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Modified Newtonian Dynamics Effects in a Region Dominated by Dark Matter and a Cosmological Constant Λ respectively

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Abstract:

We study the motion of a secondary celestial body under the influence of a corrected gravitational potential in a modified Newtonian dynamics scenario. Furthermore we look within the Milky-way where the first correction to the potential results from a modified Poisson equation, and includes two mew terms one of which is of the form $\ln(r/r_{max})$ and the other is associated with the cosmological constant lambda Λ added to the Newtonian potential. The regions of influence of the two potentials are associated with regions of interested bounded by the conditions $r < r_{max}$ for the Newtonian potential, $r > r_{max}$ the logarithmic correction to the potential relating to the term $\left(\nabla\phi\right)^2$ in the Poisson equation for the gravitational field that has matter density ρ, and finally, the domain where $r >> r_{max}$ the potential scales as $c^2\Lambda\, r^2$ and the cosmological constant lambda dominates. Next using an average disturbing potential we integrate Lagrange's planetary equations and we obtain analytical expressions for the average time rates of change of the orbital elements using our sun as an example. We find that both dark matter and cosmological constant affects the argument of the perigalaktikon point as well as the mean anomaly.

1. Introduction

Early work done by Zwicky (1937) as well as Vera Rubin (1970) resulted to a surprising result that galactic dynamics and the dynamics of galactic clusters is not in agreement with the predictions of Newtonian and Einsteinian gravity. In an effort to explain the discrepancy between theory and observation the ΛCDM model predicts a dark component of matter. With the help of this dark researchers are trying to explain why the masses of galaxies and galactic clusters resulting from dynamics were calculated to exceed the baryon mass of the corresponding systems. Furthermore, halos composed of dark matter were postulated such that the dynamical discrepancy can be resolved.

Dark matter (DE) and the corresponding dark energy (DE) is supposed to make up more than 95% of the energy density of the universe, and provides an explanation for the power spectrum of the Cosmic Microwave Background according to Ade et al. (2016), and also to the formation of various astrophysical structures (Nuza et al. 2013). At present, the nature of this dark matter it's not known, and none of the proposed candidates from stable particles in extensions of the Standard Model, to primordial Black Holes Klasen et al. (2015) and Bird et al. (2016) has not been at the moment detected beyond any doubt.

An alternative approach, that may resolve these problems, is to treat the dark matter phenomenology as an indication of a gravity modification in the very weak field regime. This way the theory of Modified Newtonian Dynamics (MOND) was first proposed in (Milgrom, 1983), in which the standard Newtonian gravitational force is enhanced for accelerations below in which the standard Newtonian gravitational force is enhanced for accelerations below an empirically determined value of $a_0 \sim 1.2 \times 10^{-10}$ m/s^2, which when applied to the galactic rotation curves its results are very effective (Sanders and Noordermeer, 2007). Furthermore, this proposal has been developed in into a full relativistic theory, a tool that is necessary for the construction of a cosmological model (Skordis, 2008). But so far none of these theories has pass all cosmological tests. Moreover, at cluster scales and beyond has become known that it might be that and additional type of unseen matter is required in the above MOND theory (Sanders, 2003). Finally, we say that the success of the MOND at galactic scales suggests it might be worthwhile to investigate the possible consequences to galaxy dynamics and its various related phenomena. In this contribution we are using the resulting corrections to the Newtonian potential resulting in a modified Poisson equation for various galactic regions, and from that we calculate the time rate of change of the mean motion of a body orbiting in this particular region. Furthermore, we calculate any possible anomalistic time changes in the same orbiting body.

2. MOND theory formulation

Over the years various theories which are able to reproduce the MOND phenomenology in the weak field limit have been proposed. In this paper we will mention two types of MOND formulation namely the AQUAL (Bekenstein and Milgrom, 1984) and the QUMOND formulation (Milgrom, 2010). The first one AQUAL is a theory of gravity based on Modified Newtonian Dynamics (MOND), but using a Lagrangian of the form:

$$L_G = -\frac{1}{8\pi G} a_0^2 F\left(\frac{|\nabla\phi|^2}{a_0^2}\right), \tag{1}$$

Results to the following associated Poisson equation that can be written as follows:

$$\nabla \cdot \left[\mu\left(\frac{|\nabla\phi|}{a_0}\right)\nabla\phi\right] = 4\pi G\rho \,. \tag{2}$$

