

A probabilistic approach to the concept of Probable Maximum Precipitation

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Abstract. The concept of Probable Maximum Precipitation (PMP) is based on the assumptions that (a) there exists an upper physical limit of the precipitation depth over a given area at a particular geographical location at a certain time of year, and (b) that this limit can be estimated based on deterministic considerations. The most representative and widespread estimation method of PMP is the so-called moisture maximization method. This method maximizes observed storms assuming that the atmospheric moisture would hypothetically rise up to a high value that is regarded as an upper limit and is estimated from historical records of dew points. In this paper, it is argued that fundamental aspects of the method may be flawed or inconsistent. Furthermore, historical time series of dew points and “constructed” time series of maximized precipitation depths (according to the moisture maximization method) are analyzed. The analyses do not provide any evidence of an upper bound either in atmospheric moisture or maximized precipitation depth. Therefore, it is argued that a probabilistic approach is more consistent to the natural behaviour and provides better grounds for estimating extreme precipitation values for design purposes.

1 Introduction

The Probable Maximum Precipitation (PMP) is defined as “theoretically the greatest depth of precipitation for a given duration that is physically possible over a given size storm area at a particular geographical location at a certain time of year” (WMO, 1986). Even though, the PMP approach has been widely proposed and used as design criterion of major flood protection works (Schreiner and Reidel, 1978; Collier and Hardaker, 1996), severe criticism has been made by hydrologists not only to the concept of the PMP, which practically assumes a physical upper bound of precipitation amount, but also to the fact that this limit can be estimated

based on deterministic considerations (Benson, 1973; Kite, 1988; Dingman, 1994; Shaw, 1994; Koutsoyiannis, 1999).

The main scope this paper is to apply the PMP estimation method and to make a probabilistic analysis of its results. An additional objective was to find an appropriate probabilistic model capable of describing the empirical distribution of the monthly maximum daily dew points, in order to be used in the application of the method. Last, but not least, an exclusively probabilistic approach was applied to the annual maximum rainfall depths.

The methodology was applied to four stations in Netherlands (De Bilt, Den Helder, Groningen, Maastricht) and the station of the National Observatory of Athens, Greece (NOA).

2 Method overview

Techniques used for estimating PMP have been listed by Wiesner (1970), as follows: (1) the storm model approach; (2) the maximisation and transposition of actual storms; (3) the use of generalised data or maximised depth, duration and area data from storms; these are derived from thunderstorms or general storms; (4) the use of empirical formulae determined from maximum depth duration and area data, or from theory; (5) the use of empirical relationships between the variables in particular valleys (only if detailed data are available); (6) statistical analyses of extreme rainfalls.

In this study, the most representative and widely used estimation method of PMP, the so-called moisture maximization method, is examined. The method is based on the simple formula

$$h_m = \frac{W_m}{W} h, \quad (1)$$

where h_m is the maximized rainfall depth, h is the observed precipitation, W is the precipitable water in the atmosphere during the day of rain, estimated by the corresponding daily dew point T_d , and W_m is the maximized precipitable water,

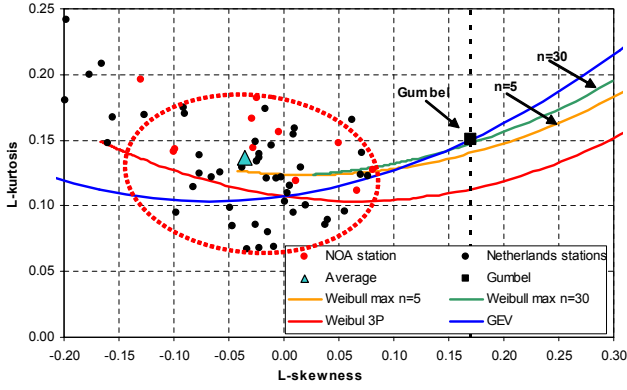


Fig. 1. L-moment ratio diagram of maximum daily dew points for each month.

