UDC 620.015

# Application of the Martel dynamic hardness to the penetration problem

Yu. V. Milman, V. A. Goncharuk, L. V. Mordel

*Penetration of the undeformable kinetic energy projectile (KEP) into the target is considered as the "deep" indentation. It was shown by the example of aluminum alloys that the Martel dynamic hardness* $HMR_d$ *can be used for description of this process.* $HMR_d$ *for the target can be calculated from the relation* $HMR_d = \frac{mv^2}{2V}$, *where m is the mass of the KEP, υ is its rate before impact collision with the target, and V is the volume of the penetration channel. The ballistic limit* $v_c$ *of the target with a given thickness l can be calculated by equation* $\upsilon_c = \sqrt{\frac{HMR_d \pi d^2 l}{2m}}$ *for the KEP with a given mass m and diameter d.*

***Keywords:*** *Penetration, Martel hardness, hardness from penetration depth, dynamic hardness.*

## Introduction

Dynamic hardness was introduced by Martel in 1895 [1]. In experiments, Martel used a steel ball that dropped from a height $h_1$ onto a smooth metallic surface and made a spherical indentation on the surface of the sample. It was shown that $A/V$ = const, where $A$ is the kinetic energy of the ball and $V$ is the volume of the indentation. Since this relation has the same dimensionality as pressure [Pa], it can be considered as the dynamic hardness of metals [2, 3].

Thus, the Martel hardness *HMR* is determined from the relation

$$HMR = \frac{A}{V}, \tag{1}$$

where $A$ is the work to create the indentation and $V$ is the volume of this indentation.

Since *HMR* has the same dimensionality as the Meyer hardness (the mean contact pressure during indentation) and characterizes the same process, it can be thought that

$$HMR = KHM, \tag{2}$$

where $K$ is a dimensionless parameter.

Martel [1] had calculated hardness from equation:

$$HMR = \frac{mgh_1}{V}. \tag{3}$$

But in experiments performed by Martel, after impact of a ball, the elastic recovery of an indentation occurred, and the ball rebounded to a height $h_2$ ($h_2 < h_1$). Since, in these experiments, it was impossible to measure the volume of indentation under load (to calculate the unrecovered hardness), the recovered hardness must be calculated. The energy that caused plastic (residual) strain can be calculated by subtracting the energy of elastic recovery from the kinetic energy of the ball.

The mathematical description of the process of dynamic indentation was given by Tabor [2].

Tabor had obtained equation

$$HMR = \frac{mg\left(h_1 - 3/8h_2\right)}{V}, \tag{4}$$

that take into account the energy of elastic recovery. Equation (3) may be used if the rebound is not very large, so that $h_2$ is small.

At present, the Martel hardness can also be calculated in static indentation by a pyramidal indenter by using the instrumental hardness with recording the “load on the indenter *P*–displacement of the indenter *h*” curve. In this case, the work expended on the formation of the hardness indentation is equal to the area under the *P*—*h* curve, and the volume of the indentation can be determined from the contact depth of penetration of the indenter $h_c$; the technique of determination of $h_c$ was developed in [4]. The value of $h_c$ can also be determined *by standard microscopy methods* via determination the size of the indentation diagonal (in the assumption that the size of diagonal is not changed during recovery) and calculation of the height of the indentation pyramid, using the value of the center line to the face angle of pyramidal indenter.

In the present work, we consider the possibility of applying the Martel hardness to the problem of penetration a target by a kinetic energy projectile (KEP) if the KEP is undeformable. It is assumed that, in this case, the process of penetration of the KEP can be considered as a “deep” indentation. The ratio of the kinetic energy expended by the KEP on the formation of the penetration channel in the target to the volume of the penetration channel must correspond to the Martel hardness according to eq. (1).

The check of this proposition has been performed for several aluminum alloys. In static indentation, the Martel hardness was determined by the instrumental indentation method in the microhardness region. The possibility to use Martel hardness to calculate the ballistic limit for targets from aluminum alloys is shown.

