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Transition vector measures and multimeasures and parametric set-valued integrals. (English summary)

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In the 1960s R. J. Aumann and G. Debreu developed the theory of multi- (= set-valued) measures; this was continued by C. Castaing, M. Valadier and others. At first these arose probably out of a natural curiosity to see how much of the usual (i.e. classical) theory of (vector) measures extended to the set-valued situation. However, multifunctions/multimeasures are also useful in applied areas such as mathematical economics, statistics, optimization and optimal control. Let us note (from the author's introduction) that transition multimeasures are useful in Markov temporary equilibrium processes in dynamic economics. For some of the theory we refer the reader to, e.g., C. Castaing and M. Valadier [*Convex analysis and measurable multifunctions*, Lecture Notes in Math., 580, Springer, Berlin, 1977; MR0467310].

Let (Ω,Σ,μ) be a finite positive measure space, X a real Banach space and $F\colon\Omega\to X$ a multi- (= nonempty set-valued) function; F is said to be measurable if for each x in X the (associated) nonnegative distance function $\omega\mapsto d(x,F(\omega))$ is measurable. Recall that d is the distance of x in X from the nonempty set $F(\omega)$. Further, let $S^1_F=\{f\in L^1(\mu)\colon\ f(\omega)\in F(\omega)\ \mu\text{-a.e.}\}$ = the set of all $L^1(\mu)$ selections of F. Although this set can be empty, under reasonable conditions on F it is not, and it is used to define the set-valued integral $\int_\Omega F(\omega)d\mu(\omega)$. Now let $C_c(X)$ be the family of nonempty convex, closed sets in X (denoted by $P_{f(C)}(X)$ in the paper), $M\colon\Sigma\to C_c(X)$ a set-valued set function, and let σ be the support function of elements of $C_c(X)$; then M is called a multimeasure if the associated set function $A\mapsto\sigma(x^*,M(A))$ on Σ is a signed measure for each x^* in the continuous dual X^* of X. Next, let T be a Polish space, $(T,\mathcal{T})$ a second measurable space, X separable; then the map $m\colon\mathcal{T}\times\Omega\to X$ is called a transition vector measure if (i) the function $\omega\mapsto m(A,\omega)$ is measurable for each A in $\mathcal{T}$, and (ii) $m(A,\omega)$ is a measure for each ω in Ω. Next, transition multimeasures M are defined with nonempty closed-set values with the difference that in condition (ii) we have multi- (vector) measures; transition selectors are transition vector measures.

In the present paper the author first proves a Radon-Nikodým (RN) theorem for transition vector measures of bounded variation; of course X is assumed to have the RN property. It should be remarked that the proof is not just an extension of classical (or modern) RN theorems found, e.g., in [J. Diestel and J. J. Uhl, Jr., *Vector measures*, Amer. Math. Soc., Providence, R.I., 1977; MR0453964]. There are some difficulties. handled (in part) by a convergence theorem for vector-valued conditional expectations, due to Neveu and Ionescu Tulcea [see, e.g., M. Métivier, *Semimartingales*, de Gruyter, Berlin, 1982; MR0688144(Theorem 11.2)], and by a theorem K. R. Parthasarathy [*Introduction to probability and measure*, Springer-Verlag New York Inc., New York, 1978; MR0651013(Proposition 48.1, Remark 48.3)]. Next, under some (reasonable) hypotheses on the measure spaces, the author establishes existence of RN derivatives for transition multimeasures with respect to (nonnegative) transition measures. This result extends a previous one proved by the author under additional hypotheses; the present, different proof is rather nontrivial. In addition, if X is finite-dimensional, a weakening of a hypothesis is possible. Finally, a curious result is proved: Given a measurable function μ on Ω whose values lie in the space of all bounded positive Radon measures on (a Polish space) T with the narrow topology, the author associates a transition measure $\mu^\frown$ (on a suitable cross product) with μ by a natural process—and a converse too, provided T is compact.

Although X necessitates use of some Banach space techniques, the proofs in this paper are for the most part measure-theoretic.

For precise details and references please see the paper under review.

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