УДК 539.3

Elastic Buckling of Selected Flanges of Cold-Formed Thin-Walled Beams

Krzysztof Magnucki

Professor, Doctor Hab. Engineer, Institute of Applied Mechanics, Poznan University of Technology, ul. Piotrowo, 3, PL 60-965, Poznan, Poland, e-mail: krzysztof.magnucki@put.poznan.pl

The paper is devoted to five different flanges of cold-formed thin-walled beams. Mathematical models of each of the flanges are formulated and solved. The theory of elastic stability of plates and cylindrical shallow shells is applied for this purpose. Critical stress for each flange of the beam is determined. Results of analytical solutions are discussed and compared with numerical (FEM) and experimental investigations. The formulas of critical stresses may be used in practical applications.

Key words: thin-walled beams, cold-formed beams, elastic buckling, thin plates, thin shells

Introduction. Cold-formed thin-walled beams of flat walls may be modeled by long rectangular plates joined with adjacent edges. Investigation of local stability of such beams converts itself to studying buckling of long rectangular plates, taking into account appropriate conditions of supporting. Timoshenko and Gere (1961) and Volmir (1967) presented a discussion of the elastic buckling problems. Detailed results of contemporary analytical, numerical — FEM and experimental investigations of selected problems of strength, buckling and optimization of cold-formed thin-walled beams are presented for example by: Bradford and Ge (1997), Bradford (1998), Put, Pi and Trahair (1999), Davies (2000), Pi and Trahair (2000), Magnucki and Monczak (2000), Rasmussen (2001), Magnucki (2002), Hancock (2003), Mohri F., Brouki A., Roth (2003), Corte *et al.* (2004), Dinis P. B., Camotim D., Silvestre (2004), Stasiewicz *et al.* (2004), Trahair and Hancock (2004), Magnucki *et al.* (2004), Magnucka-Blandzi and Magnucki (2004), Magnucki (2005), Magnucki and Ostwald (2005), Magnucki and Maćkiewicz (2005). Elastic buckling problems of five selected flanges of cold-formed thin-walled beams are presented based on the referred papers. Beams with these flanges are under a pure bending state. The upper flange of each of the beams is compressed and the lower flange is subject to tension.

1. A flat flange with a bend of the channel beam

Channel beams with flat flanges are reckoned among typical cold-formed thin-walled beams. One edge of such a flange is free or stiffened by bends. Buckling of these flanges was studied by Hancock (1997), Rogers and Schuster (1997), Bambach and Rasmussen (2001, 2004), Corte G. D. *et al.* (2004). They proposed some formulas designed for determining critical loads. Stasiewicz et. al. (2004), Magnucki and Ostwald (2005) analytically determined critical loads of flat flanges with bends. Their analytical solution was subject to numerical verification by means of the finite element method.

Approximate model of a flat flange with bends is considered as a long rectangular plate with three simply supported edges. The fourth edge with the bends is free (fig. 1). Moreover, the flange-web joint of the beam is assumed as a hinged one.

The flange width b is small as compared to its length L. A cross section of the flange rotates with respect to its supporting point (the flange-web joint) by the angle $\psi(x)$.

Angle of rotation of a flange cross section is assumed in the form

$$\psi(x) = \psi_1 \sin\frac{\pi x}{L}, \tag{1}$$

where ψ_1 — parameter;
hence, the corresponding displacement

$$v(x,z) = z\cdot\psi(x) = \psi_1\cdot z\sin\frac{\pi x}{L}. \tag{2}$$

Potential energy of elastic strain for the flange

$$U_\varepsilon = \frac{GJ_t}{2}\int_0^L\left(\frac{d\psi}{dx}\right)^2 dx + \frac{EJ_{zp}}{2}\int_0^L\left(\frac{d^2v}{dx^2}\right)^2\Bigg|_{z=b} dx, \text{ or}$$

$$U_\varepsilon = \frac{\pi^2}{4L}\left[GJ_t + \pi^2\left(\frac{b}{L}\right)^2 EJ_{zp}\right]\cdot\psi_1^2, \tag{3}$$

where

$y_p = \dfrac{1}{b+c+d+e}\left[c^2 + dc - \dfrac{1}{2}(c-e)^2\right]$ is location of central axis of the flange cross section,

