teleoperators under constant time-delay," in *IFAC Proceedings*, vol. 38, no. 1, 2005, pp. 109–114. (Cited on page 61.)

[94] L. Ni and D. W. L. Wang, "A gain-switching control scheme for position-error-based bilateral teleoperation: Contact stability analysis and controller design," *The International Journal of Robotics Research*, vol. 23, no. 3, pp. 255–274, 2004. (Cited on page 62.)

[95] H. Kazerooni, T.-I. Tsay, and K. Hollerbach, "A controller design framework for telerobotic systems," *IEEE Transactions on Control Systems Technology*, vol. 1, no. 1, pp. 50–62, 1993. (Cited on page 63.)

[96] G. M. H. Leung, B. A. Francis, and J. Apkarian, "Bilateral controller for teleoperators with time delay via mu synthesis," *IEEE Transactions on Robotics and Automation*, vol. 11, no. 1, pp. 105–116, 1995. (Cited on page 63.)

[97] J. Sheng and M. W. Spong, "Model predictive control for bilateral teleoperation systems with time delays," in *IEEE Canadian Conference on Electrical and Computer Engineering*, vol. 4, 2004, pp. 1877–1880. (Cited on page 63.)

[98] S. Sirouspour and A. Shahdi, "Model predictive control for transparent teleoperation under communication time delay," *IEEE Transactions on Robotics*, vol. 22, no. 6, pp. 1131–1145, 2006. (Cited on page 63.)

[99] T. Slama, A. Trevisani, D. Aubry, R. Oboe, and F. Kratz, "Experimental analysis of an internet-based bilateral teleoperation system with motion and force scaling using a model predictive controller," *IEEE Transactions on Industrial Electronics*, vol. 55, no. 9, pp. 3290–3299, 2008. (Cited on page 64.)

[100] N. Chopra and M. W. Spong, "Adaptive coordination control of bilateral teleoperators with time delay," in *IEEE Conference on Decision and Control , CDC*, vol. 5, 2004, pp. 4540–4547. (Cited on page 64.)

[101] L. Chan, F. Naghdy, and D. Stirling, "Application of adaptive controllers in teleoperation systems: A survey," *IEEE Transactions on Human-Machine Systems*, vol. 44, no. 3, pp. 337–352, 2014. (Cited on page 64.)

[102] A. Jazayeri and M. Tavakoli, "Absolute stability analysis of sampled-data scaled bilateral teleoperation systems," *Control Engineering Practice*, vol. 21, no. 8, pp. 1053–1064, 2013. (Cited on page 64.)

[103] A. Jazayeri, "Stability of bilateral teleoperation systems: Effect of sampled-data control and non-passivity or strict-passivity of terminations," PhD dissertation, University of Alberta, 2013. (Cited on page 64.)

[104] N. Young, *An introduction to Hilbert space*. Cambridge University Press, 1988. (Cited on page 65.)

[105] U. Jönsson, "Lecture notes on integral quadratic constraints," Department of Mathematics, KTH, Stockholm, Tech. Rep., 2001. (Cited on pages 65 and 70.)

[106] ——, "Robustness analysis of uncertain and nonlinear systems," PhD dissertation, Lund Institute of Technology, 1996. (Cited on pages 66 and 69.)

[107] J. C. Willems, *The Analysis of Feedback Systems*. The MIT press, 1971. (Cited on pages 67 and 69.)

[108] C. J. Harris and J. M. A. Valenca, *The Stability of Input-Output Dynamical Systems*. Elsevier, 1983. (Cited on page 67.)

[109] B. Brogliato, R. Lozano, B. Maschke, and O. Egeland, *Dissipative systems analysis and control: theory and applications*. Springer, 2006. (Cited on page 67.)

[110] J. Carrasco, W. P. Heath, and A. Lanzon, "Factorization of multipliers in passivity and IQC analysis," *Automatica*, vol. 48, no. 5, pp. 909–916, 2012. (Cited on pages 68 and 72.)

