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AdS/CFT Correspondence and the Reheating of the Brane-Universe

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Abstract

We present a mechanism for exit from inflation and reheating using the AdS/CFT correspondence. A cosmological evolution is induced on a probe D3-brane as it moves in a black D-brane background of type-0 string theory. If the tachyon field is non zero, inflation is induced on the brane-universe, with the equation of state parameter in the range $-1 < w < -1/3$, depending on the position of the probe brane in the bulk. As the probe brane approaches the horizon of the background black hole, the inflation rate decreases and the value of w gets larger. At some critical distance away from the horizon, inflation ends. When the brane-universe reaches the horizon, the conformal invariance is restored, the background geometry becomes $AdS_5 \times S^5$, and the brane-universe feels the CFT thermal radiation and reheats.

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1. Introduction

All recent observational data supports the idea that our universe in its early cosmological evolution had an inflationary phase. The most conventional inflationary dynamics is described by a scalar field, the inflaton, with a self-interacting potential [1] and the exit from inflation and reheating of the universe happens when the inflaton field reaches the minimum of the potential and starts oscillating around that minimum. This inflationary scenario can cure the problems of the standard cosmology, like the horizon, flatness and monopole problems, and the inflationary dynamics parametrized by the inflaton field is adequate to describe most of the observational data. However, it is very difficult to produce a complete theoretical model where all the inflationary dynamics comes out in a consistent way from the theory [2]. Specifically, understanding the mechanism of the exit from inflation and reheating is of fundamental interest in the inflationary scenario.

In this direction, it is important to apply new theoretical ideas for understanding the inflationary scenario and the mechanisms involved. Recently, motivated by string theory, the idea of AdS/CFT correspondence was proposed. The AdS/CFT correspondence is build on Maldacena conjecture [3], that the (3+1)-dimensional world-volume of N coinciding D3-branes, in the large N limit, is dual to type IIB superstrings, propagating on the near-horizon $AdS_5 \times S^5$ background geometry [4]. A very important consequence of this conjecture is that, because the (3+1)-dimensional world-volume of N coinciding D3-branes give rise to $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills (SYM) theory [5, 6, 7, 8], the thermodynamics of Yang-Mills theory is linked with the thermodynamics of Schwarzschild black holes embedded in the AdS space [9].

Many ideas about AdS/CFT correspondence were influenced by the intriguing concept of "holography" [10, 11]. The underlying principle, which was originated in the Bekenstein bound [12], is based on the notion that the maximal entropy that can be stored within a given volume will be determined by the largest black hole fitting inside that volume. Since the entropy of a black hole is essentially given by its surface area, it follows directly that all the relevant degrees of freedom of any system must in some sense live on the boundary enclosing that system.

Recently, using the holographic principle, the entropy bounds in a radiation dominated closed Friedmann-Robertson-Walker universe was analyzed [13]. It was found a surprising similarity between Cardy's entropy formula for 1+1 dimensional CFT and the Friedmann equation governing the evolution of the universe. After a suitable identification, it was shown that actually the Cardy's formula [14] maps to the Friedmann equation. In a further development [15] this correspondence between Cardy's formula and the Friedmann equation was tested in the Randall-Sundrum type model [16]. In the case where the bulk is a Schwarschild-AdS background and there is no matter on the brane, the correspondence between Cardy's formula and the Friedmann equation is recovered when the brane crosses the black hole horizon.

We generalized these results in [17] where, using the AdS/CFT correspondence we studied the holographic principle in the near-horizon $AdS_5 \times S^5$ geometry with a probe D3-brane playing the rôle of the boundary to this space. The motion of the probe D3-brane in the bulk, induces a cosmological evolution on the brane. As the brane crosses the horizon of the bulk Schwarzschild-AdS_5 black hole, it probes the holography of the dual CFT. We tested the holographic principle and we found corrections to the entropy relations in various physical cases: for radially moving, spinning and electrically charged D3-brane and for a NS/NS B-field in the bulk.

The cosmological evolution on the brane due to its motion in higher dimensional spacetimes was studied in [18] where the Israel matching conditions were used to relate the bulk to the domain wall (brane) metric, and some interesting cosmological solutions were found. In [19] a universe three-brane is considered in motion in ten-dimensional space in the presence of a gravitational field of other branes. It was shown that this motion in ambient space induces cosmological expansion (or contraction) on our universe, simulating various kinds of matter. This is known as Mirage Cosmology. Using the technics of Mirage cosmology we showed in [20, 21], that if a probe D3-brane moves in a non-conformal background of type-0 string theory, with constant values of dilaton and tachyon fields, a cosmological evolution is induced on the brane-universe, which for some range of the parameters has an inflationary phase and there is also an exit from this phase [22]. In [23], using similar technics the cosmological evolution of the three-brane in the background of type IIB string theory was also considered.

