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Probing the Holography of Near-Horizon $\mathbf{AdS_5 \times S^5}$ Geometry

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Abstract

Using the AdS/CFT correspondence we study the holographic principle and the CFT/FRW relations in the near-horizon $AdS_5 \times S^5$ geometry with a probe D3-brane playing the rôle of the boundary to this space. The motion of the probe D3-brane in the bulk, induces a cosmological evolution on the brane. As the brane crosses the horizon of the bulk Schwarzschild-AdS$_5$ black hole, it probes the holography of the dual CFT. We test the holographic principle and we find corrections to CFT/FRW relations in various physical cases: for radially moving, spinning and electrically charged D3-brane and for a NS/NS B-field in the bulk.

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1. Introduction

The D-brane solutions of type II supergravity [1] and their near-horizon geometry [2] are the central geometrical objects upon which the AdS/CFT correspondence is build [3, 4, 5]. In particular, according to Maldacena conjecture [3], the (3+1)-dimensional world-volume of N coinciding extremal D3-branes, which give rise to $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills (SYM) theory, in the large N limit, is dual to type IIB superstrings, propagating on the near-horizon $AdS_5 \times S^5$ background geometry. In a further proposal [6] the thermodynamics of large N, $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory is linked with the thermodynamics of Schwarzschild black holes embedded in the AdS space [7]. This allows to relate, the Bekenstein-Hawking entropy, in the Maldacena limit, to the entropy of Yang-Mills gas at N$\longrightarrow \infty$ and large 't Hooft coupling g_{YM}^2N.

Many ideas about AdS/CFT correspondence were influenced by the intriguing concept of "holography"[8, 9]. The underlying principle, which was originated in the Bekenstein bound [10], is based on the notion that the maximal entropy that can be stored within a given volume will be determined by the largest black hole fitting inside that volume. Since the entropy of a black hole is essentially given by its surface area, it follows directly that all the relevant degrees of freedom of any system must in some sense live on the boundary enclosing that system.

The holographic principle imposes on generic field theories coupled to gravity "holographic area bounds" which limit the number of physical degrees of freedom. These "holographic area bounds" were elegantly applied to cosmology [11] where it was shown that the entropy which crosses the lightlike boundary of an observable region of the universe, the particle horizon, should not exceed the horizon area in Planck units.

Recently, using the holographic principle, the entropy bounds in a radiation dominated closed Friedmann-Robertson-Walker universe was analyzed [12]. It was found a surprising similarity between Cardy's entropy formula for 1+1 dimensional CFT and the Friedmann equation governing the evolution of the universe. After a suitable identification, it was shown that actually the Cardy's formula [13] maps to the Friedmann equation. In a further development [14] this correspondence between Cardy's formula and the Friedmann equation was tested in the Randall-Sundrum type model [15]. In the case where the bulk is a Schwarschild-AdS background and there is no matter on the brane, the correspondence between Cardy's formula and the Friedmann equation is recovered when the brane crosses the black hole horizon.

Motivated by the work in [12, 14] the cosmological holographic principle was applied to various black hole backgrounds under additional physical assumptions. Generalizations to include a non-vanishing cosmological constant on the brane with AdS or dS background were studied in [16, 17, 18, 19, 20, 21, 22, 23]. The case of having stiff matter on the brane was studied in [25, 26]. Extensions to charged black hole background were analyzed in [27, 28, 29, 30], to Kerr-Newmann rotated black holes in [31, 32, 33]. Extension to Gauss-Bonnet background geometry was considered in [24, 34, 35, 36], while the holography of AdS-Taub-Bolt spacetimes was studied in [37].

In this work we come back to the original ideas developed in [14] and we study in a systematic way the cosmological holographic principle as this is expressed through the AdS/CFT correspondence and the CFT/FRW-cosmologies relations in a generic static spherically symmetric background geometry with a boundary simulated by a probe D3-brane moving in this background. An important issue is that these backgrounds are consistent vacuum solutions of ten-dimensional string theory, like the $AdS_5 \times S^5$ background geometry and its near-horizon limit. Our approach will enable us to test the AdS/CFT correspondence and the CFT/FRW-cosmologies relations and find corrections to these relations in various physical situations, like a spinning or electrically charged D3-probe brane moving in a near-horizon $AdS_5 \times S^5$ background with or without a NS/NS B-field.

The necessary machinery for such an investigation has already been developed in [38] where it was shown that the motion of the probe D3-brane in this generic background induces on the brane a cosmological evolution [39], by generating on the brane an effective energy density and an effective pressure. However, the induced equation of state on the brane corresponds in most of the cases, depending on the energy and angular momentum of the probe D3-brane, to "Mirage" or stiff matter with $|w| > 1/3$. Nevertheless, by choosing particular spherically symmetric backgrounds sensible cosmological evolution can be generated on the brane and also brane inflation [40, 41, 42] and exit from it [43].

The paper is organized as follows. After the introduction in section one, we review the main results of Mirage cosmology needed to our work, in section two. In section three we study the holography of near-horizon $AdS_5 \times S^5$ geometry as it is probed by the D3-brane in various physical cases: the probe D3-brane moving in this particular background radially, spinning or being electrically charged. In section four we study the more general problem of a probe D3-brane moving in the field of other D3-black branes. After reviewing the relevant formalism, we study the holography of this geometry introducing also an NS/NS B-field in the background. Finally in section five are our conclusions.

2. A Probe D3-Brane Moving in a Static Spherically Symmetric Background

We will consider a probe D3-brane moving in a generic static, spherically symmetric background. We assume the brane to be light compared to the background so that we will neglect the back-reaction. The background metric we consider has the general form

$$ds_{10}^2 = g_{00}(r)dt^2 + g(r)(d\vec{x})^2 + g_{rr}(r)dr^2 + g_S(r)d\Omega_5, \tag{2.1}$$

where g_{00} is negative, and there is also a dilaton field Φ as well as a RR background $C(r) = C_{0\ldots3}(r)$ with a self-dual field strength.

The dynamics on the brane will be governed by the Dirac-Born-Infeld action given by

$$\begin{aligned} S &= T_3 \int d^4\xi e^{-\Phi}\sqrt{-det(\hat{G}_{\alpha\beta} + (2\pi\alpha')F_{\alpha\beta} - B_{\alpha\beta})} \\ &+ T_3 \int d^4\xi \hat{C}_4 + anomaly\ terms. \end{aligned} \tag{2.2}$$

The induced metric on the brane is

$$\hat{G}_{\alpha\beta} = G_{\mu\nu}\frac{\partial x^\mu \partial x^\nu}{\partial\xi^\alpha \partial\xi^\beta}, \tag{2.3}$$

with similar expressions for $F_{\alpha\beta}$ and $B_{\alpha\beta}$. For an observer on the brane the Dirac-Born-Infeld action is the volume of the brane trajectory modified by the presence of the anti-symmetric two-form $B_{\alpha\beta}$, and worldvolume anti-symmetric gauge fields $F_{\alpha\beta}$. This means that, if there is for example radiation on the D3-brane, $F_{\alpha\beta} \neq 0$, the brane dynamics will be altered relative to the case of a brane with no radiation. As the brane moves the induced world-volume metric becomes a function of time, so there is a cosmological evolution from the brane point of view [38].

