
ARTÍCULOS DOCTRINALES / ARTICLES

LUCA PACIOLI AND HIS ART**Alan Sangster**

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Abstract

Much has been written about Luca Pacioli, the ‘father of accounting’, and his contributions in many fields; and much has been written about his involvement and portrayal in art, but never has what we know of his involvement in art been articulated to bring all these various facets together. This paper addresses this gap in the literature and reveals a greater impact arising from his work than has previously been recognised. In doing so, it provides insights into the mind of one of the leading Renaissance men, insights that reveal a more ‘human’ side than might be expected of someone considered to have been a master of all that he did; and it reveals him as an artist in his own right, not an artist who painted pictures, but an artist whose talent lay in creating new opportunities for others.

Keywords: *accounting history, Luca Pacioli, art.*

LUCA PACIOLI Y SU ARTE

Resumen: Mucho se ha escrito sobre Luca Pacioli, el "padre de la contabilidad", y sus contribuciones en muchos campos; y mucho se ha escrito sobre su participación y representación en el arte, pero nunca se ha articulado lo que sabemos de su participación en el arte para unir todas estas diversas facetas. Este artículo aborda esta brecha en la literatura y revela un mayor impacto derivado de su trabajo de lo que se ha reconocido anteriormente. Al hacerlo, proporciona conocimientos sobre la mente de uno de los principales hombres del Renacimiento, conocimientos que revelan un lado más "humano" de lo que cabría esperar de alguien que se consideraba un maestro en todo lo que hizo; y lo revela como un artista por derecho propio, no como un artista que pintaba cuadros, sino como un artista cuyo talento radicaba en crear nuevas oportunidades para los demás.

Palabras clave: *historia contable, Luca Pacioli, art.*

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1. Introduction

In this study, a unique approach is adopted in evaluating the life, contribution, and impact of Luca Pacioli. Rather than the conventional focus on his vocation, his occupation, or accounting, it looks at his versatility, his achievements, his lasting influence on others; and at how some, in more recent times, have paid homage to his achievements, traded on his fame, and identified him as a recognisable, trustworthy, and respected icon from accounting's past. It does so by looking at Pacioli's 'art'. This is not to be confused with the art of the portrait, the play, the book, the musician, the theatre, or the tapestry, which were among the many topics discussed under the theme, '*Storia della Ragioneria e Arti – Accounting History and Arts*', at the *14th Società Italiana di Storia della Ragioneria (SISR)* conference, held in Turin in November 2018.¹

Art can be defined as, "*a skill at doing a specified thing, typically one acquired through practice*"² which, of course, includes double entry bookkeeping; and Luca Pacioli provided the theoretical blueprint for acquiring the art of bookkeeping that is still in use today (Sangster 2018). This is why he is remembered by accountants as the 'father of accounting', the man who wrote and printed the first instructional manual on double entry bookkeeping. He is also revered by those in other disciplines for the contribution he made to their development, not least mathematics and art (Ciocci, 2011). As this paper will show, while Pacioli was primarily a mathematician by trade and a friar by calling, it was art in all its forms where he arguably had the most immediate impact. In contrast, his influence on accounting was slow, spreading across Europe as merchants purchased *Summa Arithmetica* but, without any noticeable impact on practice until after the publication of three other manuals, all based on it, in the 1540s (Yamey 1994, 22; 2004): Manzoni (1540); Oldcastle (1543); & Ympyn (1543a,b; 1547), and, in particular, Manzoni's that introduced the concept of exemplar account books that was endorsed and used by the writers that followed (Jeannin 1991, 247).

Pacioli's impact on mathematics was more immediate, revolutionising the teaching of algebra in the early 16th century and acting as the catalyst for the axiomatic foundations of that subject that were articulated for the first time by the middle of that century, largely as a result of his interpretation and presentation of the subject in *Summa Arithmetica* in 1494. However, his impact on art was even more immediate. He emancipated Leonardo da Vinci by teaching him the mathematics he needed to perfect his use of perspective, both in art and sculpture. He did the same for Albrecht Dürer and Jacopo de' Barbari, the artist to whom Pacioli's well-known portrait, currently in the Museo di Capodimonte in Naples, is attributed. Like Leonardo, he taught them the secrets of perspective (Sangster 2018); and he also taught many of the artists of his day, including the leading Florentine artists Sandro Botticelli and Antonio del Pollaiuolo (Ciocci 2009).

