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Triangular Neutrosophic Topology

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Abstract: In this current article, the primary focus is to develop the concept of neutrosophic topology as triangular neutrosophic topology. We extend the neutrosophic set operations, we newly introduce the defuzzification, examples, and basics to clear the concept and new possibilities. Finally, we also investigate some properties such as neutrosophic exterior, neutrosophic subspace, neutrosophic boundary, and neutrosophic closure.

Keywords: Neutrosophic set, Triangular neutrosophic topology, MCDM, neutrosophic interior, neutrosophic exterior, neutrosophic subspace, and neutrosophic boundary.

1. Introduction

Mathematicians, researchers, and analysts from all over the world work hard to develop new strategies to overcome decision-making problems in a different scenario. It widely uses in medicine, MCDM, engineering, and other math fields. The most challenging issue is to deal with critical situations such as theoretical stuff. To handle vagueness and uncertain situations in both practical and theoretical problems, researchers introduce theories like fuzzy, and neutrosophic sets. The neutrosophic set [1] is based on Truth, indeterminacy, and falsity, it is more effective than crisp and fuzzy.

The idea of triangular neutrosophic numbers was the new development in the line of neutrosophic. Smarandache [2,3] introduce the concept of neutrosophic sets. Un these concepts we have a degree of membership of truth, degree of indeterminacy, and degree of falsity. Many researchers [4-7] do their best and introduce new possibilities such as topology on neutrosophic sets. Other efforts of researchers [8-12] are also remarkable.

We put forward the concept of neutrosophic topology and design new operations and possibilities. MCDM is a wide and developed field and other researchers [13-32] put this concept forward. Triangular neutrosophic numbers plays important role in multi-criteria decision-making, and now it enhances the new evolutions in topology. We also introduce a decent way of defuzzification to make it useable in topological spaces. We also discuss neutrosophic interior, neutrosophic exterior, and other important things in this article.

The idea of neutrosophic is not limited and researchers in recent times proposed, Triangular neutrosophic numbers, Trapezoidal neutrosophic numbers, Pentagonal neutrosophic numbers,

Hexagonal neutrosophic numbers, Heptagonal neutrosophic numbers. Moreover, the Octagonal neutrosophic numbers, Nonagonal neutrosophic numbers were presented and published by us [33-34]. By using neutrosophic techniques, researchers overcome real life problems [35-39].

2 Preliminaries

The related definitions are given below.

Definition 2.1: We define neutrosophic set \dot{A} over universe of discourse as:

$$\dot{A} = \{(\dot{x}, \mu_A(\dot{x}), \sigma_A(\dot{x}), Y_A(x)) : \dot{x} \in \dot{X}\}$$

Where $\mu_A, \sigma_A, Y_A: X \rightarrow]0^-, 1^+[$ and $0^- \leq \mu_A(\dot{x}) + \sigma_A(\dot{x}) + Y_A(x) \leq 3^+$, the value of neutrosophic sets takes from standard and non-standard of $]0^-, 1^+[$. If we consider the value from real life problem, it will become hard to use value of neutrosophic sets takes from standard and non-standard of $]0^-, 1^+[$. For our convenient we take the value from the subset of $[0,1]$. $N(\dot{X})$.

Definition 2.2: Apply formula $\frac{a+b+c}{3}$ and here, $\dot{A}, \dot{B} \in N(\dot{X})$. So,

➤ (Inclusive) If $\mu_{\dot{A}}(\dot{x}) \leq \mu_{\dot{B}}(\dot{x})$, $\sigma_{\dot{A}}(\dot{x}) \geq \sigma_{\dot{B}}(\dot{x})$, $Y_{\dot{A}}(x) \geq Y_{\dot{B}}(x)$ for every $\dot{x} \in \dot{X}$, then

neutrosophic subset can be define as $\dot{A} \sqsubseteq \dot{B}$ where, \dot{A} neutrosophic subset of \dot{B} and then

neutrosophic subset can be define as $\dot{B} \sqsubseteq \dot{A}$ where, \dot{B} neutrosophic subset of \dot{A} .

➤ (Equality) if $\dot{A} \sqsubseteq \dot{B}$ and $\dot{B} \sqsubseteq \dot{A}$ then $\dot{A} = \dot{B}$.

