IFAC DECOM-TT 2004
Automatic Systems for Building the Infrastructure in Developing Countries
October 3 - 5, 2004 Bansko, Bulgaria

OPTIMIZING UNIT COMMITMENT TECHNIQUE FOR OPERATING AUTOMATED HYDRO-ELECTRIC POWER PLANTS

Arsen Arsenov[1] and Georgi. M. Dimirovski[1, 2]

[1] *SS Cyril and Methodius University, Faculty of Electrical Engineering*
P.O. Box 574, MK-1000 Skopje, Republic of Macedonia
Fax: +389-2-306-4262; E-mail: aarsenov@etf.ukim.edu.mk

[2] *Dogus University, Computer Engineering Department, Faculty of Engineering*
Acibadem, Zeamet Sk. 21, Kadikoy, TR-34722 Istanbul, Republic of Turkey
Fax: +90-216-327-9631; E-mail: gdimirovski@dogus.edu.tr

Abstract: A systems approach to optimized, energy efficient, hydro-electric power plant operation via commitment of optimum number of generating units has been elaborated and tested in a case study. The application case study is a hydro-electric plant comprising 4 generating units taken from the national power industry practice. This paper presents an optimization technique developed for solving the problem of economic unit commitment dispatch for automated hydro-electric power plants operating under a given power demand. It is based on minimizing a cost function which exploits operational characteristics of water and power consumption. The technique derived is an iterative one with solutions at each iteration being obtained via analytical formulae. *Copyright © 2004 IFAC*

Keywords: Economic dispatch; electrical power systems; hydro-electrical plants; operation planning; optimization.

1. INTRODUCTION

The system engineering approach to designing optimized, energy efficient, hydro-electric power plant operation by using advanced system planning and control techniques is the main focus in here. At this point, let us recall the words of Professor H. H. Rosenbrock (1977, p. 390): "My own conclusion is that engineering is an art rather than a science, and by saying this I imply a higher, not a lower status". These words are particularly relevant in modern systems engineering projects for operating automated plants where operation planning and supervision play a crucial role. Hence, the systems philosophy framework adopted is the one in which original real-world systems and their conceptual models and mathematical representations coexist along the comprehension that dynamical processes in the real world (below the speed of light) constitute a unique non-separable inter-play of the three fundamental natural quantities of energy, matter and information. Moreover, energy and matter are information carriers, but solely information has the impact capacity as to modulate and direct energy and matter transformation with the unavoidable constraints. In this paper one such applications oriented technique for systems operation planning of unit commitment in hydro-electric power plants while retaining responsive spinning reserve, which is aimed at controlling the future operation in an optimization way, is reported.

It is well-known (Miller and Malinowski, 1994; Wood and Wollenberg, 1996) that, for a given demand at hydro-power plants, the concept of optimum number of units committed in operation is important for energy efficient operation (Dimirovski and co-authors, 1995) as well as for modeling their water consumption and specific water consumption characteristics (Raabe, 1985; Warnick, 1984). These characteristics are subsequently used for the purpose of determining optimum power schedule in complex electrical power systems composed of hydro and thermal power plants (e.g., see Bobo and Mauzy, 1993; Chouwdry and Rahman, 1990; Lee and Breipohl, 1993). Moreover, it is necessary for this purpose that these characteristics be known at different levels of electric power system consumption. Typically, each plant's daily load diagram is divided into 24 time intervals and within each of them electrical load being assumed constant when determining these operating characteristics.

The technique developed in this paper is based on the concept of water consumption characteristics for each hydro-unit (not taking into account self-consumption of water and electric power for running the unit) and the functional dependencies of own-consumptions of water and power as functions of net-consumption of water and gross-production by the hydro-unit, respectively (Wood and Wolenberg, 1996). This method also requires knowledge of the plant's load diagram and the engineering features of the generating units (Arsenov, 2002). The proposed technique is an iterative one, but at each of the iterations the actual solution computation is based on analytical formulae, thus the required time for the final results is very short.

The rest of the paper is written as follows. In Section 2, there are presented the formal statement of the problem and the derivation analysis of the solution leading to the iterative algorithmic procedure. Section 3 further illustrates this technique via the application and results for a case study of typical medium size hydro-power plants 75-145 MW comprising 3 generating units. Conclusions and references are given thereafter.