As mentioned above, Pacioli also sat on the other side to the artist, acting as the sitter for his portrait, but his involvement in its creation was more than as a passive subject. As will

¹ Presentations included papers on accounting and: 15th & 16th century Flemish paintings, Giuseppe Verdi, Shakespeare, the painting of Boris Kustodiev's Merchant, Hermann Broch's 'The Sleepwalkers', Enrico Caruso, the Lorenzetti Siena Frescoes, the Teatro Massimo di Palermo, and the Bayeux Tapestry, authored, in turn by Torelli et al., Giovannoni et al., Sokolov et al., Hermann, Antonnelli et al., Servalli & Hoskin, Costa et al., and Miley & Read.

² English Oxford Living Dictionary. Available on 23 December 2019 at <https://en.oxforddictionaries.com/definition/art>

be explained later, he planned it, and he designed it. It is a painting that asks considerably more questions than it answers. Not only is the identity of the artist disputed, so is the identity of the second figure in the painting, and many objects within it have been wrongly identified. Adding to the confusion, it has been suggested that the painting contains riddles and hidden code, including the hidden story of the murder of the Duke of Milan, Gian Galeazzo Sforza, in October 1494 (Glori 2013). If that interpretation were true, some may even suggest, with good reason, that Pacioli knew what was being done when he posed for the painting, which raises the question: why did he agree to do it? However, this suggestion has not been embraced by other scholars. Other interpretations have been more acknowledged and shall be returned to later.

Figure 1. *Ritratto di Luca Pacioli* (c. 1495)³



Despite conclusions to the contrary, this portrait is not the only picture of Pacioli that has come down to us. There are at least three others, unknown to date in the accounting literature. As will become evident in this paper, what and where they are goes some way to addressing the question of who painted his portrait.

Other mysteries involve the artwork in Pacioli's *De Divina Proportione*, first published in manuscript in 1498. Some of it is so difficult to replicate, the art in the two surviving manuscript copies is not identical. Nor are they identical with the art in the printed edition of 1509. Furthermore, the printer's copy of the manuscript, dated 1508, which quite possibly contains the originals of Leonardo's drawings, was recently located in the Vatican archives, but has since disappeared from the online archive catalogue before anyone is known to have either copied it or examined it. Three further examples of Pacioli's art will be considered in this paper: his use of handmade figures to supplement his teaching of perspective, proportions, and proportionality in *De Divina Proportione*, his *Tree of Proportions and Proportionality* from *Summa Arithmetica* and *De Divina Proportione*, and his renowned design of the alphabet so aptly used by his printer in the reprinting of pages from *Summa Arithmetica* and utilised in the typesetting of *De Divina Proportione* in 1509.

³ Reproduced with the permission of the Ministry of Cultural Heritage and Activities - Museum and Real Bosco di Capodimonte, Naples.

1.1. Contribution

Previous studies linking accounting with art, such as Basil Yamey's visually compelling *Art and Accounting* (1989), have focused primarily on accounting artefacts or practices appearing in art. In adopting a less focused and more inclusive approach, this paper presents a new set of achievements and influences that we can attribute to Pacioli that are more diverse than was previously recognised in the literature. The key contribution of this paper is its identification of the long-reaching impact of Pacioli's art: his ability to communicate, how to write about what his audience needed, and in language – verbal and visual – that they knew and understood, something reflected in the longevity of the recognition of the extent of his achievements. This was the art that opened doors, created opportunities, engaged his audience, and achieved his humanist-inspired purpose: to improve what people did and what they were capable of doing.

1.2. Research methodology

The research underpinning this paper explored the literature through a hermeneutic frame, focusing on Pacioli's interaction with art and artists, high society and intellectuals, his mathematical grounding, his upbringing and education, his believe in balance through harmony, and in the divine influence on science and art and all things physical. Hermeneutics, or the theory and practice of interpretation, involves a circular spiral of analysis whereby an artefact is interpreted in the context of its time and place as revealed through what is known of its contemporary environment: society, life, culture, religion, politics, etc. Through its adoption, *“hermeneutic analysis enables you to elicit an in-depth understanding of meanings of, for example: human practices, culture, works of art and texts”* (Koppa 2010).

2. The formation of Luca Pacioli⁴

Luca Pacioli was born in Borgo San Sepolcro between 23 October 1446 and 19 June 1447 and died on 19 June 1517 (Sangster 2018). His early schooling was at the town's humanist grammar school. Orphaned in January 1459, he was taken into the home of the local merchant and politician, Folco di Giovanni di Canti Bofolci. It was from Bofolci that Pacioli began to learn *abaco* (practical mathematics for business). He enhanced his knowledge of the subject with his fellow townsman, the artist Piero della Francesca, who also introduced him to theoretical mathematics and Euclid.

