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Multiple solutions for nonlinear discontinuous strongly resonant elliptic problems. (English summary)

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The authors consider strongly resonant problems with discontinuous right-hand side: $-\mathrm{div}(|Dx(z)|^{p-2}Dx(z))-\lambda_1|x(z)|^{p-2}x(z)\in\widehat{f}(z,x(z))$ a.e. on Z, $x|_{\partial Z}=0$, $2\le p<\infty$, where $\widehat{f}(z,x(z))=\{y\in\mathbf{R}\colon\ f_1(z,x(z))\le y\le f_2(z,x(z))\}$, $Z\subseteq\mathbf{R}^N$ is a bounded domain, $\lambda_1=\min\{\|Dx\|_p^p/\|x\|_p^p\colon\ x\in W_0^{1,p}(Z),\ x\ne 0\}$. Passing to a multivalued problem by filling the gaps at the discontinuity points, the authors prove the existence of at least three solutions. The proof uses the nonsmooth critical point theory for locally Lipschitz functionals due to Chang and a generalized version of the Ekeland variational principle. A well-known result of the "smooth" case is extended by showing that the nonsmooth (PS)-condition implies the coercivity of the functional.

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