MR891162 (88k:28007) 28A20 54C60 60G99
Lucchetti, R. [Lucchetti, Roberto E.] (I-MILAN); **Papageorgiou, N. S.** (1-IL); **Patrone, F.** (I-PAVI)
Convergence and approximation results for measurable multifunctions.
Proc. Amer. Math. Soc. **100** (1987), *no. 3,* 551–556.

In the paper under review the authors employ the Kuratowski-Mosco convergence of sets to extend some of the convergence and approximation results of G. Salinetti and R. J.-B. Wets [Trans. Amer. Math. Soc. **266** (1981), no. 1, 275–289; MR0613796] to separable Banach spaces. Let (Ω, Σ) be a complete measurable space, X a separable Banach space, F and F_n, $n \geq 1$, graph-measurable multifunctions on Ω with nonempty values in X and let $F(\omega) \subset s\text{-}\underline{\lim}\, F_n(\omega)$ for all $\omega \in \Omega$. Then any Castaing representation of F can be obtained as the strong limit of Castaing representations of the multifunctions F_n.

The second main theorem of the paper is a useful approximation result for a large class of multifunctions. If F is a weakly measurable multifunction with nonempty closed convex values in a separable Banach space X, then F is the Kuratowski-Mosco limit of a sequence F_n of weakly measurable countably simple multifunctions with nonempty closed (and if so desired, also convex) values in X. In the final theorem X is a separable, reflexive Banach space and F a multifunction on Ω with nonempty closed, convex values in X. Then there is an equivalence between the weak measurability and the approximability of F by countably simple multifunctions on Ω. *P. Maritz*

References

1. H. Attouch, *Variational convergence for functions and operators*, Pitman, Boston, Mass., 1984. MR0773850
2. C. Castaing and M. Valadier, *Convex analysis and measurable multifunctions*, Lecture Notes in Math., vol. 580, Springer, Berlin, 1977. MR0467310
3. J. Diestel, *Geometry of Banach spaces—Selected topics*, Lecture Notes in Math., vol. 485, Springer, Berlin, 1975. MR0461094
4. C. Himmelberg, *Measurable relations*, Fund. Math. **87** (1975), 53-72. MR0367142
5. K, Kuratowski, *Topology.* I, Academic Press, New York, 1966. MR0217751
6. U. Mosco, *Convergence of convex sets and of solutions of variational inequalities*, Adv. in Math. **3** (1969), 510-585. MR0298508
7. G. Salinetti and R. Wets, *On the convergence of closed valued measurable multifunctions*, Trans. Amer. Math. Soc. **266** (1981), 275-289. MR0613796
8. V. Toma, *Quelgues problèmes de mésurabilité de multifonctions*, Séminaire d'Analyse Convexe, Montpellier 13, 1983, pp. 6.1-6.17. MR0746115
9. M. Tsukada, *Convergence of best approximations in smooth Banach spaces*, J. Approx. Theory **40** (1984), 301-309. MR0740641
10. D. Wagner, *Survey of measurable selection theorems*, SIAM J. Control. Optim. **15** (1977), 859-903. MR0486391
11. F. Hiai, *Convergence of conditional expectations and strong laws of large numbers for multivalued random variables*, Trans. Amer. Math. Soc. **291** (1985), 613-627. MR0800254
12. J. P. Aubin and A. Cellina, *Differential inclusions*, Springer, Berlin, 1984. MR0755330
13. E. Klein and A. Thompson, *Theory of correspondences*, Wiley, New York, 1984. MR0752692

14. M. A. Khan, *On extensions of the Cournot-Nash theorem*, Advances in Equilibrium Theory, (C. D. Aliprantis et al., eds.), Springer, New York, 1985. MR0873761

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.