Pacioli's two elder brothers were Franciscan friars resident in the local convent, a convent Pacioli was to join and subsequently lead a few years later. In 1464, it was decided that he should go to Venice to study theoretical mathematics. To finance his studies, he became apprenticed to a leather merchant, Antonio Rompiasi. There he learnt double entry bookkeeping and the legal environment of business and accounting. He also began to teach *abaco* and wrote a book on that subject which he dedicated to Rompiasi's three sons. When he left Venice in 1470, he was already an accomplished mathematician but he sought more, so he travelled to Rome and stayed with the leading architect and influential humanist educator, Leon Battista Alberti. It was here that he acquired an understanding of architectural theory and was introduced to the highest echelons and beliefs of the humanist education movement.

⁴ The principal source for much of this section is the biography of Pacioli by Sangster (2020).

After several months Pacioli, now a committed humanist, returned to Borgo San Sepolcro, where he became a novitiate in the Franciscan Order of the Friars Minor Conventual. While in Borgo San Sepolcro, he continued his study of mathematics with Piero della Francesca, at the very least helping him write his treatise, *Del Abaco*. His studies continued in 1475 when he travelled to Perugia to study philosophy and mathematics in the Faculty of Theology. While there, he taught *abaco* until he left in 1480, leaving behind a very lengthy *abaco* manuscript (Pacioli 1480) that he dedicated once again to his students, this time the youth of Perugia. After a sojourn in Zara, he returned to his convent in Borgo San Sepolcro. By September 1484, he had been awarded his degree and, reflecting both the depth of his religious calling and the level of his education, had been appointed head of his Franciscan convent. His formal education complete, in mid-1485 he left for Florence where he spent two years studying mathematics and, in particular, proportion in the Dominican Order's library of San Marco.

This was where he found much of the mathematics he used in *Summa Arithmetica*, in particular the Campanus (1482) printing of Euclid's *Elements*. He also consulted various studies of perspective; works of Piero della Francesca; and *Tractato di praticia di geometria*, a translation of Leonardo Pisano (Fibonacci)'s Latin treatise, *Pratica geometrie* (1220), written by an anonymous Florentine disciple of the abacist Domenico di Agostino Vaiaio (Ulivi 2015). This was the main source Pacioli used in *Summa Arithmetica* and the one from which he copied the statement in that book that anything to which he does not attribute a source is from Leonardo Pisano (Picutti 1989, 76). He also mingled with artists, including those from the workshop of Andrea del Verrocchio (1435-1488), and discussed art and perspective with Botticelli and Pollaiuolo (Ciocci 2009, 104-6), something he must also have discussed previously with Piero della Francesca, the acknowledged leading exponent on perspective of his day. In May 1487, he returned to teaching in Perugia, where he began writing *Summa Arithmetica* (*ibid.*).

Thus, through his studies and his networking, Pacioli acquired the knowledge and the expertise to enter the workshops of the artists, learn from them and teach them how to improve their art, something he continued to do when he returned to Borgo San Sepolcro (*ibid.*, 170). Later, he would do the same with Leonardo da Vinci in 1496, Albrecht Dürer in 1506, and Jacopo de' Barbari, the man attributed as the painter of Pacioli's portrait.

3. The paintings, busts, statues, stamps, and coin of Luca Pacioli

Pacioli vanished from public consciousness in the 17th century. In the early 19th century, he was reintroduced to Russian, then German accountants. His reintroduction to Italy was in a speech given in Milan by Professor Ernesto Lucchini in 1869. Since then, dozens of memorials have been created bearing 'his' image. In 1986, Ernest Stevelinck published an article describing many of these, and included 21 photographs of paintings, statues, and busts (Stevelinck 1986). Unfortunately, while he provided details relating to each of them, he did not conclude sufficiently on which bear true likenesses, but it is clear that many are derived from Pacioli's portrait in the Museo Capodimonte. Others bear no resemblance at all. These include two contemporary paintings by Piero della Francesca: The Polyptych of St. Anthony, currently in the Galleria Nazionale dell'Umbria, Perugia; and Madonna and Child with Saints (or the Montefeltro altar piece), currently in the Pinacoteca di Brera, Milan).

Figure 2. The Polyptych of St. Anthony (1468)⁵



Figure 3. Madonna and child with saints (1472-1474)⁶



In Figure 2, the figure on the left dressed in monk's robes holding a book has been identified as Pacioli, as has the man second from the right at the back in Figure 3 (Stevelinck 1986). Neither claim is correct. Figure 2 was painted before Pacioli became a novitiate and the person in Figure 3 is the same as the one in Figure 2. Furthermore, Pacioli would not have been allowed to pose for paintings of this type by his Order.

