PROPAGATION OF THE FAST MAGNETOSONIC WAVE THROUGH THE GENERALIZED BUDDEN BARRIER

Ye.O. Kazakov, I.V. Pavlenko, I.O. Girka
V.N. Karazin Kharkov National University, Kharkov, Ukraine
E-mail: kazakov_evgenii@mail.ru

Propagation of the fast magnetosonic wave through the generalized Budden barrier, which is formed by the ion-ion hybrid resonance and the accompanying L-cutoff, is studied. Analytical expressions for the transmission, reflection and conversion coefficients are derived. It is shown that the non-zero reflection from the barrier arises in case of the wave incidence from the resonance side, and the conversion coefficient can reach the value 48.6% for the cutoff incidence case. The obtained results generalize the formulas of the Budden theory in case of the different fast wave wavelength at the opposite sides of the ion-ion hybrid resonance.

PACS: 52.50.Qt

1. INTRODUCTION. BUDDEN THEORY

The Ion Cyclotron Resonance Heating (ICRH) is widely used in modern tokamaks [1]. The ICRH antenna, which is located either at the high field side (HFS) or at the low field side (LFS) of the trap, launches the fast magnetosonic wave (FW) into the plasma. The wave propagates to the plasma center, and is either absorbed at the fundamental and harmonic cyclotron resonance layers by ions, or is converted to the small-scale plasma mode at the ion-ion hybrid (IIH) resonance layer. The latter arises only in multicomponent plasmas with two or more ion species with the different charge-to-mass ratio. In this regime, which is known as the mode conversion, the localized electron heating is observed [2]. The effective electron Landau damping of the converted mode occurs due to the upshift of the parallel wavenumber under the presence of the toroidal current in tokamaks [3]. Mode conversion regime is extensively studied within the last years since it has a number of important applications beyond heating itself [4]. To name just a few: it is used to study electron transport, generate plasma rotation and current drive, measure the plasma composition, as a mechanism of impurity pump-out, etc. The successful performance of such a heating scenario relies on the achievement of the effective conversion conditions. Thus, the numerous efforts have been made to understand the physics of the mode conversion.

The propagation of the FW through the inhomogeneous in the x direction plasma is usually described by the wave equation

$$y'' + Q(x)y = 0, \qquad (1)$$

where y is one of the electric field components of the wave, and $Q(x)$ is the potential function which depends on the dispersion relation for the FW, $Q(x) = (\omega^2/c^2)n^2_{\perp,\mathrm{FW}}$. The latter is given by

$$n^2_{\perp,\mathrm{FW}} = \frac{(L - n^2_{||})(R - n^2_{||})}{S - n^2_{||}}. \qquad (2)$$

Here, S, L and R are the components of the cold plasma dielectric tensor in the notation of Stix [5], $n_{||}$ is the parallel (with respect to the magnetic field) refractive index. In the ion cyclotron frequency range the resonance denominator condition $S = n^2_{||}$ defines the ion-ion hybrid resonance. Its frequency lies between the ion cyclotron frequencies of the ion species, Ω_1 and Ω_2. It is located near the cyclotron resonance of the minority ions, and shifts towards majority resonance with the minority concentration increase. The IIH resonance is accompanied by the left-hand polarized L-cutoff, which is defined by the condition $L = n^2_{||}$, towards the LFS. Together they form the evanescence layer, where $n^2_{\perp,\mathrm{FW}} < 0$ (Fig.1). The hot plasma theory resolves the IIH resonance. The more sophisticated full-wave models show that at the IIH resonance layer the FW couples to the small-scale mode.