Type-0 string theory is interesting because it does not have space-time supersymmetry and as a result of GSO projection a tachyon field appears in the action [25, 26, 28]. Actually the tachyon field manifests itself in two ways: through its tachyon potential and a function f(T) which couples to the RR flux of the background. It was shown that if this function has an extremum then, because of its coupling to the RR field, stabilizes the tachyon potential driving its mass to positive values [25, 29].

Another appealing feature of type-0 string theory is that, when the tachyon field is non trivial, because of its coupling to dilaton field, there is a renormalization group flow from infrared (IR) to ultra violet (UV) [26, 27]. This corresponds to a flow of the couplings of the dual gauge theory from strong (IR region) to weak couplings (UV region) where the dilaton field gets small and the tachyon field requires a constant value. However, because the evolution equations are complicated when the tachyon and dilaton fields are not constant, the analytic evolution from IR to UV fixed points is not known, and only the behaviour of the theory near the fixed points is well understood. At these two fixed points it has been shown that the background geometry asymptotes to the near-horizon $AdS_5 \times S_5$ geometry [25, 28].

In this work we will use the AdS/CFT correspondence to propose a mechanism for exit from inflation and reheating in the case of a brane-universe moving in a type-0 string background [20, 21, 22, 24]. We will first show that as the brane moves in a conformal $AdS_5 \times S^5$ background, there is no inflation induced on the brane-universe. If the background is generated by the field of other Dp-branes then inflation is possible. In particular we will show that when a probe D3-brane is moving in a black D-brane type-0 string background, inflation is induced on the brane-universe when the tachyon field is not zero, and inflation has its maximum rate when it condenses. During inflation, the equation of state parameter w has the value $w = -1$ when the brane is away from the horizon of the background black hole, while it is increasing, as the brane is approaching the horizon. We will also show, that there is a critical distance from the horizon where inflation on the brane-universe stops and the equation of state parameter becomes $w = -1/3$. When the brane reaches the horizon, the equation of state parameter becomes $w = 1/3$, the background geometry is $AdS_5 \times S^5$ and then because of the AdS/CFT correspondence, the brane-universe feels the thermal radiation and reheats.

The paper is organized as follows. After the introduction in Sect. I, we briefly describe the technics of Mirage cosmology in Sect. II and we apply these technics to a conformal near-horizon $AdS_5 \times S^5$ background. In Sect. III we discuss the case of a probe D3-brane moving in a general background generated by other Dp-branes and in particular the brane motion in the D-brane type-0 string background. In Sect. IV we discuss the inflation generated on the brane-universe and the mechanism to exit from it and reheating, and finally in Sect. V are our conclusions.

2. A Probe D3-Brane Moving in a Near-Horizon $\mathbf{AdS_5 \times S^5}$ Black Hole Background

Consider a probe D3-brane moving in a generic static, spherically symmetric background. We assume the brane to be light compared to the background so that we will neglect the back-reaction. The background metric we consider has the general form

$$ds^2_{10} = g_{00}(r)dt^2 + g(r)(d\vec{x})^2 + g_{rr}(r)dr^2 + g_S(r)d\Omega_5, \tag{2.1}$$

where g_{00} is negative, and there is also a dilaton field Φ as well as a RR background $C(r) = C_{0\ldots3}(r)$ with a self-dual field strength.

The dynamics on the brane will be governed by the Dirac-Born-Infeld action given by

$$\begin{aligned} S &= T_3 \int d^4\xi e^{-\Phi}\sqrt{-det(\hat{G}_{\alpha\beta} + (2\pi\alpha')F_{\alpha\beta} - B_{\alpha\beta})} \\ &+ T_3 \int d^4\xi \hat{C}_4 + anomaly\ terms. \end{aligned} \tag{2.2}$$

The induced metric on the brane is

$$\hat{G}_{\alpha\beta} = G_{\mu\nu}\frac{\partial x^\mu \partial x^\nu}{\partial\xi^\alpha\partial\xi^\beta}, \tag{2.3}$$

with similar expressions for $F_{\alpha\beta}$ and $B_{\alpha\beta}$. For an observer on the brane the Dirac-Born-Infeld action is the volume of the brane trajectory modified by the presence of the anti-symmetric two-form $B_{\alpha\beta}$, and worldvolume anti-symmetric gauge fields $F_{\alpha\beta}$. As the brane moves the induced world-volume metric becomes a function of time, so there is a cosmological evolution from the brane point of view [19].