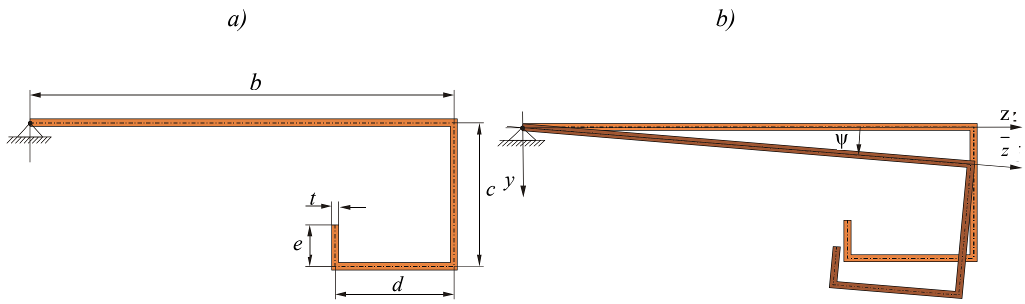

Fig. 1. Schemes: *a)* cross section of the flange, *b)* displacement of a buckled flange

$J_{zp} = t\left\{\frac{2}{3}c^3 + c^2 d - \frac{1}{3}(c-e)^3 - \frac{1}{b+c+d+e}\left[c^2 + dc - \frac{1}{2}(c-e)^2\right]^2\right\}$ is moment of inertia,

$J_t = \frac{1}{3}t^3(b+c+d+e)$ is geometric torsion stiffness of the cross section.

The upper flange is compressed and the work of the load

$$W = \frac{1}{2}t\sigma_0\left[\int_0^b\int_0^L\left(\frac{dv}{dx}\right)^2 dxdz + (c+d+e)\int_0^L\left(\frac{dv}{dx}\right)^2\Bigg|_{z=b} dx\right],$$

and, upon integrating

$$W = \frac{\pi^2 b^2 t}{12L}\sigma_0\left[b + 3(c+d+e)\right]\cdot\psi_1^2, \tag{4}$$

where

$\sigma_0 = \left(1 - 2\frac{y_p}{H}\right)\sigma_{max}$ is compressive stress in the flange, H depth of the beam.

Maximal stress in the bent beam $\sigma_{max} = \frac{M}{2J_z}H$, where M_{max} is maximal bending moment.

The principle of stationary total potential energy $\delta(U_\varepsilon - W) = 0$ enabled determining critical stress of the beam flange subject to pure bending

$$\sigma_{0,KR}^{(1)} = \frac{3}{b^2 t\left[b + 3(c+d+e)\right]}\left[GJ_t + \pi^2\left(\frac{b}{L}\right)^2 EJ_{zp}\right]. \tag{5}$$

In particular case of a flange without bends $(c = d = e = 0)$ the critical stress

$$\sigma_{KR} = G\left(\frac{t}{b}\right)^2 = \frac{E}{2(1+\nu)}\left(\frac{t}{b}\right)^2.$$

Example calculation of critical stress has been carried out (5) for the channel beam flange, assuming the following numerical data: $t = 1{,}5$ mm, $b = 100$ mm, $0 \le c \le 20$ mm, $0 \le d \le 10$ mm, $0 \le e \le 4$ mm, $L = 800$ mm, $\nu = 0{,}3$. Results of the calculation are shown in fig. 2. Enlarging the length of any bend results, of course, in growing critical stress. Very effective growth of the stress occurs while extending the first bend up to a certain minimal value $(c/b)_{min} = 0{,}05$. Smaller growth of the stress, slightly below linear, corresponds to extending the second bend $(0 \le d/b \le 0{,}1)$. On the other