Equation (2) is a highly non-linear equation, where $\mu(x)=\mathrm{d}F(z)/\mathrm{d}z$, $z = x^2$, ρ is the matter density distribution, φ is the gravitational potential and a_0 is the MOND acceleration scale and $\mu(x)$ is the interpolation function, This function is common to all current MOND formulations. This function interpolates between the

Newtonian regime in the presence of large accelerations, to the deep MOND regime when the accelerations are small, and therefore the form of $\mu(x)$ is only constrained at the following limits

$$\begin{aligned}&\mu(x)\rightarrow 1 \quad \text{if } \ x>>1\\&\mu(x)\rightarrow x \quad \text{if } \ x<<1\end{aligned}\quad . \tag{3}$$

An alternative formulation, suitable to a numerical treatment, is the quasi-linear QUMOND formulation. In this case the gravitational Lagrangian involves an auxiliary potential and may be written as follows

$$L_G=-\frac{1}{8\pi G}\left[2\nabla\phi\cdot\nabla\phi^N-a_0^2Q\left(\frac{\left|\nabla\phi^N\right|^2}{a_0^2}\right)\right]. \tag{4}$$

A variation of the above action involving this Lagrangian results to the following equations for the ϕ and ϕ^N fields, results to: η

$$\nabla^2\phi^N=4\pi G\rho\,, \tag{5}$$

and

ρwhere $\eta(y)$=dQ(z)/dz, y = z^2, and the field ϕ^N satisfies the standard Newtonian Poisson equation. The function η (y) again performs an interpolation between Newtonian and MOND regimes, depending on the system acceleration. The limiting behaviour in this formulation goes as follows:

$$\begin{aligned}&\eta(y)\rightarrow 1 \qquad\quad y>>1\\&\eta(y)->y^{-1/2} \quad y<<1\end{aligned}\quad . \tag{7}$$

3. Modifications of Newtonian gravity

So far there is enough research done, and galactic rotation curves can be explained by invoking the modification of Newtonian gravity (MONG). Following Sivaram et al., (2020) the authors a Poisson equation in which a gravitational self-energy is taken into account and the equations reads:

$$\nabla^2\phi+K\left(\nabla\phi\right)^2=4\pi G\rho\,, \tag{8}$$

where $\phi\approx-\dfrac{GM}{r}$ is the gravitational potential, and the constant has the value of $K\approx G^2/c^2$ that also contributes to the gravitational field along together with the matter density ρ. In the outskirts of galaxies the matter density is small and therefore Eq. (8) can be simplified as follows (ibid, 2020):

$$\nabla^2\phi+K\left(\nabla\phi\right)^2=0\,. \tag{9}$$

Equation has solution of the form:

$$\phi(r)=K'\ln\left(\frac{r}{r_{\max}}\right), \tag{10}$$

Where the new constant $K' = \dfrac{GM}{r_{\max}}$. Moreover, if the dark energy is given by the cosmological constant Λ the Newtonian modification of the Poisson equations takes the form:

$$\nabla^2 \phi - \Lambda c^2 = 0 . \tag{11}$$

Finally, if we include both gravitational self-energy and dark energy densities the modified Poisson equation takes the form:

$$\nabla^2 \phi + K\left(\nabla \phi\right)^2 - \Lambda c^2 = 0 . \tag{12}$$

Equation (12) has a general solution for the potential $\phi\left(r\right)$ of the form:

$$\phi\left(r\right) = -\frac{GM}{r} + K' \ln\left(\frac{r}{r_{\max}}\right) + c^2 \Lambda\, r^2 . \tag{13}$$

Next, quoting Sivaram et al. (2020) we say that we can apply the general solution as given by eq. (13) in three different regions present in every galaxy. In the case when matter density dominates i.e. the region $r < r_{\max}$ the potential is $\phi(r) \approx -\dfrac{GM}{r}$, and the potential results from the solution of the Poisson equation $\nabla^2 \phi = 4\pi G \rho$, and as a result the velocity varies linearly with distance. Furthermore, in the region where $r > r_{\max}$ the term $(\nabla \phi)^2$ dominates, the potential is given by $\phi(r) \approx K' \ln\left(\dfrac{r}{r_{\max}}\right)$ resulting to a constant velocity that accounts for the constant velocity and is solution to the following modified Poisson equation $\nabla^2 \phi + K\left(\nabla \phi\right)^2 = 0$. Finally, in the region where $r >> r_{\max}$ the potential is $\phi(r) \approx \Lambda c^2 r^2$ the dark energy is dominant via the cosmological constant lambda Λ. In the case of the Milky-Way, the velocity profile flattens out at a distance of about 2 Kpc.

