

# MATCHING OF BEAM PHASE DENSITY DISTRIBUTION WITH ION-OPTIC CHARACTERISTICS OF A PROBE-FORMING SYSTEM IN NUCLEAR MICROPROBE

*A.A. Ponomarov, V.I. Miroshnichenko*

*Institute of Applied Physics, National Academy of Sciences of Ukraine, Sumy, Ukraine*

*E-mail: ponomart@yandex.ru*

The nonuniform phase density distribution of the beam particles and its nonlinear dynamic simulation in the probe-forming system used to obtain distribution profiles of the beam current density on the target subject to the conditions of a probe formation. These conditions are determined by the change of initial distribution parameters of the phase density due to the variation of position and lens excitations of a doublet of electrostatic quadrupoles. This approach allows us to find probe-forming modes when parameters of initial distribution of the beam phase density are matching up to ion optical characteristics of the probe-forming system of a microprobe. Such matching gives increase of the current density and decrease of the beam size on a spot.

PACS: 41.85.P; 13 41.85.Gy; 14 41.85.Lc; 15 41.85.Si

## 1. INTRODUCTION

Ion optical characteristics of a probe-forming system (PFS) of a nuclear microprobe include demagnifications and aberrations that together determine non linear processes of the beam focusing on a target. Chromatic and spherical aberrations related to the angular divergence of a beam play a leading role in these processes. To limit influence of these aberrations an aperture collimator separating ions according to the angle of the traveling direction is used. PFS with small demagnifications  $D_x \times D_y \sim 10^3$  have moderate aberrations permitting to use the aperture collimator of considerably big sizes. From the other side, as was shown in works [1,2] in the electrostatic accelerators the ion beam of a few MeV energies has small angular spread in comparison with permissible sizes of the aperture collimator. As a result, parameters of a beam phase density distribution are not match up to ion optical characteristics of the PFS and focusing process in such system is practically linear. In works [2, 3] we showed that choice of the sizes of object and aperture collimators can significantly influence on the profile of the beam current density distribution on the target at the cost of the nonlinear effects related to the aberrations. This influence is of particular interest because probe sizes in many applications are determined by the value of the full width at half maximum of the beam current density distribution on the target. This paper describes matching method of the parameters of a nonuniform phase density distribution with ion optical characteristics of the PFS to change profile of the beam current density distribution on the target and hence to improve nuclear microprobe resolution.

## 2. PARAMETER OPTIMIZATION OF THE INITIAL DISTRIBUTION OF THE BEAM PHASE DENSITY

In plasma RF sources phase density of the beam ions can be presented through the normal distribution in the phase space  $(x, x', y, y')$ , where  $x, y$  – are ion transverse coordinates,  $x', y'$  – are angles of the speed vector projection to the corresponding plane with the optical axis  $z$ . Transformation of the phase coordinates of the beam ions during their motion in an accelerator and transportation system is close to linear. Therefore at the entrance to the PFS in the plane of the object collimator nature of

the initial phase density distribution does not change obeying the normal law. A phase density of particles is proportional to a beam brightness distribution, hence phase density parameters coincide with brightness distribution parameters. In work [2] a reconstruction method of the brightness distribution parameters was proposed. This method is based on the measurement of a beam current passing through the object and aperture collimators with change in their sizes. As a result, beam brightness distribution in the plane of the object collimator can be presented as follows:

$$b(x, x', y, y') = b_0 \cdot b_x(x, x') \cdot b_y(y, y'), \quad (1)$$

where

$$b_\tau(\tau, \tau') = \exp \left[ -\frac{1}{2(1 - \kappa_\tau^2)} \left( \frac{\tau^2}{\sigma_\tau^2} - 2\kappa_\tau \frac{\tau\tau'}{\sigma_\tau\sigma_{\tau'}} + \frac{\tau'^2}{\sigma_{\tau'}^2} \right) \right],$$

$\tau = (x, y)$ ,

reconstructed distribution parameters:

$b_0 = 7 \text{ pA}/(\mu\text{m}^2 \cdot \text{mrad}^2)$  – axial beam brightness,

$\sigma_x = 146 \mu\text{m}$ ,  $\sigma_y = 144 \mu\text{m}$  – linear dispersion,

$\sigma_{x'} = 0.046 \text{ mrad}$ ,  $\sigma_{y'} = 0.161 \text{ mrad}$  – angular dispersion,

$\kappa_x = 0.54$ ,  $\kappa_y = 0.31$  – covariance.