[111] A. Rantzer and A. Megretski, "Stability criteria based on integral quadratic constraints," in *Proceedings of the 35th IEEE Conference on Decision and Control*, vol. 1, 1996, pp. 215–220. (Cited on page 70.)

[112] A. Megretski and A. Rantzer, "System analysis via integral quadratic constraints," *IEEE Transactions on Automatic Control*, vol. 42, no. 6, pp. 819–830, 1997. (Cited on pages 70, 71, and 79.)

[113] M. Fu, S. Dasgupta, and C. S. Yeng, "Integral quadratic constraint approach vs. multiplier approach," in *7th International Conference on Control, Automation, Robotics and Vision*, vol. 1, 2002, pp. 144–149. (Cited on pages 71 and 72.)

[114] M. Fu, S. Dasgupta, and Y. C. Soh, "Integral quadratic constraint approach vs. multiplier approach," *Automatica*, vol. 41, no. 2, pp. 281–287, 2005. (Cited on page 71.)

[115] R. H. Gielen and M. Lazar, "Non-conservative dissipativity and small-gain conditions for stability analysis of interconnected systems," in *51st IEEE Conference on Decision and Control (CDC)*, 2012, pp. 4187–4192. (Cited on page 72.)

[116] A. Ohara, K. Matsumoto, and M. Ide, "Robust controller synthesis for plants satisfying a certain class of IQC," in *American Control Conference*, vol. 5, no. 3. IEEE, 2001, pp. 25–27. (Cited on page 73.)

[117] A. Rantzer, "On the Kalman Yakubovich Popov lemma," *Systems & Control Letters*, vol. 28, no. 1, pp. 7–10, 1996. (Cited on page 74.)

[118] T. Iwasaki and S. Hara, "Generalized KYP lemma: Unified frequency domain inequalities with design applications," *IEEE Transactions on Automatic Control*, vol. 50, no. 1, pp. 41–59, 2005. (Cited on page 74.)

[119] E. Feron, "Analysis of robust H_2 performance using multiplier theory," *SIAM Journal on Control and Optimization*, vol. 35, no. 1, pp. 160–177, 1997. (Cited on page 75.)

[120] J. Carrasco, M. C. Turner, and W. P. Heath, "Zames-Falb multipliers for absolute stability : from O'Shea's contribution to convex searches," in *European Control Conference, ECC15*, Linz, 2015, pp. 1261–1278. (Cited on page 76.)

[121] Y. N. Kyrychko and S. J. Hogan, "On the use of delay equations in engineering applications," *Journal of Vibration and Control*, vol. 16, no. 7-8, pp. 943–960, 2010. (Cited on page 76.)

[122] K. Gu and S.-I. Niculescu, "Survey on recent results in the stability and control of time-delay systems," *Journal of Dynamic Systems, Measurement, and Control*, vol. 125, no. 2, pp. 158–165, 2003. (Cited on page 76.)

[123] M. Fu, H. Li, and S. Niculescu, "Robust stability and stabilization of time-delay systems via integral quadratic constraint approach," in *Stability and control of time-delay systems*. Springer Berlin Heidelberg, 1998, pp. 101–116. (Cited on pages 76 and 93.)

[124] M. Jun and M. G. Safonov, "Stability analysis of a system with time-delayed states," in *Proceedings of the American Control Conference*, vol. 2, 2000, pp. 949–952. (Cited on pages 76 and 93.)

[125] ——, "IQC robustness analysis for time-delay systems," *International Journal of Robust and Nonlinear Control*, vol. 11, no. 15, pp. 1455–1468, 2001. (Cited on pages 76, 93, 144, and 146.)

[126] ——, "Rational multiplier IQCs for uncertain time-delays and LMI stability conditions," *IEEE Transactions on Automatic Control*, vol. 47, no. 11, pp. 1871–1875, 2002. (Cited on pages 76 and 93.)

[127] C.-Y. Kao and A. Rantzer, "Stability analysis of systems with uncertain time-varying delays," *Automatica*, vol. 43, no. 6, pp. 959–970, 2007. (Cited on pages 76, 80, and 82.)