4. Lagrange’s planetary equations for the dark matter and cosmological constant effects

The differential equations describing the time variations for the osculating elements as a function of a perturbing acceleration resolve in three different directions in space. Therefore we can write the equations to be (Vallado, 2007):

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2}{na}\frac{\partial R}{\partial M} , \tag{14}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{\left(1-e^2\right)}{na^2 e}\frac{\partial R}{\partial M} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega} , \tag{15}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{1}{na^2 \sin i \sqrt{1-e^2}}\left[\cos i \frac{\partial R}{\partial \omega} - \frac{\partial R}{\partial \Omega}\right], \tag{16}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t}=\frac{\sqrt{1-e^2}}{na^2e}\frac{\partial R}{\partial e}-\frac{\cot i}{na^2\sqrt{1-e^2}}\frac{\partial R}{\partial i}, \tag{17}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t}=-\frac{1}{na^2\sin i\sqrt{1-e^2}}\frac{\partial R}{\partial i}, \tag{18}$$

$$\frac{\mathrm{d}M}{\mathrm{d}t}=n-\frac{2}{na}\frac{\partial R}{\partial a}-\frac{\left(1-e^2\right)}{na^2e}\frac{\partial R}{\partial e}, \tag{19}$$

where, R = is the perturbing potential per unit mass that causes the perturbation of the orbital elements of the orbiting body which in general can be a function of the orbital elements, and where a is the orbital semi-major axis of the orbiting body, e its eccentricity, i is its inclination, ω is the argument of the perigalaktikon point, Ω the argument of the node with respect to the plane of the galaxy, and M is the mean anomaly of the orbiting body.

Given the corrections to the Newtonian potential in Eq. (13) we will consider the corrections to the Newtonian potential separately per galactic region. Thus in the region where $r>r_{\max}$ the term $(\nabla\phi)^2$ dominates, the perturbing potential is given by (Sivaram, et al., 2020):

$$\phi(r)\simeq -K'\ln\left(\frac{r}{r_{max}}\right)=-\frac{GM_{gal}}{r_{max}}\ln\left(\frac{r}{r_{max}}\right). \tag{20}$$

In order to solve Lagrange's planetary equations let us use eq. (20) and calculate an average perturbing potential acting on the orbiting body in one revolution around the galactic plane. Therefore using the transformation relating r to the eccentric anomaly E namely $r=a\left(1-e\cos E\right)$ (Murray and Dermott 1999) we can write that

$$\begin{aligned}\left\langle R\right\rangle &=\frac{1}{2\pi}\int_0^{2\pi}R\left(\frac{r}{a}\right)\mathrm{d}E=-\frac{1}{2\pi}\int_0^{2\pi}K'\ln\left(\frac{r}{r_{max}}\right)\left(\frac{r}{a}\right)\mathrm{d}E\\ &=-\frac{K'}{2\pi}\int_0^{2\pi}\ln\left(a\left(1-e\cos E\right)-\ln r_{max}\right)\left(1-e\cos E\right)\mathrm{d}E\end{aligned}. \tag{21}$$

The integral results in the following solution:

$$\left\langle R_{DM}\right\rangle=\frac{GM}{r_{\max}}\left[1-\left(\sqrt{1-e^2}\right)+\ln\left(\frac{a\left(1+\sqrt{1-e^2}\right)}{2r_{max}}\right)\right]. \tag{22}$$

Similarly, in the region of influence of the cosmological constant lambda constant lambda Λ, where $r>>r_{\max}$ the potential is given by the expression $\phi(r)\simeq\Lambda c^2r^2$. Averaging again over one revolution we obtain:

$$\left\langle R\right\rangle=\frac{1}{2\pi}\int_0^{2\pi}R\left(\frac{r}{a}\right)\mathrm{d}E=-\frac{1}{2\pi}\int_0^{2\pi}\Lambda c^2r^2\left(\frac{r}{a}\right)\mathrm{d}E=-\frac{\Lambda c^2}{2\pi}\int_0^{2\pi}a^2\left(1-e\cos E\right)^3\mathrm{d}E\,, \tag{23}$$

we find that:

$$\left\langle R_\Lambda\right\rangle=\Lambda\,c^2a^2\left(1+\frac{3}{2}e^2\right). \tag{24}$$