2. FORMAL STATEMENT OF THE PROBLEM AND SOLUTION DERIVATION

We consider a typical multi-machine hydro-electric plant close to reservoir damp (see Figures I and II) and assume each of the generating units has own water inflow and outflow piping, hence penstocks may differ. As is the case in practice, it is assumed that the generating units have their own consumptions covered in part by special small hydro-electric generators and in part from the transformers for plant's own local needs. Also, it is assumed that that hydro-electric plant is fully automated and equipped with an industrial computer process control system (e.g., see Fasol and Pohl, 1990; Dimirovski and co-authors, 1995; Fasol, 1997) as well as that the respective power system dispatch centre is also fully automated. Then the higher-level optimization tasks are feasible to practical implementation and exploitation (Miller and Malinowski, 1994; Wood and Wollenberg, 1996) in automatic generation operating the power plants.

Now, let consider the schematics of hydro-electric plants above. It is readily noted that the local and pipe losses in each of the inflows are denoted as $\Delta h_{d,i}$ ($i=1,N$), and in the respective outflows as $\Delta h_{o,i}$ ($i=1,N$). Of course, N denotes the number of generating units in the hydro-plant. Notice in general, from the point of view of this technique, these may differ one from the other in general case, albeit this is seldom a practice.

Following the operational practice of power industry (Miller and Malinowski, 1994; Raabe, 1985; Warnick, 1984), hydro-unit characteristics of water consumption and the respective specific water consumptions can be approximated with high accuracy by means of polynomial models of fifth and fourth degrees, respectively. It is therefore that we can write:

$$Q_{ni} = f(P_{bi}) = \\ = \sum_{k=0}^{n_p} a_{i,n_p-k} \cdot P_{bi}^{n_p-k}, \; i = 1,..., N \quad (1)$$

In here, we have: Q_{ni} and P_{bi} are the net-flow and gross-power, respectively, of the i-th unit; n_p is the polynomial degree; and $a_0, a_1, ... a_{n_p}$ se the polynomial coefficients by means of which these characteristics are tune fitted to approximate the real-world ones.

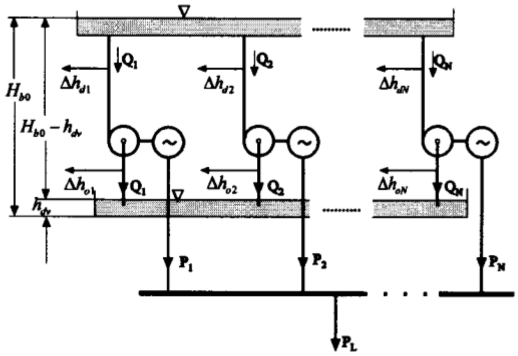

Fig. I. Conceptual model of the hydro-electrical plant in the considered optimal unit commitment operation defining water and power consumptions penstocks heads (separate inflow and outflow of water)

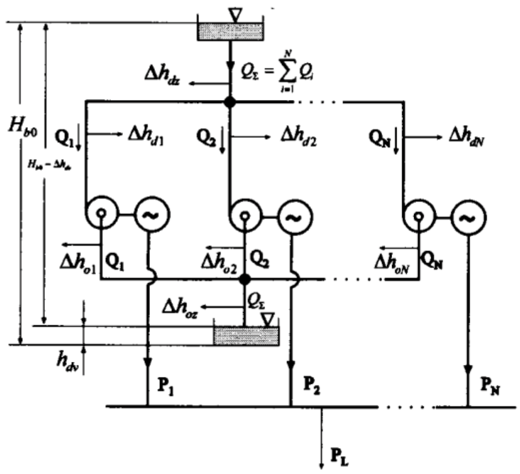

Fig. II. Conceptual model of the hydro-electrical plant in the considered optimal unit commitment operation defining water and power consumptions penstocks heads (common inflow and outflow of water).

Further, it is also known (Raabe, 1985; Warnick, 1984) that the characteristics of own water, $S_{qi} = f(Q_{ni})$, and electrical power, $S_{pi} = f(P_{bi})$, consumptions of hydro-units may well be approximated by second-degree polynomials. Hence, we can write:

$$S_{qi} = a_{qi} \cdot Q_{ni}^2 + b_{qi} \cdot Q_{ni} + c_{qi}, \quad i = 1, N, \quad (2)$$

$$S_{pi} = a_{pi} \cdot P_{bi}^2 + b_{pi} \cdot P_{bi} + c_{pi}, \quad i = 1, N, \quad (3)$$

where a_{qi}, b_{qi}, c_{qi} and a_{pi}, b_{pi}, c_{pi} are the respective polynomial coefficients which are experimentally determined for each given plant at the beginning of its life-cycle of operation (see Table 1 in Section 3).