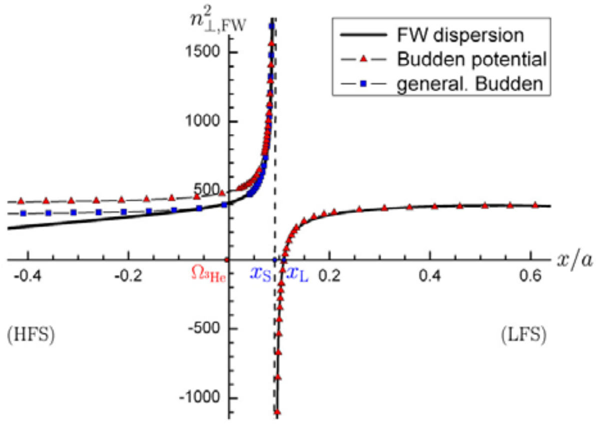

Fig.1. The typical spatial dependence of the FW-refractive index for the two-ion component plasma. The evanescence layer is formed by the ion-ion hybrid resonance and the accompanying L-cutoff

The classical theory which describes the propagation of the FW through the isolated IIH cutoff-resonance pair is the Budden theory [6]. In this case the potential function is modeled by the following expression:

$$Q_{\mathrm{B}}(x) = k^2_{\mathrm{A}}(1 - \Delta/(x - x_{\mathrm{S}})), \qquad (3,a)$$

where k_{A} is the wavenumber of the FW far from the resonance, Δ is the width of the evanescence layer, x_{S} is the location of the IIH resonance. If we normalize all the spatial variables on the FW wavelength, i.e. introduce new variable $\xi = k_{\rm A}(x - x_{\rm S})$, then the potential (3,a) is simplified and given by

$$Q_{\rm B}(\xi) = 1 - \eta/\xi. \qquad (3,b)$$

The dimensionless parameter $\eta = k_{\rm A}\Delta$ is called as the tunneling factor. Within the Budden theory it entirely defines the scattering coefficients. The important feature of the considered barrier is the asymmetry of the scattering coefficients with respect to the side of the wave incidence. While the transmission coefficient equals to $T_{\rm B} = e^{-\pi\eta}$ regardless of the incidence side, the dependence of the reflection coefficient is essentially different for the HFS and LFS cases. For the HFS incidence (resonance side) the wave is transmitted through the layer without any reflection. The rest of the energy is converted to the small-scale mode. In such a way providing the evanescence layer enough thick (by increasing the concentration of the minority ions) the effective mode conversion is obtained. This is not appropriate for the case of the antenna location at the LFS (as for most of present-say tokamaks). In this case the reflection coefficient is equal to $R_{\rm B} = (1 - T_{\rm B})^2$. For the thick evanescence layers it is the dominant process. The mode conversion coefficient, $C_{\rm B} = T_{\rm B}(1 - T_{\rm B})$ reaches its maximal value 25%, when the evanescence layer is semi-transparent one, $\eta = \ln(2)/\pi \approx 0.22$.

The Budden theory implies the inhomogeneity of the magnetic field. In a real situation due to the decrease of the plasma density to the edge, the dispersion of the FW is more complicated than that described by (3a). The FW wavenumber at the HFS decreases (Fig.1), and even the additional R-cutoff at the HFS can appear. In the present paper the generalized Budden barrier is considered, for which the FW is assumed to have different wavelength at the cutoff and resonance sides. The analytical formulas for the scattering coefficients are derived. Comparison with the Budden results is presented.

2. GENERALIZED BUDDEN POTENTIAL

This section describes the scattering properties of the generalized Budden barrier. It is convenient to normalize all the spatial variables to the FW wavelength at the LFS (cutoff) side. Then, the potential is written similarly to (3b):

$$Q(\xi) = \begin{cases} 1 - \eta/\xi, & \xi > 0 \\ \gamma^2 - \eta/\xi, & \xi < 0 \end{cases}. \qquad (4)$$

Its spatial dependence is shown in Fig.2. The parameter $\gamma = \lambda_{\rm LFS}/\lambda_{\rm HFS}$ is the ratio of the FW wavelength at the cutoff and resonance sides.