Obtained brightness distribution parameters characterize distribution value of the particle phase density at the entrance to the PFS of the probe in the form

$$n(x, x', y, y') = n_0 \cdot b_x(x, x') \cdot b_y(y, y'), \quad (2)$$

where  $n_0$  is axial particle phase density,  $n_0 = b_0 \cdot \Delta t / e$ ,  $\Delta t$  – is the time of probe stay on the defined position on the target,  $e$  – is an elementary charge.

Angular dispersion  $\sigma_{x'}$  and  $\sigma_{y'}$  in distribution (1) has very small values. At the same time linear dispersion  $\sigma_x$  and  $\sigma_y$  is characterized by the big values. Thus to optimize it is necessary to decrease linear dispersion on the account of the increase of the angular one using doublet of quadrupole electrostatic lenses. The role of the electrostatic doublets is similar to condenser lens in a scanning electron microscope that adjust beam parameters during the final focusing of the objective lens. Therefore, by analogy, doublet of electrostatic quadrupole lenses is called here condenser system. The scheme of the arrangement of the condenser and PFS of the nuclear microprobe is shown on Fig.1, where the voltage of lens poles  $V_1$ ,  $V_2$  and distance  $a$  between the con-

denser system and object collimator plane can change in the limit ranges. Geometrical sizes of the condenser system and its base location in the channel of a nuclear scanning microprobe available at the IAP NAS of

Ukraine [4] have the following values: lens aperture radius is  $r_a = 0.02$  m, length of the effective field  $l = 0.14$  m, distance between the effective field boundaries  $s = 0.01$  m,  $a = 2.4$  m.

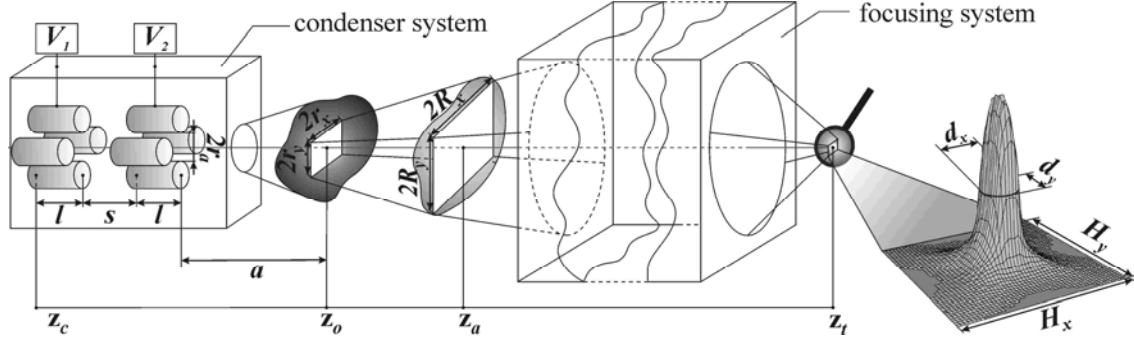


Fig.1. Scheme of the arrangement of the condenser and probe-forming systems in the nuclear microprobe

To solve the problem of parameter optimization of the initial distribution of the beam phase density it is necessary to determine a dependence of these parameters on the voltage of the lens poles  $V_1$ ,  $V_2$  and distance  $a$ , describing location of the condenser system along the ion optical axis. To do this, transformation of the phase coordinates of the particles from the plane  $z_o$  to plane  $z_c$  was done using the inverse transformation of a drift gap with the length  $z_o - z_c$  with turned off lens voltage of the condenser system. Then obtained phase coordinates were transformed into the plane of object collimator  $z_o$  with condenser system lenses turned on and new parameters of the initial distribution of the beam phase density were determined. Described above transformation of the phase coordinates was done using transformation matrices in the form of:

$$\mathbf{R}_\tau = \mathbf{T}(a)\mathbf{R}_{2\mu}\mathbf{T}(s)\mathbf{R}_{1\tau}[\mathbf{T}(d)]^{-1}, \quad \tau=(x,y), \mu=(y,x), \quad (3)$$

$$\text{where } \mathbf{T}(z) = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \text{ – is the transformation matrix}$$

of free intervals with lengths  $z$ ,  $d = a + 2l + s$ ,

$$\mathbf{R}_{i\theta} = \begin{bmatrix} \cos(\beta_i l) & \sin(\beta_i l)/\beta_i \\ -\beta_i \sin(\beta_i l) & \cos(\beta_i l) \end{bmatrix},$$

$$\mathbf{R}_{iy} = \begin{bmatrix} ch(\beta_i l) & sh(\beta_i l)/\beta_i \\ \beta_i sh(\beta_i l) & ch(\beta_i l) \end{bmatrix},$$

are transformation matrices of the electrostatic quadrupole lens with number  $i$ ;  $\beta_i = \sqrt{V_i / (r_a^2 U)}$  – is the lens excitation;  $r_a$  – is the radius of the lens aperture;  $U = 1$  MV – is accelerating voltage passed by the beam particle.