In the static gauge, $x^\alpha = \xi^\alpha, \alpha = 0,1,2,3$ using (2.3) we can calculate the bosonic part of the brane Lagrangian which reads

$$\mathcal{L} = \sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j} - C(r), \tag{2.4}$$

where $h_{ij}d\varphi^i d\varphi^j$ is the line element of the unit five-sphere, and

$$A(r) = g^3(r)|g_{00}(r)|e^{-2\Phi}, B(r) = g^3(r)g_{rr}(r)e^{-2\Phi}, D(r) = g^3(r)g_S(r)e^{-2\Phi}, \tag{2.5}$$

and $C(r)$ is the RR background. The problem is effectively one-dimensional and can be solved easily. The momenta are given by

$$\begin{aligned} p_r &= -\frac{B(r)\dot{r}}{\sqrt{A(r) - B(r)\dot{r}^2}}, \\ p_i &= -\frac{D(r)h_{ij}\dot{\phi}^j}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\phi}^i\dot{\phi}^j}}. \end{aligned} \tag{2.6}$$

Since (2.4) is not explicitly time dependent and the ϕ-dependence is confined to the kinetic term for $\dot{\phi}$, for an observer in the bulk, the brane moves in a geodesic parametrised by a conserved energy E and a conserved angular momentum l^2 given by

$$\begin{aligned} E &= \frac{\partial\mathcal{L}}{\partial\dot{r}}\dot{r} + \frac{\partial\mathcal{L}}{\partial\dot{\phi}^i}\dot{\phi}^i - \mathcal{L} = p_r\dot{r} + p_i\dot{\phi}^i - \mathcal{L}, \\ l^2 &= h^{ij}\frac{\partial\mathcal{L}}{\partial\dot{\phi}^i}\frac{\partial\mathcal{L}}{\partial\dot{\phi}^j} = h^{ij}p_ip_j. \end{aligned} \tag{2.7}$$

Solving these expressions for $\dot{r}$ and $\dot{\phi}$ we find

$$\dot{r}^2 = \frac{A}{B}(1 - \frac{A}{(C+E)^2}\frac{D+\ell^2}{D}),\ h_{ij}\dot{\varphi}^i\dot{\varphi}^j = \frac{A^2\ell^2}{D^2(C+E)^2}. \tag{2.8}$$

The allowed values of r impose the constraint that $C(r) + E \geq 0$. The induced four-dimensional metric on the brane, using (2.3) in the static gauge, is

$$d\hat{s}^2 = (g_{00} + g_{rr}\dot{r}^2 + g_S h_{ij}\dot{\varphi}^i\dot{\varphi}^j)dt^2 + g(d\vec{x})^2. \tag{2.9}$$

In the above relation we substitute $\dot{r}^2$ and $h_{ij}\dot{\varphi}^i\dot{\varphi}^j$ from (2.8), and using (2.5) we get

$$d\hat{s}^2 = -\frac{g_{00}^2 g^3 e^{-2\phi}}{(C+E)^2}dt^2 + g(d\vec{x})^2. \tag{2.10}$$

We can define the cosmic time η as

$$d\eta = \frac{|g_{00}|g^{\frac{3}{2}}e^{-\Phi}}{|C+E|}dt, \tag{2.11}$$

so the induced metric becomes

$$d\hat{s}^2 = -d\eta^2 + g(r(\eta))(d\vec{x})^2, \tag{2.12}$$

The induced metric on the brane (2.12) is the standard form of a flat expanding universe. We can derive the analogue of the four-dimensional Friedmann equations by defining $g = \alpha^2$

$$\Big(\frac{\dot{\alpha}}{\alpha}\Big)^2 = \frac{(C+E)^2 g_S e^{2\Phi} - |g_{00}|(g_S g^3 + \ell^2 e^{2\Phi})}{4|g_{00}|g_{rr}g_S g^3}\Big(\frac{g'}{g}\Big)^2, \tag{2.13}$$

where the dot stands for derivative with respect to cosmic time while the prime stands for derivatives with respect to r. The right hand side of (2.13) can be interpreted in terms of an effective matter density on the probe brane

$$\frac{8\pi G}{3}\rho_{eff} = \frac{(C+E)^2 g_S e^{2\Phi} - |g_{00}|(g_S g^3 + \ell^2 e^{2\Phi})}{4|g_{00}|g_{rr}g_S g^3}\Big(\frac{g'}{g}\Big)^2, \tag{2.14}$$

where G is the four-dimensional Newton's constant. We can also calculate

$$\begin{aligned}\frac{\ddot{\alpha}}{\alpha} &= \Big(1 + \frac{g}{g'}\frac{\partial}{\partial r}\Big)\frac{(C+E)^2 g_S e^{2\Phi} - |g_{00}|(g_S g^3 + \ell^2 e^{2\Phi})}{4|g_{00}|g_{rr}g_S g^3}\Big(\frac{g'}{g}\Big)^2 \\ &= \Big[1 + \frac{1}{2}\alpha\frac{\partial}{\partial\alpha}\Big]\frac{8\pi G}{3}\rho_{eff}.\end{aligned} \tag{2.15}$$

If we set the above equal to $-\frac{4\pi G}{3}(\rho_{eff} + 3p_{eff})$ we can define the effective pressure p_{eff}.