For both sides of the problem the analytical solution of the wave equation (1) in terms of the confluent hypergeometric (Whitakker) functions can be written. For region $\xi > 0$ the solution is written as

$$y(\xi) = C_1 f_1(\xi) + C_2 f_2(\xi), \qquad (5)$$

where the functions $f_1(\xi)$ and $f_2(\xi)$ are given by

$$f_1(\xi) = e^{i\xi}\xi\, M(1 + ik; 2; -2i\xi), \qquad (6)$$
$$f_2(\xi) = e^{i\xi}\xi\, U(1 + ik; 2; -2i\xi),$$

and $k = \eta/2$. $M(a;b;z)$ is the Kummer's function, $U(a;b;z)$ is the second independent solution of the Kummer's equation. The definition and properties of these functions can be found in [7].

In region $\xi < 0$, the solution of (4) is represented as follows:

$$y(\xi^*) = B_1 F_1(\xi^*) + B_2 F_2(\xi^*), \qquad (7)$$

where $\xi^* = \gamma\xi$, $\eta^* = \eta/\gamma$, $k^* = \eta^*/2$, and

$$F_1(\xi^*) = e^{i\xi^*}\xi^*\, M(1 + ik^*; 2; -2i\xi^*), \qquad (8)$$
$$F_2(\xi^*) = e^{i\xi^*}\xi^*\, U(1 + ik^*; 2; -2i\xi^*).$$

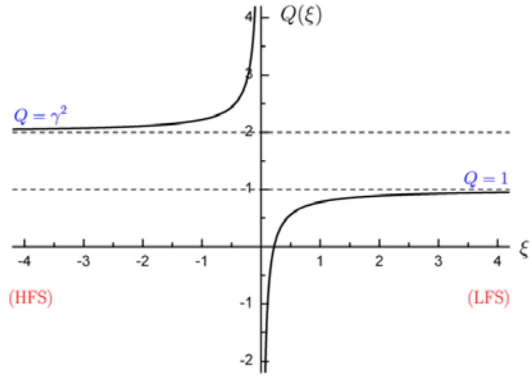

Fig.2. Plot of the generalized Budden potential, which describes the propagation of the FW through the isolated cutoff-resonance pair

In order to find the global solution of (4) one needs to match the coefficients C_1, C_2 and B_1, B_2 of the solutions (5) and (7). Therefore, three conditions, which connect the coefficients, should be formulated. The first two are obtained from the solution matching at the point $\xi = 0$. It implies the continuity of the solution function $y(x)$ and its first derivative $y'(x)$. Using the expansion of the Kummer's functions for small arguments one obtains:

$$C_2 = \gamma\frac{\Gamma(ik)}{\Gamma(ik^*)}B_2, \qquad (9)$$
$$\frac{C_1}{\gamma} + \frac{B_2}{\Gamma(ik^*)}\left[\psi(ik) - \psi(ik^*) + \frac{i(\gamma-1)}{\eta} - \ln\gamma\right] = B_1,$$

where $\psi(z) = \Gamma'(z)/\Gamma(z)$ is the Digamma function [7]. The last matching equation depends on the considered side of the wave incidence. For the HFS incidence, condition

$$C_1 = 0 \qquad (10,a)$$

ensures that at the opposite side only the transmitted wave exists. Similarly, for the case of the LFS incidence, condition

$$\frac{B_1}{\eta^*}\frac{e^{-\pi k^*/2}}{\Gamma(-ik^*)} + \frac{iB_2}{2}e^{\pi k^*/2} = 0 \qquad (10,b)$$

suppresses the non-physical right-travelling term (proportional to $e^{i\xi^*}$) at the HFS. Condition (10,a) or (10,b) is called as the radiating boundary condition. Its explicit form is derived using the asymptotic expansion of (6) and (8) for large arguments of the independent variable.

Using the matching conditions (9) and (10), the scattering coefficients can be easily calculated. For convenience, we introduce the following parameters:

$$\upsilon = \psi\left(\frac{i\eta}{2}\right) - \psi\left(\frac{i\eta}{2\gamma}\right) + \frac{i(\gamma-1)}{\eta} - \ln\gamma, \qquad (11)$$
$$s = \frac{2\pi i}{1 - e^{-\pi\eta/\gamma}}\cdot\frac{1}{\upsilon}. \qquad (12)$$

In case of the Budden barrier with $\gamma = 1$, the parameter s is infinitely large.