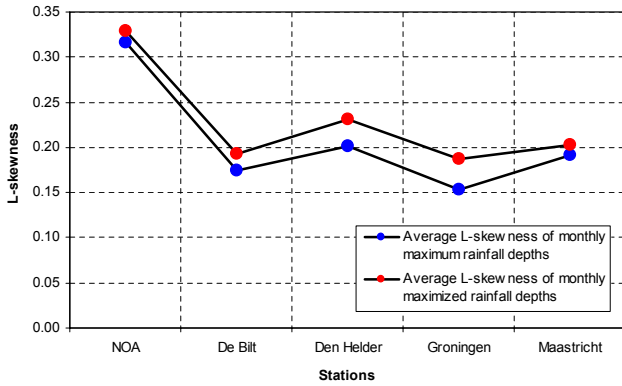


Fig. 2. Average L-skewness of monthly maximum and maximized daily rainfall depths.

estimated by the maximum daily dew point $T_{d,m}$ of the corresponding month.

The term $T_{d,m}$ is estimated either as the maximum historical value from a sample of at least 50 years length, or as the value corresponding to a 100-years return period, for samples smaller than 50 years (WMO, 1986).

3 Statistical analysis of dew points

The Gumbel distribution (Gumbel, 1958), which is the most common probabilistic model for hydrological extremes, proved inadequate for describing the empirical distribution of the monthly maximum daily dew points (Papalexiou, 2005). Therefore, the probability theory of maxima was applied. According to that theory, given a number of n independent identically distributed random variables, the largest of them (more precisely, the largest order statistic), i.e. $X = \max(Y_1, \dots, Y_n)$, has probability distribution function

$$H_n(x) = [F(x)]^n, \quad (2)$$

where $F(x) = P(Y_i \leq x)$ is the probability distribution function of each Y_i , referred to as the parent distribution.

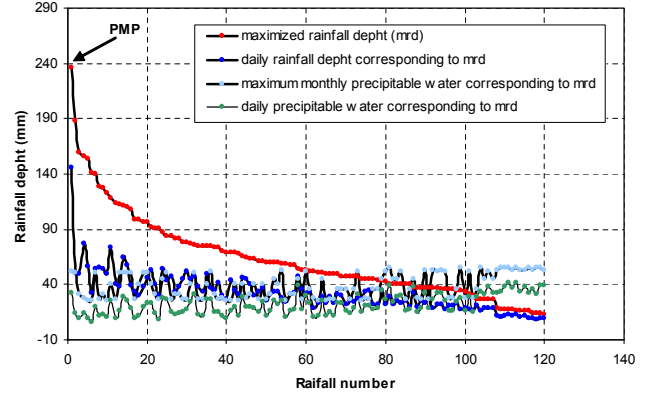


Fig. 3. Maximized rainfall depths of the NOA station and related quantities.

The frequency analysis for the daily dew points indicated the three-parameter Weibull model as a sufficient probabilistic model for describing their empirical distribution (Papalexiou, 2005). As a result, it can be used as parent distribution, so that the theoretical maximum distribution of the monthly maximum daily dew point can be described by Eq. (2), where n stands for the days of each month.

Since the condition of independence of random variables does not hold, as shown from the high values of autocorrelation coefficients, the exponent n was expected and proved, indeed, lower than the number of days in a month (Papalexiou, 2005). Moreover, given the uncertainties related to the estimation of the three parameters of the parent distributions, as well as the uncertainty of the value of the exponent n , we implemented a “parallel” optimization approach, by simultaneously fitting the theoretical models $F(x)$ and $H_n(x)$ to the empirical distributions of daily and monthly maximum daily dew points. The objective function is written as

$$LSE_{Total} = LSE(F(x)) + [LSE(H_n(x))]^2, \quad (3)$$

where LSE is the least square error between the theoretical and the empirical distributions. This strategy helped to better fit the theoretical maximum distribution derived by the parent distribution.