In the static gauge, $x^\alpha = \xi^\alpha, \alpha = 0, 1, 2, 3$ using (2.3) we can calculate the bosonic part of the brane Lagrangian which reads

$$\mathcal{L} = \sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j} - C(r), \tag{2.4}$$

where $h_{ij}d\varphi^i d\varphi^j$ is the line element of the unit five-sphere, and

$$A(r) = g^3(r)|g_{00}(r)|e^{-2\Phi}, B(r) = g^3(r)g_{rr}(r)e^{-2\Phi}, D(r) = g^3(r)g_S(r)e^{-2\Phi}, \tag{2.5}$$

and $C(r)$ is the RR background. The problem is effectively one-dimensional and can be solved easily. The momenta are given by

$$\begin{aligned} p_r &= -\frac{B(r)\dot{r}}{\sqrt{A(r) - B(r)\dot{r}^2}}, \\ p_i &= -\frac{D(r)h_{ij}\dot{\phi}^j}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\phi}^i\dot{\phi}^j}}. \end{aligned} \tag{2.6}$$

Since (2.4) is not explicitly time dependent and the ϕ-dependence is confined to the kinetic term for $\dot{\phi}$, for an observer in the bulk, the brane moves in a geodesic parametrised by a conserved energy E and a conserved angular momentum l^2 given by

$$\begin{aligned} E &= \frac{\partial\mathcal{L}}{\partial\dot{r}}\dot{r} + \frac{\partial\mathcal{L}}{\partial\dot{\phi}^i}\dot{\phi}^i - \mathcal{L} = p_r\dot{r} + p_i\dot{\phi}^i - \mathcal{L}, \\ l^2 &= h^{ij}\frac{\partial\mathcal{L}}{\partial\dot{\phi}^i}\frac{\partial\mathcal{L}}{\partial\dot{\phi}^j} = h^{ij}p_ip_j. \end{aligned} \tag{2.7}$$

Solving these expressions for $\dot{r}$ and $\dot{\phi}$ we find

$$\dot{r}^2 = \frac{A}{B}(1 - \frac{A}{(C+E)^2}\frac{D+\ell^2}{D}),\ h_{ij}\dot{\varphi}^i\dot{\varphi}^j = \frac{A^2\ell^2}{D^2(C+E)^2}. \tag{2.8}$$

The allowed values of r impose the constraint that $C(r) + E \geq 0$. The induced four-dimensional metric on the brane, using (2.3) in the static gauge, is

$$d\hat{s}^2 = (g_{00} + g_{rr}\dot{r}^2 + g_S h_{ij}\dot{\varphi}^i\dot{\varphi}^j)dt^2 + g(d\vec{x})^2. \tag{2.9}$$

In the above relation we substitute $\dot{r}^2$ and $h_{ij}\dot{\varphi}^i\dot{\varphi}^j$ from (2.8), and using (2.5) and we get

$$d\hat{s}^2 = -\frac{g_{00}^2 g^3 e^{-2\phi}}{(C+E)^2}dt^2 + g(d\vec{x})^2. \tag{2.10}$$

We can define the cosmic time η as

$$d\eta = \frac{|g_{00}|g^{\frac{3}{2}}e^{-\Phi}}{|C+E|}dt, \tag{2.11}$$

so the induced metric becomes

$$d\hat{s}^2 = -d\eta^2 + g(r(\eta))(d\vec{x})^2, \tag{2.12}$$

The induced metric on the brane (2.12) is the standard form of a flat expanding universe. The relation (2.11) will play a central rôle in the following. It relates the cosmic time η, the time an observer on the brane uses, with the time t that it is used by an observer in the bulk. We can derive the analogue of the four-dimensional Friedmann equations by defining $g = \alpha^2$

$$\Big(\frac{\dot{\alpha}}{\alpha}\Big)^2 = \frac{(C+E)^2 g_S e^{2\Phi} - |g_{00}|(g_S g^3 + \ell^2 e^{2\Phi})}{4|g_{00}|g_{rr}g_S g^3}\Big(\frac{g'}{g}\Big)^2, \tag{2.13}$$

where the dot stands for derivative with respect to cosmic time while the prime stands for derivatives with respect to r. The right hand side of (2.13) can be interpreted in terms of an effective matter density on the probe brane

$$\frac{8\pi G}{3}\rho_{eff} = \frac{(C+E)^2 g_S e^{2\Phi} - |g_{00}|(g_S g^3 + \ell^2 e^{2\Phi})}{4|g_{00}|g_{rr}g_S g^3}\Big(\frac{g'}{g}\Big)^2, \tag{2.14}$$

where G is the four-dimensional Newton's constant. We can also calculate

$$\begin{aligned} \frac{\ddot{\alpha}}{\alpha} &= \Big(1 + \frac{g}{g'}\frac{\partial}{\partial r}\Big)\frac{(C+E)^2 g_S e^{2\Phi} - |g_{00}|(g_S g^3 + \ell^2 e^{2\Phi})}{4|g_{00}|g_{rr}g_S g^3}\Big(\frac{g'}{g}\Big)^2 \\ &= \Big[1 + \frac{1}{2}\alpha\frac{\partial}{\partial\alpha}\Big]\frac{8\pi G}{3}\rho_{eff}. \end{aligned} \tag{2.15}$$

If we set the above equal to $-\frac{4\pi G}{3}(\rho_{eff} + 3p_{eff})$ we can define the effective pressure p_{eff}.

Therefore, the motion of a D3-brane on a general spherically symmetric background had induced on the brane an energy density and a pressure. Then, the first and second Friedmann equations can be derived giving a cosmological evolution of the brane universe in the sense that an observer on the brane measures a scale factor $\alpha(\eta)$ of the brane-universe evolution. This scale factor depends on the position of the brane in the bulk. This cosmological evolution is known as "Mirage Cosmology" [38]: the cosmological evolution is not due to energy density on our universe but on the energy content of the bulk. In the next section we will also describe the case where the cosmological evolution can be triggered by the motion of a probe D3-brane moving in the field of other Dp-branes.

The formalism developed so far allows also for the probe D3-brane to have a non-zero angular momentum. We can assume also that there is an electric field on the probe D3-brane. In this case the action for the D3-brane is given by (2.2) and in the background metric (2.1), the Lagrangian takes the form

$$\mathcal{L} = \sqrt{A - B\dot{r}^2 - \mathcal{E}^2 g^2} - C, \tag{2.16}$$

where $\mathcal{E}^2 = 2\pi a' E_i E^i$ and $E_i = -\partial_t A_i(t)$ in the $A_0 = 0$ gauge and A and B are given by (2.5). The equations of motion for the electric field are

$$\partial_t\Big(\frac{g^2 E_i}{\sqrt{A - B\dot{r}^2 - \mathcal{E}^2 g^2}}\Big) = 0 \tag{2.17}$$

and one can find [38]

$$E_i = \frac{\mu_i}{g}\sqrt{\frac{A - B\dot{r}^2}{\mu^2 + g^2}},\ \mathcal{E}^2 = \frac{\mu^2}{g^2}\frac{A - B\dot{r}^2}{g^2 + \mu^2}, \tag{2.18}$$

where μ_i are integration constants and $\mu^2 = (2\pi a')\mu_i\mu^i$. In the case $\dot{r} = 0$, E_i is constant as it is required by ordinary Maxwell equations. From (2.17) we can calculate $\dot{r}$,

$$\dot{r}^2 = \frac{A}{B}\Big(1 - \frac{A}{(C+E)^2(1 + \mu^2 g^{-2})}\Big), \tag{2.19}$$

from which we obtain $\mathcal{E}^2$ after substitution in (2.18)

$$\mathcal{E}^2 = \mu^2 \frac{A^2}{(C+E)^2(g^2 + \mu^2)^2}. \tag{2.20}$$

The induced metric on the probe D3-brane turns out to be

$$d\hat{s}^2 = -\frac{g_{00}^2 g^5 e^{-2\phi}}{(C+E)^2(\mu^2 + g^2)}dt^2 + g(d\vec{x})^2, \tag{2.21}$$

and by defining the cosmic time as

$$d\eta = \frac{|g_{00}| g^{\frac{5}{2}} e^{-\Phi}}{|C+E|(\mu^2 + g^2)^{\frac{1}{2}}}dt, \tag{2.22}$$

the induced metric on the brane becomes

$$d\hat{s}^2 = -d\eta^2 + g(r(\eta))(d\vec{x})^2. \tag{2.23}$$

Then, the Friedmann equations with an electric field on the probe D3-brane in a radial motion is

$$\Big(\frac{\dot{\alpha}}{\alpha}\Big)^2 = \frac{(C+E)^2(1+\mu^2 g^{-2}) - |g_{00}|g^3 e^{-2\Phi}}{4|g_{00}|g_{rr}g^3 e^{-2\Phi}}\Big(\frac{g'}{g}\Big)^2. \tag{2.24}$$

The dominant contribution to the induced energy density from the electric field, as can be seen from (2.20), is of the order $\mathcal{E}^2$.