5. Calculation of the orbital time rate of changes

Using Eqs. (22) – (24) Lagrange's planetary equations give the following time rates of change:

$$\left\langle\frac{\mathrm{d}a}{\mathrm{d}t}\right\rangle_{DM}=0\,, \tag{25}$$

$$\left\langle\frac{\mathrm{d}e}{\mathrm{d}t}\right\rangle_{DM}=0\,, \tag{26}$$

$$\left\langle\frac{\mathrm{d}i}{\mathrm{d}t}\right\rangle_{DM}=0\,, \tag{27}$$

$$\left\langle\frac{\mathrm{d}\omega}{\mathrm{d}t}\right\rangle_{DM}=\frac{GM_{gal}}{r_{max}na^2}\left[1-\frac{1}{\left(1+\sqrt{1-e^2}\right)}\right]=n\left(\frac{a}{r_{max}}\right)\left[1-\frac{1}{\left(1+\sqrt{1-e^2}\right)}\right], \tag{28}$$

$$\left\langle\frac{\mathrm{d}M}{\mathrm{d}t}\right\rangle_{DM}=n-\frac{2GM_{gal}}{na^2r_{max}}=n\left(1-\frac{2a}{r_{max}}\right). \tag{29}$$

Similarly, in the region of the cosmological constant Lagrange's planetary equations result to the following time rates of change for the orbital elements:

$$\left\langle\frac{\mathrm{d}a}{\mathrm{d}t}\right\rangle_\Lambda=0 \tag{30}$$

$$\left\langle\frac{\mathrm{d}e}{\mathrm{d}t}\right\rangle_\Lambda=0 \tag{31}$$

$$\left\langle\frac{\mathrm{d}i}{\mathrm{d}t}\right\rangle_\Lambda=0 \tag{32}$$

$$\left\langle\frac{\mathrm{d}\omega}{\mathrm{d}t}\right\rangle_\Lambda=\frac{3c^2\Lambda}{n}\sqrt{1-e^2} \tag{33}$$

$$\left\langle\frac{\mathrm{d}M}{\mathrm{d}t}\right\rangle_\Lambda=n-\frac{7c^2\Lambda}{n}\left(1+\frac{3e^2}{7}\right). \tag{34}$$

6. Discussion and Numerical Results

First, we proceed with the calculation of the parameters necessary in the numerical evaluation of the orbital element time rates of change. Using results from Martinez et al. (2002) we use that the bulge mass of the Milky Way is $M_B = 1.41\times10^{10} M_{\odot} = 2.805\times10^{40}$ kg, To proceed with our numerical evaluation let us assume our sun of mass M=1$M_{\odot} = 1.99\times10^{30}$ kg at a distance of r = 8.32 kpc = 2.468×10^{20}m (Gillessen et al., 2016) from the center of the Milky-Way, and $v_{\odot} = 220$ km/s. The corresponding mass at the sun's distance in the galactic plane can be found from Kepler's third law according to the formula:

$$T_{\odot} = 2\pi\left[\frac{a^3}{G\left(M_{\odot} + M_{Gal}\right)}\right]^{1/2}, \tag{35}$$

From which we have that

$$\left(M_{gal} + M_{\odot}\right) \approx M_{gal} \approx \frac{4\pi^2}{G}\left(\frac{a^3}{T^2}\right) = \frac{av_{\odot}^2}{G}. \tag{36}$$

Substituting the numerical values above we find that $M_{gal} \approx 1.863\times10^{41}$ kg = 9.360×10^{10} $M_{\odot}$ which agrees with the mass given in Merrifield, (2004). Moreover, following (Sivaram et al., 2020) and taking into account that the orbital velocity beyond r_{max}, is independent of r something that is consistent with observation. In the the case of the Milky Way, this is constant and approximately ∼300 km/s which is the same order as that observed. Using equation (6) as it is given in (Sivaram et al., 2020) namely:

$$v = \left(GMa_{min}\right)^{1/4}. \tag{37}$$

Using v= 300 km/s and solving for $r_{\max}$ we obtain that:

$$a_{max} = \frac{v^4}{GM_{gal}} = 6.518\times10^{-10}\ \text{m/s}^2. \tag{38}$$

To be in the region of the logarithmic correction must have that $r > r_{\max}$ and therefore we find that:

$$r_{\max} = \sqrt{\frac{GM_B}{a_{\min}}} = 1.380\times10^{20}\ \text{m} \quad , \tag{38}$$

And the condition $r > r_{\max}$ is satisfied, and thus:

$$K' = \frac{GM_B}{r_{\max}} = 9.000\times10^{10}\,\text{m}^2/\text{s}^2. \tag{39}$$