As a result of the transformation (3), new parameters of the phase density distribution  $\tilde{\sigma}_\tau$ ,  $\tilde{\sigma}_{\tau'}$ ,  $\tilde{\kappa}_\tau$  in the plane  $z_o$  and depending on  $V_1$ ,  $V_2$  and distance  $a$ , and expressed through the known distribution parameters (1) were obtained in the form:

$$\tilde{\sigma}_\tau^2 = (1 - \kappa_\tau^2) / (A - C^2/B),$$

$$\tilde{\sigma}_{\tau'}^2 = (1 - \kappa_{\tau'}^2) / (B - C^2/A), \quad (4)$$

$$\tilde{\kappa}_\tau^2 = C^2 / AB, \quad \tau = (x, y),$$

where

$$A = \frac{r_{22}^2}{\sigma_\tau^2} + \frac{r_{21}^2}{\sigma_{\tau'}^2} + \frac{2\kappa_\tau r_{21} r_{22}}{\sigma_\tau \sigma_{\tau'}}, \quad B = \frac{r_{12}^2}{\sigma_\tau^2} + \frac{r_{11}^2}{\sigma_{\tau'}^2} + \frac{2\kappa_{\tau'} r_{11} r_{12}}{\sigma_\tau \sigma_{\tau'}},$$

$$C = \frac{r_{12} r_{22}}{\sigma_\tau^2} + \frac{r_{11} r_{21}}{\sigma_{\tau'}^2} + \frac{\kappa_\tau (r_{11} r_{22} + r_{12} r_{21})}{\sigma_\tau \sigma_{\tau'}},$$

$r_{ij}$  – are elements of square transformation matrix of the phase coordinates (3);  $i, j = 1, 2$ .

The problem of the parameter optimization of the beam phase density distribution in the plane  $z_o$  consisted in the following. For different  $a$  values, due to the voltage change on the lens poles of the condenser system, new values of the angular dispersion  $\tilde{\sigma}_{x'}$ ,  $\tilde{\sigma}_{y'}$ , at the minimal root-mean-square value of the linear dispersion  $\tilde{\sigma}$  were found. This task can be formulated in the form:

$$\tilde{\sigma}(a) = \min_{V_1, V_2} \sqrt{(\tilde{\sigma}_x^2 + \tilde{\sigma}_y^2)}, \quad (5)$$

$$0 < V_1 \leq 50 \text{ kV}, 0 < V_2 \leq 50 \text{ kV}, 0.4 \text{ m} \leq a \leq 2.4 \text{ m}.$$

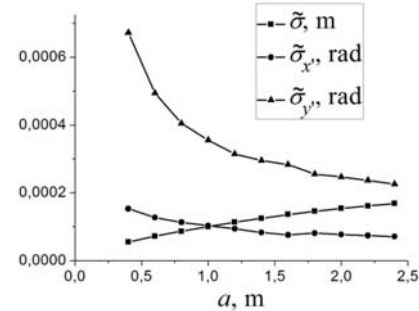


Fig.2. The optimal parameters of the beam phase density distribution versus the location of the condenser system

Results of the task solving under the optimal  $V_1$ ,  $V_2$  are presented in Fig.2, where it can be seen that with a decrease of distance  $a$  the root-mean-square value of the linear dispersion  $\tilde{\sigma}$  decreases and angular dispersion  $\tilde{\sigma}_{x'}$ ,  $\tilde{\sigma}_{y'}$  increases.