[128] H. Pfifer and P. Seiler, "An overview of integral quadratic constraints for delayed nonlinear and parameter-varying systems," *arXiv:1504.02502*, 2015. (Cited on page 77.)

[129] ——, "Integral quadratic constraints for delayed nonlinear and parameter-varying systems," *Automatica*, vol. 56, no. 6, pp. 36–43, 2015. (Cited on pages 78 and 79.)

[130] M. Jun and M. G. Safonov, "Multiplier IQCs for uncertain time delays," *Journal of the Franklin Institute*, vol. 338, no. 2, pp. 335–351, 2001. (Cited on page 79.)

[131] C.-Y. Kao and A. Rantzer, "Stability criteria for systems with bounded uncertain time-varying delay," in *European Control Conference, ECC*, 2003, pp. 1525–1530. (Cited on page 80.)

[132] ——, "Robust stability analysis of linear systems with time-varying delays," in *IFAC Proceedings Volumes 38.1*, 2005, pp. 173–178. (Cited on page 80.)

[133] X. Chen and J. T. Wen, "Robustness analysis for linear time-invariant systems with structured incrementally sector bounded feedback nonlinearities," *Applied Mathematics and Computer Science*, vol. 6, no. 4, pp. 625–648, 1996. (Cited on pages 83, 84, and 139.)

[134] J. Veenman, C. W. Scherer, and H. Köroglu, "Robust stability and performance analysis based on integral quadratic constraints," *European Journal of Control*, vol. 31, no. 9, pp. 1–32, 2016. (Cited on pages 83, 84, and 139.)

[135] H. Kwakernaak and R. Sivan, *Modern Signals and Systems*. Prentice Hall, 1991. (Cited on page 83.)

[136] J. Veenman and C. W. Scherer, "IQC synthesis with general dynamic multipliers," *International Journal of Robust and Nonlinear Control*, vol. 24, no. 17, pp. 3027–3056, 2014. (Cited on page 84.)

[137] C.-Y. Kao, A. Megretski, U. T. Jönsson, and A. Rantzer, "A MATLAB toolbox for robustness analysis," in *International Symposium on Computer Aided Control Systems Design*. IEEE, 2004, pp. 297–302. (Cited on pages 84 and 139.)

[138] E. Fridman, "Tutorial on Lyapunov-based methods for time-delay systems," *European Journal of Control*, vol. 20, no. 6, pp. 271–283, 2014. (Cited on page 91.)

[139] A. Seuret, F. Gouaisbaut, and L. Baudouin, "D1.1 Overview of Lyapunov methods for time-delay systems," PhD dissertation, LAAS-CNRS, 2016. (Cited on page 91.)

[140] J. Lofberg, "YALMIP: A toolbox for modeling and optimization in MATLAB," in *IEEE International Symposium on Computer Aided Control Systems Design*, Taipei, 2004, pp. 284–289. (Cited on page 92.)

[141] S.-I. Niculescu, "On the stability and stabilization of linear systems with delayed state (in French)," Ph.D. dissertation, Grenoble Institute of Technology, 1996. (Cited on page 93.)

[142] Q. L. Han, "Absolute stability of time-delay systems with sector-bounded nonlinearity," *Automatica*, vol. 41, no. 12, pp. 2171–2176, 2005. (Cited on pages 94 and 95.)

[143] M. Wu, Z. Y. Feng, and Y. He, "Improved delay-dependent absolute stability of Lur'e systems with time-delay," *International Journal of Control, Automation and Systems*, vol. 7, no. 6, pp. 1009–1014, 2009. (Cited on pages 94 and 95.)

[144] A. Kazemy and M. Farrokhi, "Robust absolute stability analysis of multiple time-delay Lure systems with parametric uncertainties," *Asian Journal of Control*, vol. 15, no. 1, pp. 203–213, 2013. (Cited on pages 94 and 95.)

