
A.A. VLASOV AND N.N. BOGOLYUBOV, THE FORERUNNERS OF QUANTUM ELECTRODYNAMICS

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PACS 52,57K
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All written below is the child of my brain, although the meetings and discussions of these two great physicists actually took place, and these discussions resulted in the substantiation of the famous Vlasov equation in the seminal work by N.N. Bogolyubov “Dynamic Methods in the Statistical Physics” published in 1946. This work, together with Vlasov’s one, not only proved the statistical physics of many-particle systems with Coulomb interaction, but also laid the corner stone of the method of electrodynamic perturbation theory, which brought about the creation of quantum electrodynamics at the end of the 1940s.

At the beginning of the 1940s, after the famous work by A.A. Vlasov had been published (Zh. Eksp. Teor. Fiz., 1938) – most likely, after N.N. Bogolyubov had studied it – the latter became a frequent visitor from Kyiv to Moscow and spent a lot of time at the Faculty of Physics of the Moscow State University, in hot discussions with A.A. Vlasov. N.N. Bogolyubov was interested in the essence and substantiation of the Vlasov kinetic equation – or, as it is conventionally referred to, the equation with a self-consistent field. In fact, one may assert that A.A. Vlasov ingeniously foresaw this equation, and his arguments in its favor did not satisfy N.N. Bogolyubov, except for that it is an explanation of a large cycle of experiments carried out by I. Langmuir and his collaborators. Really, before A.A. Vlasov, only the Boltzmann kinetic equation – written down as early as at the end of the 19-th century and not recognized by the scientific community, in particular, by H. Poincaré, for a long time – was known in the kinetic theory of gases. H. Poincaré perfectly understood the meaning of a small parameter introduced by L. Boltzmann,

$$a_0 n_0^{1/3} \ll 1, \quad (1)$$

where a_0 is the size of a gas atom or molecule, and n_0 is the atomic concentration (the number of particles in a unit volume). This condition which is referred to as the ideal-gas one says that the average distance between

atoms in the gas is much longer than the atomic size, so that the atoms spend much more time in a free (thermal) flight, being engaged in collision processes for only a small time fraction; or, to put it in other words,

$$\tau_{cm} \sim a_0/v_T \ll n_0^{-1/3}/v_T \sim 1, \quad (2)$$

where $v_T = \sqrt{T/m}$ is the average thermal velocity of an atom, m is its mass, and T is the gas temperature measured in energy units. At the same time, H. Poincaré could not understand altogether how an equation that describes the dissipation can be obtained for a Hamiltonian system that conserves energy. It seems that N.N. Bogolyubov had an answer to H. Poincaré’s question even before his discussions with A.A. Vlasov started. At the beginning of the 1940s, he had already been developing a time hierarchy of correlation functions for a gas of short-range interacting particles (the famous Bogolyubov chains), and he understood that, under condition (1) – or, equivalently, condition (2) – this chain of equations can be cut to obtain a closed finite system of equations. In this case, in the zeroth-order approximation with respect to parameter (1), the Liouville equation, which describes a gas of noninteracting particles (the ideal gas), appears. In the first approximation with respect to parameter (1), only the pair correlation functions are taken into account, with ternary and higher-order correlations being rejected. As a result, a kinetic equation is obtained, which describes a gas, where only pair collisions between particles are made allowance for, and which is known as the Boltzmann equation.

But what can we do with a gas with the Coulomb interaction between particles? Such a gas, with the blessing of I. Langmuir in 1929, was called plasma. I. Langmuir not only gave a name to the ionized gas (the overwhelming number of its particles are in the charged state), but also carried out its fundamental experimental and theoretical researches, which won him the Nobel Prize in 1932. The high award evidenced the importance of the object under investigation, i.e. plasma. Plasma is too widespread in the nature; it is a lightning, atmospheric and laboratory discharges in gases and

solids, the ionosphere of the Earth, an interplanetary gas, stars, nebulae, and, at last, plasma in solids (metals and semiconductors). Not accidentally did D.A. Frank-Kamenetskii, who wrote one of the first textbooks on plasma physics, call plasma the fourth aggregate state of the substance. Meanwhile, a simple hydrodynamical model of plasma proposed by I. Langmuir for the explanation of his experiments, although having a bright success in some cases, was totally out of place in the others.