➤ (Intersection) The intersection in neutrosophic sense can be defined as: $\dot{A} \cap \dot{B}$ and defined:

$$\dot{A} \cap \dot{B} = \{\dot{x}, \mu_{\dot{A}}(\dot{x}) \wedge \mu_{\dot{B}}(\dot{x}), \sigma_{\dot{A}}(\dot{x}) \vee \sigma_{\dot{B}}(\dot{x}), Y_{\dot{A}}(x) \vee Y_{\dot{B}}(x) : \dot{x} \in \dot{X}\}$$

➤ (Union) The union in neutrosophic sense can be define as

$$\dot{A} \sqcup \dot{B} = \{\dot{x}, \mu_{\dot{A}}(\dot{x}) \vee \mu_{\dot{B}}(\dot{x}), \sigma_{\dot{A}}(\dot{x}) \wedge \sigma_{\dot{B}}(\dot{x}), Y_{\dot{A}}(x) \wedge Y_{\dot{B}}(x) : \dot{x} \in \dot{X}\}$$

➤ (Compliment) The compliment \dot{A}^c in neutrosophic sense can be defined as:

$$\dot{A}^c = \{\dot{x}, Y_{\dot{A}}(x), 1 - \sigma_{\dot{A}}(\dot{x}), \mu_{\dot{A}}(\dot{x}) : \dot{x} \in \dot{X}\}$$

➤ (Universal set) It can be defined as: $\mu_{\dot{A}}(\dot{x}) = 1, \sigma_{\dot{A}}(\dot{x}) = 0, Y_{\dot{A}}(x) = 0$ for all $\dot{x} \in \dot{X}$.

➤ (Empty set) It can be defined as: $\mu_{\dot{A}}(\dot{x}) = 0, \sigma_{\dot{A}}(\dot{x}) = 1, Y_{\dot{A}}(x) = 1$ for all $\dot{x} \in \dot{X}$. We can denote it as \emptyset .

Example 1: A triangular neutrosophic problem is given below:

$$\begin{aligned} \dot{A} \\ = \{ \langle \dot{x}, (0.2, 0.4, 0.6)(0.3, 0.5, 0.8)(0.2, 0.8, 0.8) \rangle, \langle \dot{y}, (0.4, 0.7, 0.2)(0.6, 0.7, 0.2)(0.4, 0.5, 0.6) \rangle \} \end{aligned}$$

$$\begin{aligned} \dot{B} \\ = \{ \langle \dot{x}, (0.8, 0.2, 0.3)(0.4, 0.6, 0.1)(0.7, 0.3, 0.5) \rangle, \langle \dot{y}, (0.1, 0.2, 0.4)(0.4, 0.2, 0.7)(0.1, 0.2, 0.4) \rangle \} \end{aligned}$$

$$\begin{aligned} \dot{C} \\ = \{ \langle \dot{x}, (0.9, 0.3, 0.1)(0.2, 0.6, 0.3)(0.7, 0.5, 0.6) \rangle, \langle \dot{y}, (0.1, 0.9, 0.5)(0.2, 0.4, 0.6)(0.5, 0.4, 0.9) \rangle \} \end{aligned}$$

Apply formula $\frac{\dot{a} + \dot{b} + \dot{c}}{3}$,

Let $\dot{X} = \{\dot{x}, \dot{y}\}$ and $\dot{A}, \dot{B}, \dot{C} \in \dot{N}(\dot{x})$ then:

$$\dot{A} = \{ \langle \dot{x}, 0.4, 0.5, 0.6 \rangle, \langle \dot{y}, 0.4, 0.3, 0.5 \rangle \}$$

$$\dot{B} = \{ \langle \dot{x}, 0.4, 0.3, 0.6 \rangle, \langle \dot{y}, 0.2, 0.4, 0.3 \rangle \}$$

$$\dot{C} = \{ \langle \dot{x}, 0.4, 0.3, 0.6 \rangle, \langle \dot{y}, 0.5, 0.4, 0.6 \rangle \}$$

We have $\dot{A} \sqsubseteq \dot{B}$.