[145] H. Zeng, S. Xiao, B. Qin, and H. Xiao, "Improved conditions for absolute stability of Lur'e systems with time-delay," in *IEEE Third International Conference on Intelligent System Design and Engineering Applications*, 2013, pp. 862–865. (Cited on pages 94 and 95.)

[146] M. Zheng, Z. Mao, K. Li, and M. Fei, "Quadratic separation framework for stability analysis of a class of systems with time delays," *Neurocomputing*, vol. 174, no. 1, pp. 466–474, 2016. (Cited on pages 97, 99, and 139.)

[147] J. J. Hopfield, "Neural networks and physical systems with emergent collective computational abilities," in *Proceedings of the national academy of sciences*, vol. 79, no. 8, 1982, pp. 2554–2558. (Cited on page 97.)

[148] S. Mou, H. Gao, J. Lam, and W. Qiang, "A new criterion of delay-dependent asymptotic stability for Hopfield neural networks with time delay," *IEEE Transaction of Neural Networks*, vol. 19, no. 3, pp. 532–5, 2008. (Cited on pages 97 and 99.)

[149] X. Lou and B. Cui, "New LMI conditions for delay-dependent asymptotic stability of delayed Hopfield neural networks," *Neurocomputing*, vol. 70, no. 13-15, pp. 2566–2571, 2007. (Cited on page 99.)

[150] X. M. Zhang and Q. L. Han, "New Lyapunov-Krasovskii functionals for global asymptotic stability of delayed neural networks," *IEEE Transactions on Neural Networks*, vol. 20, no. 3, pp. 533–539, 2009. (Cited on page 101.)

[151] H. Zhang, S. Member, Z. Liu, and G.-b. Huang, "Novel weighting-delay-based stability criteria for recurrent neural networks with time-varying delay," *IEEE Transaction of Neural Networks*, vol. 21, no. 1, pp. 91–106, 2010. (Cited on page 101.)

[152] Z. Wang, L. Liu, Q. H. Shan, and H. Zhang, "Stability criteria for recurrent neural networks with time-varying delay based on secondary delay partitioning method," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 26, no. 10, pp. 2589–2595, 2015. (Cited on page 101.)

[153] Y. Shen, L. Zhang, and Y. Zhang, "New globally asymptotic stability criteria for delayed cellular neural networks," *IEEE Transtactions on Circuits and Systems-II: Express Briefs*, vol. 5551, no. 8, pp. 347–356, 2009. (Cited on page 101.)

[154] H.-B. Zeng, Y. He, M. Wu, and C.-F. Zhang, "Complete delay-decomposing approach to asymptotic stability for neural networks with time-varying delays," *IEEE Transactions on Neural Networks*, vol. 22, no. 5, pp. 806–812, 2011. (Cited on page 101.)

[155] C. Ge, C. Hua, and X. Guan, "New delay-dependent stability criteria for neural networks with time-varying delay using delay-decomposition approach," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 25, no. 7, pp. 1378–1384, 2014. (Cited on page 101.)

[156] T. H. Lee, J. H. Park, M.-J. Park, O.-M. Kwon, and H.-Y. Jung, "On stability criteria for neural networks with time-varying delay using Wirtinger-based multiple integral inequality," *Journal of the Franklin Institute*, vol. 352, no. 12, pp. 5627–5645, 2015. (Cited on page 101.)

[157] H.-B. Zeng, Y. He, M. Wu, and S.-P. Xiao, "Stability analysis of generalized neural networks with time-varying delays via a new integral inequality," *Neurocomputing*, vol. 161, no. 8, pp. 148–154, 2015. (Cited on page 101.)

[158] C. Hua, C. Long, and X. Guan, "New results on stability analysis of neural networks with time-varying delays," *Physics Letters A*, vol. 352, no. 4, pp. 335–340, 2006. (Cited on page 101.)

[159] Y. He, G. P. Liu, D. Rees, and M. Wu, "Stability analysis for neural networks with time-varying interval delay," *IEEE Transactions on Neural Networks*, vol. 18, no. 6, pp. 1850–1854, 2007. (Cited on page 101.)