The interesting feature of the generalized Budden barrier is the fact that the transmission coefficient does not depend on the incidence side like for the classical Budden case. This feature represents the fundamental reciprocity principle [8]. The transmission coefficient is equal to

$$T_{\rm HFS} = T_{\rm LFS} = e^{-\pi\eta} \cdot \frac{1-e^{\pi\eta/\gamma}}{1-e^{-\pi\eta}} \cdot \left|\frac{s}{1+s}\right|^2. \quad (13)$$

For $\gamma = 1$, the formula (13) reduces to the famous Budden result, $T_{\rm B} = e^{-\pi\eta}$.

In contrast to the Budden theory the non-zero reflection occurs for the HFS incidence case. The reflection coefficient is equal to

$$R_{\rm HFS} = \frac{1}{|1+s|^2}. \quad (14)$$

Fig.3 shows the reflection coefficient $R_{\rm HFS}$ as a function of γ for different values of the tunneling factor. In the vicinity of $\gamma = 1$ the parabolic dependence of $R_{\rm HFS}$ is clearly seen. This part of the curve is described by the following approximate formula:

$$R_{\rm HFS} \approx \frac{(1-e^{-\pi\eta})^2}{4\pi^2} \cdot \left|1 - \frac{i}{\eta} - \frac{i\eta}{2}\psi'(i\eta/2)\right|^2 \cdot \epsilon^2, \quad (15)$$

where the small parameter $\epsilon = \gamma - 1$ is introduced. Thus, the Budden case with zero reflection is the exceptional one. For any $\gamma \neq 1$ the non-zero reflection from the barrier occurs.

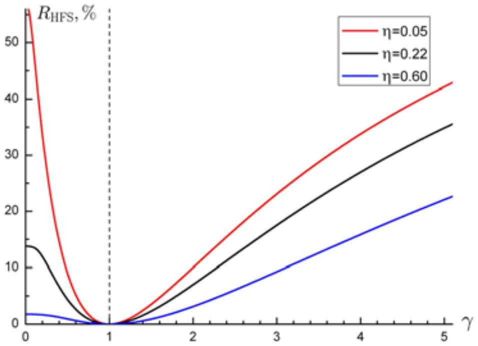

Fig.3. Dependence of the reflection coefficient $R_{\rm HFS}$ versus γ for different values of the tunneling factor

The reflection coefficient for the LFS incidence case is given by

$$R_{\rm LFS} = \left|1 - T_{\rm LFS}\left(1 + \frac{1}{s^*}\right)\right|^2, \quad (16)$$

where s^* stands for the complex conjugate of s.

Another distinctive feature of the Budden barrier is the upper limitation of the conversion coefficient for the LFS incidence at the level $C_{\max} = 25\%$. The conversion coefficient is calculated from the energy conservation law, $C = 1 - R - T$, using (13) and (16). Fig.4 shows the dependence of the conversion coefficient $C_{\rm LFS}$ as a function of the tunneling factor for different values of the parameter γ. It is clearly seen that for $\gamma > 1$ the conversion coefficient is less than the Budden result. Vice versa, for $\gamma < 1$ the mode conversion coefficient exceeds the Budden level. After some algebraic manipulations, the approximate analytical formula for the conversion coefficient $C_{\rm LFS}$ is derived. It can be presented as a sum of two terms

$$C_{\rm LFS} \approx C_{\rm B}(\eta) - F(\eta)\cdot\epsilon, \quad (17)$$

where the correction function $F(\eta)$ is defined as

$$F(\eta) = T_{\rm B}^2 \left[\frac{\pi\eta(1-2T_{\rm B})}{1-T_{\rm B}} - \frac{1-T_{\rm B}}{\pi\eta}\left(1 + (\eta^2/2)\,{\rm Re}\,\psi'(i\eta/2)\right)\right]. \quad (18)$$