Neutrosophic union of \dot{B} and \dot{C} as:

$$\begin{aligned} \dot{B} \sqcup \dot{C} = \{ \langle \dot{x}, (0.4 \vee 0.4), (0.3 \wedge 0.3), (0.6 \wedge 0.6) \rangle, \\ \langle \dot{y}, (0.2 \vee 0.5), (0.4 \wedge 0.4), (0.3 \wedge 0.6) \rangle \} \end{aligned}$$

$$\dot{B} \sqcup \dot{C} = \{ \langle \dot{x}, 0.4, 0.3, 0.6 \rangle, \langle \dot{y}, 0.5, 0.4, 0.3 \rangle \}$$

The intersection in neutrosophic sense of \dot{A} and \dot{C}

$$\begin{aligned} \dot{A} \sqcap \dot{C} = \{ \langle \dot{x}, (0.4 \wedge 0.4), (0.5 \vee 0.3), (0.6 \vee 0.6) \rangle, \\ \langle \dot{y}, (0.4 \wedge 0.5), (0.3 \vee 0.4), (0.5 \vee 0.6) \rangle \} \end{aligned}$$

$$\dot{A} \sqcap \dot{C} = \{ \langle \dot{x}, 0.4, 0.5, 0.6 \rangle, \langle \dot{y}, 0.4, 0.4, 0.6 \rangle \}$$

The complement in neutrosophic sense of \dot{C} is

$$\dot{C}^c = \{ \langle \dot{x}, 0.4, 0.3, 0.6 \rangle, \langle \dot{y}, 0.5, 0.4, 0.6 \rangle \}^c$$

$$\dot{C}^c = \{ \langle \dot{x}, 0.6, 1 - 0.3, 0.4 \rangle, \langle \dot{y}, 0.6, 1 - 0.4, 0.5 \rangle \}$$

$$\dot{C}^c = \{\langle x, \dot{0}.6, 0.7, 0.4 \rangle, \langle y, 0.6, 0.6, 0.5 \rangle\}$$

Theorem 1 Let $\dot{A}, \dot{B} \in N(\dot{X})$. Then

- $\dot{A} \cap \dot{A} = \dot{A}$ and $\dot{A} \cup \dot{A} = \dot{A}$
- $\dot{A} \cap \dot{B} = \dot{B} \cap \dot{A}$ and $\dot{B} \cup \dot{A} = \dot{A} \cup \dot{B}$
- $\dot{A} \cap \varphi = \varphi$ and $\dot{A} \cap \dot{X} = \dot{A}$
- $\dot{A} \cup \varphi = \varphi$ and $\dot{A} \cup \dot{X} = \dot{X}$
- $\dot{A} \cap (\dot{B} \cap \dot{C}) = (\dot{A} \cap \dot{B}) \cap \dot{C}$ and $\dot{A} \cup (\dot{B} \cup \dot{C}) = (\dot{A} \cup \dot{B}) \cup \dot{C}$
- $(\dot{A}^c)^c = \dot{A}$

Theorem 2 Let $\dot{A}, \dot{B} \in N(\dot{X})$. Then

- $(\prod_{i \in I} \dot{A}_i)^c = \prod_{i \in I} \dot{A}_i^c$
- $(\cup_{i \in I} \dot{A}_i)^c = \prod_{i \in I} \dot{A}_i^c$

Theorem 3 Let $\dot{A}, \dot{B} \in N(\dot{X})$. Then

- $\dot{B} \cap (\cup_{i \in I} \dot{A}_i) = \cup_{i \in I} (\dot{B} \cap \dot{A}_i)$
- $\dot{B} \cup (\prod_{i \in I} \dot{A}_i) = \prod_{i \in I} (\dot{B} \cup \dot{A}_i)$

3 Triangular neutrosophic topological spaces

Definition 3.1 Let $\dot{\tau} \subseteq N(\dot{X})$, then $\dot{\tau}$ as neutrosophic topology on \dot{X}

- \dot{X} and $\varphi \in \dot{\tau}$.
- The union and intersection of any number of neutrosophic sets in $\dot{\tau}$ belong to $\dot{\tau}$.

The pair $(\dot{X}, \dot{\tau})$ mentioned as neutrosophic topology space over \dot{X} .

Definition 3.2 If $(\dot{X}, \dot{\tau})$ be neutrosophic topological space over \dot{X} then,

- $\dot{\phi}$ and \dot{X} as neutrosophic closed sets over \dot{X} .