Therefore, the motion of a D3-brane on a general spherically symmetric background had induced on the brane an energy density and a pressure. Then, the first and second Friedmann equations can be derived giving a cosmological evolution of the brane-universe in the sense that an observer on the brane measures a scale factor $\alpha(\eta)$ of the brane-universe evolution. This scale factor depends on the position of the brane in the bulk. This cosmological evolution is known as "Mirage Cosmology" [19]: the cosmological evolution is not due to energy density on our universe but on the energy content of the bulk.

We will apply the above described formalism to the near-horizon conformal geometry $AdS_5 \times S^5$. There are Schwarzschild-AdS$_5$ black hole solutions in this background with metric

$$ds^2 = \frac{r^2}{L^2}\Big(- f(r)dt^2 + (d\vec{x})^2\Big) + \frac{L^2}{r^2}\frac{dr^2}{f(r)} + L^2 d\Omega_5^2, \tag{2.16}$$

where $f(r) = 1 - \left(\frac{r_0}{r}\right)^4$. The RR field is given by $C = C_{0\ldots3} = \left[\frac{r^4}{L^4} - \frac{r_0^4}{2L^4}\right]$.

Using (2.1) we find in this background

$$\begin{aligned} g_{00}(r) &= -\frac{r^2}{L^2}\Big(1 - (\frac{r_0}{r}\Big)^4\Big) = -\frac{1}{g_{rr}} \\ g(r) &= \frac{r^2}{L^2} \\ g_s(r) &= L^2, \end{aligned} \tag{2.17}$$

and the brane-universe scale factor is $\alpha = r/L$. Substituting the above functions to equation (2.14) we find the analogue of Friedmann equation on the brane which is [19]

$$H^2 = \frac{8\pi G}{3}\rho_{eff} = \frac{1}{L^2}\Big[\Big(1 + \frac{1}{\alpha^4}\Big(E - r_0^4/2L^4\Big)\Big)^2 - \Big(1 - \Big(\frac{r_0}{L}\Big)^4\frac{1}{\alpha^4}\Big)\Big(1 + \frac{l^2}{L^2}\frac{1}{\alpha^6}\Big)\Big], \tag{2.18}$$

where E is a constant of integration of the background field equations, expressing the conservation of energy, and it is related to the black hole mass of the background [24], while $r_0^4/2L^4$ is the constant part of the RR field, expressing essentially electrostatic energy, and it can be absorbed into the energy $\tilde{E} = E - r_0^4/2L^4$. This Friedmann equation describes the cosmological evolution of a contracting or expanding universe depending on the motion of the probe brane. This motion in turn depends on two parameters the energy $\tilde{E}$ and the angular momentum l^2. These two parameters specify various trajectories of the probe brane. The scale factor α comes in various powers, indicating that (2.18) describes the cosmological evolution of various kind of Mirage or stiff cosmological matter.

Defining the dimensionless parameter $a = l^2/L^2\alpha^6$, equation (2.18) becomes

$$H^2 = \frac{8\pi G}{3}\rho_{eff} = \frac{1}{L^2}\Big[\Big(1 + \frac{\tilde{E}}{\alpha^4}\Big)^2 - \Big(1 - \Big(\frac{r_0}{L}\Big)^4\frac{1}{\alpha^4}\Big)\Big(1 + a\Big)\Big]. \tag{2.19}$$

Using equations (2.15) and (2.19), the second Friedmann equation in this background reads

$$\dot{H} = -\frac{2}{L^2}\Big[2\frac{\tilde{E}}{\alpha^4}\Big(1 + \frac{\tilde{E}}{\alpha^4}\Big) + \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big(1 + a\Big) - \frac{3}{2}a\Big(1 - \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big], \tag{2.20}$$

and the effective pressure, using again (2.15), is

$$p_{eff} = \frac{1}{8\pi GL^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + 5\frac{\tilde{E}^2}{\alpha^8} + 2\frac{\tilde{E}}{\alpha^4} + 7a\Big(\frac{\alpha_0}{\alpha}\Big)^4 - 3a\Big]. \tag{2.21}$$

From (2.19) we also have the effective energy density

$$\rho_{eff} = \frac{3}{8\pi GL^2}\Big[\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{E}^2}{\alpha^8} + 2\frac{\tilde{E}}{\alpha^4} - a\Big(1 - \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)\Big]. \tag{2.22}$$