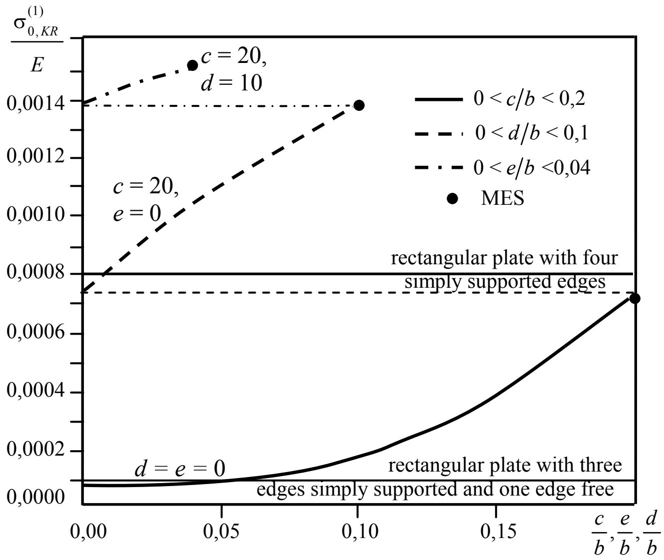

Fig. 2. Critical stress of the flange as a function
of bending ratio of its free edge

hand, extending the third bend $(0 \le e/b \le 0{,}04)$ appears to be of small meaning, as it results only in insignificant growth of the flange critical stress. It should be noticed that a free edge complemented with the bends of growing area must be finally considered as a supported one. Hence, the value of critical stress corresponding to the flange with bends exceeds the value for the flange/plate supported at four edges. For example, such support of the considered flange was obtained when the value of the second bend $d/b = 0{,}01$ (fig. 2).

Moreover, critical stress of the flange was determined by means of FEM (The COSMOS/M System). The computation was confined to the variant of the flange with maximal bends. Hence, the following numerical data have been assumed: $t = 1{,}5$ mm, $b = 100$ mm, $c = 20$ mm, $d = 10$ mm, $e = 4$ mm, $L = 800$ mm, $\nu = 0{,}3$, $E = 2{,}05 \cdot 10^5\ MPa$ with uniformly distributed compressive stress. Values of the stresses obtained this way are marked with the points of the diagram (fig. 2). Differences between numerical results obtained with both methods do not exceed 1,5%.

2. A flat rectangularly corrugated channel beam

Flat flanges of thin-walled beams may be also stiffened by their corrugation. In such a case the flange takes, in practice, a form of a long orthotropic rectangular plate, with three edges simply supported and the fourth edge free (fig. 2). Magnucki and Ostwald (2005) analytically determined critical loads of flat rectangularly corrugated flange. Magnucki and Maćkiewicz (2005) the critical loads of flat cosinusoidally corrugated flange determined.

Similarly, like for the single-bend flange, a simplified scheme of displacements shown in fig. 3 is adopted here. The angle of rotation of the flange was assumed in the form (1) and, consequently, the displacement in the form (2).

Potential energy of elastic strain (Volmir (1967))

$$U_\varepsilon = \frac{1}{2}\int_0^b\int_0^L \left[D_x\left(\frac{\partial^2 v}{\partial x^2}\right)^2 + 2D_{xz}\left(\frac{\partial^2 v}{\partial x \partial z}\right)^2 + D_z\left(\frac{\partial^2 v}{\partial z^2}\right)^2 \right] dxdz\,, \tag{6}$$

where

$v(x,z) = z \cdot \psi(x) = \psi_1 \cdot z \sin\dfrac{\pi x}{L}$ is the displacement, $D_x = \dfrac{EJ_z}{c_x} = E\dfrac{c^2 t}{4b}(b+c)$,

$J_z = \dfrac{c^2 t}{12}(b+c)$, $c_x = \dfrac{b}{3}$, $2D_{xz} = \dfrac{Gt^3}{3}\dfrac{s}{c_x} = \dfrac{Gt^3}{3}\left(1+3\dfrac{c}{b}\right)$,

$s = \dfrac{b}{3} + c$, $D_z = \dfrac{Et^3}{12(1-\nu^2)}\dfrac{c_x}{s}$.