The induced cosmological evolution of a brane moving in a Schwarzschild-AdS background was also discussed in [14]. Nevertheless the Mirage Cosmology allows for more general backgrounds and more general physical requirements on the bulk-brane system, allowing to test the AdS/CFT correspondence and the CFT/FRW-cosmologies relations in various cases. The formalism which we reviewed in this section can be generalized to a curved D3-probe brane. This will induce a spatial curvature on the brane-universe.

3. The Holographic Description of Near-Horizon $\mathbf{AdS_5 \times S^5}$ Geometry

We will apply the above described formalism first to the near-horizon geometry $AdS_5 \times S^5$. There are Schwarzschild-AdS$_5$ black hole solutions in this background with metric

$$ds^2 = \frac{r^2}{L^2}\Big(-f(r)dt^2 + (d\vec{x})^2\Big) + \frac{L^2}{r^2}\frac{dr^2}{f(r)} + L^2 d\Omega_5^2, \tag{3.1}$$

where $f(r) = 1 - \left(\frac{r_0}{r}\right)^4$. The RR field is given by $C = C_{0\ldots3} = \left[\frac{r^4}{L^4} - \frac{r_0^4}{2L^4}\right]$.

Using (2.1) we find in this background

$$\begin{aligned} g_{00}(r) &= -\frac{r^2}{L^2}\Big(1-(\frac{r_0}{r}\Big)^4\Big) = -\frac{1}{g_{rr}} \\ g(r) &= \frac{r^2}{L^2} \\ g_s(r) &= L^2, \end{aligned} \tag{3.2}$$

and the brane-universe scale factor is $\alpha = r/L$. Substituting the above functions to equation (2.14) we find the analogue of Friedmann equation on the brane which is [38]

$$H^2 = \frac{8\pi G}{3}\rho_{eff} = \frac{1}{L^2}\Big[\Big(1+\frac{1}{\alpha^4}\Big(E - r_0^4/2L^4\Big)\Big)^2 - \Big(1-\Big(\frac{r_0}{L}\Big)^4\frac{1}{\alpha^4}\Big)\Big(1+\frac{l^2}{L^2}\frac{1}{\alpha^6}\Big)\Big], \tag{3.3}$$

where E is a constant of integration of the background field equations, expressing the conservation of energy, and it is related to the black hole mass of the background [45], while $r_0^4/2L^4$ is the constant part of the RR field, expressing essentially electrostatic energy, and it can be absorbed into the energy $\tilde{E} = E - r_0^4/2L^4$. This Friedmann equation describes the cosmological evolution of a contracting or expanding universe depending on the motion of the probe brane. This motion in turn depends on two parameters the energy $\tilde{E}$ and the angular momentum l^2. These two parameters specify various trajectories of the probe brane. The scale factor α comes in various powers, indicating that (3.3) describes the cosmological evolution of various kind of Mirage or stiff cosmological matter.

III-A. A Probe D3-brane Moving Radially in a Near-Horizon $\mathbf{AdS_5 \times S^5}$ Black Hole Background

We will follow first the motion of the probe D3-brane in the case of $l^2 = 0$. Defining the dimensionless parameter $a = l^2/L^2\alpha^6$, equation (3.3) becomes

$$H^2 = \frac{8\pi G}{3}\rho_{eff} = \frac{1}{L^2}\Big[\Big(1+\frac{\tilde{E}}{\alpha^4}\Big)^2 - \Big(1-\Big(\frac{r_0}{L}\Big)^4\frac{1}{\alpha^4}\Big)\Big(1+a\Big)\Big]. \tag{3.4}$$

Using equations (2.15) and (3.4), the second Friedmann equation in this background reads

$$\dot{H} = -\frac{2}{L^2}\Big[2\frac{\tilde{E}}{\alpha^4}\Big(1+\frac{\tilde{E}}{\alpha^4}\Big) + \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big(1+a\Big) - \frac{3}{2}a\Big(1-\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big], \tag{3.5}$$

and the effective pressure, using again (2.15), is

$$p_{eff} = \frac{1}{8\pi G L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + 5\frac{\tilde{E}^2}{\alpha^8} + 2\frac{\tilde{E}}{\alpha^4} + 7a\Big(\frac{\alpha_0}{\alpha}\Big)^4 - 3a\Big]. \tag{3.6}$$

From (3.4) we also have the effective energy density

$$\rho_{eff} = \frac{3}{8\pi G L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{E}^2}{\alpha^8} + 2\frac{\tilde{E}}{\alpha^4} - a\Big(1-\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big]. \tag{3.7}$$

It is instructive to consider the equation of state $p_{eff} = w\rho_{eff}$ where w is given by

$$w = \frac{1}{3}\Big[\frac{\Big(\frac{\alpha_0}{\alpha}\Big)^4 + 5\frac{\tilde{E}^2}{\alpha^8} + 2\frac{\tilde{E}}{\alpha^4} + 7a\Big(\frac{\alpha_0}{\alpha}\Big)^4 - 3a}{\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{E}^2}{\alpha^8} + 2\frac{\tilde{E}}{\alpha^4} - a\Big(1-\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)}\Big]. \tag{3.8}$$

As the brane moves in the Schwarzschild-AdS$_5$ black hole background, the equation of state is parametrized by the energy of the bulk and the angular momentum of the brane. However, when $\tilde{E}=0$ and $a=0$ the brane-universe is radiation dominated at any position in the bulk, as can be seen from (3.8). This is expected, because the only scale in the theory is the energy scale and putting it to zero the theory is scale invariant, while a non-zero angular momentum induces on the brane all kind of Mirage matter. For $\tilde{E}=0$ and $a=0$ the equations (3.4), (3.5), (3.6) and (3.7) become

$$H^2 = \frac{1}{L^2}\Big(\frac{\alpha_0}{\alpha}\Big)^4 \tag{3.9}$$

$$\dot{H} = -\frac{2}{L^2}\Big(\frac{\alpha_0}{\alpha}\Big)^4 \tag{3.10}$$

$$\rho_{eff} = \frac{3}{8\pi G L^2}\Big(\frac{\alpha_0}{\alpha}\Big)^4 \tag{3.11}$$

$$p_{eff} = \frac{1}{8\pi G L^2}\Big(\frac{\alpha_0}{\alpha}\Big)^4 \tag{3.12}$$

In [12] it was shown that for a radiation dominated universe, the first and second Friedmann equations can be written in a way similar to Cardy and Smarr formulae respectively. This in turn means that the Cardy and Smarr formulae can be expressed in terms of cosmological quantities of a radiation dominated universe. The Hubble entropy is defined by

$$S_H = HV/2G, \tag{3.13}$$

while the Bekenstein-Hawking energy is $E_{BH} = 3V/4\pi G r^2$. We can also define the Hubble temperature T_{Hubble} [1] from

$$T_{Hubble} \equiv -\frac{\dot{H}}{2\pi H}. \tag{3.14}$$

Using the above definitions, we can verify that the first Friedmann equation (3.9) can be written as the Cardy-Verlinde formula

$$S_H = \frac{2\pi r}{3}\sqrt{E_{BH}(2E - kE_{BH})}\ , \tag{3.15}$$

[1] where the minus sign is necessary to get a positive result, since in a radiation dominated universe the expansion always slows down. Further, to avoid the danger of dividing by zero, we assume that we are in a strongly self-gravitating phase with $Hr \geq 1$.

where $kE_{BH} = 0$, because our brane-universe is flat, while the second Friedmann equation (3.10) can be written as the Smarr formula

$$kE_{BH} = 3(E + pV - T_{Hubble}S_H). \tag{3.16}$$

We can also express the Cardy-Verlinde and Smarr formulae in terms of thermodynamical quantities of the dual CFT theory. The Bekenstein-Hawking entropy of the AdS black hole is given by the area of the horizon measured in bulk Planckian units. For spherically symmetric backgrounds the entropy is defined by

$$S = \frac{V_H}{4G_{Bulk}}, \tag{3.17}$$

where G_{Bulk} is the bulk Newton's constant and V_H is the area of the horizon $V_H = r_H^n Vol(S^n)$ with $Vol(S^n)$ the volume of a unit n-sphere. The total entropy is constant during the evolution, but the entropy density varies with the time. If we define $s = S/V$, the entropy density is

$$s = \Big(\frac{r_0}{r}\Big)^n \frac{(n-1)}{4GL}, \tag{3.18}$$

where we have used the relation $G_{Bulk} = GL/(n-1)$ [46].