It is instructive to consider the equation of state $p_{eff} = w\rho_{eff}$ where w is given by

$$w = \frac{1}{3}\Big[\frac{\Big(\frac{\alpha_0}{\alpha}\Big)^4 + 5\frac{\tilde{E}^2}{\alpha^8} + 2\frac{\tilde{E}}{\alpha^4} + 7a\Big(\frac{\alpha_0}{\alpha}\Big)^4 - 3a}{\Big(\frac{\alpha_0}{\alpha}\Big)^4 + \frac{\tilde{E}^2}{\alpha^8} + 2\frac{\tilde{E}}{\alpha^4} - a\Big(1 - \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big)}\Big]. \tag{2.23}$$

As the brane moves in the Schwarzschild-AdS$_5$ black hole background, the equation of state is parametrized by the energy of the bulk and the angular momentum of the brane. However, when $\tilde{E} = 0$ and $a = 0$ the brane-universe is radiation dominated at any position in the bulk [17], as can be seen from (2.23). This is expected, because the only scale in the theory is the energy scale and putting it to zero the theory is scale invariant, while a non-zero angular momentum induces on the brane all kind of Mirage matter. This can also be understood with the use of the AdS/CFT correspondence. The moving probe D3-brane is playing the rôle of the boundary of the $AdS_5 \times S^5$ geometry. Because the background space is an AdS space, the dual theory on the brane is CFT, therefore the brane at any position would be in a thermal radiation state.

We can also investigate if an inflationary phase can be generated on the brane-universe as it moves in the near-horizon $AdS_5 \times S^5$ geometry. Defining the acceleration parameter I as

$$I = H^2 + \dot{H} \tag{2.24}$$

and using (2.19) and (2.20), and putting $a = 0$, we get

$$I = -\frac{1}{L^2}\Big[3\frac{\tilde{E}}{\alpha^4} + 2\frac{\tilde{E}^2}{\alpha^8} + \frac{\alpha_0^4}{\alpha^4}\Big]. \tag{2.25}$$

Therefore for positive energy and no angular momentum on the brane, there is no inflation induced on the brane-universe. Observe that for $\tilde{E} = 0$, I is always negative irrespective of the brane position. To have inflation on the brane the background should not be conformal as we will discuss in the following.

3. The Non-Conformal Type-0 String Background

In this section we will consider more general non-conformal backgrounds. Such backgrounds are described by Dp-black branes with a metric [5]

$$ds^2_{10} = \frac{1}{\sqrt{H_p}}\Big(- f(r)dt^2 + (d\vec{x})^2\Big) + \sqrt{H_p}\,\frac{dr^2}{f(r)} + \sqrt{H_p}\,r^2 d\Omega^2_{8-p}, \tag{3.1}$$

where $H_p = 1 + \left(\frac{L}{r}\right)^{7-p}$, and $f(r) = 1 - \left(\frac{r_0}{r}\right)^{7-p}$. In this background the RR form is

$$C_{012\ldots p} = \sqrt{1 + \Big(\frac{r_0}{L}\Big)^{7-p}}\,\frac{1 - H_p(r)}{H_p(r)}, \tag{3.2}$$

and the dilaton field takes the form $e^{\Phi} = H_p^{(3-p)/4}$. Taking the near horizon limit of the above geometry, we recover the Schwarzschild-Ad$S_5 \times S^5$ black hole geometry discussed in the previous section. The technics of Mirage cosmology can be applied to these general backgrounds [19] and the Friedmann equation on the brane can be derived. In particular, we will discuss the D-brane background of type-0 string theory.

The action of the type-0 string is given by [25]

$$\begin{aligned} S_{10} \; = \; & \int d^{10}x\sqrt{-g}\Big[e^{-2\Phi}\Big(R + 4(\partial_\mu\Phi)^2 - \frac{1}{4}(\partial_\mu T)^2 - \frac{1}{4}m^2T^2 - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}\Big) \\ & -\frac{1}{4}(1 + T + \frac{T^2}{2})|F_5|^2\Big]. \end{aligned} \tag{3.3}$$

The tachyon is coupled to the RR field through the function

$$f(T) = 1 + T + \frac{1}{2}T^2. \tag{3.4}$$

The tachyon field appearing in (3.3) is a result of GSO projection and there is no spacetime supersymmetry in the theory. The tachyon field appears in (3.3) through its kinetic term, its potential and through the tachyon function $f(T)$. The potential term is giving a negative mass squared term which however can be shifted to positive values if the function $f(T)$ has an extremum. This happens in the background where the tachyon field acquires vacuum expectation value $T_{vac} = -1$ and $f'(T_{vac}) = 0$ [25, 29]. In this background the dilaton equation is

$$\nabla^2\Phi = -\frac{1}{4\alpha'}e^{\frac{1}{2}\Phi}T^2_{vac}. \tag{3.5}$$

This equation is giving a running of the dilaton field which means that the conformal invariance is lost, and $AdS_5 \times S^5$ is not a solution. Therefore, the tachyon condensation is responsible for breaking the 4-D conformal invariance of the theory and as we will see in the next section it also induces inflation on a probe-brane moving in this background. However, the conformal invariance is restored in two conformal points, corresponding to IR and UV fixed points, when the tachyon field gets a constant value. The flow from IR to UV as exact solutions of the equations of motion derived from action (3.3) is not known, but when the dilaton and tachyon fields are constant such a solution can be found [25, 28] which can be extended to a black D-brane [30]