Integrating of the expression (6) provides

$$U_\varepsilon = \frac{\pi^2}{4}\frac{b}{L}\left[\frac{\pi^2}{3}\left(\frac{b}{L}\right)^2 D_x + 2D_{xz}\right]\cdot \psi_1^2\,.$$

Work of the load

$$W = \frac{1}{2}N_{xx}\int_0^b\int_0^L \left(\frac{\partial v}{\partial x}\right)^2 dxdz = \frac{\pi^2}{12}\frac{b^3}{L}N_{xx}\cdot\psi_1^2\,, \text{ where } N_{xx} = (b+3c)\frac{t}{b}\sigma_0^{(2)}\,. \tag{7}$$

The principle of stationary total potential energy $\delta(U_\varepsilon - W) = 0$ was a basis for determining critical stress

$$\sigma_{0,KR}^{(2)} = \frac{E}{4(1+\nu)}\left[2\left(\frac{t}{b}\right)^2 + \pi^2(1+\nu)\frac{b+c}{b+3c}\left(\frac{c}{L}\right)^2\right]. \tag{8}$$

In particular case of a flat flange without corrugated ($c = 0$) the critical stress

$$\sigma_{KR} = G\left(\frac{t}{b}\right)^2 = \frac{E}{2(1+\nu)}\left(\frac{t}{b}\right)^2.$$

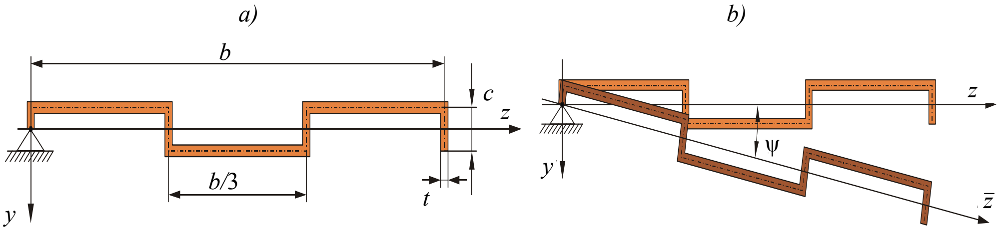

Fig. 3. Schemes: *a)* cross section of corrugated flange, *b)* displacement of a buckled flange

3. A flat three layer flange of channel beam

Stiffness of flat flange of a thin-walled beam may be increased by its double bending and filling the space created this way with polyurethane foam. In such a case the flange is considered, in practice, as a long three-layer rectangular plate with three simply supported edges and one edge free (Fig. 4). Magnucki and Ostwald (2001, 2005) presented the problems of stability of three-layer structures.

A simplified scheme of displacements is applied here, similarly as for the case of the bent or rectangularly corrugated flanges. The angle of rotation of a cross section was assumed in the form (1) and, consequently, the displacement as (2).

Potential energy of elastic strain

$$U_\varepsilon = \frac{1}{2}\int_0^L\int_0^b \left[D_x \left(\frac{\partial^2 v}{\partial x^2}\right)^2 + 2D_{xz}\left(\frac{\partial^2 v}{\partial x \partial z}\right)^2 + D_z\left(\frac{\partial^2 v}{\partial z^2}\right)^2 \right] dxdz\,, \tag{9}$$

where

$v(x,z) = z \cdot \psi(x) = \psi_1 \cdot z \sin\dfrac{\pi x}{L}$ is displacement,

$c = t + 2t_0\,,\quad D_x = \dfrac{1}{2}Ec^2 t\,,$

$D_{xz} = Gc^2 t\left[\left(1+\dfrac{c}{b}\right)^{-1} + \left(1+\dfrac{c}{L}\right)^{-1}\right] \cong Gc^2 t\,\dfrac{2+c/b}{1+c/b}\,,\quad D_z = \dfrac{1}{2}Ec^2 t\,,$

it was taken into account that $c/L \ll 1$.