- The union and intersection of any two neutrosophic closed sets is a neutrosophic closed sets over \dot{X} .

Example 2: Let $\dot{X} = \{a, b\}$ and $\dot{A} \in \dot{N}(\dot{X})$ so,

$$\dot{A} = \{\langle a, 0.4, 0.6, 0.8 \rangle, \langle b, 0.3, 0.5, 0.7 \rangle\}$$

Hence, $\dot{\tau} = \{\emptyset, \dot{X}, \dot{A}\}$ is neutrosophic topology on \dot{X} .

Example 3: Let $\dot{X} = \{a, b\}$ and $\dot{A} \in \dot{N}(\dot{X})$ so,

$$\begin{aligned} \dot{A} \\ &= \{\langle x, (0.2, 0.4, 0.6)(0.3, 0.5, 0.8)(0.2, 0.8, 0.8) \rangle, \langle y, (0.4, 0.7, 0.2)(0.6, 0.7, 0.2)(0.4, 0.5, 0.6) \rangle\} \end{aligned}$$

$$\begin{aligned} \dot{B} \\ &= \{\langle x, (0.8, 0.2, 0.3)(0.4, 0.6, 0.1)(0.7, 0.3, 0.5) \rangle, \langle y, (0.1, 0.2, 0.4)(0.4, 0.2, 0.7)(0.1, 0.2, 0.4) \rangle\} \end{aligned}$$

Apply formula $\frac{\dot{a} + \dot{b} + \dot{c}}{3}$,

$$\dot{A} = \{\langle x, 0.4, 0.5, 0.6 \rangle, \langle y, 0.4, 0.3, 0.5 \rangle\}$$

$$\dot{B} = \{\langle x, 0.4, 0.3, 0.6 \rangle, \langle y, 0.2, 0.4, 0.3 \rangle\}$$

Then, $\dot{\tau}_1 = \{\emptyset, \dot{X}, \dot{A}\}$ and $\dot{\tau}_2 = \{\emptyset, \dot{X}, \dot{B}\}$ are neutrosophic topology on \dot{X} . Here,

$\dot{\tau}_1 \cup \dot{\tau}_2 = \{\emptyset, \dot{X}, \dot{A}, \dot{B}\}$ is not neutrosophic on \dot{X} . The reason is that: $\dot{A} \cap \dot{B} \notin \dot{\tau}_1 \cup \dot{\tau}_2$. Hence,

it's not a neutrosophic topological space over \dot{X} .

Theorem 4: If $(\dot{X}, \dot{\tau})$ be neutrosophic topological space over \dot{X} and $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$ then:

- $\text{int}(\emptyset) = \emptyset$ and $\text{int}(\dot{X}) = \dot{X}$
- $\text{int}(\dot{A}) \subseteq \dot{A}$
- \dot{A} is neutrosophic open if and only if $\dot{A} = \text{int}(\dot{A})$.
- $\text{int}(\text{int}(\dot{A})) = \text{int}(\dot{A})$.
- $\dot{A} \subseteq \dot{B}$ implies $\text{int}(\dot{A}) \subseteq \text{int}(\dot{B})$.
- $\text{int}(\dot{A}) \cup \text{int}(\dot{B}) \subseteq \text{int}(\dot{A} \cup \dot{B})$.

vii. $int(\dot{A} \cap \dot{B}) = int(\dot{A}) \cap int(\dot{B})$.

Proof: i. and ii. are obvious.

iii. \dot{A} is neutrosophic open set over \dot{X} , as well as, \dot{A} is itself a neutrosophic set over \dot{X} which also contain \dot{A} . The largest neutrosophic open set contain in \dot{A} is \dot{A} and $int(\dot{A})=\dot{A}$. Conversely, $int(\dot{A})=\dot{A}$ hence, $\dot{A} \in \dot{\tau}$.

iv. If $int(\dot{A})=\dot{B}$. so, $int(\dot{B}) = \dot{B}$ from above, $int(int(\dot{A})) = int(\dot{A})$.