$$ds_{10}^2 = \frac{1}{\sqrt{H}}\Big(-\phi(r)dt^2 + (d\vec{x})^2\Big) + \sqrt{H}\,\frac{dr^2}{\phi(r)} + \sqrt{H}\,r^2 d\Omega_5^2, \tag{3.6}$$

where $H = 1 + \left(\frac{e^{\Phi_0}Q}{2r^4}\right)$, and $\phi(r) = 1 - \left(\frac{r_0}{r}\right)^4$. Q is the electric RR charge and Φ_0 denotes a constant value of the dilaton field. If we define $L = \left(e^{\Phi_0}Q/2\right)^{1/4}$ we can write $H = 1 + \left(\frac{L}{r}\right)^4$.

In this non-conformal background inflation can be induced on the brane-universe. The case of a constant dilaton and tachyon field was studied in [20], while in [21] approximate solutions of the equations of motion were used to study the induced on the brane inflationary phase.

4. AdS/CFT Correspondence, Exit from Inflation and Reheating

In this section we will study the way the inflation ends and reheating starts on the brane-universe using the AdS/CFT correspondence. In the background metric (3.6) using (2.1) we find

$$\begin{aligned}
g_{00} &= -H^{-1/2}\phi(r) \\
g &= H^{-1/2} = \alpha^2 = \Big(1 + \Big(\frac{L}{r}\Big)^4\Big)^{-1/2} \\
g_{rr} &= H^{1/2}\phi^{-1}(r) \\
g_s &= H^{1/2}r^2
\end{aligned} \tag{4.1}$$

and the RR field is given by $C = e^{-\Phi_0}f^{-1}(T)\Big(1 + \left(\frac{L}{r}\right)^4\Big)^{-1} + Q_1$ where Q_1 is an integration constant. Substituting (4.1) into (2.14) we get the induced Friedmann equation on the brane

$$H^2 = \frac{(1-\alpha^4)^{5/2}}{L^2}\Big[\Big(f^{-1}(T) + \frac{\tilde{E}}{\alpha^4}\Big)^2 - \Big(1 - \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big(\frac{1-\alpha^4}{1-\alpha_0^4}\Big)\Big)\Big(1 + a\Big(1-\alpha^4\Big)^{1/2}e^{2\Phi_0}\Big)\Big], \tag{4.2}$$

where $\tilde{E} = (Q_1 + E)e^{\Phi_0}$. To simplify the discussion we consider the case where the constant value of the dilaton field is zero and also the angular momentum of the probe brane is also zero, $a = 0$. In this case from (4.2) we can calculate the second Friedmann equation and then the equation of state

$$
\begin{aligned}
w = \frac{p_{eff}}{\rho_{eff}} &= \frac{10}{3}\alpha^4(1-\alpha^4)^{-1} + \frac{1}{3}\Big\{\Big[8\frac{\tilde{E}}{\alpha^4}\Big(f^{-1}(T) + \frac{\tilde{E}}{\alpha^4}\Big) - 3\Big(f^{-1}(T) + \frac{\tilde{E}}{\alpha^4}\Big)^2 \\
&+ 3\Big(1 - \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big(\frac{1-\alpha^4}{1-\alpha_0^4}\Big)\Big) + 4\Big(\frac{\alpha_0}{\alpha}\Big)^4\Big(1-\alpha_0^4\Big)^{-1}\Big] \\
&\Big/ \Big[\Big(f^{-1}(T) + \frac{\tilde{E}}{\alpha^4}\Big)^2 - \Big(1 - \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big(\frac{1-\alpha^4}{1-\alpha_0^4}\Big)\Big)\Big]\Big\}.
\end{aligned} \tag{4.3}
$$

This equation depends on the energy parameter $\tilde{E}$. We know that if we take the near-horizon limit of (3.6) the geometry becomes exact $AdS_5 \times S^5$ and then because of the AdS/CFT correspondence the dual theory on the brane must be CFT. We use this fact to fix the free parameter $\tilde{E}$. Considering that the bulk black hole is near extremal and taking the near-horizon limit $\alpha << 1$ of (4.3) we find

$$
w = \frac{1}{3}\Big(\frac{5\frac{\tilde{E}^2}{\alpha^8} + 2\frac{\tilde{E}}{\alpha^4}f^{-1}(T) + 3(1 - f^{-2}(T)) + \frac{\alpha_0^4}{\alpha^4}}{f^{-2}(T) + 2f^{-1}(T)\frac{\tilde{E}}{\alpha^4} + \frac{\tilde{E}^2}{\alpha^8} + \frac{\alpha_0^4}{\alpha^4} - 1}\Big). \tag{4.4}
$$