An observer in the bulk is using the AdS time t and measures the Hawking temperature from the equation

$$T_H = \frac{h'(r_0)}{4\pi}, \tag{3.19}$$

where the differentiation is with respect to r, which is the distance variable in the bulk. In our background $h(r) = \frac{r^2}{L^2}\Big[1 - \Big(\frac{r_0}{r}\Big)^4\Big]$, and the Hawking temperature becomes

$$T_H = \frac{r_0}{\pi L^2}. \tag{3.20}$$

An observer on the brane is using the cosmic time η defined by (2.11). Using equations (3.2) and the fact that the probe brane is moving in a geodesic where $\tilde{E} = 0$, equation (2.11) becomes

$$d\eta = \frac{r}{L}\Big(1 - \Big(\frac{r_0}{r}\Big)^4\Big)dt. \tag{3.21}$$

It was argue in [6] that the energy, entropy and temperature of a CFT at high temperatures can be identified up to a conformal factor with the mass, entropy and Hawking temperature of the AdS black hole. To fix the conformal factor, according to the AdS/CFT correspondence the CFT lives on a space-time which can be identified with the asymptotic boundary of the AdS black hole. The asymptotic form of the metric in our case, using (3.1), is

$$\lim_{r\to\infty}\Big[\frac{L^2}{r^2}ds^2\Big] = \lim_{r\to\infty}\Big[-f(r)dt^2 + (d\vec{x})^2 + \frac{L^4}{r^4}\frac{dr^2}{f(r)} + \frac{L^4}{r^2}d\Omega_5^2\Big]. \tag{3.22}$$

Taking the limit of $f(r)$, relation (3.22) becomes

$$\lim_{r\to\infty}\Big[\frac{L^2}{r^2}ds^2\Big] = -dt^2 + (d\vec{x})^2 + L^4 d\Omega_3^2. \tag{3.23}$$

Therefore the CFT time and the AdS time are related through the conformal factor r/L . Note that the same result can be obtained considering relation (3.21) for large r. Both procedures give the same result because our probe brane is flat as it is discussed in [49].

Having fixed the conformal factor we can relate CFT and the Hawking temperature

$$T_{CFT} = \frac{1}{\alpha}T_H = \frac{L}{r}T_H, \tag{3.24}$$

and then the Cardy-Verlinde and Smarr formulae can be derived using the AdS/CFT correspondence. Using (3.18) for $n = 3$ the CFT entropy density is

$$s_{CFT} = \frac{1}{2GL}\Big(\frac{r_0}{r}\Big)^3, \tag{3.25}$$

while the Casimir energy is defined by (Smarr formula)

$$E_C = 3\Big(E + pV - T_{CFT}S_{CFT}\Big). \tag{3.26}$$

Equation (3.26) can be written as $\rho_C = 3\Big(\rho_{eff} + p_{eff} - T_{CFT}s_{CFT}\Big)$ from which after substitution of the relevant quantities we get $E_C = 0$ as expected in a flat radiation dominated brane-universe. Then, the Cardy-Verlinde formula

$$S_{CFT} = \frac{2\pi r}{3}\sqrt{\frac{3}{2\pi r}S_C(2E - E_C)}, \tag{3.27}$$

with $S_C = \frac{V}{2Gr}\Big(\frac{r_0}{r}\Big)^2$ is trivially satisfied.

At the special moment at which the brane crosses the bulk black hole horizon, the Hubble temperature (3.14) becomes $T_{Hubble} = 1/\pi L$. On the other hand the CFT temperature T_{CFT} using

(3.19) and (3.24) on the horizon becomes $T_{CFT} = 1/\pi L$, therefore on the horizon $T_{CFT} = T_{Hubble}$. It is also easy to check that on the horizon $S_{CFT} = S_H$ and $E_C = kE_{BH}$. Therefore as the brane crosses the black hole horizon equation (3.27) is equivalent to (3.15) and equation (3.26) is equivalent to (3.16). This is known as CFT/FRW-cosmologies correspondence, expressing a special interrelation between thermodynamical and geometrical quantities in a radiation dominated universe.

These results are in agreement with various studies of a brane moving in a Schwarzschild-AdS$_5$ black hole background [14, 17, 47, 48]. In the next sections we will test the AdS/CFT correspondence and the CFT/FRW-cosmologies relations under various physical conditions on the brane and the bulk.

III-B. A Spinning Probe D3-Brane Moving in a Near-Horizon AdS$_5 \times$ S^5 Black Hole Background

In this section we will study the motion of the probe D3-brane carrying a non-trivial angular momentum in a near-horizon $AdS_5 \times S^5$ black hole background. If $a \neq 0$, we can see from (3.8) that the non-zero angular momentum induces Mirage matter on the brane with w taking any value. We will set $\tilde{E} = 0$ and we will find the corrections of the various quantities involved due to the angular momentum. Using (3.4) and (3.5) for $\tilde{E} = 0$, the Hubble entropy, Bekenstein-Hawking energy and Hubble temperature become

$$S_H = \frac{V}{2GL}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 - a\Big(1 - \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big]^{1/2} \tag{3.28}$$

$$E_{BH} = \frac{3V}{4\pi G L^2 \alpha^2} \tag{3.29}$$

$$T_{Hubble} = \frac{1}{\pi L}\Big[\frac{\Big(\big(\frac{\alpha_0}{\alpha}\big)^4 - \frac{a}{2}\Big(3 - 5\big(\frac{\alpha_0}{\alpha}\big)^4\Big)\Big)}{\Big(\big(\frac{\alpha_0}{\alpha}\big)^4 - a\Big(1 - \big(\frac{\alpha_0}{\alpha}\big)^4\Big)\Big)^{1/2}}\Big]. \tag{3.30}$$

Using the above relations we can verify that equations (3.15) and (3.16) are satisfied, suggesting that despite the brane-universe is not radiation dominated, the first and second Friedmann equations can still be written as the cosmological Cardy-Verlinde and Smarr formulae respectively, having the additional information of the spinning probe brane.

The angular momentum does not appear in the definition of the cosmic time (2.11). On the other hand the asymptotic limit of the metric is the same as in the case of $a = 0$, because the bulk metric in independent of the angular momentum of the brane. Therefore, the conformal factor is the same r/L as before and the CFT and Hawking temperatures are related the same way as in (3.24). Hence, the presence of a non-trivial angular momentum on the probe D3-brane maintains the exact AdS/CFT correspondence.

The Casimir energy, using (3.6) and (3.7), becomes

$$E_C = \frac{3Va}{4\pi G L^2}\Big[5\Big(\frac{\alpha_0}{\alpha}\Big)^4 - 3\Big]. \tag{3.31}$$

It is proportional to the angular momentum parameter a, while the CFT entropy is $S_{CFT} = V s_{CFT}$ where s_{CFT} is given by (3.25). At the moment the probe D3-brane crosses the bulk black hole horizon, the Hubble temperature from equation (3.30) becomes $T_{Hubble} = \frac{1}{\pi L}(1+a)$ while $T_{CFT} = 1/\pi L$ as before. Therefore at the horizon $T_{Hubble} \neq T_{CFT}$. One can easily check, using (3.28) and (3.31), that at the horizon $\alpha = \alpha_0$, we have

$$S_H = S_{CFT} \tag{3.32}$$

$$E_C \neq k E_{BH}. \tag{3.33}$$

Therefore, as the spinning probe brane crosses the black hole horizon the CFT/FRW-cosmologies relations break down.