It should be noted that the considered optimization problem can be formulated as a minimization problem of Q_Σ with respect to its variables under equality constraints as

$$Q_\Sigma = \sum_{i=1}^{N} Q_{bi} \Rightarrow \min$$
$$\sum_{i=1}^{N} P_{bi} = P_L \quad (4)$$

The quantity Q_Σ represents the total water consumption of the hydro-electric plant for a given demand load P_L of the plant. The minimum extreme of problem (4) is equivalent to the minimum of the following augmented function (Luenberger, 1984) using Lagrangean multipliers:

$$F = \sum_{i=1}^{N} Q_{bi} - \lambda (P_{ni} - P_L) \Rightarrow \min \quad (5)$$

Therefore, the solution sought has to satisfy (Luenberger, 1984) the following conditions:

$$\frac{\partial Q_{bi}}{\partial Q_{ni}} \cdot \frac{\partial Q_{ni}}{\partial P_{bi}} \cdot \frac{\partial P_{bi}}{\partial P_{ni}} = \lambda = const,$$
$$i = 1, \ldots, N, \quad (6)$$
$$\sum_{i=1}^{N} (P_{bi} - S_{pi}) - P_L = 0.$$

These relations further yield

$$\frac{\partial (Q_{ni} + S_{qi})}{\partial Q_{ni}} \cdot \frac{\partial Q_{ni}}{\partial P_{bi}} \cdot \frac{\partial P_{bi}}{\partial (P_{bi} - S_{pi})} =$$
$$= \lambda = const, \quad i = 2, \ldots, N, \quad (7)$$
$$\sum_{i=1}^{N} (P_{bi} - S_{pi}) - P_l = 0,$$

and finally one can derive

$$\frac{\partial Q_{ni}}{\partial P_{bi}} \cdot \frac{1 + \dfrac{\partial S_{qi}}{\partial Q_{ni}}}{1 - \dfrac{\partial S_{pi}}{\partial P_{bi}}} = \lambda = const, \quad i = 2, \ldots, N, \quad (8)$$
$$\sum_{i=1}^{N} (P_{bi} - S_{pi}) - P_L = 0.$$

The system of equations (8) can be re-written in the following convenient form:

$$f_1 = \sum_{i=1}^{N} (P_{bi} - S_{pi}(P_{bi})) - P_L = 0, \quad (9)$$

$$f_i = \frac{dQ_{n1}}{dP_{b1}} \cdot \left(1 + \frac{dS_{q1}}{dQ_{n1}}\right) \cdot \left(1 + \frac{dS_{pi}}{dP_{bi}}\right) -$$
$$- \frac{dQ_{ni}}{dP_{bi}} \cdot \left(1 + \frac{dS_{qi}}{dQ_{ni}}\right) \cdot \left(1 + \frac{dS_{p1}}{dP_{b1}}\right) = 0, \quad (10)$$
$$i = 2, \ldots, N.$$

Notice that the system of equations (9)-(10) is inherently nonlinear. Hence, numerical iterative computing methods such as Newton-Raphson (Kreyszig, 1999) are the only methodological approach to solve it. Upon the linearization of the system (9)-(10), the following matrix equation is obtained

$${}_N[J]^N \cdot {}_N[\Delta P_{bi}]^1 = -{}_N[f]^1 \quad (11)$$

where term ${}_N[J]^N$ represents the system Jacobian of Eqs. (9)-(10), and term ${}_N[\Delta P_{bi}]^1$ represents the vector of increments of gross-powers for each of the generating units.

However, should the following substitutions

$$\xi_i = 1 + \frac{dS_{qi}}{dQ_{ni}}, \quad i = 1, \ldots, N,$$
$$\varphi_i = 1 + \frac{dS_{pi}}{dP_{bi}}, \quad i = 1, \ldots, N, \quad (12)$$

have been introduced first, then for the Jacobian elements one obtains the following expressions:

$$a_{1i} = 1 - \frac{dS_{pi}}{dP_{bi}}, \quad (13)$$

$$a_{i1} = \left[\frac{d^2 Q_{n1}}{dP_{b1}^2} \cdot \xi_1 + \left(\frac{dQ_{n1}}{dP_{b1}}\right)^2 \cdot \frac{d^2 S_{q1}}{dQ_{n1}^2}\right] \cdot \varphi_i -$$
$$- \frac{dQ_{ni}}{dP_{bi}} \cdot \xi_i \cdot \left(-\frac{d^2 S_{p1}}{dP_{b1}^2}\right), \quad i = 2, \ldots, N, \quad (14)$$

