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**Power series expansions for Mathieu functions with small arguments. (English summary)**

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The authors in this paper derive some power series expansions for the even and odd angular Mathieu functions with small arguments. It is well known that these functions are solutions of Mathieu's differential equation $(*)$ $d^2y/d\theta^2 + (b - (h^2/2)\cos 2\theta)y = 0$, where $b$ and $h^2$ are constants, as well as that only for certain discrete values (eigenvalues) $\mathrm{be}_m$ or $\mathrm{bo}_m$, $m$ integer, the corresponding Mathieu eigenfunctions, denoted by $\mathrm{Se}_m$ or $\mathrm{So}_m$, are periodic, even or odd, respectively.

The authors first deal with the even case. By substituting the following expression representing the even functions given by the series $(**)$ $\mathrm{Se}_m(h, \cos\theta) = \sum_{n=0}^{\infty} B_n^{\mathrm{e}}(h, m)\cos n\theta$ $(m \geq 0,$ $m$ and $n$ being both even or odd) into $(*)$, they obtain some recurrence relations for the coefficients $B_n^{\mathrm{e}}(h, m)$ whose expansions in power series in $h$ lead to approximations for $\mathrm{Se}_m$, valid for small values of $h$.

The calculated expansion coefficients turn out also to be useful for the evaluation of the even radial functions of any kind. Since these functions are solutions of the equation $(*)$ with $\theta = i\mu$ instead of $\theta$, they can be expressed in terms of the cylindrical Bessel functions of the same kind.

The calculation of the expansion coefficients for the odd angular functions of integral order given by the series $\mathrm{So}_m(h, \cos\theta) = \sum_{n=1}^{\infty} B_n^{\mathrm{o}}(h, m)\sin n\theta$, $m \geq 1$ $(m$ and $n$ being both even or odd) follows a similar treatment to the even case.

Some relations among the expansion coefficients for even and odd functions, along with a comparison with other known results, are given also. *N. Hayek Calil*

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*