v. As, $\dot{A} \subseteq \dot{B}$. As $int(\dot{A}) \subseteq \dot{A} \subseteq \dot{B}$. as $int(\dot{A})$ is a neutrosophic subset of \dot{B} . So, $int(\dot{A}) \subseteq int(\dot{B})$.

vi. It's clear $\dot{A} \subseteq \dot{A} \cup \dot{B}$ and $\dot{B} \subseteq \dot{A} \cup \dot{B}$ thus, $int(\dot{A}) \subseteq int(\dot{A} \cup \dot{B})$ and $int(\dot{B}) \subseteq int(\dot{A} \cup \dot{B})$ hence $int(\dot{A}) \cup int(\dot{B}) \subseteq int(\dot{A} \cup \dot{B})$ by above.

vii. If $(\dot{A} \cap \dot{B}) \subseteq int(\dot{A})$ and $(\dot{A} \cap \dot{B}) \subseteq int(\dot{B})$ by above, so, $(\dot{A} \cap \dot{B}) \subseteq int(\dot{A}) \cap int(\dot{B})$. Also, $int(\dot{A}) \subseteq \dot{A}$ and $int(\dot{B}) \subseteq \dot{B}$ we have, $int(\dot{A}) \cap int(\dot{B}) \subseteq \dot{A} \cap \dot{B}$. These make $(\dot{A} \cap \dot{B}) = int(\dot{A}) \cap int(\dot{B})$.

Example 4: Let $\dot{X} = \{\dot{x}, \dot{y}\}$ and $\dot{A}, \dot{B}, \dot{C} \in \dot{N}(\dot{x})$ then:

$$\dot{A} = \{(\dot{x}, (0.3,0.3,0.3)(0.3,0.3,0.3)(0.3,0.3,0.3)), (\dot{y}, (0.5,0.5,0.5)(0.5,0.5,0.5)(0.5,0.5,0.5))\}$$

$$\dot{B} = \{(\dot{x}, (0.4,0.4,0.4)(0.4,0.4,0.4)(0.4,0.4,0.4)), (\dot{y}, (0.7,0.7,0.7)(0.7,0.7,0.7)(0.7,0.7,0.7))\}$$

$$\dot{C} = \{(\dot{x}, (0.2,0.2,0.2)(0.2,0.2,0.2)(0.2,0.2,0.2)), (\dot{y}, (0.6,0.6,0.6)(0.6,0.6,0.6)(0.6,0.6,0.6))\}$$

Apply formula $\frac{\dot{a}+\dot{b}+\dot{c}}{3}$,

$$\dot{A} = \{(\dot{x}, 0.3,0.3,0.3), (\dot{y}, 0.5,0.5,0.5)\}$$

$$\dot{B} = \{\langle \dot{x}, 0.4, 0.4, 0.4 \rangle, \langle \dot{y}, 0.7, 0.7, 0.7 \rangle\}$$

$$\dot{C} = \{\langle \dot{x}, 0.2, 0.2, 0.2 \rangle, \langle \dot{y}, 0.6, 0.6, 0.6 \rangle\}$$

Then, $\dot{\tau} = \{\dot{\emptyset}, \dot{X}, \dot{A}\}$ is soft topological space over \dot{X} . $\text{int}(\dot{B}) = \dot{\phi}, \text{int}(\dot{C}) = \dot{\phi}$ and $(\dot{B} \sqcup \dot{C}) = \dot{A}$. Moreover, $\text{int}(\dot{B}) \sqcup \text{int}(\dot{C}) \neq \text{int}(\dot{B} \sqcup \dot{C})$.

Theorem 5: If $(\dot{X}, \dot{\tau})$ be neutrosophic topological space over \dot{X} and $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$ then:

- 1) $cl(\dot{\emptyset}) = \dot{\emptyset}$ and $cl(\dot{X}) = \dot{X}$.
- 2) $\dot{A} \sqsubseteq cl(\dot{A})$.
- 3) \dot{A} can be consider as neutrosophic closed set if and only if $\dot{A} = cl(\dot{A})$.
- 4) $cl(cl(\dot{A})) = cl(\dot{A})$.
- 5) $\dot{A} \sqsubseteq \dot{B}$ implies as $cl(\dot{A}) \sqsubseteq cl(\dot{B})$
- 6) $cl(\dot{A} \sqcup \dot{B}) = cl(\dot{A}) \sqcup cl(\dot{B})$.
- 7) $cl(\dot{A} \cap \dot{B}) \sqsubseteq cl(\dot{A}) \cap cl(\dot{B})$.