### 3. OPTIMIZATION OF DISTRIBUTION PARAMETERS OF THE BEAM IONS ON THE TARGET

Distribution parameters of the beam ions on a target are determined by the outline sizes on the full width at half maximum of the beam current density distribution  $d_x \times d_y$  (FWHM), current  $I_{FWHM}$  in the spot limited by that outline, and beam sizes  $H_x \times H_y$  at the bottom of the distribution that characterize beam halo (see Fig.1). These parameters for every PFS depend on the distribution of the particle phase density in the object plane  $z_o$ . There-

fore, solving the task of finding the spot size of minimal area  $S^* = d_x^* \times d_y^*$  at given initial distribution of the phase coordinates allows us to find theoretical limit of a spatial resolution of a microprobe. Such optimization task can be formalized in the form

$$S^* = \min_{R_x, R_y} S(R_x, R_y), \quad (6)$$

$$0 < R_x \leq R_{x \max}, \quad 0 < R_y \leq R_{y \max};$$

$$I_0 = b_0 \int_{-r_x - (R_x + x)/A}^{r_x - (R_x - x)/A} \int_{-r_y - (R_y + y)/A}^{r_y - (R_y - y)/A} b_x(x, x') dx' dx \cdot$$

$$\cdot \int_{-r_y - (R_y + y)/A}^{r_y - (R_y - y)/A} \int_{-r_x - (R_x + x)/A}^{r_x - (R_x - x)/A} b_y(y, y') dy' dy = const, \quad (7)$$

where  $r_x, r_y$  and  $R_x, R_y$  are sizes of the object and aperture collimators, respectively (see Fig.1),  $A$  is the object distance ( $A = z_a - z_0$ ),  $R_{x \max} = A \tilde{\sigma}_{x'}$ ,  $R_{y \max} = A \tilde{\sigma}_{y'}$ ,  $I_0$  – is the total beam current.

Sizes of the object collimator are determined from the relations

$$r_x = r \sqrt{D_x / D_y}, \quad r_y = r \sqrt{D_y / D_x}, \quad (8)$$

where  $D_x, D_y$  are demagnifications of the PFS,  $r$  is the size of equivalent object collimator of the square form with the area  $S_0 = r^2 = r_x r_y$ . The value of  $r$  is determined from the equality (7) under the given values  $R_x, R_y$  and distribution parameters of the phase density in the object plane in the form of (4).

Initial phase set at the entrance of the PFS is formed using rectangular object and aperture collimators (Fig.1) positioned with axial symmetry. Transformation object-target has the following form

$$x_t = \sum_{j=1}^{38} \mathfrak{R}_{x1,j} \cdot \Phi_{x0,j}, \quad y_t = \sum_{j=1}^{38} \mathfrak{R}_{y3,j} \cdot \Phi_{y0,j}, \quad (9)$$

where

$$\{\Phi_{x0,j}\} = (x_0, x'_0, x_0 \delta, x'_0 \delta, x_0^3, x_0^2 x'_0, x_0 x_0'^2, x_0'^3, x_0 y_0^2, x_0 y_0 y'_0, x_0 y_0'^2, x_0 y_0' y'_0, x_0 y_0'^2)^T, \quad j = 1, \dots, 14,$$

$\{\Phi_{y0,j}\}$  is obtained from  $\{\Phi_{x0,j}\}$  as a result of substitution  $x \leftrightarrow y$ ;  $\mathfrak{R}_{x(y)}$  are the transformation

matrices of the phase moments of the beam particles in the object plane ( $z_0$ ) to the target plane ( $z_t$ ). Elements of the first lines of this matrix present linear properties and aberrations of the PFS including chromatic aberrations and intrinsic third order aberrations. Transformation matrices  $\mathfrak{R}_{x(y)}$  were calculated using matrizant method

[5]. The main ion optical characteristics of the PFS of the IAP NASU microprobe have the following values: demagnifications  $D_x = D_y = 23$ ; chromatic aberrations:  $\langle x/x' \delta \rangle = -525$ ,  $\langle y/y' \delta \rangle = -160 \mu\text{m}/(\text{mrad}\%)$ ; spherical aberrations:  $\langle x/x'^3 \rangle = 932$ ,  $\langle x/x' y'^2 \rangle = 264$ ,  $\langle y/y'^3 \rangle = 38$ ,  $\langle y/y' x'^2 \rangle = 264$ ,  $\mu\text{m}/(\text{mrad}^3)$ ;  $\delta$  – is a momentum spread,  $|\delta| \leq \delta_{\max}$ .

Sequence of operations in the formulated optimization task consists in the following. For every pair of the values  $R_x, R_y$  specifying dimensions of the aperture col-

limator value  $r$  satisfying the equation (7) with due regard for (4) and (8) is determined. After that for different parameters of initial distribution of the phase density (4) and sizes of the collimators  $r_x, r_y, R_x, R_y$  phase set of the particles in the object plane that transforms onto the target plane with using of transformation (9) is determined.