## Experimental Results

The penetration of three aluminum alloys by the KEP was investigated in the present work, to check the possibility of applying the Martel hardness to the problem of penetration. The chemical compositions of the alloys, Martel’s hardness *HMR*, and Meyer’s hardness *HM* are presented in table.

Targets of aluminum alloys in the form of sheets 25 mm in thickness were used. The mass of the KEP was $m$ = 9,6 g, and the diameter of the penetration channel practically coincided with the diameter of the KEP $d$ = 7,62 mm.

The volume of the penetration channel was calculated as follows:

$$V = \frac{\pi d^2}{4} l, \tag{5}$$

where $l$ is the depth of penetration of the KEP.

Typical instrumental indentation curves obtained in a Micron Gamma unit [5] in the *P*—*h* coordinates for three aluminum alloys are shown in fig. 1.

**Chemical compositions of aluminum alloys investigated in the work, Meyer hardness *HM* and Martel hardness *HMR*; the subscript *s* corresponds to static indentation; the subscript *d* corresponds to the mean hardness determined in penetration of the target by the KEP, and calculated by (1). *P* is the maximum load on indenter**

| Number compo-sition | Chemical composition | *HM*, GPa $P = 150$ g | $HMR_s$, GPa $P = 150$ g | $HMR_s$, GPa $P = 10$ kg | $HMR_d$, GPa |
|---|---|---|---|---|---|
| 1 | Al—4,45Mg—0,7Mn—0,13Cr | 0,99 | 1,13 | 1,02 | 1,18 |
| 2 | Al—4,45Mg—0,4Mn—0,3Sc—0,1Zr | 1,48 | 1,68 | 1,37 | 1,42 |
| 3 | Al—6Zn—2,3Mg—1,5Cr—0,3Sc—0.1Zr | 2,07 | 2,3 | 2,16 | 2,0 |

These curves were used to determine the work of indentation. The volume of a hardness indentation was calculated by the formula for calculation of a trihedral pyramid volume.

$V = \frac{1}{3} S h_c$, where $S$ is the projection area of indentation and $h_c$ is the contact depth of penetration of the indenter.

The kinetic energy of the KEP was calculated as $E = \frac{m\upsilon^2}{2}$, where $\upsilon$ is the velocity of the KEP before the impact collision with the target. The energy expended on the formation of a unit volume of the penetration channel was considered as the dynamic hardness $HMR_d$ and calculated by the formula

$$HMR_d = \frac{m\upsilon^2}{2V}, \tag{6}$$

The values of $HMR_d$ are shown in fig. 2 as a function of the velocity of the KEP v for 3 aluminum alloys.

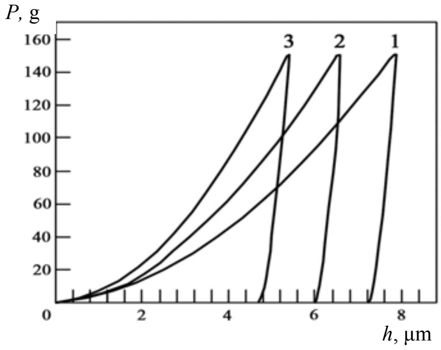

Fig. 1. Instrumental indentation curves of aluminum alloys (the numbers of the alloys correspond to those in table).

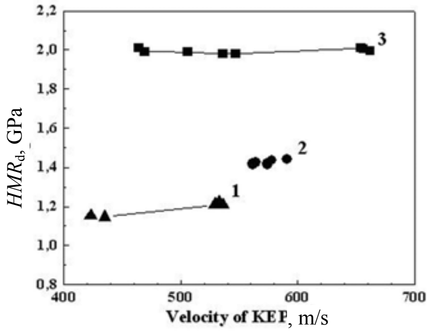

Fig. 2. Dependence of the dynamic Martel hardness $HMR_d$ , determined in penetration of KEP into the targets according to (6), on the velocity $\upsilon$.