The L-moment ratio diagram (Vogel and Fennessey, 1993; Stedinger et al., 1993) shown in Fig. 1, illustrates that the theoretical maximum distribution derived from the parent three-parameter Weibull distribution is more appropriate than the asymptotic ones (Gumbel, Generalized Extreme Value or GEV).

4 Application of the PMP estimation method

The maximized precipitation time series were analysed in comparison to the observed ones. It was concluded that the maximization process causes sometimes a disproportional increase in the range of the values of recorded rainfall and that the maximized samples exhibit higher skewness than

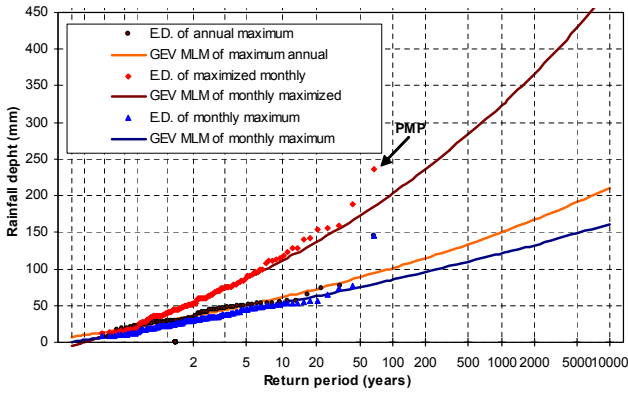


Fig. 4. Probability plot of annual maximum, monthly maximum and monthly maximized daily rainfall depths of NOA (on Gumbel probability paper).

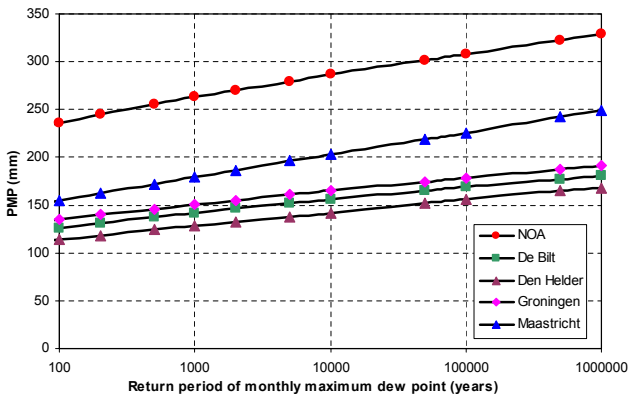


Fig. 5. PMP estimations for various return periods of monthly maximum daily dew point.

the recorded ones (Fig. 2), especially when the sample L-skewness values are low.

Figure 3 illustrates the values of maximized rainfall depths of the station of NOA in descending order, the concurrent daily-recorded rainfall depths, the concurrent daily precipitable water and the maximum monthly precipitable water. The 120 maximized rainfall depths that are illustrated in Fig. 3 (the highest is the estimated PMP), are the result of the merging of the 10 maximized rainfall events of each month, produced by the 10 maximum rainfall events of the respective month. It is evident that the estimated PMP point is located in a very uncertain area of the curve, where the slope is very high.

In addition, as shown in Fig. 3, if the record was shorter or had a missing data point (the most left in Fig. 3) the PMP value would be downgraded from 240 to about 190 mm.

Moreover, the sample of the 120 maximized rainfall depths was analyzed in the same probabilistic manner as the maximum monthly and annual rainfall depths were analyzed. Figure 4 depicts the relevant data of NOA. The fitted distributions (GEV) to the maximum monthly and annual rainfall depths, as Fig. 4 suggests, have no upper bound, so it is very

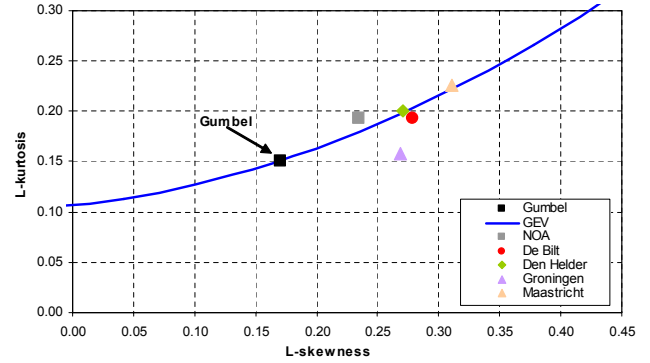


Fig. 6. L-moment ratio diagram for annual maximum daily rainfall depths.