Demanding to have $w = 1/3$ we get

$$
\frac{\tilde{E}}{\alpha^4} = \pm\Big(f^{-2}(T) - 1\Big)^{1/2}. \tag{4.5}
$$

Note that if the tachyon field is zero, also $\tilde{E} = 0$, then from (4.4), $w = 1/3$. This is because with $T = 0$ we recover the near-horizon $AdS_5 \times S^5$ geometry and then the results of Sect. II are valid. We know that we do not have inflation induced on the brane in the near-horizon $AdS_5 \times S^5$ geometry, therefore we parametrize the departure from the near-horizon limit by

$$
\frac{\tilde{E}}{\alpha^4} = \pm\Big(f^{-2}(T) - 1\Big)^{1/2}\frac{\alpha_0^4}{\alpha^4}\ . \tag{4.6}
$$

In the above relation we have used the characteristic scale in the bulk which is set by the horizon of the black hole. The departure from the near-horizon limit is then equivalent to how far away the probe brane is from the horizon of the bulk black hole. We substitute (4.6) into (4.2) (for $\Phi_0 = a = 0$) and we get

$$
H^2 = \frac{1}{L^2}\Big[f^{-2}(T)\Big(1 + \Big(\frac{\alpha_0}{\alpha}\Big)^8\Big) \pm 2\Big(\frac{\alpha_0}{\alpha}\Big)^4 f^{-1}(T)\Big(f^{-2}(T) - 1\Big)^{1/2} - 1 - \Big(\frac{\alpha_0}{\alpha}\Big)^8 + \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big]. \tag{4.7}
$$

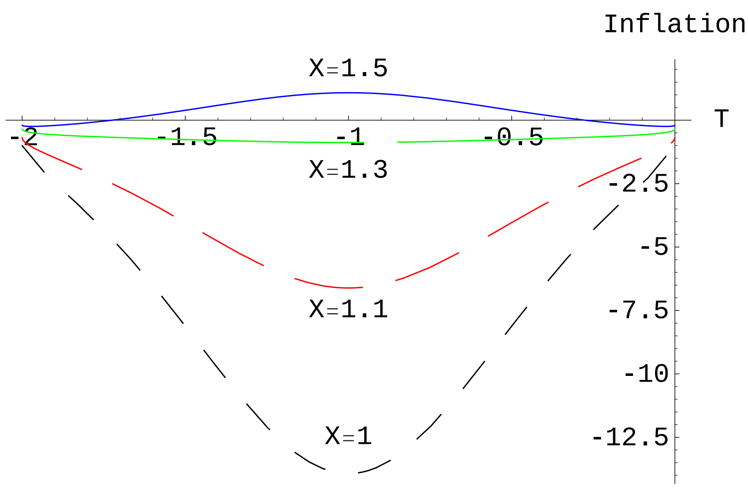

Figure 1: Inflation as a function of the tachyon field T, in various distances from the horizon, where X= α/α_0.

Using the above Friedmann equation we can calculate the effective energy density and pressure on the brane and then the equation of state becomes $p_{eff} = w\rho_{eff}$, where

$$w = \frac{1}{3}\Big\{\frac{f^{-2}(T)\Big(-3+5\big(\frac{\alpha_0}{\alpha}\big)^8\Big) \pm 2\big(\frac{\alpha_0}{\alpha}\big)^4 f^{-1}(T)\Big(f^{-2}(T)-1\Big)^{1/2} + 3 - 5\big(\frac{\alpha_0}{\alpha}\big)^8 + \big(\frac{\alpha_0}{\alpha}\big)^4}{f^{-2}(T)\Big(1+\big(\frac{\alpha_0}{\alpha}\big)^8\Big) \pm 2\big(\frac{\alpha_0}{\alpha}\big)^4 f^{-1}(T)\Big(f^{-2}(T)-1\Big)^{1/2} - 1 - \big(\frac{\alpha_0}{\alpha}\big)^8 + \big(\frac{\alpha_0}{\alpha}\big)^4}\Big\}. \quad (4.8)$$

Observe that on the horizon $\alpha_0 = \alpha$, we get $w = 1/3$, independently of the value of the tachyon field, as expected. The acceleration parameter (2.24), for positive energy, using (4.7) is

$$I = \frac{1}{L^2}\Big[f^{-2}(T)\Big(1-3\Big(\frac{\alpha_0}{\alpha}\Big)^8\Big) - 2\Big(\frac{\alpha_0}{\alpha}\Big)^4 f^{-1}(T)\Big(f^{-2}(T)-1\Big)^{1/2} - 1 + 3\Big(\frac{\alpha_0}{\alpha}\Big)^8 - \Big(\frac{\alpha_0}{\alpha}\Big)^4\Big]. \quad (4.9)$$