Similarly, the mean motion of the orbiting star the sun in this case is given by the equation:

$$n_{\odot} = \frac{2\pi}{T_{\odot}} = \frac{2\pi}{\left(6.940\times10^{15} s\right)} = 9.056\times10^{-16}\ \text{rad/s} \tag{40}$$

Next, using equations 28, 29, 33 and 34 we can obtain expressions for the time rates of changes of the orbital elements as a function of eccentricities, and therefore we have:

$$\left\langle \frac{\mathrm{d}\omega}{\mathrm{d}t} \right\rangle_{DM} = 1.2206 \times 10^{-16} \left[1 - \frac{1}{\left(1 + \sqrt{1 - e^2}\right)} \right] \text{ rad/s,} \tag{41}$$

$$\left\langle \frac{\mathrm{d}M}{\mathrm{d}t} \right\rangle_{DM} = -1.00002\ n \text{ rad/s} \quad . \tag{42}$$

Similarly, in the cosmological constant regime we obtain that:

$$\left\langle \frac{\mathrm{d}\omega}{\mathrm{d}t} \right\rangle_{\Lambda} = 2.98145 \times 10^{-20} \sqrt{1 - e^2} \text{ rad/s} \tag{43}$$

$$\left\langle \frac{\mathrm{d}M}{\mathrm{d}t} \right\rangle_{\Lambda} = n - \frac{6.30 \times 10^{-35}}{n} \left(1 + \frac{3e^2}{7} \right) \text{ rad/s.} \tag{44}$$

In the case of circular orbits i.e. $e = 0$ we have that:

$$\left\langle \frac{\mathrm{d}\omega}{\mathrm{d}t} \right\rangle_{DM} = 1.2206 \times 10^{-16} \left(\frac{1}{2} \right) = 6.103 \times 10^{-17} \text{ rad/s,} \tag{45}$$

$$\left\langle \frac{\mathrm{d}\omega}{\mathrm{d}t} \right\rangle_{\Lambda} = 2.98145 \times 10^{-20}. \text{ rad/s} \tag{46}$$

$$\left\langle \frac{\mathrm{d}M}{\mathrm{d}t} \right\rangle_{\Lambda} = n - \frac{6.30 \times 10^{-35}}{n} \text{ rad/s} \tag{47}$$

In fig. 1 we plot the effect of the time rate of the argument of the *perigalaktikon* point ω of an orbiting body due to presence of dark matter effect in the region of the Milky Way bounded by the condition $r > r_{max}$. This is for the galactic regime in which the correction due to the dark matter effect is given by the logarithmic term in equation 13. We find that the dark matter effect is much higher for orbits of low eccentricity e, where the effect becomes zero for parabolic orbits. In the dark matter regime equation (29) becomes identically zero if the involved parameters have the following values respectively:

$$r_{max} = \frac{2GM}{an^2}, \tag{48}$$

$$a = \pm \frac{1}{n} \sqrt{\frac{2GM}{r_{\max}}}, \tag{49}$$

$$n = \frac{1}{a} \sqrt{\frac{2GM}{r_{max}}}, \tag{50}$$

$$M = \frac{a^2 n^2}{2G} r_{max}. \tag{51}$$

In a similar way the mean anomaly time rate effect in the region of the effect of the cosmological becomes identically zero, if the involved parameters have the following values:

$$\Lambda = \frac{n^2}{7c^2\left(1+\frac{3e^2}{7}\right)}, \tag{52}$$

$$n = \pm c\sqrt{\Lambda\left(7+3e^2\right)}, \tag{53}$$

$$e = \pm\frac{1}{c}\sqrt{\frac{n^2}{3\Lambda}-\frac{7c^2}{3}}. \tag{54}$$

In fig. 2 plot of the time rate of the argument of the mean anomaly M as a function of mean motion of an orbiting body due to presence of dark matter effect. We find that the mean anomaly time rate of change increases as the mean motion due the dark matter effect increases. In fig 3 we plot of the time rate of the argument of the perigalaktikon point ω of an orbiting body orbiting in the region of influence of cosmological constant Λ. We find that its effect of the time rate of change also increases for decreasing eccentricities e. Furthermore, in fig. 4 plot of the time rate of the argument of the mean anomaly time rate of change of an orbiting body orbiting in a region of influence of cosmological constant Λ. We find that this is much slower change which for a whole range of eccentricities namely eccentricities in the range $0 \le e \le 1.2$ is of the order $\mathrm{d}M / \mathrm{d}t = O\left(10^{-16}\right)$ being smaller for orbits of larger eccentricities and varying in the range 9.0549×10^{-16} rad/s $\le \mathrm{d}M / \mathrm{d}t \le 9.0553\times10^{-16}$ rad/s. Finally, in fig. 5 we plot of argument of the mean anomaly time rate of change of an orbiting body as a function of semi-major axis in the region of the Milky Way where dark matter might be present. We find that $\mathrm{d}M / \mathrm{d}t$ that at distances close to the center of the Milky Way the effect it's much larger and reduces in magnitude as we move away from the center.