III-C. Electric Field on the Probe D3-brane

In this section we will consider an electric field on the probe D3-brane. To simplify the discussion we will assume $l^2 = 0$. As we can see from (2.24) the electric field introduces a radiation term on the probe brane. Substituting the functions (3.2) into (2.24) and defining $\tilde{\mathcal{E}} = \mu^2$, the first Friedmann equation becomes

$$H^2 = \frac{8\pi G}{3}\rho_{eff} + \frac{8\pi G}{3}\rho_{rad} = \frac{1}{L^2}\Big[\Big(1+\frac{\tilde{E}}{\alpha^4}\Big)^2 - \Big(1-\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big] + \frac{1}{L^2}\Big(1+\frac{\tilde{E}}{\alpha^4}\Big)^2\frac{\tilde{\mathcal{E}}}{\alpha^4}, \tag{3.34}$$

while the second Friedmann equation can be easily calculated

$$\dot{H} = -\frac{2}{L^2}\Big[2\frac{\tilde{E}}{\alpha^4}\Big(1+\frac{\tilde{E}}{\alpha^4}\Big) + \Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{\mathcal{E}}}{\alpha^4}\Big(1+\frac{\tilde{E}}{\alpha^4}\Big)\Big(1+3\frac{\tilde{E}}{\alpha^4}\Big)\Big]. \tag{3.35}$$

From (3.35) using (2.15) the pressure can be calculated

$$p = p_{eff} + p_{rad} = \frac{1}{8\pi G L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{E}}{\alpha^4}\Big(5\frac{\tilde{E}}{\alpha^4} + 2\Big)\Big] + \frac{1}{8\pi G L^2}\Big[\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big(1 + 10\frac{\tilde{E}}{\alpha^4} + 9\frac{\tilde{E}^2}{\alpha^8}\Big)\Big]. \quad (3.36)$$

Demanding to have a radiation dominated brane-universe ($w = 1/3$), we get two solutions for the parameters $\tilde{E}$ and $\tilde{\mathcal{E}}$

$$\tilde{E} = 0 \quad (3.37)$$

$$\tilde{E} = -\alpha^4 \frac{2\tilde{\mathcal{E}}/\alpha^4}{1 + 2\tilde{\mathcal{E}}/\alpha^4}. \quad (3.38)$$

The parameter $\tilde{\mathcal{E}}$ being proportional to the energy density of the electric field (relation (2.20)), introduces another energy scale in the theory and we expect to affect the motion of the probe D3-brane. For $\tilde{\mathcal{E}} = 0$ we recover the results of a radially moving probe D3-brane. We will study first the solution $\tilde{E} = 0$. The two Friedmann equations and the energy density and pressure on the brane become

$$H^2 = \frac{1}{L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{\mathcal{E}}}{\alpha^4}\Big] \quad (3.39)$$

$$\dot{H} = -\frac{2}{L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{\mathcal{E}}}{\alpha^4}\Big] \quad (3.40)$$

$$\rho = \frac{3}{8\pi G L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{\mathcal{E}}}{\alpha^4}\Big] \quad (3.41)$$

$$p = \frac{1}{8\pi G L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{\mathcal{E}}}{\alpha^4}\Big]. \quad (3.42)$$

The above relations can be represented as corrections to relations (3.9)-(3.12), due to the presence of an electric field on the probe D3-brane. Using (3.39)-(3.42), the Hubble entropy, Bekenstein-Hawking energy and Hubble temperature can be calculated to be

$$S_H = \frac{V}{2GL}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{\mathcal{E}}}{\alpha^4}\Big]^{1/2} \quad (3.43)$$

$$E_{BH} = \frac{3V}{4\pi G L^2 \alpha^2} \quad (3.44)$$

$$T_{Hubble} = \frac{1}{\pi L}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{\mathcal{E}}}{\alpha^4}\Big]^{1/2}. \quad (3.45)$$

Observe that the Hubble entropy and the Hubble temperature have been modified by the same extra radiation term. The two Friedmann equations (3.39) and (3.40) can be written as the cosmological Cardy-Verlinde and Smarr formulae respectively, modified by the radiation term due to the electric field

$$S_H = \frac{2\pi r}{3}\sqrt{E_{BH}\Big(2(E+E_{rad})-kE_{BH}\Big)}\ , \tag{3.46}$$

$$kE_{BH} = 3\Big((E+E_{rad})+(p+p_{rad})V-T_{Hubble}S_H\Big). \tag{3.47}$$

To calculate the thermodynamic quantities of the dual theory we have to find the conformal factor. The conformal time in presence of the electric field, using (2.22), becomes

$$d\eta = \alpha\Big(1-\Big(\frac{r_0}{r}\Big)^4\Big)\Big(1+\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{-1/2}dt. \tag{3.48}$$

Then the conformal factor is

$$\lim_{r\to\infty}\frac{dt}{d\eta} = \frac{1}{\alpha}\Big(1+\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{1/2}. \tag{3.49}$$

If we calculate the conformal factor using the asymptotic form of the metric we will find a different result. The reason is that the bulk metric does not "see" the electric field on the brane. This case is similar to the case of a brane having a non-zero tension discussed in the literature [20, 48, 49].

The CFT and Hawking temperature are now related by $T_{CFT} = \Big(\big(1+\frac{\tilde{\mathcal{E}}}{\alpha^4}\big)^{1/2}/\alpha\Big)T_H$ and using the Hawking temperature (3.20), which does not change because it is a bulk quantity, it becomes

$$T_{CFT} = \frac{1}{\pi L}\Big(\frac{r_0}{r}\Big)\Big(1+\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{1/2}. \tag{3.50}$$

The Casimir energy can be calculated from the first law of thermodynamics $Tds = d\rho + 3(\rho + p - Ts)dr/r$ and we get

$$E_C = \frac{3V}{2\pi G L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{\mathcal{E}}}{\alpha^4} - \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big(1+\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{1/2}\Big]. \tag{3.51}$$

Using this expression for the Casimir energy we can check that the Cardy-Verlinde formula (3.27) is not satisfied.

As the brane crosses the bulk black hole horizon, the AdS/FRW-cosmologies relations break down because $S_H \neq S_{CFT}$ and $kE_{BH} \neq E_C$.

The second solution (3.38) has similar behavior like the first one. The first and second Friedmann equations become

$$H^2 = \frac{1}{L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 - \frac{\tilde{\mathcal{E}}}{\alpha^4}\Big(3-4\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big) - 4\Big(\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^2\Big(1-\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big]\Big(1+2\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{-2} \tag{3.52}$$

$$\dot{H} = -\frac{2}{L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 - \frac{\tilde{\mathcal{E}}}{\alpha^4}\Big(3-4\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big) - 4\Big(\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^2\Big(1-\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big]\Big(1+2\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{-2}. \quad (3.53)$$

The Hubble entropy, Bekenstein-Hawking energy and Hubble temperature for this solution become

$$S_H = \frac{V}{2GL}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 - \frac{\tilde{\mathcal{E}}}{\alpha^4}\Big(3-4\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big) - 4\Big(\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^2\Big(1-\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big]^{1/2}\Big(1+2\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{-1} \quad (3.54)$$

$$E_{BH} = \frac{3V}{4\pi GL^2\alpha^2} \quad (3.55)$$

$$T_{Hubble} = \frac{1}{\pi L}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 - \frac{\tilde{\mathcal{E}}}{\alpha^4}\Big(3-4\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big) - 4\Big(\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^2\Big(1-\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big]^{1/2}\Big(1+2\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{-1}. \quad (3.56)$$

Using (3.54)-(3.56) it is easy to verify the cosmological Cardy-Verlinde and Smarr formulae (3.15) and (3.16).

The conformal factor for this solution is

$$d\eta = \alpha\Big(1-\Big(\frac{r_0}{r}\Big)^4\Big)\Big(1+2\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)\Big(1+\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{-1/2}dt. \quad (3.57)$$

From which we calculate the conformal factor

$$\lim_{r\to\infty}\frac{dt}{d\eta} = \frac{1}{\alpha}\Big(1+\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{1/2}\Big(1+2\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{-1}. \quad (3.58)$$

Using this conformal factor the CFT temperature becomes

$$T_{CFT} = \frac{1}{\pi L}\Big(\frac{r_0}{r}\Big)\Big(1+\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{1/2}\Big(1+2\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{-1}. \quad (3.59)$$

while the Casimir energy is

$$\begin{aligned} E_C &= \frac{3V}{2\pi GL^2}\Big(1+2\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{-2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 - \frac{\tilde{\mathcal{E}}}{\alpha^4}\Big(3-4\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big) \\ &- 4\Big(\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^2\Big(1-\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big) - \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big(1+\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)^{1/2}\Big(1+2\frac{\tilde{\mathcal{E}}}{\alpha^4}\Big)\Big] \end{aligned} \quad (3.60)$$

As the brane crosses the bulk black hole horizon, the CFT/FRW-cosmologies relations break down because $S_H \neq S_{CFT}$ and $kE_{BH} \neq E_C$.