Proof: 1, 2, 6, and 7 are clear, as well as done previously above.

3) Suppose that \dot{A} is neutrosophic closed set over \dot{X} , here \dot{A} contain \dot{A} and it is itself closed set over \dot{X} . \dot{A} can be consider as smallest neutrosophic closed set contains \dot{A} such as $\dot{A} = cl(\dot{A})$. Conversely, $\dot{A} = cl(\dot{A})$ as \dot{A} is small one neutrosophic closed set over \dot{X} contains \dot{A} .

4) by above case, $\dot{A} = cl(\dot{A})$, \dot{A} is neutrosophic closed set.

5) $\dot{A} \sqsubseteq \dot{B}$. We can clearly see every neutrosophic closed super set of \dot{B} is also neutrosophic closed super set of \dot{A} . Hence, $cl(\dot{A}) \sqsubseteq cl(\dot{B})$.

Example 5: Let $\dot{X} = \{\dot{x}, \dot{y}\}$ and $\dot{A}, \dot{B}, \dot{C} \in \dot{N}(\dot{x})$ then:

$$\dot{A} = \{\langle \dot{x}, (0.3, 0.3, 0.3)(0.3, 0.3, 0.3)(0.3, 0.3, 0.3) \rangle, \langle \dot{y}, (0.5, 0.5, 0.5)(0.5, 0.5, 0.5)(0.5, 0.5, 0.5) \rangle\}$$

$$\dot{B} = \{\langle \dot{x}, (0.2, 0.2, 0.2)(0.2, 0.2, 0.2)(0.2, 0.2, 0.2) \rangle, \langle \dot{y}, (0.6, 0.6, 0.6)(0.6, 0.6, 0.6)(0.6, 0.6, 0.6) \rangle\}$$

Apply formula $\frac{a+b+c}{3}$,

$$\dot{A} = \{\langle \dot{x}, 0.3, 0.3, 0.3 \rangle, \langle \dot{y}, 0.5, 0.5, 0.5 \rangle\}$$

$$\dot{B} = \{\langle \dot{x}, 0.2, 0.2, 0.2 \rangle, \langle \dot{y}, 0.6, 0.6, 0.6 \rangle\}$$

Then,

$$\dot{\tau} = \{\dot{\emptyset}, \dot{X}, \dot{A}, \dot{B}, \dot{A} \cap \dot{B}, \dot{A} \cup \dot{B}\}$$

After taking the compliment,

$$\{\dot{\emptyset}^c, \dot{X}^c, \dot{A}^c, \dot{B}^c, (\dot{A} \cap \dot{B})^c, (\dot{A} \cup \dot{B})^c\}$$

Therefore,

$$\dot{A}^c = \{\langle \dot{x}, 0.6, 0.7, 0.6 \rangle, \langle \dot{y}, 0.5, 0.5, 0.5 \rangle\}$$

$$\dot{B}^c = \{\langle \dot{x}, 0.2, 0.8, 0.2 \rangle, \langle \dot{y}, 0.6, 0.4, 0.6 \rangle\}$$

$$(\dot{A} \cap \dot{B})^c = \{\langle \dot{x}, 0.2, 0.7, 0.3 \rangle, \langle \dot{y}, 0.6, 0.5, 0.5 \rangle\}$$

$$(\dot{A} \cup \dot{B})^c = \{\langle \dot{x}, 0.2, 0.8, 0.3 \rangle, \langle \dot{y}, 0.5, 0.5, 0.6 \rangle\}$$

$$\dot{A} \cap \dot{B} = \{\langle \dot{x}, 0.2, 0.8, 0.3 \rangle, \langle \dot{y}, 0.5, 0.5, 0.6 \rangle\}$$

$$cl(\dot{A}) = \dot{X}$$

$$cl(\dot{B}) = \dot{X}$$

$$cl(\dot{A} \cap \dot{B}) = (\dot{A} \cup \dot{B})^c$$

$$cl(\dot{A} \cap \dot{B}) \subseteq cl(\dot{A}) \cap cl(\dot{B}).$$

Theorem 6: Let, $(\dot{X}, \dot{\tau})$ be neutrosophic topological space over \dot{X} and $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$ then:

