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The periodic Euler-Bernoulli equation. (English summary)

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The author continues the study of the Floquet spectral theory of the infinite periodic beam equation, namely the spectral problem on the axis

$$(a(x)u'')'' = \lambda\rho(x)u, \quad -\infty < x < \infty,$$

where $\rho(x)$ and $a(x)$ are periodic and strictly positive. He develops a theory analogous to the theory of the Hill operator using the concept of pseudospectrum introduced in [V. G. Papanicolaou and D. Kravvaritis, J. Differential Equations **150** (1998), no. 1, 24–41; MR1660270]. *Vyacheslav N. Pivovarchik*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.