L.D. Landau was the first who understood the necessity of plasma description making use of the kinetic equation. In 1937 (*Zh. Eksp. Teor. Fiz.*, 1937), he paid attention that the ideal-gas condition (1) is not suitable in the plasma case, since the typical radius of interaction between particles in plasma, the Debye radius, is much larger than the average interparticle distance, i.e. the inverse inequality, as compared with inequality (1), takes place:

$$r_D n^{1/3} \approx \left(\frac{T}{e^2 n^{1/3}} \right)^{1/2} \gg 1. \quad (3)$$

Here, e is the particle charge, and T is the plasma temperature. However, it is condition (2) that says that the average potential energy of interaction between charged particles is much lower than their average kinetic (thermal) energy, i.e.

$$\eta = \frac{e^2 n^{1/3}}{T} \ll 1, \quad (4)$$

which is an equivalent to inequality (3). Just this condition was introduced by L.D. Landau as the ideal-gas criterion for plasma.

But his next step – namely, he followed L. Boltzmann and wrote a Boltzmann equation (a Liouville equation that makes allowance for pair collisions) for plasma as a gas of charged particles – was, strictly speaking, incorrect. As a by-product, he masterfully overcame the well-known Coulomb divergence, by introducing the well-known Coulomb logarithm (in essence, the logarithm of the inverse gas parameter η (see Eq. (4)) when writing down the final Coulomb collision integral.

Exactly in a year, A.A. Vlasov published his famous equation with a self-consistent field in the article cited above and substantiated it making use of literally the same words as L.D. Landau did. In particular, the interaction sphere had to include plenty of particles, i.e. condition (3) had to be satisfied. However, the following words were quite different. Following A.A. Vlasov's speculations, if it is so, every particle interacts, in the

first approximation, with all other particles simultaneously or, in other words, with the electromagnetic field created by all particles in the plasma. As a result, we obtain, in the first approximation, a system of equations that consists of the Liouville kinetic equation, in which the Lorentz force is presented, and the Maxwell equations describing the fields under the action of the Lorentz force. The field sources in the Maxwell equations are the densities of charges and currents created by all charged particles in the plasma. Together, those equations comprise the Vlasov–Maxwell system of equations, or the equations with a self-consistent field.

Well, what can we do next? How would the equations look like, if the next approximation order is taken into account? This issue excited N.N. Bogolyubov, and it was a subject of his hot discussions with A.A. Vlasov, at the beginning of the 1940s, in a university audience on Mokhovaya Street in Moscow, where N.N. Bogolyubov came several times from Kyiv. The hot discussions between N.N. Bogolyubov and A.A. Vlasov resulted in the monography by N.N. Bogolyubov mentioned above. In this monography, he applied the quantum-mechanical electrodynamic method to the statistical physics problems for the first time. N.N. Bogolyubov proceeded from a Hamiltonian consisting of a sum of Hamiltonians of free particles and the field, and a Hamiltonian of interaction between them (but not more). By applying the perturbation theory method (the expansion into a power series of ϵ^2) to his famous chain of equations for correlation functions, N.N. Bogolyubov obtained – as the first approximation in ϵ^2 – the Vlasov kinetic equation and – in the next approximation (with an accuracy to ϵ^4) – the Vlasov equation with the Landau collision integral. Thus, a consecutive derivation of kinetic equations for gases by N.N. Bogolyubov had been completed. Surprisingly, this method is known as the Bogolyubov–Bhatnagar–Gross–Krook (BBGK) one, although the works of the others appeared independently and, as the saying is, some later!

It was much later, when the quantum electrodynamics had already been created, that R. Balescu, using the Feynman diagram method, showed the following:

- 1) if only a vertex diagram is taken into consideration (the particle radiates or absorbs the field), the Vlasov equation is obtained;
- 2) if both a vertex and an exchange diagram are taken into consideration (one particle radiates the field, and another particle absorbs it), the Vlasov equation with the Landau collision integral is obtained;

3) if all crossed exchange diagrams are summed up (the ladder approximation), the Vlasov equation with the Lenard–Balescu collision integral (which takes a plasma polarization into account due to the particle interaction) is obtained.

Hence, there is a good reason to refer to the system of equations that describes the plasma kinetics as the Vlasov–Landau–Bogolyubov–Maxwell system, in the ascending order of contribution made by each of the participants into the physics as a whole!

Received 00.00.09.

Translated from Ukrainian by A.I. Voitenko

А.О. ВЛАСОВ І М.М. БОГОЛЮБОВ –
ПОПЕРЕДНИКИ КВАНТОВОЇ ЕЛЕКТРОДИНАМІКИ

A.A. Pughadze

Резюме

Все, що написано – плід моєї фантазії, хоча зустрічі та дискусії цих двох великих фізиків дійсно мали місце. Ці дискусії привели до обґрунтування знаменитого рівняння Власова у видатній роботі М.М. Боголюбова “Динамічні методи в статистичній фізиці”, яку було опубліковано в 1946 році. Ця робота разом із роботою Власова не тільки обґрунтувала статистичну фізику багаточастинкових систем із кулонівською взаємодією, але й заклала основу методу електродинамічної теорії збурень, яка в кінці 40-х років привела до створення квантової електродинаміки.