The instrumental Martel hardness $HMR_s$ and Meyer hardness $HM$ were determined at a maximum load on the indenter equal to $P = 150$ g. The obtained results (see Table 1) showed that for the investigated aluminum alloys, relation (2) is satisfied for $K \approx 1{,}13$. The Meyer hardness $HM$ was also determined with a Vickers hardness tester under a load of $P = 10$ kg, and the Martel hardness $HMR_s$ was calculated by formula (2) under the assumption that $K = 1{,}13$ as in the case of a smaller load on the indenter.

## Discussion of Results

The Meyer hardness $HM$ is the force approach to the *indentation* problem when one determines a maximum value of the mean contact pressure at which the penetration of the indenter terminates. The Martel hardness can be considered as the energy approach to the indentation problem, for which the work expended for the formation of the unit of a hardness indentation volume is determined.

In physics of strength, both the force and the energy approaches are often applied to the same problem and enable one to reveal more completely the essence of the process. For instance, in the problem of development of a crack and the fracture toughness, the energy approach was developed by Griffith [6] and Orovan [7], and the force approach was developed by Irvin [8].

The force approach is based on the laws of mechanics, and the energy approach uses the notions of the energy balance.

The use of both the force and the energy approaches extends the possibility of using the notion of hardness in different physical processes.

For instance, it is practically impossible to compute the Martel hardness by using the standard methods of measuring the static hardness by a rigid indenter (the Vickers method, Brinnel method, etc.) because these methods do not enable one to determine the energy $A$ expended on the formation of an indentation. An estimate of the quantity $A$ in static indentation (under the assumption that $P \sim d^n$, where $d$ is the diameter of the indentation of a spherical indenter or a diagonal of the indentation of a pyramidal indenter) given in [9], does not take into account that, in the indicated methods, the loading of the indenter by a maximum load $P$ is followed by a hold, during which the hardness indentation increases at $P = \text{const}$, and some work is expended on this process as well. This is why the conclusion that $HMR = HM$ (i. e., the conclusion that the constant $K$ is equal to 1 in formula (2)) made in [9] does not agree with results presented in table. In other words, $K$ obtained in instrumental indentation is somewhat larger than 1 precisely due to holding indenter under $P = \text{const}$. As is seen from table, for aluminum alloys, we have $K \approx 1{,}13$. It is clear, that in penetration of the KEP into a target, the notion of Meyer hardness loses its meaning because this hardness is determined by the value of residual (plastic) strain at the moment when the indenter under a load $P$ stops, but in the penetration process the KEP does not stop in the surface layer.

However, the efficiency of using the fairly simple technique of measuring the Meyer hardness by different rigid indenters (spheres, trihedral or tetrahedral regular pyramids, cones, etc.) for the characterization of properties of the materials is beyond any doubt.

Static indentation by a rigid indenter with the determination of the Meyer hardness makes it possible to determine the average contact stress and calculate not only the flow stress from it, but also determine a number of other mechanical characteristics of materials, e. g., to construct stress-strain curves, determine the strain hardening and plasticity characteristic, estimate the fracture toughness, etc. [10, 11]. In [12], it was shown that, in aluminum alloys, around the channel of penetration, a disperse granular nonequiaxial structure and a dislocation cellular substructure, which are typical for metals deformed in compression by 70%, are formed. However, this does not enable us to determine the Meyer hardness *HM* in the case of penetration a target by a KEP.

At the same time, experimental results obtained in the present work show that the Martel hardness can be used to describe the process of penetration of the KEP.

It is seen from fig. 2 that the Martel hardness $HMR_d$, that is determined in the process of penetration the target by the KEP , is practically independent on the velocity υ of the KEP in the investigated range of used values of υ. This enables us to determine the critical velocity of penetration (ballistic limit) from the relation

$$\upsilon_c = \sqrt{\frac{HMR_d \pi d^2 l}{2m}}\,. \qquad (7)$$

The critical thickness of the target $l$, which will be penetrated at a velocity $\upsilon$ of the KEP, can be determined from the same relation at a known velocity $\upsilon$ . At the same time, it follows from fig. 2 that there exists some tendency to decrease of $HMR_d$ as the velocity $\upsilon$ of the KEP diminishes. A comparison of the value of $HMR_s$ (obtained under static loading of the indenter) and $HMR_d$ shows that for alloys 1 and 2, $HMR_d$ is somewhat higher than $HMR_s$ (table). Under static loading of the indenter, its velocity can be estimated from the relation $\upsilon_s = \frac{h}{t}$ (where $h$ is the maximum displacement of the indenter and $t$ is the loading time). The estimate shows that $\upsilon_s = 5\times10^{-7}$ m/s, i. e., it is smaller by 9 orders than that in the case of penetration of the target by the KEP.