At this point we are going to ask a hypothetical question: Is there a point in the plane of the Milky-Way where the where thεσε two average disturbing potentials might be equal in magnitude? In other words $\langle R_{DM}\rangle = \langle R_\Lambda\rangle$ or is there a point in the Milky-Way such that the effects of the two disturbing potentials cannot differentiated? In order to find possible conditions let us equate

$$\frac{GM}{r_{\max}}\left[1-\left(\sqrt{1-e^2}\right)+\ln\left(\frac{a\left(1+\sqrt{1-e^2}\right)}{2r_{max}}\right)\right] = \Lambda c^2 a^2\left(1+\frac{3}{2}e^2\right). \tag{55}$$

First of all let find out if there is a distance r_{max} value that this can be achieved. Solving for r_{max} we find that:

$$r_{max} = \frac{2GM_g}{a^2c^2\Lambda\left(2+3e^2\right)} W\left[\frac{a^3c^2\left(2+3e^2\right)\left(1+\sqrt{1-e^2}\right)\mathrm{e}^{1-\sqrt{1-e^2}}\Lambda}{4GM_g}\right] \tag{56}$$

Where W is the Lambert function of the indicated argument, which for circular orbits becomes:

$$r_{max} = \frac{GM_g}{a^2c^2\Lambda} W\left[\frac{a^3c^2\Lambda}{2GM_g}\right]. \tag{57}$$

Similarly, equation (56) can hold if the semi-major axis takes the following values:

$$a = \pm\left[\frac{GM_g}{-c^2\Lambda\, r_{max}\left(2+3e^2\right)} W\left(\frac{4c^2\left(2+3e^2\right)\left(-2+e^2+2\sqrt{1-e^2}\right)\mathrm{e}^{-2+2\sqrt{1-e^2}}\Lambda r_{max}^3}{e^4GM}\right)\right]^{1/2} \tag{58}$$

which for circular orbits becomes:

$$a = \pm\left[\frac{GM_g}{-2c^2\Lambda\, r_{max}} W\left(0\right)\right]^{1/2} = 0 \tag{59}$$

Since the Lambert function $W(0) = 0$. This implies that for circular orbits there can be a semi-major axis for which the perturbing potential of dark matter results to an effect that can be comparable to that of the cosmological constant. Next in relation to the eccentricity of the orbit let us find out if there are eccentricities for which the two perturbing potentials they will appear to the same. Restricting ourselves to e <1 equation (55) can be approximated as follows:

$$\frac{GM}{r_{\max}}\left[1-\left(1-\frac{1}{2}e^2\right)+\ln\left(\frac{a\left(1+\left(1-\frac{1}{2}e^2\right)\right)}{2r_{max}}\right)\right] = \Lambda\, c^2a^2\left(1+\frac{3}{2}e^2\right)\ , \tag{60}$$

Which can be further simplified to:

$$\frac{a}{2r_{\max}}\left(2-\frac{e^2}{2}\right) = 1+\frac{GM_g}{r_{\max}}\left(\Lambda\, c^2a^2\left(1+\frac{3}{2}e^2\right)-\frac{GM_g}{2r_{\max}}\right). \tag{61}$$

Solving for the eccentricity we obtain:

$$e = \pm\left(\frac{1-\frac{a}{r_{max}}+\frac{a^2c^2\Lambda}{GM_g}r_{max}}{2-\frac{a}{r_{max}}-\frac{6a^2c^2\Lambda}{GM_g}r_{max}}\right)^{1/2}\ , \tag{62}$$

from which we keep only the positive value since eccentricity cannot be negative. In a similar way equating similar time rates of change due to the dark matter and cosmological constant we find conditions on the orbital elements for which this can be true. First equating the time rates of the perigalaktikon equation s (28) and (33) and solving for the eccentricity we obtain:

$$e = \pm 1 \quad , \tag{63}$$

$$e = \pm \frac{\sqrt{-GM\left(GM + 6a^2c^2\Lambda\, r_{max}\right)}}{3c^2a^2\Lambda} . \tag{64}$$

