

**MR1632661 (99f:34015)** 34A60 34B15 49J45

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**Existence and relaxation results for nonlinear second-order multivalued boundary value problems in  $\mathbf{R}^N$ . (English summary)**

*J. Differential Equations* **147** (1998), no. 1, 123–154.

The authors consider the second-order boundary value problem

$$(1) \quad \begin{cases} x''(t) \in F(t, x(t), x'(t)) & \text{a.e. on } T, \\ (x'(0), -x'(b)) \in \xi(x(0), x(b)). \end{cases}$$

Here  $T = [0, b]$ ,  $F: T \times \mathbf{R}^N \times \mathbf{R}^N \rightarrow 2^{\mathbf{R}^N} \setminus \{\emptyset\}$  and  $\xi: \mathbf{R}^N \times \mathbf{R}^N \rightarrow 2^{\mathbf{R}^N \times \mathbf{R}^N}$  is a maximal monotone map. The solutions of (1) are sought in the Sobolev space  $W^{2,1}(T, \mathbf{R}^N)$ . If some hypotheses are met, then problem (1) has a solution  $x \in W^{2,1}(T, \mathbf{R}^N)$ . The existence theorems are proved for both the convex and nonconvex problems. This formulation covers general problems such as the Dirichlet, Neumann, and periodic problems. *Oleg P. Filatov*

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*