[160] A. I. Lur'e, *Some nonlinear problems in the theory of automatic control*. Her Majesty's stationery office, 1957. (Cited on page 66.)

[161] M. Vidyasagar, *Nonlinear Systems Analysis*. London: Prentice-Hall International Editions, 1993. (Cited on pages 102 and 104.)

[162] V. Popov, "Absolute stability of nonlinear systems of automatic control," pp. 857–875, 1962. (Cited on page 102.)

[163] M. A. Aizerman, "On a problem related to global stability of dynamic systems (in Russian)," *Uspehi Mat. Nauk*, vol. 4, pp. 187–188, 1949. (Cited on page 102.)

[164] R. Kalman, "Physical and mathematical mechanisms of instability in nonlinear automatic control systems," *Transactions of ASME*, vol. 79, no. 3, pp. 553–566, 1957. (Cited on pages 102 and 103.)

[165] R. E. Fitts, "Two counterexamples to Aizerman's conjecture," *IEEE Transactions on Automatic Control*, vol. 11, no. 3, pp. 553–556, 1966. (Cited on pages 103 and 104.)

[166] V. A. Pliss, "Certain problems of the theory of global stability of motion (in Russian)," Ph.D. dissertation, Leningrad State University, 1958. (Cited on page 103.)

[167] N. E. Barabanov, "On the Kalman problem," *Siberian Mathematical Journal*, vol. 29, no. 3, pp. 333–341, 1989. (Cited on pages 103 and 104.)

[168] J. Carrasco, W. P. Heath, and M. De La Sen, "Second-order counterexample to the discrete-time Kalman conjecture," in *IEEE European Control Conference, ECC*, 2015, pp. 981–985. (Cited on page 103.)

[169] V. Rasvan, "Delay-independent and delay-dependent Aizerman problem," in *Unsolved Problems in Mathematical Systems and Control Theory*, 2004, ch. 6.6, pp. 212–220. (Cited on pages 104 and 105.)

[170] D. A. Altshuller, "A partial solution of the Aizerman problem for second-order systems with delays," *IEEE Transactions on Automatic Control*, vol. 53, no. 9, pp. 2158–2160, 2008. (Cited on pages 104, 106, and 113.)

[171] A. Tesi, A. Vicino, and G. Zappa, "Clockwise property of the Nyquist plot with implications for absolute stability," *Automatica*, vol. 28, no. 1, pp. 71–80, 1992. (Cited on pages 104, 108, 109, 113, and 114.)

[172] J. C. Hamann and B. R. Barmish, "Convexity of frequency response arcs associated with a stable polynomial," *IEEE Transactions on Automatic Control*, vol. 38, no. 6, pp. 904–915, 1993. (Cited on pages 104 and 108.)

[173] K. Gu, "Comments on "Convexity of frequency response arcs associated with a stable polynomial"," *IEEE Transactions on Automatic Control*, vol. 39, no. 11, pp. 2262–2265, 1994. (Cited on pages 104 and 108.)

[174] G. Vinnicombe, "On the stability of networks operating tcp-like congestion control," in *IFAC Proceedings Volumes*, vol. 35, no. 1, 2002, pp. 217–222. (Cited on page 104.)

[175] Y. P. Tian and H. Y. Yang, "Stability of the Internet congestion control with diverse delays," *Automatica*, vol. 40, no. 9, pp. 1533–1541, 2004. (Cited on page 104.)

[176] D. A. Altshuller, "Three classes of time-delay systems for which the Aizerman conjecture is true," *Contributions to Unsolved Problems in Math. Systems & Contr. Theory*, 2009. (Cited on page 106.)

[177] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Society for industrial and applied mathematics, 1994. (Cited on page 107.)

[178] R. Brockett and J. Willems, "Frequency domain stability criteria–part I," *IEEE Transactions on Automatic Control*, vol. 10, no. 3, pp. 255–261, 1965. (Cited on page 107.)

