

MR2113927 (2005m:46018) 46B20 03E05 46B26**Argyros, Spiros A.** (GR-ATHN); **Mercourakis, Sophocles** (GR-UATH)**Examples concerning heredity problems of WCG Banach spaces. (English summary)***Proc. Amer. Math. Soc.* **133** (2005), no. 3, 773–785 (electronic).

The core of the paper has two examples about weakly compactly generated (in short WCG) Banach spaces. This important class, formed by those Banach spaces admitting weakly compact sets whose linear span is dense, can be considered as the simultaneous generalization of separable and reflexive spaces. Nevertheless, contrary to what happens in those two cases, a subspace of a WCG space need not be WCG.

This was first shown by H. P. Rosenthal [*Compositio Math.* **28** (1974), 83–111; [MR0417762](#)]. Afterwards, the general problem “when subspaces of a WCG space are WCG” remains widely open. One interesting result has been recently provided by A. Mercourakis and E. Stamati [“A new class of weakly K -analytic Banach spaces”, *Mathematika*, to appear] showing that under Martin’s axiom every WCG space with dimension strictly smaller than 2^{\aleph_0} has all its subspaces WCG.

The two examples of WCG spaces containing non-WCG subspaces of this paper feature the following additional properties:

Example 1 (Theorem 2.6) is a WCG Banach space with an unconditional basis containing a non-WCG subspace with unconditional basis. This example answers a question of V. E. Zizler [in *Handbook of the geometry of Banach spaces, Vol. 2*, 1743–1816, North-Holland, Amsterdam, 2003; [MR1999608](#)].

Example 2 (Theorem 4.3) is a WCG Banach space X whose bidual X^{**} is also WCG and contains a non-WCG subspace Y whose bidual Y^{**} is not WCG. Moreover none of them contains l_1 and the quotient X/Y is reflexive.

This second example is quite interesting because of its construction as a James-tree space over the remarkable Rezníčenko sequence of trees. Indeed, in a subsequent paper [*Comment. Math. Univ. Carolin.* **46** (2005), no. 3, 399–408] the authors show that this example answers negatively questions of Fabian, Godefroy, Hájek and Zizler, and the hope of using this construction to get further anomalous spaces has not faded.

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