$$a_{ii} = -\frac{dQ_{n1}}{dP_{b1}} \cdot \xi_1 \cdot \left(-\frac{d^2 S_{pi}}{dP_{bi}^2} \right) -$$
$$-\left[\frac{d^2 Q_{ni}}{dP_{bi}^2} \cdot \xi_i + \left(\frac{dQ_{ni}}{dP_{bi}} \right)^2 \cdot \frac{d^2 S_{qi}}{dQ_{ni}^2} \right] \cdot \varphi_i, \quad (15)$$
$$i = 2,...,N.$$

Hence, due to the natural character of these term coefficients, a nontrivial solution exists for the linearized matrix equation (11). And therefore Eq. (11) becomes analytically solvable. The solution of Eq. (11) in its analytical form, after a certain lengthy derivation, is found to be given by means of the following formulae:

$$\Delta P_{b1} = \frac{-f_1 + \sum_{i=2}^{N} a_{1i} \cdot \frac{f_i}{a_{ii}}}{a_{11} - \sum_{i=2}^{N} a_{1i} \cdot \frac{a_{i1}}{a_{ii}}}, \quad (16)$$

$$\Delta P_{bi} = -\frac{f_i}{a_{ii}} - \frac{a_{i1}}{a_{ii}} \Delta P_{b1}, \; i = 2,...,N. \quad (17)$$

Let us now address the engineering and physical background of the equations (16)-(17), which determine the gross-power increments. Since these equations have been derived under the presumption that gross-powers of the generating units were $P_{bi}^{(0)}, \; i = 1,...,N$, they should be corrected in an iterative manner by using the equation

$$P_{bi}^{(l+1)} = P_{bi}^{(l)} + \Delta P_{bi}^{(l+1)}$$
$$i = 1,...,N; \; l = 0,1,2,... \quad (18)$$

where $l = 0,1,2,...$ represents the iteration step number. Of course, the iterative evaluation goes until the respective vector norm, e.g. square-root of the sum of the squares of power increments, becomes smaller than a certain sufficiently small value chosen via observing the given engineering physical background of the problem. And this completes the derivation of proposed technique.

On the grounds of the presented algorithm a program in VISUAL FORTRAN 6 has been elaborated. This program was first validated on a specific Known example and then applied to the case-study, which is presented next. However, the algorithm was found equally efficient for hydropower plants with various schemes of water inflow and outflow to and from the hydropower plant, other than the one of the case study.

The mathematical model derived as well as the associated algorithm and computing program are our original developments albeit based on known theories. These are useful for both: identifying of the optimum number of units to be engaged in one hydropower plant under a given load; and also for determining the equivalent power characteristics of hydropower plants and their respective differential characteristics of water consumption, which are essential when determining the optimum distribution of active and reactive powers in complex hydro – thermal systems. The power characteristics can be used further for identifying of the anticipated electricity production in the case of separate hydropower plants as well as in the case of hydropower plants which are hydraulically interconnected.

3. APPLICATION EXAMPLE AND ILLUSTRATION OF THE PROPOSED TECHNIQUE

The proposed technique is illustrated by its application to hydro-electric plants having up to four installed generating units (taken from Arsenov, 2002), the operating characteristics of which $Q_{ni} = f(P_{bi})$, i=1, 2,..., N (see Eq. (1)) are assumed to be represented by 5th degree polynomials. In doing so, in order to highlight the influence of unit's own consumptions of water and power, it has been assumed that all units have the same consumption characteristics but different characteristics of own operating needs, $S_{qi} = f(Q_{ni})$ and $S_{pi} = f(P_{bi})$.

The coefficients of the 5th degree polynomial have been found elsewhere (Arsenov. 2002) and these are: 3.72E-06; 0.054; -2.032; 39.481; -1.82E+02; -241.702; notice the last one represents the polynomial's free term. The evaluation of the 2nd degree polynomial approximations of the own consumption characteristics, see Eqs. (2)-(3), has given the results presented in Table 1.