The projection of the transformed set on the plane ( $x_t, y_t$ ) determines particle density distribution on the target that is proportional to the beam current density. Processing of the obtained distribution gives values  $d_x, d_y$  (FWHM),  $I_{FWHM}$ ,  $H_x, H_y$ . Minimal area of the spot  $S^* = d_x^* \times d_y^*$  is finally determined on the set of values ( $R_x, R_y$ ).

## 4. RESULTS OF SIMULATION

As a result of solving optimization task (6) profiles of the beam current density distribution on the target at the different parameters of the initial distribution of the beam phase density in the plane of the object collimator were obtained. Fig.3 shows projections on the plane  $x_t O_j$  and  $y_t O_j$  of a two dimensional distribution of the current density on the target  $j(x_t, y_t)$  with the total beam current  $I_0 = 100 \text{ pA}$  and momentum spread  $\delta_{\max} = 5 \cdot 10^{-4}$ .

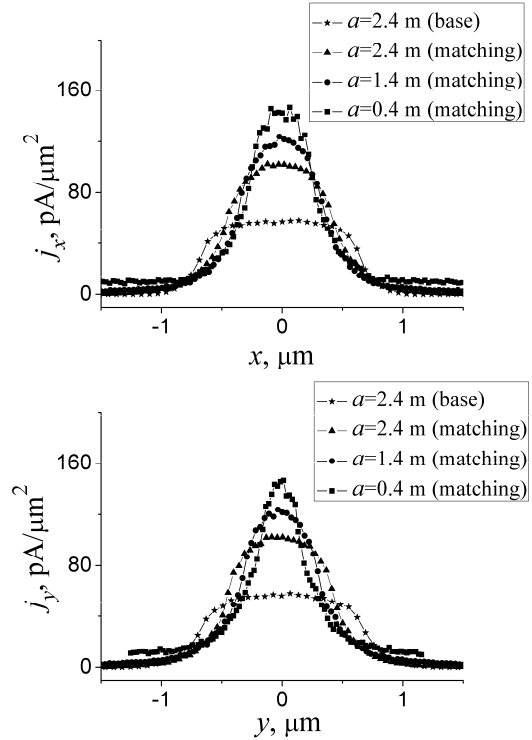


Fig.3. Distribution profiles of the beam current density on the target for different values of the distance  $a$  at corresponding optimal values  $V_1, V_2$  presented in Table

All parameter values of the distributions obtained are shown in Table.

From Table and Fig.3 one can see that the minimal area of the spot  $S^*$  (FWHM) decreases with the decrease of the distance  $a$  that corresponds to the increase of the angular dispersion of initial distribution of the beam phase density (see Fig.2). However, while this halo sizes increase significantly and current portion in the spot  $I_{FWHM}/I_0$  decreases.

*Distribution parameters of the beam current density on the target at different conditions of the probe formation*

	$a=2.4$ m (base)	$a=2.4$ m (matching)	$a=1.4$ m (matching)	$a=0.4$ m (matching)
$I_{FWHM}/I_0$	0.8	0.63	0.43	0.28
$d_x, \mu\text{m}$	1.31	0.92	0.70	0.65
$d_y, \mu\text{m}$	1.32	0.90	0.70	0.49
$H_x, \mu\text{m}$	2.80	3.98	7.08	19.43
$H_y, \mu\text{m}$	2.28	3.47	5.65	21.63
$J_{max},$ $\text{pA}/\mu\text{m}^2$	55	98	118	133
$r_x; r_y, \mu\text{m}$	32; 32	21; 21	15; 15	9; 9
$R_x; R_y, \mu\text{m}$	176; 408	230; 836	321; 1143	354; 2241
$V_1, \text{V}$	0.0	4051	6183	11823
$V_2, \text{V}$	0.0	4051	6396	14843