Integration of the expression (9) gives

$$U_\varepsilon = \frac{\pi^2}{4}\frac{b}{L}\left[\frac{\pi^2}{3}\left(\frac{b}{L}\right)^2 D_x + 2D_{xz}\right]\cdot \psi_1^2\,. \tag{10}$$

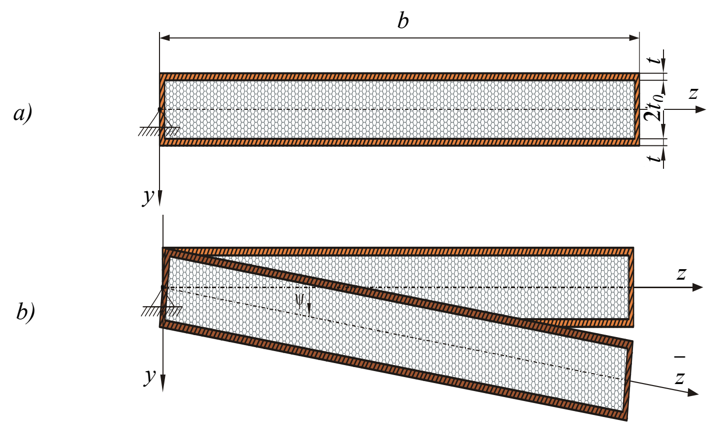

Fig. 4. Schemes: *a)* flat three-layer flange, *b)* displacements of a buckled flange

Work of the load

$$W = \frac{1}{2} N_{xx} \int_0^b \int_0^L \left(\frac{\partial v}{\partial x} \right)^2 dxdz = \frac{\pi^2}{12} \frac{b^3}{L} N_{xx} \cdot \psi_1^2 . \qquad (11)$$

The principle of stationary total potential energy $\delta(U_\varepsilon - W) = 0$ enabled determining intensity of critical load

$$N_{xx,KR} = \left(\frac{\pi}{L} \right)^2 D_x + \frac{6}{b^2} D_{xz} = \frac{1}{2} Et \left(\pi \frac{c}{L} \right)^2 + 6Gt \frac{2 + c/b}{1 + c/b} \left(\frac{c}{b} \right)^2 ,$$

hence, the critical stress of the compressed flange

$$\sigma_{x,KR} = \frac{Eb}{2(b+c)} \left[\frac{3}{1+\nu} \frac{2 + c/b}{1 + c/b} \left(\frac{c}{b} \right)^2 + \frac{1}{2} \left(\pi \frac{c}{L} \right)^2 \right]. \qquad (12)$$

4. A flat double flange of I-beam

Mathematical model for local buckling of the upper flange of the beam is assumed in the form of a beam on an elastic foundation [Magnucki and Ostwald (2005)]. Scheme of the deformation of the cross section of the beam is shown in fig. 5.

The differential equation for the beam on an elastic foundation is in the following form

$$\frac{d^4 w}{dx^4} + k^2 \frac{d^2 w}{dx^2} + \beta \cdot w(x) = 0 \qquad (13)$$

where

$k^2 = \frac{F}{EJ_{z,f}}$, $\beta = \frac{c}{EJ_{z,f}}$, $J_{z,f} = \frac{2}{3} bt^3$, $c = 8E \left(\frac{t}{b} \right)^3$ is module of the elastic foundation, F is longitudinal compression force of the upper flange.

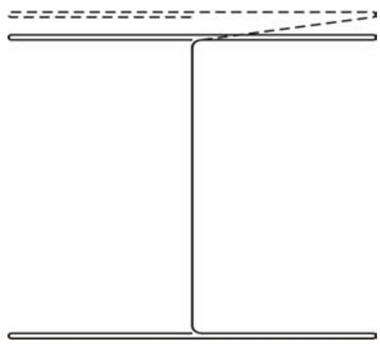

Fig. 5. Deformed cross section of the beam under pure bending

The web is rigid as compared to the flange of the beam. In consequence, the deflection function determining the buckling shape is assumed in the following form

$$w(x) = w_a \cdot \sin^2 \frac{m\pi x}{L} , \qquad (14)$$

where w_a is amplitude, m is natural number.