4. A Probe Dp-Brane Moving in the Field of a Dp′-Brane

In this section we will generalize the motion of a probe Dp-brane in the field of a Dp'-brane with $p' \geq p$ [44, 38]. In this case the Dp'-brane metric is of the form

$$ds_{10}^2 = g_{00}(r)dt^2 + g(r)(d\vec{x}_{p'})^2 + g_{rr}(r)dr^2 + g_S(r)d\Omega_{8-p'}. \tag{4.1}$$

In this background there exist in general a non-trivial dilaton field and a RR $p'+1$ form $C_{p'+1}$. The motion of the Dp-brane in this background will be determined by the Dirac-Born-Infeld action given by

$$S_p = T_p \int d^{p+1}\xi e^{-\Phi}\sqrt{-det(\hat{G}_{\alpha\beta})}. \tag{4.2}$$

Following a similar procedure as before the induced metric on the Dp-brane is $d\hat{s}^2 = -d\eta^2 + g(r(\eta))(d\vec{x})^2$ where the cosmic time is given by

$$d\eta = \frac{|g_{00}|g^{\frac{P}{2}}e^{-\Phi}}{|C+E|}dt. \tag{4.3}$$

The analogue of the p+1-dimensional Friedmann equations are determined by defining the scale factor as $\alpha^2 = g$. Then the Friedmann equation we get is given by

$$\frac{8\pi G}{3}\rho_{eff} = \frac{(C+E)^2 g_S e^{2\Phi} - |g_{00}|(g_S g^p + \ell^2 e^{2\Phi})}{4|g_{00}|g_{rr}g_S g^p}\Big(\frac{g'}{g}\Big)^2. \tag{4.4}$$

In this background there is also the possibility of having a constant NS/NS two-form which lives in the world-volume of the Dp'-brane and it can be parametrized by $B = bdx^{p-1} \wedge dx^p$. This constant B field will not affect the background field equation since it enters via its field strength $H = dB$ which is zero for a constant B. However, the probe brane will feel not H but directly the antisymmetric field $B_{\mu\nu}$ through the coupling

$$\begin{aligned} S_p &= T_p \int d^{p+1}\xi e^{-\Phi}\sqrt{-det(\hat{G}_{\alpha\beta} - \hat{B}_{\alpha\beta})} \\ &+ T_p \int d^{p+1}\xi \hat{C}_{p+1} + anomaly\ terms, \end{aligned} \tag{4.5}$$

where

$$\hat{B}_{\alpha\beta} = B_{\mu\nu}\frac{\partial x^\mu \partial x^\nu}{\partial\xi^\alpha\partial\xi^\beta}. \tag{4.6}$$

Then, the induced Friedmann equation on the brane (in case $p = p'$) can be calculated to be

$$\frac{8\pi G}{3}\rho_{eff} = \frac{(C+E)^2 g_S e^{2\Phi} - |g_{00}|\Big(g_S g^{p-2}(g^2+b^2) + \ell^2 e^{2\Phi}\Big)}{4|g_{00}|g_{rr}g_S g^{p-2}(g^2+b^2)}\Big(\frac{g'}{g}\Big)^2. \tag{4.7}$$

IV-A. The Holography Probed by a D3-Brane Moving in the Background Geometry of a D3-Black Brane

We will consider a probe D3-brane moving in the background geometry of a near-extremal black hole with a metric [1]

$$ds_{10}^2 = \frac{1}{\sqrt{H_{p'}}}\Big(-f(r)dt^2 + (d\vec{x})^2\Big) + \sqrt{H_{p'}}\,\frac{dr^2}{f(r)} + \sqrt{H_{p'}}\,r^2 d\Omega^2_{8-p'}, \tag{4.8}$$

where $H_{p'} = 1 + \left(\frac{L}{r}\right)^{7-p'}$, and $f(r) = 1 - \left(\frac{r_0}{r}\right)^{7-p'}$. In this background the RR form is

$$C_{012\ldots p'} = \sqrt{1 + \Big(\frac{r_0}{L}\Big)^{7-p'}}\,\frac{1 - H_{p'}(r)}{H_{p'}(r)}, \tag{4.9}$$

and the dilaton field takes the form $e^{\Phi} = H_{p'}^{(3-p')/4}$. Taking the near horizon limit of the above geometry, we recover the Schwarzschild-Ad$S_5 \times S^5$ black hole geometry discussed in the previous sections. There are also spherically symmetric backgrounds, like the type-0 string background, in which the Ad$S_5 \times S^5$ geometry is obtained only asymptotically. For this particular background, depending on the value of the Tachyon field, the conformal symmetry is restored in the infrared and in the ultraviolet, where the Ad$S_5 \times S^5$ geometry is recovered [50, 51]. The main motivation of our study in this section, is to follow the motion of a probe D3-brane in these backgrounds where the conformal invariance is broken and find how the thermodynamic quantities change and what is their relation to geometrical quantities. This study would be useful to determine the cosmological evolution of a brane-universe moving between conformal points [52].

Defining the parameter $\xi = \left(1 + \left(\frac{r_0}{L}\right)^{7-p'}\right)^{1/2}$ the first Friedmann equation (4.4) in this background is

$$H^2 = \frac{(7-p')}{16L^2}\,\alpha^{\frac{2(3-p')}{(7-p')}}(1-\alpha^4)^{\frac{2(8-p')}{(7-p')}}\Big[\frac{(E+\xi\alpha^4-\xi)^2}{\alpha^8} - \Big(\xi^2 - \frac{\xi^2-1}{\alpha^4}\Big)\Big(1 + \frac{l^2}{L^2}\frac{(1-\alpha^4)^{\frac{2}{(7-p')}}}{\alpha^{4+\frac{8}{(7-p')}}}\Big)\Big] \tag{4.10}$$

We will consider the case of $p' = 3$. In this case the above Friedmann equation becomes

$$H^2 = \frac{1}{L^2}(1-\alpha^4)^{5/2}\Big[\Big(\xi + \frac{\hat{E}}{\alpha^4}\Big)^2 - \Big(\frac{r_0}{L}\Big)^4\frac{1}{\alpha_0^4}\Big(1 - \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big(1 + a(1-\alpha^4)^{1/2}\Big)\Big], \tag{4.11}$$

where $\hat{E} = E - \xi$ and α is given by

$$\alpha = \Big(1 + \Big(\frac{L}{r}\Big)^4\Big)^{-1/4}. \tag{4.12}$$

The second Friedmann equation is

$$
\begin{aligned}
\dot{H} &= -\frac{2}{L^2}\Big[\frac{5}{2}\alpha^4(1-\alpha^4)^{3/2}\Big[\Big(\xi+\frac{\hat{E}}{\alpha^4}\Big)^2-\Big(\frac{r_0}{L}\Big)^4\frac{1}{\alpha_0^4}\Big(1-\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big(1+a(1-\alpha^4)^{1/2}\Big)\Big] \\
&+ (1-\alpha^4)^{5/2}\Big[2\Big(\xi+\frac{\hat{E}}{\alpha^4}\Big)^2-2\xi\Big(\xi+\frac{\hat{E}}{\alpha^4}\Big)+\Big(\frac{r_0}{L}\Big)^4\frac{1}{\alpha^4}\Big(1+a(1-\alpha^4)^{1/2}\Big) \\
&- \Big(\frac{r_0}{L}\Big)^4\frac{1}{\alpha_0^4}\Big(1-\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big(1+a(1-\alpha^4)^{-1/2}\Big)+\frac{3}{2}(1-\alpha^4)^{1/2}\Big]. \qquad (4.13)
\end{aligned}
$$