Table 1 Coefficients of 2nd degree polynomials of the characteristics for plant's own operating needs

i	a_{pi}	b_{pi}	c_{pi}
1	0.0008	0.0001	0.000
2	0.0009	0.0001	0.000
3	0.0006	0.0001	0.000
4	0.0007	0.0001	0.000
i	a_{qi}	b_{qi}	c_{qi}
1	0.00022	0.000	0.000
2	0.00024	0.000	0.000
3	0.00026	0.000	0.000
4	0.00028	0.000	0.000

The application of the proposed technique has given the results depicted in Figure 3 and presented in Table 2. In Figure 3 there are depicted graphics showing the plant's operating characteristics $Q_b = f(P_n)$ in the case of optimum number of units in operation for different demand loads on the plant. For the case of $n = 3$ generating units in operation, the impact of the different ways of satisfying own consumption needs for water and electrical power can be inferred by examining the results in Table 2.

Table 2 – Part I Results for the consumer power demand loads 75-145 MW also showing the impact of own consumptions

Cons-umer power	Generating Unit	Gross Water Inflow	Net Water Inflow
1	2	3	4
MW		m^3/s	m^3/s
75.	1	83.367	81.891
75.	2	85.078	83.408
75.	3	84.052	82.291
80.	1	83.038	81.574
80.	2	90.445	88.563
80.	3	91.909	89.812
85.	1	93.198	91.361
85.	2	91.998	90.051
85.	3	93.181	91.027
90.	1	97.542	95.534
90.	2	96.726	4.579
90.	3	97.615	95.256
95.	1	102.306	100.101
95.	2	101.548	99.187
95.	3	102.385	99.796
100.	1	107.487	105.058
100.	2	106.674	104.074
100.	3	107.529	104.680
105.	1	113.127	110.443
105.	2	112.193	109.325
105.	3	113.097	109.954
110.	1	119.264	116.288
110.	2	118.165	114.991
110.	3	119.129	115.651
115.	1	125.929	122.621
115.	2	124.638	121.117
115.	3	125.660	121.803
120.	1	133.154	129.466
120.	2	131.662	127.745
120.	3	132.731	128.442
125.	1	140.976	136.856
125.	2	139.297	134.928
125.	3	140.391	135.610
130.	1	149.458	144.842
130.	2	147.617	142.728
130.	3	148.712	143.368
135.	1	158.693	153.509
135.	2	156.719	151.230
135.	3	157.791	151.800
140.	1	168.825	162.981
140.	2	166.733	160.547
140.	3	167.764	161.023
145.	1	180.051	173.434
145.	2	177.835	170.831
145.	3	178.814	171.194

Table 2 – Part II (completion)

	Own Consum -ption	Gross Power Gen.	Net Power Gen.	Own Consum -ption
	5	6	7	8
MW	m^3/s	MW	MW	MW
75.	1.475	25.237	24.725	0.512
75.	1.670	25.867	25.262	0.605
75.	1.761	25.402	25.013	0.390
80.	1.464	25.107	24.600	0.507
80.	1.882	28.040	27.329	0.710
80.	2.097	28.563	28.071	0.492
85.	1.836	29.206	28.521	0.685
85.	1.946	28.663	27.921	0.742
85.	2.154	29.068	28.558	0.510
90.	2.008	30.889	30.122	0.766
90.	2.147	30.511	29.670	0.841
90.	2.359	30.779	30.208	0.571
95.	2.204	32.629	31.774	0.855
95.	2.361	32.289	31.348	0.942
95.	2.589	32.516	31.878	0.638
100.	2.428	34.391	33.442	0.950
100.	2.600	34.052	33.005	1.047
100.	2.849	34.261	33.554	0.708
105.	2.683	36.168	35.118	1.050
105.	2.868	35.810	34.652	1.158
105.	3.143	36.012	35.230	0.782
110.	2.975	37.954	36.798	1.156
110.	3.174	37.569	36.295	1.274
110.	3.478	37.766	36.907	0.860
115.	3.308	39.748	38.480	1.268
115.	3.521	39.334	37.938	1.396
115.	3.857	39.524	38.582	0.941
120.	3.688	41.546	40.161	1.385
120.	3.917	41.106	39.581	1.525
120.	4.289	41.285	40.258	1.027
125.	4.120	43.344	41.837	1.507
125.	4.369	42.888	41.228	1.660
125.	4.781	43.051	41.934	1.116
130.	4.615	45.143	43.508	1.635
130.	4.889	44.681	42.880	1.801
130.	5.344	44.822	43.612	1.210
135.	5.184	46.942	45.174	1.768
135.	5.489	46.484	44.534	1.949
135.	5.991	46.599	45.292	1.308
140.	5.844	48.741	46.836	1.905
140.	6.186	48.295	46.191	2.104
140.	6.741	48.383	46.973	1.409
145.	6.617	50.544	48.495	2.049
145.	7.004	50.112	47.847	2.265
145.	7.620	50.173	48.658	1.515

It should be observed, given the results in Table 2, Parts I and II, the proposed optimization technique has provided a solution with a rather balanced distribution of load demand over the plant's units.