The differential equation (13) is solved with the Galerkin method. The critical force is obtained in the following form

$$F_{CR} = 8E\frac{t^3}{b}\min_{Y}\left[\frac{2}{3}Y^2 + \frac{3}{4}\frac{1}{Y^2}\right] = 8\sqrt{2}E\frac{t^3}{b}\text{, where } Y = m\pi\frac{b}{L} \tag{15}$$

The critical stress for the compressed flange of the I-section of cold-formed beam is in the following form

$$\sigma_{CR}^{(anal)} = \frac{F_{CR}}{2bt} = 4\sqrt{2}E\left(\frac{t}{b}\right)^2. \tag{16}$$

5. A circular cylindrical flange

Stability of a compressed cylindrical shell was extensively studied and described in literature. The first solution to the problem was presented by R. Lorenz in 1908 and 1911, S. P. Timoshenko in 1910, and R. V. Southwell in 1914. They determined critical stress of an axially compressed cylindrical shell

$$\sigma_{KR}^{(L-T-S)} = \frac{E}{\sqrt{3\left(1-\nu^2\right)}}\frac{t}{R}, \tag{17}$$

where t is thickness of the shell, r is radius of the shell, E, ν are material constants.

Stability of cylindrical shells was studied very thoroughly, particularly in the latter part of the 20th century. Nevertheless, stability of an axially compressed open cylindrical shell with linear free edges was investigated only rarely. Chu *et al*. (1967), Yang and Guralnick (1976) solved this problem for open cylindrical shells with sectorial angle $\beta \le \pi/2$. Magnucka-Blandzi and Magnucki (2004) determined analytically and FEM-numerically the critical stress for open cylindrical shell of grater sectorial angles $\pi/2 \le \beta \le \pi$. Magnucki and Maćkiewicz (2005) presented an extended study of these shells. They assumed that two curvilinear edges of the shell are pivoted, while two others free. The shell is loaded at both ends with a distributed force of the intensity N_{xx} along the curvilinear edges, giving rise to the stress $\sigma_x = N_{xx}/t$ (fig. 6).

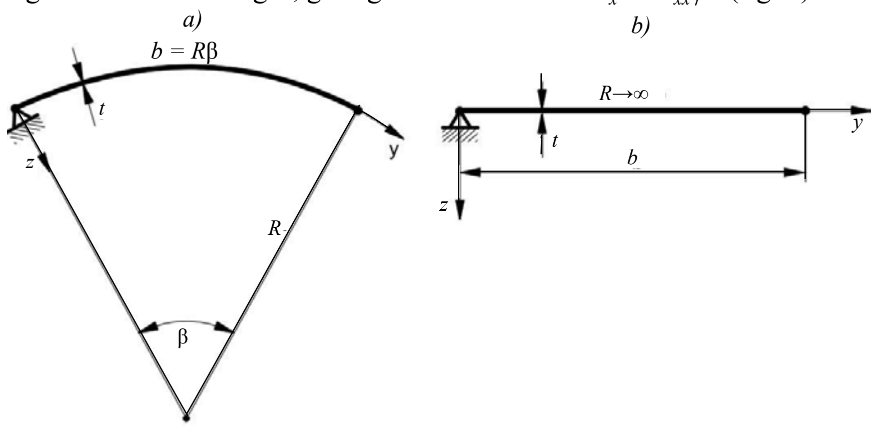

Fig. 6. Schemes: *a)* cross section of the cylindrical flange, *b)* cross section of the flat flange

Equations of stability for the circular cylindrical shell are in the following form

$$\nabla^4 F - \frac{Et}{R}\frac{\partial^2 w}{\partial x^2} = 0\,,\ D\nabla^4 w + \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + N_x^0 \frac{\partial^2 w}{\partial x^2} = 0\,, \tag{18}$$

where $w = w(\varphi, x)$ — displacement-deflection function, $\nabla^4 = \nabla^2\nabla^2$ — linear operator, $F = F(\varphi, x)$ — airy force function, $D = Et^3/\left[12\left(1-\nu^2\right)\right]$ — bending stiffness, t and R — thickness and radius of the cylindrical shell.