We say that is possible only for parabolic equations since $e = 1$ is the only accepted value. Similarly equation (64) is always negative and therefore rejected. This means that in elliptical orbits there is no suitable orbital eccentricity for which the corresponding perigalaktikon effects due to dark matter and cosmological can be the same. Similarly, we find that the r_{max} value that can result in identical time rate of change of the perigalaktikon of the orbiting star that is given by:

$$r_{max} = \frac{GM_g}{3c^2a^2\Lambda\left(-1+\sqrt{1-e^2}\right)} , \tag{65}$$

which for circular orbits $e = 0$ is undefined and negative for any other value of eccentricity that is less than one and a complex number for values of eccentricity $e > 1$. There for we conclude that this is not an accepted value and therefore this not possible. Next equating the mean anomaly time rate of change between the tw

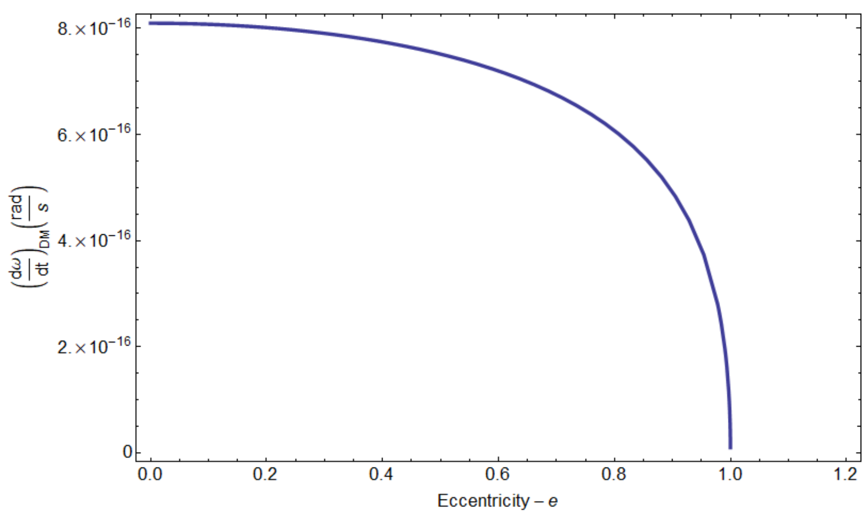

Fig.1 Plot of the time rate of the argument of the perigalaktikon point ω of an orbiting body due to presence of dark matter effect.

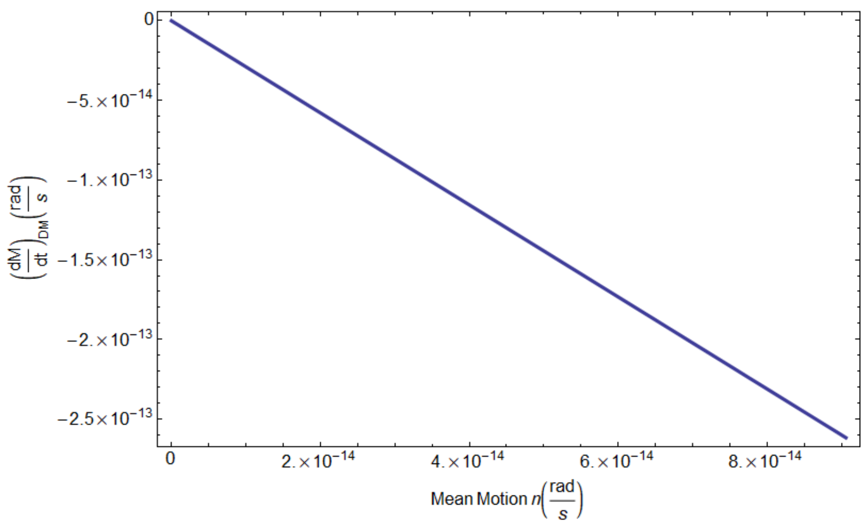

Fig.2 Plot of the time rate of the argument of the mean anomaly M as a function of mean motion of an orbiting body due to presence of dark matter effect.