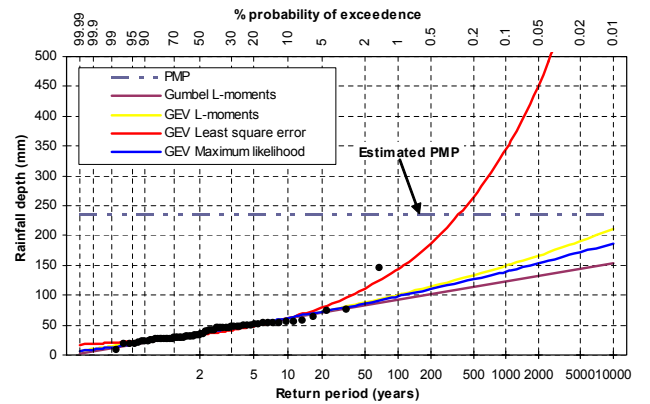


Fig. 7. Probability plot of annual maximum daily rainfall values of NOA station on Gumbel probability paper.

likely that if a longer rainfall record were available, the estimate of the PMP would be higher.

Furthermore, if solely the distribution of maximized rainfalls is taken under consideration, then the estimate of PMP is the outcome of an extremely uncertain estimation of the first order statistic.

Finally, the PMP estimation method was applied with regard to the monthly maximum daily dew point for a wide range of return periods, admitting that the WMO suggestion to use a 100 year return period is arbitrary and this return period could be well assumed greater. The distribution of the monthly maximum daily dew point was the one derived by the three-parameter Weibull parent distribution. The estimate of the PMP, as illustrated in Fig. 5, is an increasing function of the monthly maximum daily dew point.

5 A probabilistic approach for the annual maximum daily rainfall depth

If one abandons the effort to put an upper limit to precipitation and the deterministic thinking behind it, the next step is to model maximum rainfall probabilistically. As the L-moment ratio diagram of the annual maximum daily rainfall

(Fig. 6) suggests, the GEV distribution describes sufficiently the empirical distribution of the annual maximum daily rainfall depths.

The GEV model was fitted on the historical data with three different methods, the method of Least Square Error (Papoulis, 1990), the method of L-Moments (Hosking, 1990) and the method of Maximum Likelihood (Fisher, 1922). According to the fitted probabilistic model, the estimate of the PMP value is associated with a return period or probability of exceedence. It can be concluded that this probability is not negligible. The above analysis was also conducted using the Gumbel model, which obviously underestimates the exceedence probability of the PMP (Fig. 7).

6 Conclusions

In the above analysis, no evidence for an upper bound of dew point and of precipitation was found. The estimation of the PMP based on the moisture maximization concept is considerably uncertain and was proven to be too sensitive against the available data.

The study showed that the existence of an upper limit on precipitation, as implied by the PMP concept, is statistically inconsistent. Moreover, such a limit cannot be specified in a deterministic way, as the method asserts; in reality, from a statistical point-of-view, this “limit” tends to infinity.

According to the probabilistic analysis on the annual daily maximum rainfall depths, the hypothetical upper limit of the PMP method corresponds to a small, although not negligible, exceedence probability. For example, this probability for the Athens area is 0.27%, a value that would not be acceptable for the design of a major hydraulic structure.

A probabilistic approach, based on the GEV model, seems to be a more consistent tool for studying hydrological extremes.

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