IMPURITY ION DRIFT AND TOROIDAL ROTATION IN TOKAMAKS

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This is an analysis of the toroidal drifts of bulk (hydrogenic) and impurity ions, and electrons, in ohmically heated tokamaks. Experimentally observed ion drift is consistently explained by a 1-D model with Ohm's law and the charged particle equations of motion in a quiescent plasma. Calculations show that the drifts are usually decoupled, so the notion of "toroidal rotation" of the plasma as a whole is untenable in this situation.

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1. IS IT SINGLE-FLUID TOROIDAL ROTATION?

The idea that the plasma in tokamaks rotates toroidally "as a whole", i.e., as a bulk fluid, is widespread [1-3]. Spectroscopic observations of toroidal impurity ion drift have been reported for decades in ohmically heated (OH) tokamaks and in many with auxiliary heating. Bulk plasma rotation has been invoked as a measure of plasma state and stability and is proposed as a monitor for control of ITER plasmas [4].

The issue of collisional coupling of the plasma ion species is rarely addressed [5], although it was discussed in several early papers on ion heating [6,7]. Here the relative and absolute toroidal drift velocities of the charged particles in steady-state OH tokamaks are calculated and briefly compared with experiment.

The equations for the relative and absolute drift velocities of the plasma species show that decoupling of impurity ion drift from hydrogenic ion drift is normal in tokamak discharges.

2. BASIC ASSUMPTIONS AND EQUATIONS

Consider a plasma consisting of hydrogenic ions (A=H, D, or T), a single dominant impurity ion X, and electrons (e), as well as one or more trace impurities Q. n is particle number density, u is directed velocity, m, A and Z are particle mass, atomic mass and charge, T temperature, E electric field, e is the electronic charge and m_p is the a.m.u. (Units: MKS, with eV for temperature and energy. Unless noted, Greek subscripts indicate summation over all species, e, A [hydrogenic ion], and impurity ions X; Latin, sum over ions only.) The basic assumptions and equations are [8]:

Composition: mass density $\mu\equiv\Sigma n_\beta m_\beta = m_p \Sigma n_\beta A_\beta$ and effective charge Z_{eff} ranging from 1 to Z_X:

$$Z_{eff}\equiv\Sigma_k n_k Z_k^2/\Sigma_k n_k Z_k = \Sigma_k n_k Z_k^2/n_e. \quad (1)$$

Charge neutrality: charge density

$$\rho(\mathbf{r})\equiv\Sigma n_\beta Z_\beta e = 0 \text{ or } \Sigma n_k Z_k = n_e. \quad (2)$$

Zero (total, linear) momentum density:

$$\mathbf{p}\equiv\Sigma_\beta n_\beta m_\beta \mathbf{u}_\beta = m_p \Sigma_\beta n_\beta A_\beta \mathbf{u}_\beta = 0, \quad (3)$$

Quiescent plasma: the toroidal magnetic field **B** confines the plasma to 1-D motion. Moderate (toroidal) electric field **E**, and all species are thermal, i.e. have kinetic temperatures (T), with low drift velocities (no species in a runaway state), so that the current density obeys Ohm's law and equals the sum of contributions from all charged particle species:

$$\mathbf{j}\equiv\Sigma_\beta n_\beta Z_\beta e \mathbf{u}_\beta = -n_e e \mathbf{u}_e + n_A Z_A e \mathbf{u}_A + n_X Z_X e \mathbf{u}_X = \sigma\mathbf{E}, \quad (4)$$

where σ is the electrical conductivity (see below).

1-D equation of motion (momentum equation) in toroidal direction for each species, neglecting transverse terms. (Transverse motion is considered only as built-in particle loss/source, if needed.) In the steady state, this leaves electric force and elastic collisional drag terms for species α (sum over all species but α):

$$m_\alpha d\mathbf{u}_\alpha/dt = Z_\alpha e\mathbf{E} - \Sigma_{\beta\neq\alpha}\, \mu_{\alpha\beta}\nu_{\alpha\beta}(\mathbf{u}_\alpha - \mathbf{u}_\beta) = 0. \quad (5)$$