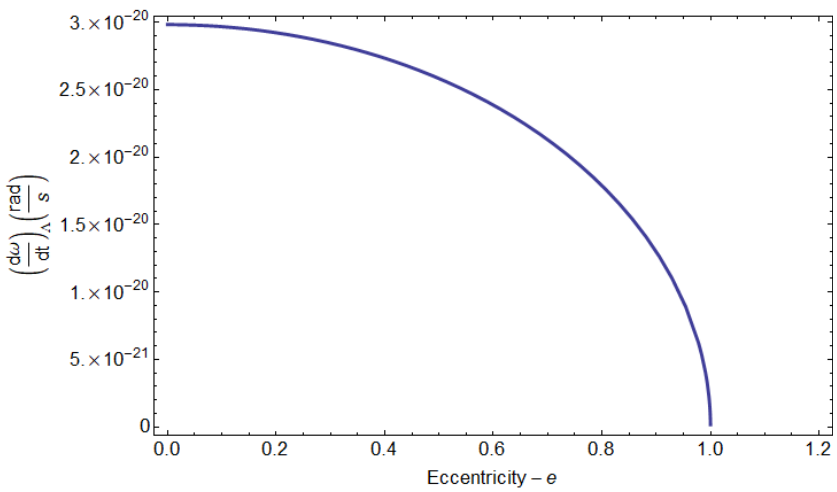

Fig.3 Plot of the time rate of the argument of the perigalaktikon point ω of an orbiting body due to presence of cosmological constant Λ.

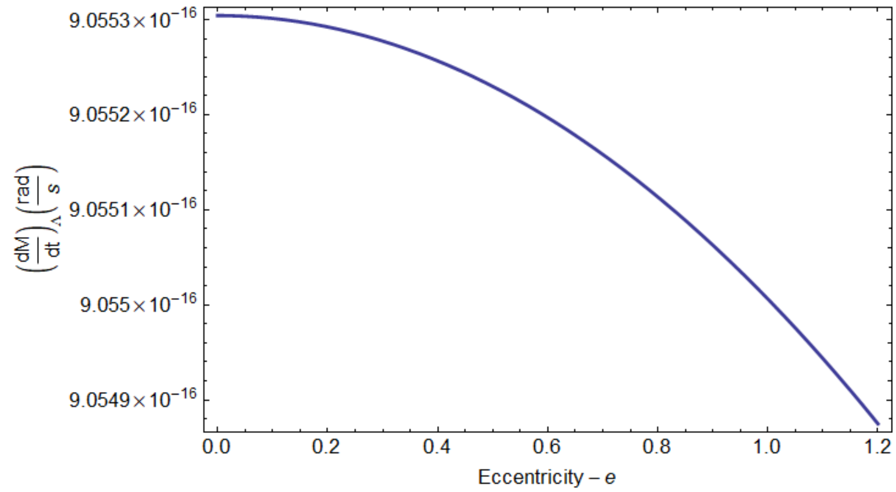

Fig. 4 Plot of the time rate of the argument of the mean anomaly time rate of change of an orbiting body due to presence of cosmological constan Λ.

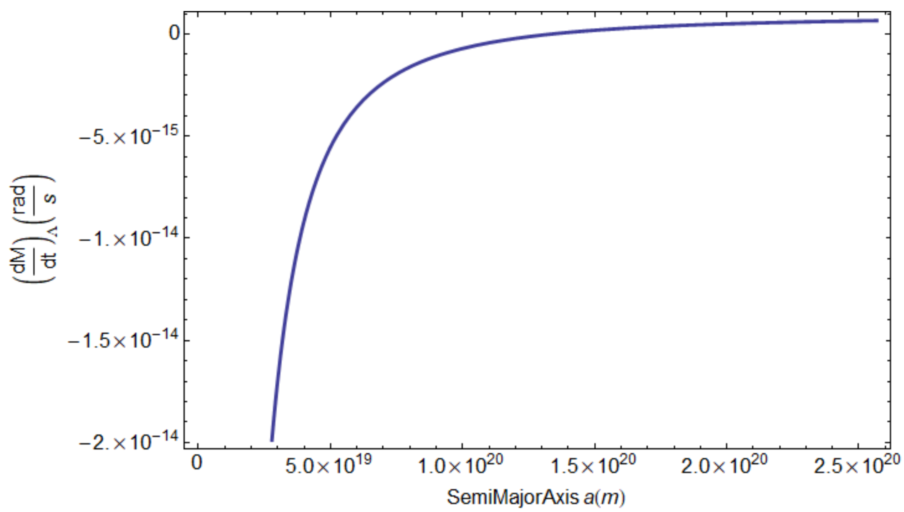

Fig. 5 Plot of argument of the mean anomaly time rate of change of orbiting body as a function of semi-major axis in the Milky Way in the presence of dark matter.

7. Conclusions

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