$$i. \quad (fr(\dot{A}))^c = ext(\dot{A}) \cup int(\dot{A}).$$

$$\text{ii. } cl(\dot{A}) = int(\dot{A}) \sqcup \dot{f}r(\dot{A}).$$

Proof. $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$. Then,

Here we have,

$$(\dot{f}r(\dot{A}))^c = (cl(\dot{A}) \cap \dot{f}r(\dot{A}^c))^c$$

$$(\dot{f}r(\dot{A}))^c = (cl(\dot{A}))^c \sqcup (\dot{f}r(\dot{A}^c))^c$$

$$(\dot{f}r(\dot{A}))^c = (cl(\dot{A}))^c \sqcup (int(\dot{A}^c))^c$$

$$ext(\dot{A}) \sqcup int(\dot{A})$$

$$int(\dot{A}) \sqcup \dot{f}r(\dot{A}) = int(\dot{A}) \sqcup (cl(\dot{A}) \cap \dot{f}r(\dot{A}^c))$$

$$int(\dot{A}) \sqcup \dot{f}r(\dot{A}) = int(\dot{A}) \sqcup (cl(\dot{A}) \cap (int(\dot{A}) \sqcup \dot{f}r(\dot{A}^c)))$$

$$int(\dot{A}) \sqcup \dot{f}r(\dot{A}) = cl(\dot{A}) \cap (int(\dot{A}) \sqcup int(\dot{A}))^c$$

$$int(\dot{A}) \sqcup \dot{f}r(\dot{A}) = cl(\dot{A}) \cap \dot{X}$$

$$int(\dot{A}) \sqcup \dot{f}r(\dot{A}) = cl(\dot{A}).$$

Theorem 7: Let, $(\dot{X}, \dot{\tau})$ be neutrosophic topological space over \dot{X} and $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$ then:

$$\text{i. } \dot{f}r(\dot{A}) \cap int(\dot{A}) = \emptyset$$

$$\text{ii. } \dot{f}r(int(\dot{A})) \subseteq \dot{f}r(\dot{A})$$

Proof. $\dot{A} \in \dot{N}(\dot{X})$. then,

i. is clear.

To prove ii. Let,

$$\dot{f}r(int(\dot{A})) = cl(int(\dot{A})) \cap cl(int(\dot{A}))$$

$$\dot{f}r(int(\dot{A})) = cl(int(\dot{A})) \cap \dot{f}r(\dot{A}^c)$$

$$\dot{f}r(int(\dot{A})) = cl(\dot{A}) \cap \dot{f}r(\dot{A}^c)$$

$$fr(int(\dot{A})) \subseteq fr(\dot{A})$$

Definition: Let, $(\dot{X}, \dot{\tau})$ be neutrosophic topological space and \dot{Y} is non empty subset of \dot{X} .

Neutrosophic relative topology as:

$$\dot{\tau}_Y = \{\dot{A} \cap \dot{Y} : \dot{A} \in \dot{\tau}\}$$

$$Y(\dot{x}) = \begin{cases} \langle 1, 0, 0 \rangle & \dot{x} \in \dot{Y} \\ \langle 0, 1, 1 \rangle & \text{otherwise} \end{cases}$$

Hence, $(\dot{X}, \dot{\tau}_Y)$ as neutrosophic subspace of $(\dot{X}, \dot{\tau})$.

Example 6: Let $\dot{X} = \{\dot{a}, \dot{b}, \dot{c}\}$, $\dot{Y} = \{\dot{a}, \dot{b}\} \subseteq \dot{X}$ and $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$ then,

$$\dot{A} = \{\langle \dot{x}, (0.3, 0.5, 0.7) \rangle, \langle \dot{y}, (0.2, 0.5, 0.9) \rangle, \langle \dot{z}, (0.4, 0.6, 0.7) \rangle, \langle \dot{w}, (0.3, 0.1, 0.8) \rangle, \langle \dot{v}, (0.5, 0.2, 0.3) \rangle\}$$

$$\dot{B} = \{\langle \dot{x}, (0.2, 0.6, 0.4) \rangle, \langle \dot{y}, (0.6, 0.2, 0.7) \rangle, \langle \dot{z}, (0.2, 0.1, 0.6) \rangle, \langle \dot{w}, (0.6, 0.8, 0.7) \rangle, \langle \dot{v}, (0.7, 0.6, 0.3) \rangle, \langle \dot{u}, (0.1, 0.5, 0.6) \rangle\}$$