Note that the hierarchy of (momentum) collision times is built into this model. Here $\mu_{\alpha\beta}$ is the reduced mass and $\nu_{\alpha\beta}$ the momentum transfer collision rate for particles of type α in field β (elastic collisions):

$$\mu_{\alpha\beta} \equiv m_\alpha m_\beta/(m_\alpha+m_\beta) = m_p A_\alpha A_\beta/(A_\alpha+A_\beta)$$

$$\nu_{\alpha\beta} = 4\pi Z_\alpha^2 Z_\beta^2 e^4/(m_{\alpha\beta}^2 v^3) n_\beta \ln\Lambda =$$

$$7.36\cdot10^{-13} Z_\alpha^2 Z_\beta^2 (A_\alpha+A_\beta)^2 n_\beta/(A_\alpha A_\beta)^{1/2}/(A_\alpha T_\beta + A_\beta T_\alpha)^{3/2}$$

where the Coulomb logarithm $\ln\Lambda\approx15$ and $v=v_{rel}=(qT_{\alpha\beta}/\mu_{\alpha\beta})^{½}$ is the average (effective) relative velocity between particles of types α and β, where q is a geometric factor, taken equal to 3 here, and $T_{\alpha\beta}$ is the effective temperature of the combined populations, $T_{\alpha\beta}\equiv(m_\alpha T_\beta + m_\beta T_\alpha)/(m_\alpha+m_\beta)$. When a species does not have a thermal distribution but is characterized by kinetic energy K, the temperature can be replaced by an effective temperature $T\sim⅔K$. The electrical conductivity is given by $\sigma=2n_e e^2/\Sigma_k\, \mu_{ek}\nu_{ek}$.

Relative drifts of electrons, hydrogenic ions, and impurity (X) ions: For the three species, i,j,k equal to some permutation of e, A, X, symmetry indicates a relation of the following form for the drift velocities of two of the three, i and j (other characteristics of k, the third species, appear as parameters in the function f):

$$\mathbf{u}_i f(i,j) = \mathbf{u}_j f(j,i); \quad (6)$$

f(i,j) can be found by writing down the momentum equations (5) for each of e, A, and X, using the equation for k≠i, j to eliminate **E**, and using the resulting two equations and the conservation of momentum (3) to express $\mathbf{u}_i$ in terms of $\mathbf{u}_j$ (eliminating $\mathbf{u}_k$; here $S_{mn}\equiv\mu_{mn}\nu_{mn}$) in the form (6), with

$$f(i,j)\equiv S_{ij}/Z_i + S_{ji}/Z_j + S_{ik}/Z_i + n_i A_i (S_{ik}/Z_i - S_{jk}/Z_j)/n_k A_k. \quad (7)$$

The absolute magnitudes of the drift velocities can be calculated in two ways. The first is to multiply the relative velocities (u_X/u_e, u_A/u_e) from Eq. (6) by the electron drift velocity ($\mathbf{u}_e = -\mathbf{j}/\{n_e e(1+\eta A_e)\}\approx -j/n_e e$, to a good approximation) to obtain the ion drift velocities. An explicit, more intuitive formula for the impurity ion drift velocity $\mathbf{u}_X$ can be obtained by solving the momentum equation (5) for species X:

$$\mathbf{u}_X = \{Z_X e\mathbf{E} + \mu_{Xe}\nu_{Xe}\mathbf{u}_e + \mu_{XA}\nu_{XA}\mathbf{u}_A\}/(\mu_{Xe}\nu_{Xe} + \mu_{XA}\nu_{XA}). \quad (8)$$

This simplifies substantially in the core of a reasonably pure hydrogen tokamak plasma (moderate Z_{eff}) with comparable electron and ion temperatures because the ratio $\mu_{XA}\nu_{XA}/\mu_{Xe}\nu_{Xe}$>>1. This leaves

$$\mathbf{u}_X \approx \{Z_X e\mathbf{E} + \mu_{Xe}\nu_{Xe}\mathbf{u}_e\}/(\mu_{XA}\nu_{XA}) + \mathbf{u}_A, \quad (9)$$

i.e., *the sum of terms owing to the electric field and the oppositely directed drag on the drifting electrons, superimposed on the hydrogenic ion drift velocity.* $\mathbf{u}_A$ (opposite to and <<$\mathbf{u}_e$) can be found indirectly using the method of the preceding section or, approximately, from the current density and Ohm's law (4).

The direction and magnitude of the impurity ion drift are essentially determined by **E** and **j**. $\mathbf{u}_X$ can be very large compared to $\mathbf{u}_A$ and opposite to it, to **j**, and to **E** (in low electric fields, as in modern tokamaks; i.e., the second term in the curly brackets in Eq. (9) is dominant), as Gurevich [7] pointed out long ago. This is the normal situation in a modern, low loop voltage, quiescent, high-temperature tokamak discharge. **E** can be expressed in terms of **j** through the equation $\mathbf{j}=\sigma\mathbf{E}$ to give $\mathbf{u}_X$ in terms of the plasma and device parameters.