Apart from incorrectly identifying Pacioli in paintings, aficionados of Pacioli have also taken a 19th century painting by Angiolo Tricca (1817-1884), which depicted Pacioli being taught by Piero della Francesco, as a true image (Figure 4). The authentic feel to the painting drives them towards it. Similar responses arise with the some of the imaginary image statues and busts.

⁵ This image is in the public domain.

⁶ This image is in the public domain.

Figure 4. Pacioli being taught by Piero della Francesca⁷

Busts of Pacioli first began to appear in the last three decades of the 19th century, but the portrait in Naples was not publicly known and no-one at that time had identified any of the figures in the paintings of della Francesca as being of Pacioli. When Florence commissioned a bust of Pacioli after Italian unification in 1870, it was created using the sculpture's imagination (Figure 5). As with Tricca's portrait, its imagery matched the imagination of many: its presentation of a thoughtful and relaxed middle-aged Florentine man appealing to the senses of many who saw it. Over the course of the 20th century, it was included and labelled or implied as being a likeness of Pacioli in or on the cover of books published on the history of accounting in France, Germany, Belgium, and Russia.

Figure 5. The Florentine bust of Luca Pacioli (1870s)⁸

Many other busts have been produced, including one that was available to order online in 2018 for under €100 (Figure 6). It soon sold-out, and demand was such that the creator of the bust recreated it in two new designs.

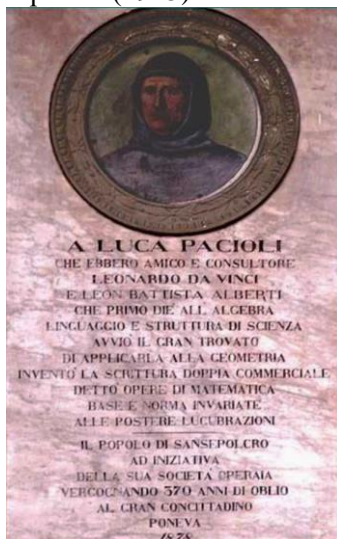
⁷ Reproduced with the permission of the Museo Civico "Piero della Francesca" – Comune di Sansepolcro.

⁸ Source: Stevelinck (1986, 12).

Figure 6. Pacioli's mass-produced bust (2018)



The portrait of Pacioli emerged from the Medici collection into the public gaze in Naples at the beginning of the 20th century and, in 1903, it was placed in the Museo di Capodimonte. Since then, it has guided the design of most memorials to Pacioli. One of the first of these was the image of Pacioli in the plaque erected in his honour in Sansepolcro. The invented image used when the plaque was originally erected in 1878 was replaced in 1925 with one based on the Capodimonte portrait.

Figure 7. Pacioli's plaque in Sansepolcro (1925)⁹

However, undoubtedly the best known of these post-discovery memorials is a statue commissioned by Japanese scholars and placed outside the entrance to the former Franciscan convent where Pacioli lived in Borgo San Sepolcro.

⁹ Photograph taken by the author

Figure 8. Luca Pacioli's statue in Sansepolcro (1995)¹⁰



Another, arguably more evocative statue is located in Manchester Convention Centre, England.

Figure 9. Luca Pacioli's statue in Manchester (1994)¹¹



Both these statues were commissioned to celebrate the 500th anniversary of the publication of *Summa Arithmetica* in 1494. Stamps and coins were also issued that year celebrating Pacioli's book. Both the stamps based their image of Pacioli on the Capodimonte portrait. The coin took a different perspective, using the woodcut of Pacioli in *Summa Arithmetica* as the basis for the image. The unassuming but insistent humanist, friar and educator had become the focus of public celebration, endorsed by governments as worthy of recognition. Elsewhere, a video had been released in 1990 presenting a (largely fictional) biography of Pacioli: *Luca Pacioli: Unsung Hero of the Renaissance* (Jackson, Tinius & Weis 1990). It was to propel Pacioli into classrooms around the world and has been viewed over 120,000 times in the 6 years since it was uploaded to *YouTube* in March 2014.

Figure 10. Stamps from Sri Lanka and Italy; Italian coin; the *Summa Arithmetica* woodcut¹²



¹⁰ Photograph taken by the author.

¹¹ Photograph taken by the author.

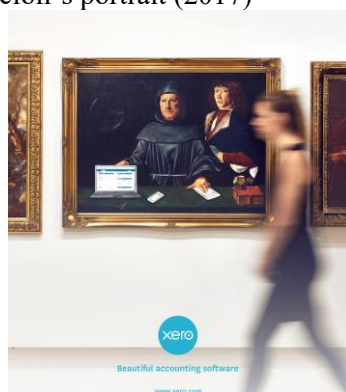
¹² Sources: stamps (Strick 2009); the woodcut is in the public domain; image of coin taken by the author.