As is seen from (2), $HMR \sim HM$. However, it is known that *HM* is proportional to the flow stress $\sigma_s$ and that $\sigma_s$ increases with the strain rate [10, 13]. For this reason, the fact that $HMR_d$ is somewhat larger than $HMR_s$ seems to be natural. The indicated relationship between $HMR_d$ and $HMR_s$ is observed for alloys 1 and 2, whereas for the hardest alloy 3, $HMR_d$ is even slightly lower than $HMR_s$. It can be assumed that, for this alloy, the condition of determination of the Martel hardness cannot be satisfied, i. e., the KEP cannot be absolutely undeformed.

At the same time, there exists one more factor that can lead to a difference between the values of $HMR_d$ and $HMR_s$, namely, the scale dependence of the hardness (indentation size effect), which manifests itself to the highest degree for nanohardness [14], but some decrease in *HM* with increasing load *P* is observed in micro- and macro hardness regions as well. For the hardest alloy 3, the scale dependence of the hardness must be stronger than that for the softer alloys 1 and 2 because the size of indents for harder alloys is smaller than that for softer alloys.

It should also be noted the important results of work [15], in which ballistic limit for some aluminum alloys was determined for two different KEPs.

The values of $HMR_\mathrm{d}$ calculated by eq. (6) for the data of work [15] were found to be practically equal for KEPs of different diameter and mass. For instance, for 5083 aluminum alloy, at $\upsilon_\mathrm{c}$ = 722 m/s, $HMR_\mathrm{d}$ appears to be equal for the KEP with $m$ = 44,9 g, $d$ = 12,9 mm and $l$ = 59,7 and the KEP with $m$ = 10,4 g, $d$ = 7,84 мм, and $l$ = 35. For both KEPs, $HMR_\mathrm{d} \approx$ 1,43 GPa. Note also that alloy 2, used in the present work, has a chemical composition close to that of 5083 alloy and its value of $HMR_\mathrm{d}$ (1,42 GPa) is practically equal to that of 5083 alloy.

## Conclusion

It has been proposed to consider the penetration of the target by the undeformable kinetic energy projectile as a process of "deep" indentation and determine the Martel dynamic hardness $HMR_\mathrm{d}$ for this target from the relation $HMR_\mathrm{d} = \dfrac{mv^2}{2V}$, where $m$ is the mass of the KEP, $\upsilon$ is its rate before impact collision with the target, and $V$ is the volume of the penetration channel. By the example of aluminum alloys, it has been shown that $HMR_\mathrm{d}$ depends slightly on the velocity of the KEP $\upsilon$. The static Martel hardness $HMR_\mathrm{s}$ is related to the Meyer hardness $HM$ by the simple relation $HMR_\mathrm{s} = KHM$, where $K$ is somewhat larger than 1.

If $HMR_\mathrm{d}$ of the target has been determined , the ballistic limit for this target with a given thickness l can be calculated by equation (7) for the KEP with a given mass and diameter.