The internal forces of the shell are as follows

$Q_x = D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{R^2\partial\varphi^2}\right)$, $Q_\varphi = D\frac{\partial}{R\partial\varphi}\left(\frac{\partial^2 w}{R^2\partial\varphi^2} + \frac{\partial^2 w}{\partial x^2}\right)$ are shear forces,

$N_{xx} = \frac{\partial^2 F}{R^2\partial\varphi^2}$, $N_{\varphi\varphi} = \frac{\partial^2 F}{\partial x^2}$, $N_{x\varphi} = -\frac{\partial^2 F}{R\partial x\partial\varphi}$ are normal and tangent forces,

$M_{xx} = D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{R^2\partial\varphi^2}\right)$, $M_{\varphi\varphi} = D\left(\frac{\partial^2 w}{R^2\partial\varphi^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$,

$M_{x\varphi} = D(1-\nu)\frac{\partial^2 w}{R\partial x\partial\varphi}$ are bending and twisting moments.

Boundary conditions of the cylindrical shell are as follows:

- two simply supported edges ($x = 0$ and $x = L$)

$$w(\varphi, x)\big|_{x=0,L} = 0\,,\quad \frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{R^2\partial\varphi^2} = 0\,, \tag{19}$$

- one simply supported edge $(\varphi = 0)$

$$w(\varphi, x)\big|_{\varphi=0} = 0\,,\quad \frac{\partial^2 w}{R^2\partial\varphi^2} + \nu\frac{\partial^2 w}{\partial x^2} = 0\,, \tag{20}$$

- one free edge ($\varphi = \beta$)

$$\frac{\partial^2 w}{R^2\partial^2\varphi} + \nu\frac{\partial^2 w}{\partial x^2} = 0\,,\quad \frac{\partial}{\partial\varphi}\left(\frac{\partial^2 w}{R^2\partial^2\varphi} + \frac{\partial^2 w}{\partial x^2}\right) = 0\,,\quad \frac{\partial^2 F}{\partial x^2} = 0\,,\quad \frac{\partial^2 F}{\partial x\partial\varphi} = 0\,. \tag{21}$$

The system of two differential equations (18) includes two unknown functions $w(\varphi, x)$ and $F(\varphi, x)$. Confining the solution only to asymmetric buckling the unknown functions are assumed in the following forms

$$w(\varphi, x) = w_1\left[\sin\left(\frac{\pi}{2\beta}\varphi\right) + \alpha_3\sin\left(\frac{3\pi}{2\beta}\varphi\right)\right]\sin\frac{m\pi x}{L}\,,$$

$$F(\varphi, x) = -EtRf_1\left[\sin\left(\frac{\pi}{\beta}\varphi\right) + \frac{1}{2}\sin\left(\frac{2\pi}{\beta}\varphi\right)\right]\sin\frac{m\pi x}{L},$$

where $\alpha_3 = \dfrac{1+4\nu k^2}{9+4\nu k^2}$, $k = m\beta\dfrac{R}{L}$, m is natural number, w_1, f_1 are function parameters.

The functions satisfy the boundary conditions (19), (20) and (21), while the equations of stability (18) are not met. The Bubnov-Galerkin method enabled formulating two orthogonal conditions

$$\int_0^{\beta}\int_0^{L}\left(\nabla^4 F - \frac{Et}{R}\frac{\partial^2 w}{\partial x^2}\right)\left[\sin\left(\frac{\pi}{\beta}\varphi\right) + \frac{1}{2}\sin\left(\frac{2\pi}{\beta}\varphi\right)\right]\sin\frac{m\pi x}{L}d\varphi dx = 0,$$

$$\int_0^{\beta}\int_0^{L}\left(D\nabla^4 w + \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + N_x^o\frac{\partial^2 w}{\partial x^2}\right)\left[\sin\left(\frac{\pi}{2\beta}\varphi\right) + \alpha_3\sin\left(\frac{3\pi}{2\beta}\varphi\right)\right]\sin\frac{m\pi x}{L}d\varphi dx = 0.$$