[179] Y.-S. Cho and K. Narendra, "An off-axis circle criterion for stability of feedback systems with a monotonic nonlinearity," *IEEE Transactions on Automatic Control*, vol. 13, no. 4, pp. 413–416, 1968. (Cited on pages 107 and 108.)

[180] J. Carrasco, M. C. Turner, and W. P. Heath, "Zames-Falb multipliers for absolute stability: from O'Shea's contribution to convex searches," *European Journal of Control*, vol. 28, no. 3, pp. 1–19, 2016. (Cited on pages 107, 108, and 114.)

[181] I. Horowitz and S. Ben-Adam, "Clockwise nature of Nyquist locus of stable transfer functions," *International Journal of Control*, vol. 49, no. 4, pp. 1433–1436, 1989. (Cited on page 112.)

[182] A. C. Bartlett, "Correspondence Counter-example to "Clockwise nature of Nyquist locus of stable transfer functions"," *International Journal of Control*, vol. 51, no. 6, pp. 1479–1483, 1990. (Cited on page 112.)

[183] J. M. Maciejowski, *Multivariable Feedback Design*. Electronic Systems Engineering Series, 1989. (Cited on page 113.)

[184] M. G. Safonov and G. Wyetzner, "Computer-aided stability analysis renders Popov criterion obsolete," *IEEE Transactions on Automatic Control*, vol. 32, no. 12, pp. 1128–1131, 1987. (Cited on page 114.)

[185] M. Chang, R. Mancera, and M. Safonov, "Computation of Zames-Falb multipliers revisited," *IEEE Transactions on Automatic Control*, vol. 57, no. 4, pp. 1024–1029, apr 2012. (Cited on page 114.)

[186] H. Tugal, J. Carrasco, and M. Maya-Gonzalez, "Teleoperation with memoryless, monotone, and bounded environments: A Zames-Falb multiplier approach," in *European Control Conference, ECC15*, 2015, pp. 1968–1973. (Cited on page 116.)

[187] H. Tugal, J. Carrasco, P. Falcon, and A. Barreiro, "Stability analysis of bilateral teleoperation with bounded and monotone environments via Zames-Falb multipliers," *IEEE Transactions of Control System Technology*, 2016. (Cited on page 116.)

[188] H. Tugal and J. Carrasco, "Bilateral teleoperation with nonlinear environments: multiplier approach," in *4th IFAC Symposium on Telematics Application*, Porto Alegre, 2016, pp. 308–313. (Cited on page 116.)

[189] R. Adams and B. Hannaford, "Control law design for haptic interfaces to virtual reality," *IEEE Transactions on Control Systems Technology*, vol. 10, no. 1, pp. 3 – 13, 2002. (Cited on page 117.)

[190] J. M. Dolan, M. B. Friedman, and M. L. Nagurka, "Dynamic and loaded impedance components in the maintenance of human arm posture," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 23, no. 3, pp. 698–709, 1993. (Cited on page 117.)

[191] M. S. Erden and A. Billard, "Hand impedance measurements during interactive manual welding with a robot," *IEEE Transactions on Robotics*, vol. 31, no. 1, pp. 168–179, 2015. (Cited on page 117.)

[192] N. Hogan, "Controlling impedance at the man/machine interface," in *IEEE International Conference on Robotics and Automation (ICRA)*, vol. 3, 1989, pp. 1626–1631. (Cited on page 117.)

[193] A. Haddadi, K. Razi, and K. Hashtrudi-Zaad, "Operator dynamics consideration for less conservative coupled stability condition in bilateral teleoperation," *IEEE/ASME Transactions on Mechatronics*, vol. 20, no. 5, pp. 2463–2475, 2015. (Cited on page 117.)

[194] M. Dyck, A. Jazayeri, and M. Tavakoli, "Is the human operator in a teleoperation system passive?" in *World Haptics Conference (WHC)*, 2013, pp. 683–688. (Cited on page 117.)