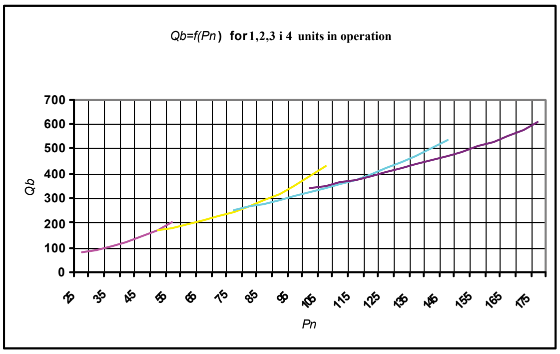

Fig. III. Operating characteristics $Q_b = f(P_n)$ for 1, 2, 3, and 4 units in the optimized operation of the hydro-electric power plant depending on demand loads.

4. CONCLUSION

The concept of optimum number of units committed in operation is important for both energy efficient operation as well as for modelling plant's operating characteristics of water consumption and specific water consumption characteristics. This paper presents one applications oriented technique for optimum system operation planning by means of unit commitments in hydro-electric power plants while retaining responsive spinning reserve. The mathematical model derived as well as the associated algorithm and computing program are our original developments albeit based on known theories.

The technique derived is an iterative one with solutions at each of the iterations being obtained via analytical formulae. This technique enables to determine numerically the so-called operating consumption characteristics of hydro-electric power plants as well as to find out the impact of the variants of covering unit own needs of water and power, which in turn is to be observed determining the operating schedule of plants in the electrical power system.

The presented technique allows for units to have different specific characteristics of water and electrical power consumption for own needs. It is aimed at controlling the future operation via an optimization strategy, which can be part of the supervisory control level of the plant or the area computer control system. Future research is envisaged in improving further the representation model so as to encompass some additional technical specifications.

REFERENCES

Arsenov, A. (2002), Determining the Optimal Number of Operating Units Hydro-Electric Power Plant Operation. *ECRP Technical Report OPPO-CIGRE/02*, Faculty of Electrical Engineering, SS Cyril and Methodius University, Skopje (in Macedonian).

Bobo, D. R. and D. M. Mauzy (1993). Economic generation dispatch with responsive spinning reserve. In: *Proceedings of the IEEE Conference on Power Industry Computer Applications,* pp. 299-303. The IEEE, New York.

Chouwdhury, B. H. and S. Rahman (1990). A review of recent advances in economic dispatch. *IEEE Transactions on Power Systems*, **5**, no. 4 (Nov.), pp. 1248-1259.

Fasol, K. H. (1997), Stabilization and re-engineering of a hydro-electric power plant – A case study. *Control Engineering Practice,* **5**, no. 1, pp. 109-115.

Fasol, K. H. and G. M. Pohl (1990), Simulation, controller design and field tests for a hydro-power plant – A case study. *Automatica,* 26, no. 1, pp. 475-484.

Dimirovski, G. M., J. D. Stefanovski, R. Hanus, and M. J. Stankovski (1995). Optimum supervisory control of low-power multi-machine hydroelectric plants. In: *Preprints IFAC Symposium on Large Scale Systems Theory and Applications* (P. D. Roberts and J. E. Ellis (Eds)), vol. 2, pp. 911-916. Pergamon Elsevier Science Ltd, Oxford.

Lee, F. N. and A. M. Breipohl (1993). Reserve constrained economic dispatch with prohibited operating zones. *IEEE Transactions on Power Systems*, **8**, no. 1 (Feb.), pp. 246-254.

Luenberger, D. G. (1984), *Linear and Nonlinear Programming.* Addison-Wesley, Reading, MA.

Miller, R. H. and J. H. Malinowski (1994). *Power System Operation (3rd ed.)*. McGraw-Hill, New York.

Raabe, J. (1985), *Hydro Power.* VDI-Verlag, Duesseldorf.

Rosenbrok, H. H. (1977). The future of control. *Automatica*, **13**, pp. 388-392.

Warnick, C. C. (1984). *Hydropower Engineering*. Prentice Hall, Englewood Cliffs, NJ.

Wood, A. J. and B. F. Wollenberg (1996). *Power Generation, Operation, and Control.* J. Wiley & Sons, New York.