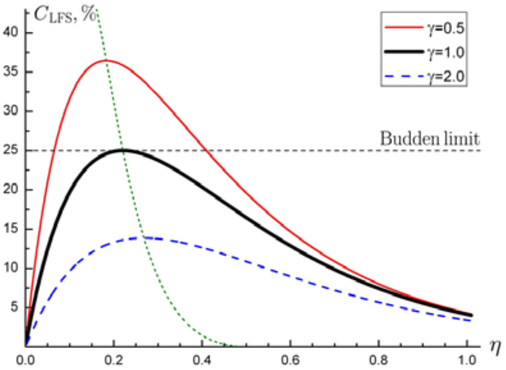

Fig.4. Mode conversion coefficient $C_{\rm LFS}$ as a function of the tunneling factor for different values of γ

The first term in (17) is the Budden result. The sign of the second term is determined by the sign of ϵ. As shown in Fig.5, the correction function is positively defined. Thus, for $\gamma < 1$ the correction term in (17) is positive, and the mode conversion coefficient exceeds the result of Budden.

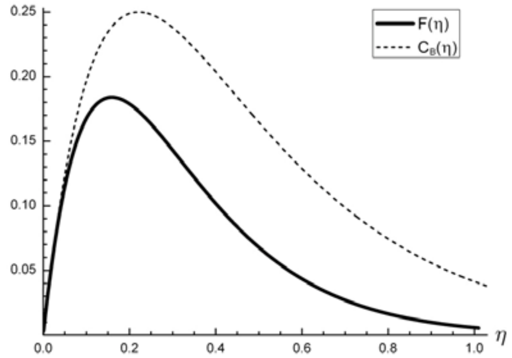

Fig.5. Plot of the correction function $F(\eta)$ defined by (18)

Next, we are interested in the question, what is the highest level of the conversion coefficient that can be achieved for the arbitrary γ value. We have calculated numerically the value of the maximal conversion coefficient $C_{\max}$ for the given value of γ. The results are shown in Fig.6.

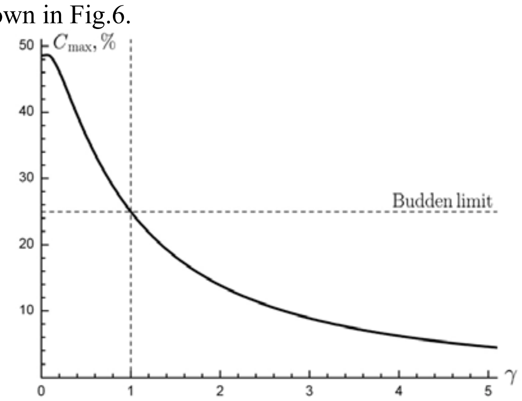

Fig.6. Dependence of the maximal conversion coefficient $C_{\max}$ as a function of γ (LFS incidence). For $\gamma < 1$, $C_{\max}$ exceeds the classical Budden result

The highest value $C_{\max} \approx 48.6\%$ is reached for $\gamma \approx 0.06$ at $\eta \approx 0.13$. It is nearly twice greater than the classical Budden result.

The dispersion of the FW, which is shown in Fig.1, is calculated for (^{3}He)H plasma with the concentration of ^{3}He ions X[^{3}He]=4%. The parameters chosen are typical for the JET tokamak ^{3}He heating experiments: f=37 MHz, B_0=3.6 T, k_z=3.5 m^{-1}, n_{e0}=2.5·10^{13} cm^{-3}, R_0=2.96 m, a=0.9 m. For the conditions considered the parameter γ is equal to $\gamma \approx 0.9$. The scattering coefficients calculated using the formulas for the generalized Budden barrier differ from the results of the classical theory just by a few percent. Thus, the presented approximate formulas (17) and (18) give the value of the conversion coefficient C_{LFS} with a high accuracy for the wide range of experimental parameters.