These images are only a small selection of those that have been created over the past 150 years. They all celebrate the genius of Pacioli and, in particular, his publication of the first printed manual on Venetian double entry bookkeeping in 1494. Most are interesting in their own way and, as mentioned above, some continue to be used to illustrate books on Pacioli even though it is known that they are not a true likeness. Outside the world of caricature, it must be rare indeed that images of imaginary likeness are used in place of a readily available image known to be of the subject. Perhaps this can be explained by the gravity of Pacioli's portrait, the clear implication within it that this is a teacher of importance to the upper class, and at the highest level: *"the objects are not intended merely to suggest intellect, but to be the very stuff of that intellect"* (Mackinnon 1993, 195)". Pacioli does not appear in it as a man of the people, whereas, for example, the painting by Angiolo Tricca unambiguously presents a man seeking to help his friend and his students.

4. Pacioli's portrait

Pacioli's impact on society goes far beyond his deeds and far beyond the contribution he made to accounting, business, mathematics, architecture, and art. His name has been used to name colleges, business schools, research institutes, research societies, an algebraic relationship, email addresses, streets, and t-shirts have been mass-produced featuring his portrait, his alphabet,¹³ and other artwork from *De Divina Proportione* and *Summa Arithmetica*. His character has appeared in novels and spoof biographies, films, and plays. However, he sat for only one portrait, and it too has developed a life of its own, spawning dozens of memorials in his name to the point that it is so well known that it has been redrawn so that it can be used in advertising, for example, as shown in Figure 11, of accounting software.

Figure 11. Xero advert using Pacioli's portrait (2017)¹⁴



Yet, the portrait, while undoubtedly of Pacioli teaching mathematics, is a riddle that has challenged scholars ever since it was rediscovered. Franciscan friars did not sit for paintings out of vanity, and they could not sit for any purpose without gaining permission from their Order to do so. For that to be granted, the purpose of the painting had to justify Pacioli's participation in the eyes of the Church. Why would the Friars Minor have endorsed his involvement? This is a question yet to be addressed. It could not have been for his own satisfaction. It must have been as a favour to someone or at the behest of someone to whom he

¹³ A selection of these t-shirts can be seen at <https://fineartamerica.com/shop/tshirts/luca+pacioli>

¹⁴ Reproduced with permission from Xero Ltd. *Digital or die: How digital accounting has revived a classic.* (2017). Available on 5 February 2020 at <https://www.xero.com/content/dam/xero/pdf/xero-digital-or-die-report.pdf>

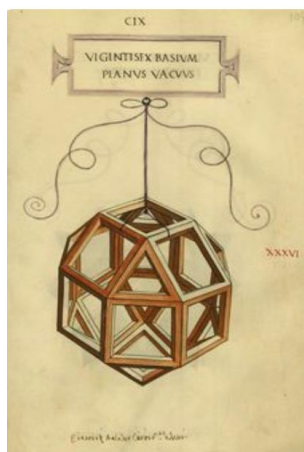
could not say ‘no’. Did he even know it was being painted? And, if not, how can we be sure that this is a true likeness? The clues that provide answers to these questions must lie in the painting itself. But, what are they?

The identity of the second figure is disputed, though its provenance and dedication tell us that it must be Guidobaldo da Montefeltro, the Duke of Urbino (1472-1508),¹⁵ a man to whom Pacioli certainly owed a considerable debt: he had supported Pacioli’s endeavour to research and then publish *Summa Arithmetica*; had provided Pacioli with a copy of Euclid’s *Elements* in its original Greek; had lent him the services of his court mathematician, Paulo di Middelburgo, to proof read *Summa Arithmetica*; and had permitted Pacioli to use his name in rhetorical statements in *Summa Arithmetica* and, in particular, in the Introduction to his treatise on double entry bookkeeping (Sangster 2018).

Other aspects of the painting have also raised questions about its purpose, including the presence of a fly; the identity of the books and the shape – a rhombicuboctahedron, made of glass – suspended in the left of the picture; the reflections in the sides of that shape; the writing on the label; and the identity of the artist. The books have been identified, as has the suspended shape, but the artist and the designer of the painting remain a mystery. In a recent study, Renzo Baldasso and John Logan (2017) address many of the mysteries. They conclude from the imagery in the painting that it may have been painted for Pacioli as a gift to Guidobaldo (ibid., 135), which would have been sufficient motivation for him to have received permission from his Order, and is consistent with his having dedicated the painting to the Duke (Mackinnon 1993, 181); and that (Baldasso & Logan 2017, 130):

The iconography of the Portrait was carefully composed by Pacioli, at a moment – retrospectively – that marked a turning point in his life: after 1495, the friar will transform himself from itinerant teacher of introductory and classical mathematical knowledge into courtier and creator of new and exciting mathematical insights, which eventually bore fruit in the pages of the *Divina proportione*.