Thus, for OH discharges in D^+ the steady state drift velocity of a single impurity ion (m/s), assuming n_x<<n_A (Z_{eff}~1), m_x>>m_A, is given by

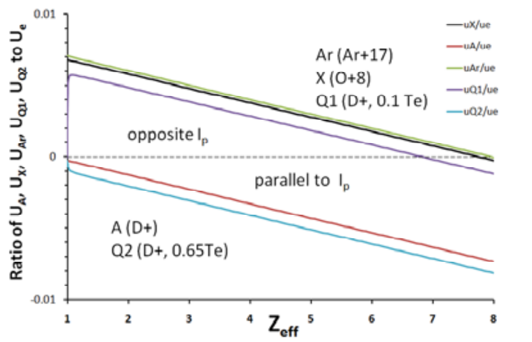
$$\mathbf{u}_x \approx \frac{1.5\cdot 10^{19} V_L \mathbf{e}_z/R - 3.3\cdot 10^{16}\pi Z_x \mathbf{j}/T_e^{3/2}}{Z_x n_e/T_i^{3/2}} + \mathbf{u}_A. \quad (10)$$

Here V_L is the loop voltage, R is the torus major radius, $\mathbf{e}_z$ is the unit vector in the direction of the toroidal field **E** (and **j**), Z_x is the impurity ion charge, **j** is the (local) plasma current density, T_e and T_i are the electron and ion (for convenience assumed equal for all ions) temperatures, and n_e is the electron density. Eq. (10) is the result for a moderate amount of impurity in a D plasma; the general result for arbitrary Z_{eff} follows from Eq. (8), etc. Relative drift velocities of trace (n_Q<<n_e) ions can be found using Eq. (5), given the ratios (6) u_A/u_e and u_X/u_e.

3. SOME CALCULATIONS

Fig. 1 shows the calculated (Eq. (6)) ratios of ion drift velocities to electron drift velocity $\mathbf{u}_e$ in an ohmically heated discharge composed of D^+ (A), O^{8+} (X), and trace Ar^{17+}, each with $T=0.6T_e$, and for trace groups of D^+ with characteristic energies $T=0.1T_e$ [Q1] and $T=0.65T_e$ [Q2]. The O^{8+} and Ar^{17+} drift velocities are essentially equal, regardless of the plasma purity (amount of oxygen; Z_{eff}). Thus, agreement among measured drift velocities of different nonhydrogenic impurity ions is no evidence of bulk rotation of a plasma. The drift velocities for A, X, and trace heavy impurities (here Ar) are approximately linear functions of Z_{eff}, while the change from a pure D plasma to one with Z_{eff}>1 is quite sharp for trace D^+ ion groups as impurity ions, even at very low concentrations, begin to collide with them.

A few experimental attempts have been made to show that the drift velocities of hydrogenic bulk ions are the same as those of impurity ions in tokamaks. Charge exchange measurements on Tore Supra [9] were aimed at confirming the identity of D^+ and impurity (Cr^{22+}) ion drifts in OH plasmas. However, that study did not deal with the fact that the deuterium neutral population in the plasma core is almost certainly dominated by the influx of lower energy atomic hydrogen from the plasma edge [10], rather than by an ionization-recombination balance in the core. Convected neutrals from the edge will have energies significantly lower than the core ion temperature, and correspond roughly to group Q1 of figure (Thermalized D^+ will lie along the u_A/u_e curve and D^+ groups with energies >T_A will, as does group Q2, have a higher drift velocity parallel to E than the nominal u_A.)



Calculated drift velocities of ions relative to electron drift in an OH plasma (see text for details)

Thus, the Z_{eff}>1 discharge of Tore Supra has a large group of low energy, nonthermal D^+ ions with drift velocities closer to that of heavy impurity ions (u_X) than to the drift velocity u_A of the thermal D^+ population (imposed by the toroidal E and momentum balance). This is probably the source of the charge exchange neutrals leading to an apparent hydrogen ion drift comparable to the heavy impurity ion drift velocity reported in these very careful charge exchange measurements [9].