The energy parameter $\hat{E}$ can be written as

$$
\hat{E}=\tilde{E}+\Big(\frac{r_0}{L}\Big)^4-\Big(1+\Big(\frac{r_0}{L}\Big)^4\Big)^{1/2}. \qquad (4.14)
$$

In the near horizon limit $r_0 \ll L$, the two Friedmann equations (4.11) and (4.13) become identical to (3.4) and (3.5) respectively, with $\hat{E}=\tilde{E}-1$ as can be seen from (4.14). For simplicity we will consider the case of $a=0$. Calculating the effective energy density and pressure from (4.11) and (4.13) and demanding to have a radiation dominated brane-universe, from the equation of state $w=p_{eff}/\rho_{eff}=1/3$ we get

$$
\frac{\hat{E}}{\alpha^4}=-\frac{5}{2}\xi\alpha^4(1+\frac{3}{2}\alpha^4)^{-1}\Big[1\pm[1-\frac{2}{5}\Big(\frac{\alpha_0}{\alpha}\Big)^4\frac{1}{\alpha^4}-\frac{3}{5}\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big]^{1/2}\Big]. \qquad (4.15)
$$

The only real solution of this equation is $\hat{E}/\alpha^4=0$ for $r\to 0$. This is consistent with our previous discussion, because in this limit we find a radiation dominated brane-universe and then our results of a radially moving probe D3-brane are recovered. To find corrections to our thermodynamic quantities we will consider the case of $\hat{E}/\alpha^4=0$ but with $\alpha_0 << 1$. In this limit the first and second Friedmann equations become

$$
\begin{aligned}
H^2 &= \frac{1}{L^2}(1-\alpha^4)^{5/2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4+\alpha_0^4\Big] \qquad (4.16)\\
\dot{H} &= -\frac{2}{L^2}(1-\alpha^4)^{5/2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4+\frac{5\alpha^4}{2(1-\alpha^4)}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4+\alpha_0^4\Big]\Big]. \qquad (4.17)
\end{aligned}
$$

The equation of state can be calculated from the above expressions

$$
p_{eff}=\frac{1}{3}\Big(\frac{1+9\alpha^4}{1-\alpha^4}\Big)\rho_{eff}. \qquad (4.18)
$$

Therefore, as the probe D3-brane moves in the background of D3-black brane, all sorts of Mirage matter is induced on the brane-universe and when $r\to 0$, the brane should pass from a conformal point where the Schwarzschild-Ad$S_5\times S^5$ black hole geometry is restored.

In this limit one can show that still the first and second Friedmann equations (4.16) and (4.17) can be written as the cosmological Cardy-Verlinde and Smarr formulae with the Hubble entropy, Bekenstein-Hawking energy and Hubble entropy given by

$$S_H = \frac{V}{2GL}(1-\alpha^4)^{5/4}\Big(\frac{\alpha_0}{\alpha}\Big)^2\Big[1+\alpha^4\Big]^{1/2} \tag{4.19}$$

$$E_{BH} = \frac{3V}{4\pi G L^2\alpha^2} \tag{4.20}$$

$$T_{Hubble} = \frac{1}{\pi L}\Big(\frac{\alpha_0}{\alpha}\Big)^2(1-\alpha^4)^{5/4}\Big[\frac{1+\frac{5\alpha^4}{2(1-\alpha^4)}(1+\alpha^4)}{\Big(1+\alpha^4\Big)^{1/2}}\Big]. \tag{4.21}$$

An observer in the bulk, measuring distances with the variable r and time with the AdS time t, uses equation (3.19) with $h(r)$ given by

$$h(r) = \Big(1-\Big(\frac{r_0}{r}\Big)^{7-p'}\Big)\Big(1+\Big(\frac{L}{r}\Big)^{7-p'}\Big)^{-1/2} \tag{4.22}$$

and finds the Hawking temperature

$$T_H = \frac{(7-p')}{4\pi}\frac{r_0^{\frac{(5-p')}{2}}}{\sqrt{r_0^{(7-p')}+L^{(7-p')}}}, \tag{4.23}$$

where for $p'=3$ becomes

$$T_H = \frac{1}{\pi}\frac{r_0}{\sqrt{r_0^4+L^4}}. \tag{4.24}$$

An observer on the brane, measures the scale factor α of the brane-universe using the cosmic time η. The cosmic time η is related to the AdS time t through the relation (4.3) which in this background becomes

$$d\eta = \alpha\Big(1+\frac{\hat{E}}{\alpha^4}\Big)^{-1}\Big(1-\Big(\frac{r_0}{r}\Big)^4\Big)dt. \tag{4.25}$$

In the limit we considered, the conformal factor is

$$\lim_{r\to\infty}\frac{dt}{d\eta} = \frac{1}{\alpha}. \tag{4.26}$$

Then using the AdS/CFT relation $T_{CFT} = 1/\alpha T_H$ the CFT temperature is

$$T_{CFT} = \frac{1}{\pi L}\Big(\frac{r_0}{r}\Big)\Big(1+\Big(\frac{L}{r}\Big)^4\Big)^{-1/4}. \tag{4.27}$$

The Casimir energy in this limit can be calculated to be

$$E_C = \frac{3V}{8\pi G L^2}\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big[\Big(1+\Big(\frac{r}{L}\Big)^4\Big)^{-3/2}\Big(1+\alpha_0^4\Big)\Big(\frac{4+6\alpha^4}{1-\alpha^4}\Big)-4\Big(1+\Big(\frac{r}{L}\Big)^4\Big)^{1/2}\Big]. \tag{4.28}$$

The CFT/FRW-cosmologies relations are valid only when the probe D3-brane passes from a conformal point ($r \to 0$) as easily can be checked.

IV-B. A Probe D3-Brane Moving in the Background of Near-Horizon Geometry with a Constant B-field

In this section we will consider the motion of a probe D3-brane in the near-horizon geometry of a D3-brane with a constant B-field. Because we are mainly interested for the corrections of our thermodynamics quantities which are due to the presence of the B-field, we will take first the near-horizon limit of the metric (4.8) and then use (4.7) to find the Friedmann equations.

The near-horizon limit of (4.8) is (3.1) and in this background (4.7) becomes

$$H^2 = \frac{1}{L^2}\Big[\Big(1+\frac{\hat{E}}{\alpha^4}\Big)^2\Big(1+\frac{b^2}{\alpha^4}\Big)^{-1} - \Big(1-\Big(\frac{r_0}{L}\Big)^4\frac{1}{\alpha^4}\Big)\Big(1+a\Big(1+\frac{b^2}{\alpha^4}\Big)^{-1}\Big)\Big]. \qquad (4.29)$$

We are mainly interested for the effect of the B-field so we take $a = 0$ and then the above equation can be rewritten

$$H^2 = \frac{1}{L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + 2\frac{\hat{E}}{\alpha^4} + \frac{\hat{E}^2}{\alpha^8} - \frac{b^2}{\alpha^4}\Big(1+\frac{\hat{E}}{\alpha^4}\Big)^2\Big(1+\frac{b^2}{\alpha^4}\Big)^{-1}\Big]. \qquad (4.30)$$

The presence of the B-field in (4.30) acts effectively as a radiation term on the brane-universe like the electric field we already considered. The second Friedmann equation in this background becomes

$$\dot{H} = -\frac{1}{L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + 2\frac{\hat{E}}{\alpha^4} + 2\frac{\hat{E}^2}{\alpha^8} - \frac{b^2}{\alpha^4}\Big(1+\frac{b^2}{\alpha^4}\Big)^{-2}\Big[1+4\frac{\hat{E}}{\alpha^4}+3\frac{\hat{E}^2}{\alpha^8}+2\frac{b^2}{\alpha^4}\frac{\hat{E}}{\alpha^4}+2\frac{b^2}{\alpha^4}\frac{\hat{E}^2}{\alpha^8}\Big]\Big]. \qquad (4.31)$$