¹⁵ Basil Yamey (1989, 133) expressed doubt of its being the Duke, stating that it bore no resemblance to authentic likenesses of him. The was also the conclusion of Mackinnon (1993).

Figure 12. Leonardo's drawing of a rhombicuboctahedron for *De Divina Proportione*¹⁶

Baldasso and Logan criticise the quality of the imagery and the inclusion of impossible features, such as the reflections shown in the rhombicuboctahedron. But, if Pacioli planned it as carefully as their analysis reveals, would it not be reasonable to assume these imperfections were part of the rhetoric? If so, the artist was an exceptional expert. If not, the painter was still learning his art. For all their criticism of the artwork, Baldasso and Logan end their article implying the imperfections were intentional (ibid., 143):

Both the friar and his patron [Guidobaldo] are reflected in the crystal [rhombicuboctahedron], a marvellous object that projects the beauty and timeliness of mathematical truth and principles, while its semiperfect nature – one seemingly incommensurate to phi – is an apt reminder that the Portrait's scene reflects the imperfect, corruptible reality of the figures and objects that live in it.

If it is the case that the artist was an expert, in 1495, the date attributed to the painting, few had mastered the art of perspective to the extent needed to execute the intricacies in the painting. Certainly, the artist to whom it is attributed, Jacopo de' Barbari, *was not* one of them. Another potential candidate, Albrecht Dürer, may have been, and he did seek help in mastering perspective. However, it is thought that this was in 1506 (Mackinnon 1993, 147), which suggests he was not capable of the intricacies of this work in 1495. But, if the date in the painting (which is obscured by a fly, making the final number illegible) is not the date of the painting but something else, it is possible that he may have been the artist in 1506, not 1495. If so, this painting could have been a practical exercise that Pacioli used to teach him about perspective; and the painting could have been commissioned by Pacioli to serve as a reminder to Guidobaldo that he was Pacioli's patron, a sound and timely tactic given the publishing planned by Pacioli in 1509. Tempting as it is, this hypothesis must be dismissed: the first two numbers in the date are clearly '14', indicating that the painting was done before 1500.

¹⁶ Image reproduced with permission of Editorial Akai from Pacioli, L. (1991) *La divina proporción*, Spanish Castellano edition.

Figure 13. The fly obscuring the date on Pacioli's portrait¹⁷

However, there is a much more plausible candidate for whom a date in the 15th century does make sense, one who we are aware prepared many works of art for Pacioli in 1498 to be used in *De Divina Proportione*: Leonardo da Vinci. Leonardo would not only have been capable of all the artwork in the painting, he would also have been capable of including all the imperfections. Furthermore, as a man who revelled in puzzles and codes, he would have been the perfect foil to put Pacioli's intentions into effect. In 2013, Carla Glori published a paper with her analysis of the portrait (Glori 2013). Her conclusion was that Leonardo was the artist. For part of her analysis she relied on the work of Giovanni Barca (Barca n.d.). In particular, Barca, writing about Pacioli's portrait, presented drawings of a man from Leonardo's notebooks that he demonstrates are consistent with Pacioli's facial features. Barca also concludes that Leonardo painted Pacioli's portrait.

Figure 14. Leonardo da Vinci's three drawings of a man believed to be Luca Pacioli and Pacioli from his portrait¹⁸

5. Pacioli's Tree of Proportions and Proportionality

For all his other talents, Pacioli himself was no artist. This made his friendship with Leonardo a perfect combination – he helped Leonardo with roots and perspective, Leonardo helped Pacioli with his pen and ink. The result was the 59 figures in *De Divina Proportione*. But, one thing Leonardo did not do for Pacioli was to redraw his Tree of Proportions from *Summa Arithmetica* for use in the later book. Pacioli did his best when he first drew the tree for

¹⁷ Image from Glori (2013, 5)

¹⁸ Images from Barca (N.D., XXXII-XXXIII).