1. *Martell R.* Commision de Methodes d'Essai des Materiaux de Construction. — Paris. — 1895. — **3**. — 261 p.
2. *Tabor D.* The hardness of metals. — Oxford: Clarendon Press, 1951.
3. *Kohlhöfer W., Penny R. K.* Dynamic hardness testing of metals // Ynt. Y. Pres. Ves. and Piping 61. — 1995. — P. 65—75.
4. *Oliver W. C., Pharr G. M.* A new improved technique for determining hardness and elastic modulus using load and sensing indentation experiments // J. of Mater. Res. — 1992. — **7**, No. 6. — P. 1564—1582.
5. *Gnatowicz S. R., Shmarov V. N., Zakiev I. M., Maistrenko Y. N.* Control of physical and mechanical properties of solid surfaces by methods of scanning local and micro / nano indentation // Nanotechnology: Sat. Reports. Kharkov Nanotechnology Assembly 2008. — Kharkov: KIPT, 2008. — **1**. — P. 243—246.
6. *Griffith A. A.* The phenomena of rupture and flows in solids // Phil. Trans., Roy. Soc. — 1920. — **A 221**. — P. 163.
7. *Orovan E.* Fracture and strength of solids rep // Progr. Phys. — 1949. — **12**. — P. 185.
8. *Irvin G. R.* Fracturing of Metals. — Cleveland. Ohio: ASM, 1948. — 147 p.
9. *Grigorovich V. K.* Hardness and Microhardness of Metals [in Russian]. — Moscow: Nauka, 1976.
10. *Milman Yu. V.* Physics of the hardness of materials and new possibilities of the indentation technique // Sintering. — 2000. — P. 133—146.
11. *Milman Yu. V.* New methods of micromechanical testing of materials by local loading with a rigid indenter // Advanced Mater. Science: 21st century. — Cambridge international science publishing, 1998. — P. 638—660.
12. *Milman Yu. V., Chugunova S. I., Goncharova I. V. et al.* Physics of deformation and fracture at impact loading and penetration // Int. J. Impact Engineering. — 2006. — **33**. — P. 452—462.
13. *Trefilov V. I., Milman Yu. V.* The influence of temperature and structure factors on mechanical properties of refractory materials // Proc. of the 10th Plansee Seminar, 1981. — **2**. — P. 15—26.

14. *Milman Yu. V., Golubenko A. A., Dub S. N.* Indentation size effect in nanohardnes // Acta Mater. — 2011. — **59**. — P. 7480—7487.
15. *Tyrone L. Jones and Richard D. DeLorme*. Development of a ballistic specification for magnesium alloy AZ31B // Army Res. Laboratory USA—ARL—TR—4664. — December 2008.

---

## Застосування динамічної твердості за Мартелем до проблеми проникнення

Ю. В. Мільман, В. А. Гончарук, Л. В. Мордель

*Проникнення недеформівним кінетичним індентором (КЕР) в цілому можна розглядати як "глибоке" індентування. На прикладі алюмінієвих сплавів було показано, що динамічна твердість за Мартелем* $HMR_d$ *може бути використана для опису цього процесу.* $HMR_d$ *може бути розрахована із співвідношення* $HMR_d = \frac{mv^2}{2V}$*, де m — маса КЕР, υ — швидкість перед зіткненням з мішенню і V — об'єм каналу проникнення. Балістичну межу* $V_C$ *мішені із заданою товщиною l можна розрахувати по рівнянню* $\upsilon_c = \sqrt{\frac{HMR_d \pi d^2 l}{2m}}$ *для КЕР з даної масою m і діаметром d.*

***Ключові слова:*** *проникнення, твердість по Мартелю, твердість від глибини проникнення, динамічна твердість.*

## Применение динамической твердости по Мартелю к проблеме проникновения

Ю. В. Мильман, В. А Гончарук, Л. В. Мордель

*Проникновение недеформируемым кинетическим индентором (КЕР) в целом можно рассматривть как "глубокое" индентирование. На примере алюминиевых сплавов показано, что динамическая твердость по Мартелю* $HMR_d$ *может быть использована для описания этого процесса.* $HMR_d$ *может быть рассчитана из соотношения* $HMR_d = \frac{mv^2}{2V}$ *где m — масса КЕР, υ — скорость перед столкновением с мишенью и V — объем канала проникновения. Балистический предел* $V_C$ *мишени с заданной толщиной l можно рассчитать по уравнению* $\upsilon_c = \sqrt{\frac{HMR_d \pi d^2 l}{2m}}$ *для КЕР с данной массой m и диаметром d.*

***Ключевые слова:*** *проникновение, твердость по Мартелю, твердость от глубины проникновения, динамическая твердость.*