CONCLUSIONS

The paper describes the propagation of the FW through the generalized Budden barrier. The assumption that the wavelength of the FW is equal to both sides of the barrier is neglected. The analytical solution of the wave equation in terms of the confluent hypergeometric functions is derived. The scattering coefficients are found for both cases of the wave incidence. The detailed analysis of the scattering coefficients is performed. It is shown that the obtained results generalize the formulas of the classical Budden theory. Particularly, it is shown that the non-zero reflection from the barrier occurs for the HFS incidence. For the LFS incidence the conversion coefficient can reach the value 48.6% that is nearly twice greater than the maximum within the Budden theory.

REFERENCES

1. A.V. Longinov and K.N. Stepanov. *High-Frequency Plasma Heating* / Ed. A.G. Litvak. New York: “American Institute of Physics”, 1992, p.93-238.
2. M.J. Mantsinen, et al. Localized bulk electron heating with ICRF mode conversion in the JET tokamak // *Nuclear Fusion*. 2004, v.44, №1, p.33-46.
3. A.K. Ram and A. Bers. Propagation and damping of mode converted ion-Bernstein waves in toroidal plasmas // *Phys. Fluids B*. 1991, v.3, №4, p.1059-1069.
4. M.J. Mantsinen, et al. Application of ICRF waves in tokamaks beyond heating // *Plasma Physics and Controlled Fusion.* 2003, v.45, №12A, p.A445-A456.
5. T.M. Stix. *The Theory of Plasma Waves*. New York: “McGraw-Hill”, 1962, 283 p.
6. K.G. Budden. *The Propagation of Radio Waves*. Cambridge: “Cambridge University Press”, 1985, 684 p.
7. M. Abramowitz and I.A. Stegun. *Handbook of Mathematical Functions*. New York: “Dover”, 1970, 1046 p.
8. V.L. Ginzburg. *The Propagation of Electromagnetic Waves in Plasmas.* Oxford: “Pergamon Press”, 1970, 615 p.

Статья поступила в редакцию 31.05.2010 г.

РАСПРОСТРАНЕНИЕ БЫСТРОЙ МАГНИТОЗВУКОВОЙ ВОЛНЫ ЧЕРЕЗ ОБОБЩЕННЫЙ БАРЬЕР БАДДЕНА

Е.А. Казаков, И.В. Павленко, И.А. Гирка

Решена задача распространения быстрой магнитозвуковой волны через обобщенный барьер Баддена, который образован ион-ионным гибридным резонансом и связанной с ним L-отсечкой. Получены аналитические выражения для коэффициентов прохождения, отражения и конверсии. Показано, что имеет место ненулевое отражение от барьера в случае падения волны со стороны резонанса, а коэффициент конверсии может достигать величины 48.6% при падении волны со стороны отсечки. Полученные результаты обобщают формулы теории Баддена на случай различной длины волны по разные стороны от ион-ионного гибридного резонанса.

ПОШИРЕННЯ ШВИДКОЇ МАГНІТОЗВУКОВОЇ ХВИЛІ КРІЗЬ УЗАГАЛЬНЕНИЙ БАР’ЄР БАДДЕНА

Є.О. Казаков, І.В. Павленко, І.О. Гірка

Розв’язано задачу про поширення швидкої магнітозвукової хвилі крізь узагальнений бар’єр Баддена, який утворений іон-іонним гібридним резонансом та L-відсічкою, що пов’язана з ним. Здобуто аналітичні вирази для коефіцієнтів проходження, відбиття та конверсії. Показано, що має місце ненульове відбиття від бар’єру для випадку падіння хвилі зі сторони резонансу, а коефіцієнт конверсії може сягати величини 48.6% за умови падіння хвилі зі сторони відсічки. Здобуті результати узагальнюють формули теорії Баддена на випадок різної довжини хвилі по різні боки від іон-іонного гібридного резонансу.