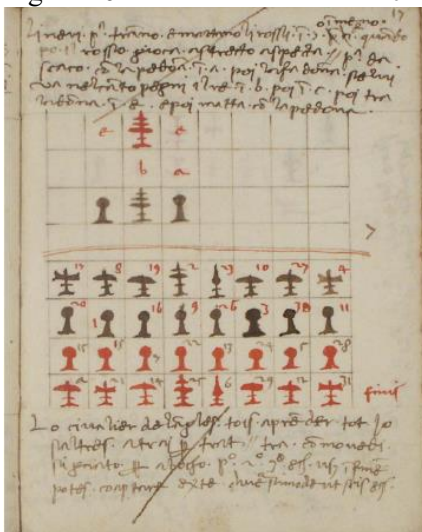
Summa Arithmetica, but it is hardly a fine piece of art, showing the roots of the tree growing down and out from the base. It does, however, express well Pacioli's belief that everything could be expressed in mathematics and that mathematics was the tool God used in the Creation.

Figure 15. Pacioli's *Tree of Proportions and Proportionality* (1494 and 1509)¹⁹



The low quality of Pacioli's artistry is no better evidenced than in his book on chess, *De Ludo Scachorum*.

Figure 16. Pacioli's artwork in *De Ludo Scachorum* (c. 1500)²⁰



6. Pacioli's solids

Pacioli designed and commissioned 60 geometric figures with faces made of glass that he presented to Guidobaldo in Naples in 1488. These were the same shapes drawn by Leonardo da Vinci for *De Divina Proportione*. He mentions in that book that:

I have with my own hands here in Milan arranged, coloured and decorated, and formed to the number of sixty among regulars and their dependants. I arranged a set of as many others for my patron Sir

¹⁹ Image courtesy History of Science Collections, University of Oklahoma Libraries; copyright the Board of Regents of the University of Oklahoma.

²⁰ Reproduced with the permission of the Museo Civico "Piero della Francesca" – Comune di Sansepolcro.

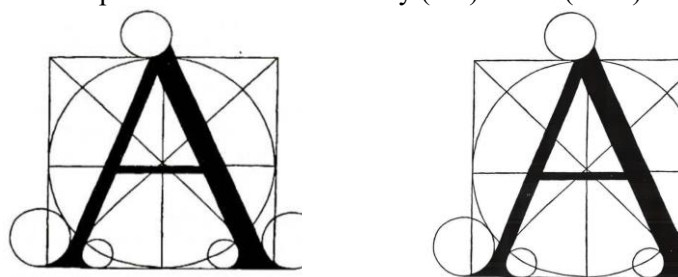
Galeazzo Sansevrino in that place; and as many more again in Florence after the example of my perpetual Confalonier Petro Soderini, which are at present found in his palace. (Pacioli 1509, c. 28v)

The solids were to be used as an instructional device in conjunction with the book and are referred to in the text when being discussed. The three sets mentioned by Pacioli were probably made of wood and paper, rather than the glass used in the original set given to Guidaboldo. No pieces from these four sets have survived, though they presumably assisted Leonardo in his preparation of the drawings contained in *De Divina Proportione*. Whatever the quality of Pacioli's artistry, in 1502 Soderini paid him 52 *lire* 9 *soldi* for the solids (more than 7 florins), the equivalent of a labourer's wages for 3.5 months (Goldthwaite 2009, 612-613).

7. Pacioli's design of the alphabet

After the three main sections of *De Divina Proportione* are two appendices, the second contains the drawings of the solids by Leonardo da Vinci. The first contains 23 images of capital letters drawn by Luca Pacioli. It has been described as, "a creation of pure artistic form" (Smedresman 2006, 21) and "a model of perfection for carving epitaphs and inscriptions" (McKinney 2016). Pacioli adopted the same square and circle approach as Leon Battista Alberti, though he attributes it to Vitruvius (Magnaghi-Delfino & Norando 2018, 2205), but it was also the approach used by others, including Damiano da Moile who published the first printed manual on the design of letters in 1480. Comparison of the designs of Pacioli and Miole reveals some strong similarities, as in the example shown in Figure 17 that compares their designs of the letter 'A'. However, Pacioli's was unique, using a proportion within the letters of 1:9 – Miole used 1:10, which was the norm at that time. This ratio refers to the thickness of the major stroke in a letter compared to its height. (Swetz 1996) Pacioli described this saying, "the right limb (of A) must be one-ninth part as thick as the height" (Magnaghi-Delfino & Norando 2015: 562). By using a smaller ratio than his predecessors, Pacioli made the letters clearer from a distance, and thus more suitable for his declared audience of stonemasons, sculpturers, and architects. To the end, Pacioli, the humanist, presented what he believed would be of most use to his audience.