### CONCLUSIONS

Using the results of analysis of a compatibility of the phase density distribution of the beam particles with ion optical characteristics of the nuclear microprobe PFS which has the demagnifications  $D_x \times D_y \sim 10^3$  and moderate aberrations the following conclusions can be made. The base parameters of initial distribution of the beam phase density in the electrostatic accelerator are not match up to considered type of the PFS that expressed in practically linear process of the probe formation. Therefore, distribution profile of the beam current density on the target with the minimal spot size (FWHM) has flat maximum shape with 80% of the beam current concentrated in the spot. Application of the condenser system consisting of the doublet of the electrostatic quadrupole lenses allows us to match up parameters of initial distribution of the beam phase density with ion optical characteristics of the PFS. Matching is expressed in the increase of the angular dispersion of initial distribution on account for the decrease of the linear disper-

sion. This gives a possibility to decrease spot size (FWHM) more than two times and to increase current density in the spot. However, while this, halo sizes increase significantly and contribution of the spot current to the total beam current decreases before 30%. Such distributions with a pronounced peak shape are needed in some applications where it is necessary to obtain image of the target surface or structure.

### ACKNOWLEDGEMENT

The authors acknowledge the assistance of Ms. A.S. Belska with the preparation of this paper for publication.

### REFERENCES

1. D.N. Jamieson. New generation nuclear microprobe systems // *Nucl. Instr. and Meth. B.* 2001, №181, p.1-9.
2. A.A. Ponomarev, V.I. Miroshnichenko and A.G. Ponomarev. Influence of the beam current density distribution on the spatial resolution of a nuclear microprobe // *Nucl. Instr. and Meth. B.* 2009, №267, p.2041-2045.
3. V.I. Miroshnichenko, A.A. Ponomarev. Transformation of the ion beam phase density by nonlinear probe-forming systems // *Problems of Atomic Science and Technology. Series "Plasma Electronics and New Acceleration Methods"* (6). 2008, №4, p.264-268.
4. D.V. Magilin, A.G. Ponomarev, V.A. Rebrov, et al. Performance of the Sumy nuclear microprobe with the integrated probe-forming system // *Nucl. Instr. and Meth. B.* 2009, №267, p. 2046-2049.
5. A. Dymnikov, R. Hellborg. Matrix theory of the motion of a charged particle beam in curvilinear space-time. Part I. General theory. Part II. Nonlinear theory // *Nucl. Instr. and Meth. A.* 1993, №330, p.323-362.

*Статья поступила в редакцию 31.05.2010 г.*

### СОГЛАСОВАНИЕ ПАРАМЕТРОВ РАСПРЕДЕЛЕНИЯ ФАЗОВОЙ ПЛОТНОСТИ ЧАСТИЦ ПУЧКА С ИОННО-ОПТИЧЕСКИМИ ХАРАКТЕРИСТИКАМИ ЗОНДОФОРМИРУЮЩЕЙ СИСТЕМЫ ЯДЕРНОГО МИКРОЗОНДА

*А.А. Пономарев, В.И. Мирошніченко*

Основной задачей моделирования нелинейной динамики пучка в зондоформирующей системе с неравномерным распределением фазовой плотности частиц является получение профилей распределения плотности тока на мишени в зависимости от условий формирования зонда. Эти условия определяются изменением параметров начального распределения фазовой плотности в плоскости объектного коллиматора за счет выбора расположения и питания дублета электростатических квадрупольных линз. Такой подход позволяет найти режимы формирования зонда, при которых параметры начального распределения фазовой плотности пучка согласованы с ионно-оптическими характеристиками ЗФС микрозонда. Результатом согласования является увеличение плотности тока и уменьшение размеров пучка на мишени.

### УЗГОДЖЕННЯ ПАРАМЕТРІВ РОЗПОДІЛУ ФАЗОВОЇ ЩІЛЬНОСТІ ЧАСТИНОК ПУЧКА З ІОННО-ОПТИЧНИМИ ХАРАКТЕРИСТИКАМИ ЗОНДОФОРМУЮЧІЙ СИСТЕМИ ЯДЕРНОГО МІКРОЗОНДА

*А.А. Пономарьов, В.І. Мирошніченко*

Головним завданням моделювання нелінійної динаміки пучка в зондоформуєчій системі з нерівномірним розподілом фазової щільності частинок є одержання профілів розподілу щільності струму на мішені у залежності від умов формування зонда. Ці умови визначаються зміною параметрів початкового розподілу фазової щільності в площині об'єктного коліматора за рахунок вибору розташування та живлення дублету електростатичних квадрупольних лінз. Такий підхід дозволяє знайти режими формування зонда, при яких параметри початкового розподілу фазової щільності пучка узгоджені з іонно-оптичними характеристиками ЗФС мікрозонда. Результатом узгодження є збільшення щільності струму і зменшення розмірів пучка на мішені.