4. CONCLUSION AND DISCUSSION

In OH tokamaks, impurity ion drift is usually, as shown here, the result of the forces on those ions owing to the toroidal electric field and to drag on the plasma electrons, superimposed on the bulk (hydrogenic) ion drift velocity. The magnitude and direction of the impurity drift can readily change (even becoming parallel to the toroidal current) when (i) the toroidal electric field approaches (roughly) the Dreicer limit, i.e., the electrons begin to run away and Ohm's law fails [11]; or (ii) the conductivity mechanism changes, as during RF current drive (e.g., elevated hydrogenic ion drift or "ion tail" formation in ICH) [12,13]. A third cause of modified impurity drifts is neutral beam injection, during which the fresh beam ion population easily dominates the force terms in the momentum equation (5), and once a collision term for the fast beam ions is included, the major observed features can be calculated readily with this collisional model [14].

The above discussion concerns OH plasmas in tokamaks where the current obeys Ohm's law ($\mathbf{j}=\sigma\mathbf{E}$, i.e., low loop voltage, so the particle distribution functions are only slightly perturbed by **E**). The calculations show that the ion species (hydrogenic and impurity) are generally decoupled. Equal $\mathbf{u}_x$ and $\mathbf{u}_A$ occur in OH plasmas only when the electron temperature T_e>>T_{ion} (by a factor of order 15). As noted above, reversal of the direction of impurity ion drift (to parallel to I_p) is often observed when auxiliary heating is applied.

Neglect of the hierarchy of collision times and of the incomplete coupling of the plasma ions in published papers on tokamaks casts serious doubt on the interpretation of the observed ion drift as representing a toroidal rotation of these plasmas "as a whole." The use of this "rotation" effect (more precisely, these observations) as an indicator of plasma state is incorrect as posited. Beyond this, the notion of plasma rotation occurring in OH tokamaks with "no external momentum source" [15] is meaningless or, with proper interpretation (as here), trivial. Newton's third law applies, even in tokamaks. The consequences are evident in figure An interpretation of the observations in terms of detached ion drifts with momentum balance may, however, offer a number of interesting and important diagnostic possibilities.

The observed OH impurity drift is sensitive to various plasma and device parameters; however, as the above equations show (cf. Eq. (10) for a simple case), measurements of $\mathbf{u}_X$ might provide information on two parameters which cannot ordinarily be measured locally in these plasmas: **E** and **j**. Thus, given near-thermal distributions for the particles (with known, local temperatures and densities), and localized measurements of impurity ion drifts, it should be possible to derive local values of E and j, an important advance in plasma diagnostics. The uncoupled velocities of hydrogenic and impurity ions may also produce signals with a variety of periods in various types of detectors which appear to indicate "toroidal rotation" dependent phenomena. This may simply be an analog of the Strouhal number ($St \equiv fL/U$) for ordinary fluids in swirl and circular flow environments (characteristic frequency $f_S \approx 0.2u/D$ for circular motion at velocity u along a path of diameter D) [16]. In fact, there seem to be many toroidal flows in a tokamak (cf. figure), but these are flows of different species or subgroups in a multifluid plasma, rather than of the plasma as a whole.

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ДРЕЙФЫ ПРИМЕСНЫХ ИОНОВ И ТОРОИДАЛЬНОЕ ВРАЩЕНИЕ В ТОКАМАКАХ

Д.Х. Макнилл

Анализируются тороидальные дрейфы основных (водородных) и примесных ионов и электронов в токамаках с омическим нагревом. Экспериментально обнаруженные дрейфы ионов самосогласованно объясняются одномерной моделью с законом Ома и уравнениями движения заряженных частиц в стационарной плазме. Расчеты показывают, что, как правило, дрейфы разных сортов частиц развязаны, поэтому понятие "тороидальное вращение" плазмы в целом здесь несостоятельно.

ДРЕЙФИ ДОМІШКОВИХ ІОНІВ І ТОРОЇДАЛЬНЕ ОБЕРТАННЯ В ТОКАМАКАХ

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Аналізуються тороїдальні дрейфи основних (водневих) та домішкових іонів і електронів у токамаках з омічним нагріванням. Експериментально виявлені дрейфи іонів самоузгоджено пояснюються одномірною моделлю з законом Ома і рівняннями руху заряджених часток у стаціонарній плазмі. Розрахунки показують, що, як правило, дрейфи різних сортів часток розв'язані, тому поняття "тороїдальне обертання" плазми в цілому тут неспроможне.