This, upon integrating, some simple transformations, and consideration of the $\sigma_x^o = N_x^o / t$ expression, enabled to express the critical stress in the form

$$\frac{\sigma_{x,KR}^o}{E} = \min_k\left[\frac{C_1(k) + C_2(k)}{C_3(k)}\right],$$

where

$$C_1(k) = \frac{1}{192(1-\nu^2)}\left[(1+4k^2)^2 + \alpha_3^2(9+4k^2)^2\right]\left(\frac{t}{R}\right)^2\left(\frac{\pi}{\beta}\right)^2,$$

$$C_2(k) = \frac{4k^2}{20+16k^2+5k^4}\left[\frac{32}{15\pi}\left(1+\frac{9}{7}\alpha_3\right)\right]^2\left(\frac{\beta}{\pi}\right)^2,$$

$C_3(k) = k^2(1+\alpha_3^2)$ are coefficients for the case $N_{xx} = N_x^o = \text{const}$.

Numerical study of the expression shows two local minima. The first one is related to classical shell buckling, being compatible with the value resulting from the expression (17). The other is related to local buckling of the free shell edge that is considerably smaller than the first classical one. Calculation shows that in this case the value of the angle β only slightly affects the level of critical stress. Consideration of the Lorenz, Timoshenko and Southwell formula (17) enabled to propose the following form of the critical stress of an open cylindrical shell subject to axial compression $(N_{xx} = N_x^o = \text{const})$

$$\sigma_{KR}^{(s1)} = \alpha_{c1}\frac{E}{\sqrt{3(1-\nu^2)}}\frac{t}{R}, \text{ where } \alpha_{c1} = \frac{1}{8{,}11}\left(1 - 0{,}0146\frac{\beta}{\pi}\right), \ \frac{\pi}{2} \le \beta \le \pi. \qquad (22)$$

It should be noticed that buckling of an open cylindrical shell is of local character and concentrates at its free edge. Hence, critical stress of an open cylindrical shell subject to axial compression with the force varying at its length should be assumed to be equal to two above described cases with lengthwise constant force.

Moreover, buckling of the cylindrical shells was numerically studied with FEM. The following data have been adopted: $t = 1$ mm, $r = 302{,}6$ mm, $\beta = \langle \pi/2, 2\pi/3, 5\pi/6, \pi \rangle$, $\nu = 0{,}3$, $E = 2{,}05 \cdot 10^5$ MPa. Comparison of the values of critical stress obtained from analytical and FEM-numerical solutions indicates that the difference between them does not exceed 6,5%.

Conclusions. The buckling problems of flat flanges of cold-formed thin-walled beams are extensively investigated and described in many publications. Basic models of these flanges are isotropic or orthotropic rectangular plates under longitudinal compression. Nevertheless, the buckling problem of axially compressed cylindrical panel with three simply supported edges and one edge free were only weakly recognized. The critical stresses of thin-walled elements with free edges are significantly smaller than the critical stresses of simply supported elements.

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Пружне випучування вибраних профілів холоднокатаних тонкостінних балок

Кшиштоф Магнуцкі

У роботі розглянуто п'ять різних профілів холоднокатаних тонкостінних балок. Сформульовано та досліджено математичні моделі для кожного з п'яти профілів. У представлених моделях використано теорію пружної стійкості пластин та пологих циліндричних оболонок. Визначено критичні напруження для кожного типу профілю. Проаналізовано результати аналітичних досліджень та проведено їх порівняння із результатами числових досліджень методом скінченних елементів, а також експериментальними даними. Отримані у роботі формули для критичних напружень можуть бути використані на практиці.

Упругое выпучивание некоторых профилей холоднокатаных тонкостенных балок

Кшиштоф Магнуцки

В роботе рассмотрено пять разных профилей холоднокатаных тонкостенных балок. Сформулированы и исследованы математические модели для каждого из пяти профилей. В представленных моделях использовано теорию упругой устойчивости пластин и цилиндрических пологих оболочек. Определены критические напряжения для каждого типа профиля. Проведен анализ аналитических результатов и представлено сравнение их с результатами численного исследования методом конечных элементов, а также экспериментальными данными. Представленные формулы для критических напряжений могут быть использованы в практике.

Отримано 18.10.05