In Fig. 1 we plotted the acceleration parameter I as a function of the tachyon field. Inflation can start at a critical distance $\alpha \sim 1.37\alpha_0$ from the horizon and depends on the value of the tachyon field. During inflation the tachyon field gives the maximum rate of inflation when it condenses at $T_{vac} = -1$. In Fig. 2 we plotted the acceleration parameter as a function of equation of state w parameter. At the critical value $\alpha = 1.37\alpha_0$, when tachyon condenses, $w = -1/3$ and inflation starts. For distances $\alpha > 1.37\alpha_0$ w is decreasing and for larger values of the distance goes asymptotically to $w = -1$. For $\alpha < 1.37\alpha_0$ where there is no inflation, w is increasing and on the horizon takes the value $w = 1/3$.

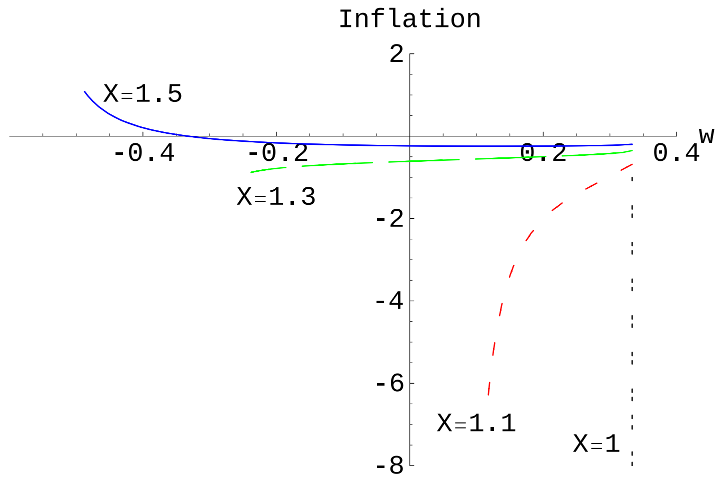

Figure 2: Inflation as a function of the equation of state parameter w. Again here X= α/α_0.

The physical picture we get from our previous analysis is as follows. We can trust the theory only near the conformal points where the $AdS_5 \times S^5$ geometry is restored. We parametrized the departure from the exact $AdS_5 \times S^5$ geometry by the ratio α/α_0 where α_0 is the horizon of the bulk black hole. As the inflating probe brane approaches the black hole horizon the inflation rate decreases and the equation of state parameter w increases departing from its asymptotic value $w = -1$. At some critical distance from the horizon, depending on the value of the tachyon field, the probe brane stops inflating and at that moment $w = -1/3$. As the probe brane comes nearer to the horizon, w increases, and on the horizon, where the near-horizon $AdS_5 \times S^5$ geometry is restored, $w = 1/3$ and the brane-universe reheats.

5. Conclusions

We have presented a mechanism for exit from inflation and reheating using the AdS/CFT correspondence. We followed the motion of a probe D3-brane moving in a black type-0 string background. For constant values of the dilaton and tachyon fields, inflation is induced on the brane-universe. At relative large distances from the horizon of the background black hole, the equation of state parameter w has its minimum value $w = -1$, while inflation has its maximum rate when the tachyon field condenses. As the brane comes nearer to the horizon, the inflation rate decreases and the value of w gets larger. At some critical distance away from the horizon, inflation ends. When the brane-universe reaches the horizon, the conformal invariance is restored, the background geometry becomes $AdS_5 \times S^5$, the dual theory on the brane is CFT and the brane-universe reheats.

The presence of the closed tachyon field in the background, is crucial to the generation of inflation on the brane-universe. If $T = 0$ the theory has an exact conformal invariance, the background geometry is $AdS_5 \times S^5$ and there is no inflation induced on the brane-universe. The tachyon field couples to the RR flux of the background with the function $f(T)$. Because of this coupling, when the tachyon field condenses, the theory is stabilized. It also couples to the dilaton field and because of that, the tachyon condensation is responsible for breaking the 4-D conformal invariance of the theory. When the tachyon field condenses, the induced inflation on the brane-universe has its maximum rate. However, the tachyon function $f(T)$ is arbitrary and it is not predicted by the theory. In this work we considered a polynomial form of the function, but it would be interesting to consider other forms, like exponential, and study its effect to the cosmological evolution on the brane-universe.

We have considered the simplest case of a probe brane moving radially in the D-brane type-0 string background. A non zero angular momentum on the probe brane is another source of breaking the conformal invariance of the theory and it would be interesting to see what is the effect of a non zero angular momentum on the brane inflation. Another possible extension of our work is to consider an electric field on the brane. The electric field on the brane acts effectively as radiation term on the brane-universe and it is interesting to see its effect on the reheating.

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