[195] I. Polat, "A note on passivity based stability conditions for bilateral teleoperation," in *Proceedings of 18th IFAC World Congress*, vol. 18, Milano, 2011, pp. 320–325. (Cited on page 119.)

[196] B. Willaert, B. Corteville, D. Reynaerts, H. Van Brussel, and E. B. Vander Poorten, "A mechatronic analysis of the classical position-force controller based on bounded environment passivity," *The International Journal of Robotics Research*, vol. 30, no. 4, pp. 444–462, 2010. (Cited on pages 122 and 124.)

[197] D. E. Rivera, M. Morari, and S. Skogestad, "Internal model control: PID controller design," *Industrial & engineering chemistry process design and development*, vol. 25, no. 1, pp. 252–265, 1986. (Cited on page 122.)

[198] E. Delgado, A. Barreiro, P. Falcon, and M. Diaz-Cacho, "Robust stability of scaled-four-channel teleoperation with internet time-varying delays," *Sensors*, vol. 16, no. 5, 2016. (Cited on page 127.)

[199] S. Byford, "The Verge," Date of access 2015-08-17. [Online]. Available: www.theverge.com (Cited on page 128.)

[200] B. Willaert, D. Reynaerts, H. Van Brussel, and E. B. Vander Poorten, "Bilateral teleoperation: quantifying the requirements for and restrictions of ideal transparency," *IEEE Transactions on Control Systems Technology*, vol. 22, no. 1, pp. 387–395, 2014. (Cited on page 132.)

[201] D. Peaucelle, D. Henrion, and D. Arzelier, "Quadratic separation for feedback connection of an uncertain matrix and an implicit linear transformation," *Automatica*, vol. 43, no. 5, pp. 795–804, 2007. (Cited on pages 139 and 140.)

[202] Y. Ariba, F. Gouaisbaut, and D. Peaucelle, "Stability analysis of time-varying delay systems in quadratic separation framework," in *ICNPAA-2008: Mathematical Problems in Engineering*. Aerospace and Sciences, 2008. (Cited on page 139.)

[203] Y. Ariba and F. Gouaisbaut, "Input-output framework for robust stability of time-varying delay systems," in *Proceedings of the 48h IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference*, 2009, pp. 274–279. (Cited on page 139.)

[204] Y. Ariba, F. Gouaisbaut, and K. H. Johansson, "Stability interval for time-varying delay systems," in *Proceedings of the IEEE Conference on Decision and Control (CDC)*, no. 2, 2010, pp. 1017–1022. (Cited on page 139.)

[205] ——, "Robust stability of time-varying delay systems: The quadratic separation approach," *Asian Journal of Control*, vol. 14, no. 5, pp. 1205–1214, 2012. (Cited on page 139.)

[206] T. Iwasaki and S. Hara, "Well-posedness of feedback systems: Insights into exact robustness analysis and approximate computations," *IEEE Transactions on Automatic Control*, vol. 43, no. 5, pp. 619–630, 1998. (Cited on page 140.)

[207] D. Luenberger, "Dynamic equations in descriptor form," *IEEE Transactions on Automatic Control*, vol. 22, no. 3, pp. 312–321, 1977. (Cited on page 141.)

[208] P. C. Müller, "Descriptor systems: pros and cons of system modelling by differential-algebraic equations," *Mathematics and Computers in Simulation*, vol. 53, no. 4-6, pp. 273–279, 2000. (Cited on page 141.)

[209] M. G. Safonov, *Stability and robustness of multivariable feedback systems*. MIT press, 1980. (Cited on page 141.)

[210] J. Carrasco and P. Seiler, "Integral quadratic constraint theorem: A topological separation approach," in *54th IEEE Conference on Decision and Control (CDC)*, 2015, pp. 5701–5706. (Cited on page 141.)

[211] Y. Ariba and F. Gouaisbaut, "Stability analysis of time-delay systems via bessel inequality : a quadratic separation approach," *Elsevier in press*, 2016. (Cited on page 144.)