Figure 17. A comparison of the letter 'A' by (left) Moile (1480)²¹ and (right) Pacioli (1509)²²



While the design of Pacioli's alphabet is easily found online or in any of the several replicas of the printed edition of *De Divina Proportione*, in 1933 it was also published in English in a critical edition by Stanley Morison, as part of a book on Luca Pacioli (Morison 1933). The section of that book depicting the letters and critical translation of Pacioli's instructions in how to draw them was published separately in 1994 (Morison 1994).

²¹ Moile image by Swetz (1996, 3).

²² Pacioli image is in the public domain:

https://commons.wikimedia.org/wiki/File:Luca_Pacioli,_De_divina_proportione,_Letter_A.jpg

Pacioli was not the only writer of a manual on double entry to publish a manual on the formation of letters. In 1524, Giovanni Tagliente followed suit, the year before he published his bookkeeping manual. Pacioli's student, Albrecht Dürer, did so in 1525, using proportions of 1:9 and 1:10.

8. The market for Pacioli's books

Pacioli's books are rare and highly valued collector's items, and this includes sections of them being sold separately, including Pacioli's alphabet which, remarkably, was recently sold, not extracted from the published book, but in the form of a printer's proof of 22 of the original 23 folios from *De Divina Proportione*. It was sold at auction in London for £18,000 in 2016 (Anon 2018). This is slightly less than the £20,000 sale price in Australia in 2018 of a copy of the first volume of the 1523 printing of *Summa Arithmetica*, but it is dwarfed by the £470,000 paid for a copy of the first edition of *Summa Arithmetica* in 2005 (ibid.). That price more than doubled when the next copy to be sold at auction (in the US in June 2019) raised US\$ 1.215 million (€1,085,000/£969,000).

9. Conclusion

Luca Pacioli was a true hero of the Renaissance but, his legacy goes well beyond what we normally read about others described in this way. He had a profound impact upon many different arts. His publication in a print run of 2,000 copies (Sangster 2007) of a book containing a treatise on double entry bookkeeping, with its portrayal of the *libro doppio* (Tagliente 1525) – the “ledger with its journal” – bookkeeping system and method of Venice, became the *de facto* standard across Europe, and the method still taught today. His manual on how to create perfectly proportioned Roman letters was the first to contain detailed instructions in how to create them. It is a design so well respected for its beauty that, until recently, the Metropolitan Museum in New York used its letter ‘M’ as its logo.

His teaching of mathematics to Leonardo da Vinci provided assistance others could not offer in improving the quality of his art, to the benefit of artists and art scholars ever since; as did his tuition in perspective of Albrecht Dürer and the countless other artists with whom he had contact. He provided those engaged in the art of trade with a manual on business mathematics that was never equalled; and his *De Divina Proportione* was instrumental in elevating theoretical mathematics to be recognised as a subject of major importance, rather than a minor tool. His impact on society can be felt today, with memorials to him across the world; major volumes published on his life and work in a multitude of disciplines, languages, and in all continents; colleges, business schools, research institutes, research societies, an algebraic relationship, email addresses, streets, and computer packages named after him; and his image is so well known that it has been used in advertising accounting software from the 1990s to the present day.

This study has contributed to our understanding of Luca Pacioli, identifying him as an artist in his own right, not an artist who painted pictures, but an artist whose talent lay in creating new opportunities for others, from all walks of life and all strata of society, whether in business, architecture, building, education, or art. Pacioli could identify what people needed, what would improve their lives and make their work and their experiences more rewarding. In doing so, if there is one ‘art’ in particular that he mastered and perfected above all others, it was the art of

communication: he knew how to write about what his audience needed, and in language – verbal and visual – that they knew and understood. In that art, he has never been surpassed.

His legacy, with its reach into several disciplines, is vast. Standing back and looking at what Pacioli did and what he achieved, it is apparent that he was also a phenomenal visionary. And not just in identifying what people needed, such as a manual on double entry, an encyclopaedia of mercantile mathematics, or a book of chess puzzles. He also realised how several things could be done better. For example, the way he changed how algebra was taught and learned, which was diametrically opposite in approach to the inductive methods of the *abaco* masters; or the benefits to architects, building designers, and artists of changing the ratio in letter design from 1:10 to 1:9. He was also the first known educator to realise that a textbook accompanied by wooden shapes might make it easier for a learner to follow and understand the text. When he did this for the manuscript copies of *De Divina Proportione*, it was an innovation that, like Leonardo's parachute and helicopter, belongs more in the 20th and 21st century than the 15th. This paper has not explored that visionary side of Pacioli, a side where a desire that people learn led him to discover new ways and new designs, sometimes centuries ahead of everyone else. Studies on Pacioli that investigate the influences that impacted on him and made him the educational visionary he became in virtually every field he touched are long overdue.

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