From (4.30) and (4.31) the ρ_{eff} and p_{eff} can be calculated and the equation of state specified. Demanding to have a radiation dominated universe, the energy parameter is fixed to $\hat{E} = b^2$. Using this value for $\hat{E}$, the two Friedmann equations and the effective energy density and pressure on the probe brane become

$$H^2 = \frac{1}{L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{b^2}{\alpha^4}\Big] \qquad (4.32)$$

$$\dot{H} = -\frac{2}{L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{b^2}{\alpha^4}\Big] \qquad (4.33)$$

$$\rho_{eff} = \frac{3}{8\pi G L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + 2\frac{b^2}{\alpha^4} + \frac{b^4}{\alpha^8} - \frac{b^2}{\alpha^4}\ \Big(1+\frac{b^2}{\alpha^4}\Big)\Big] \qquad (4.34)$$

$$p_{eff} = \frac{1}{8\pi G L^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + 2\frac{b^2}{\alpha^4} + 5\frac{b^4}{\alpha^8} - \frac{b^2}{\alpha^4}\ \Big(1+\frac{b^2}{\alpha^4}\Big)^{-2}\Big(1+7\frac{b^2}{\alpha^4}+11\frac{b^4}{\alpha^8}+5\frac{b^6}{\alpha^{12}}\Big)\Big] \qquad (4.35)$$

Observe that the two Friedmann equations (4.32) and (4.33) get a correction in first order in b^2 like the contribution they get from the electric field (in the case $\tilde{E} = 0$), while the effective energy density and pressure are receiving high order corrections in b^2. The Hubble entropy and the Hubble temperature, using (4.32)-(4.35), in this background become

$$\begin{aligned} S_H &= \frac{1}{2GL}\Big(\frac{\alpha_0}{\alpha}\Big)^2\Big(1+\Big(\frac{\alpha_0}{\alpha}\Big)^4\frac{b^2}{\alpha^4}\Big)^{1/2} && (4.36)\\ T_{Hubble} &= \frac{1}{\pi L}\Big(\frac{\alpha_0}{\alpha}\Big)^2\Big(1+\Big(\frac{\alpha_0}{\alpha}\Big)^4\frac{b^2}{\alpha^4}\Big)^{1/2}, && (4.37)\end{aligned}$$

while the Bekenstein Hawking temperature is unchanged and is given by (3.29). It is straitforward to show that the two Friedmann equations can be written in terms of (4.36) and (4.37) and E_{BH}, as the cosmological Cardy-Verlinde and Smarr formulae.

To calculate the thermodynamic quantities we have to find the conformal factor for this background. The conformal time is

$$d\eta = \alpha\Big(1-\Big(\frac{r_0}{r}\Big)^4\Big)\Big(1+\frac{b^2}{\alpha^4}\Big)^{-1/2}dt, \qquad (4.38)$$

from where the conformal factor is

$$\lim_{r\to\infty}\frac{dt}{d\eta} = \frac{1}{\alpha}\Big(1+\frac{b^2}{\alpha^4}\Big)^{1/2}. \qquad (4.39)$$

Using the Hawking temperature $T_H = r_0/\pi L^2$ the CFT temperature according to AdS/CFT correspondence is

$$\begin{aligned} T_{CFT} &= \frac{1}{\alpha}\Big(1+\frac{b^2}{\alpha^4}\Big)^{1/2}T_H\\ &= \frac{1}{\pi L}\Big(\frac{\alpha_0}{\alpha}\Big)\Big(1+\frac{b^2}{\alpha^4}\Big)^{1/2}. && (4.40)\end{aligned}$$

The CFT entropy is unchanged and it is given by (3.25), while the Casimir energy becomes

$$E_C = \frac{3V}{2\pi GL^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{b^2}{\alpha^4} - \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big(1+\frac{b^2}{\alpha^4}\Big)^{1/2}\Big]. \qquad (4.41)$$

At the moment the probe brane crosses the bulk black hole horizon, from (4.37) and (4.40) we get

$$T_{CFT} = T_{Hubble} = \frac{1}{\pi L}\Big(1+\frac{b^2}{\alpha_0^4}\Big)^{1/2}. \qquad (4.42)$$

We also have

$$S_H = \frac{1}{2GL}\Big(1+\frac{b^2}{\alpha_0^4}\Big)^{1/2} \neq S_{CFT} = \frac{1}{2GL} \tag{4.43}$$

$$E_C = \frac{3V}{2\pi GL^2}\Big[1+\frac{b^2}{\alpha_0^4}-\Big(1+\frac{b^2}{\alpha_0^4}\Big)^{1/2}\Big] \neq kE_{BH} = 0. \tag{4.44}$$

Therefore, in the near horizon limit with a constant B-field in the background, there is no CFT/FRW-cosmologies correspondence.

5. Conclusions

We studied the cosmological holographic principle in a generic static spherically symmetric background with a probe D3-brane moving in this background playing the rôle of the boundary to this space. After reviewing the necessary formalism, we followed the motion of the probe D3-brane in specific background geometries under different initial conditions of the probe D3-brane.

First we followed the motion of the probe D3-brane in the near-horizon $AdS_5 \times S^5$ background with a Schwarzschild-AdS_5 black hole [44]. Demanding to have a radiation dominated universe on the probe D3-brane, the energy parameter is fixed and for zero angular momentum we showed that the Cardy-Verlinde and Smarr formulae are equivalent to the first and second Friedmann equations respectively. Using the AdS/CFT correspondence we related the entropy and the Hawking temperature of the AdS$_5$ black hole with the entropy and temperature of the CFT on the probe D3-brane. At the special moment when the probe D3-brane crosses the horizon of the background black hole, an observer on the brane measures the CFT entropy and temperature using pure geometrical quantities, the Hubble parameter and its time derivative. Furthermore, at that particular moment, we showed that the CFT entropy is described by the Cardy-Verlinde formula, with zero Casimir energy. These results indicate that, as the D3-brane crosses the horizon of the background black hole, it probes the holography of the dual CFT theory of the $AdS_5 \times S^5$ geometry and these results are in agreement with [12, 14].

Next we followed the motion of a probe D3-brane having a non-zero angular momentum, in the same near-horizon $AdS_5 \times S^5$ background. We showed that the AdS/CFT correspondence is exact in the sense that the CFT and Hawking temperature are related in the same way as in the case of zero angular momentum. The presence of a non-zero angular momentum on the brane, modifies some of the thermodynamic quantities, and on the horizon the T_{CFT} cannot anymore be expressed in terms of the Hubble parameter and its time derivative. The modification of the second Friedmann equation results in a non-zero Casimir energy which has the consequence that the CFT entropy on the horizon is not described by the Cardy-Verlinde formula as it happens to the zero angular momentum case.

An electric field was also considered on the probe D3-brane. Its presence introduces another energy scale which is proportional to its field strength. It was shown that the electric field acts effectively as a new radiation term on the probe brane. Demanding to have a radiation dominated universe, the energy parameters were fixed and then we calculated the corrections to the thermodynamic quantities due to the electric field. We showed that as the brane crosses the bulk black hole horizon it is no possible any more to relate thermodynamic with geometrical quantities through the CFT/FRW-cosmologies relations. Finally, we considered a more general problem of a probe D3-brane moving in the field of other D3-branes. In the general case, the AdS/CFT correspondence and the CFT/FRW relations break down as expected, but the previous results can be recovered in the near-horizon limit.

It would be interesting to investigate the AdS/CFT correspondence and the CFT/FRW relations in the case of a non-zero ρ^2 term on the brane. This is the term generated in early cosmological evolution in the Randall-Sundrum model. We know that the trace anomaly of the energy momentum tensor is proportional to this ρ^2 term [53]. Then one expects to have a broken conformal theory [54], and it would be interesting to see what kind of holographic description it is possible for such a theory which has a trace anomaly [55, 56, 57].

Another interesting line of investigation is to apply these ideas and formalism to a realistic inflationary brane model. The AdS/CFT correspondence for example can help us to solve the problem of exit from inflation, because the moment the moving brane-universe crosses the horizon of the bulk black hole, it enters a thermal bath and the reheating process of the universe can start. Actually such a model can be realized for a particular background geometry [52].

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