Apply formula $\frac{\dot{a} + \dot{b} + \dot{c}}{3}$,

$$\dot{A} = \{\langle \dot{x}, 0.5, 0.3, 0.6 \rangle, \langle \dot{y}, 0.5, 0.4, 0.3 \rangle\}$$

$$\dot{B} = \{\langle \dot{x}, 0.4, 0.5, 0.3 \rangle, \langle \dot{y}, 0.7, 0.5, 0.4 \rangle\}$$

Thus,

$$\dot{\tau} = \{\dot{\emptyset}, \dot{X}, \dot{A}, \dot{B}, \dot{A} \cap \dot{B}, \dot{A} \cup \dot{B}\}$$

As neutrosophic topology on \dot{X} . As well as,

$$\dot{\tau}_Y = \{\dot{\emptyset}, \dot{Y}, \dot{C}, \dot{M}, \dot{L}, \dot{K}\} \quad \text{such that, } \dot{C} = \dot{Y} \cap \dot{A}, \dot{M} = \dot{Y} \cap \dot{B}, \dot{L} = \dot{Y} \cap (\dot{A} \cap \dot{B}) \text{ and}$$

$$\dot{K} = \dot{Y} \cap (\dot{A} \cup \dot{B}).$$

Conclusion

In this current article, we rearrange the neutrosophic set operations and design triangular neutrosophic topology on the structure of neutrosophic topology with defuzzification. We introduce some properties linked to operations. We also clear the neutrosophic topology structure of neutrosophic sets. Moreover, we believe, with these new approaches, the researcher will able to enhance new possibilities in neutrosophic topology.

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References

1. F. Smarandache, Neutrosophic set, a generalization of the intuitionistic fuzzy sets, *Inter. J. Pure Appl. Math.* Vol. 24, pp. 287–297, 2005.
2. F. Smarandache, Neutrosophic set - a generalization of the intuitionistic fuzzy set, *International Journal of Pure and Applied Mathematics*, 24(3) (2005) 287–297.
3. F. Smarandache, Neutrosophy and neutrosophic logic, first international conference on neutrosophy, neutrosophic logic, set, probability, and statistics, *University of New Mexico*, Gallup, NM 87301, USA (2002).
4. F. G. Lupia'nez, On neutrosophic topology, *The International Journal of ~ Systems and Cybernetics*, 37(6) (2008), 797–800.
5. F. G. Lupia'nez, Interval neutrosophic sets and topology, *The International ~ Journal of Systems and Cybernetics*, 38(3/4) (2009), 621–624.
6. F. G. Lupia'nez, On various neutrosophic topologies, *The International ~ Journal of Systems and Cybernetics*, 38(6) (2009), 1009–1013.
7. F. G. Lupia'nez, On neutrosophic paraconsistent topology, *The Interna- ~ tional Journal of Systems and Cybernetics*, 39(4) (2010), 598–601.
8. S. Broumi and F. Smarandache, Intuitionistic neutrosophic soft set, *Journal of Information and Computing Science*, 8(2) (2013), 130–140.
9. S. Broumi and F. Smarandache, More on intuitionistic neutrosophic soft set, *Computer Science and Information Technology*, 1(4) (2013), 257–268.
10. S. Broumi, Generalized neutrosophic soft set, arXiv:1305.2724.
11. C. L. Chang, Fuzzy topological spaces, *Journal of Mathematical Analysis and Applications*, 24 (1968), 182–190.
12. D. C, oker, An introduction to intuitionistic fuzzy topological spaces, *Fuzzy Sets and Systems*, 88(1) (1997), 81–89.
13. Abdel-Baset, M., Chang, V., & Gamal, A. Evaluation of the green supply chain management practices: *A novel neutrosophic approach. Computers in Industry*, vol 108, pp. 210-220, 2019.
14. Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Applied Soft Computing*, vol 77, pp. 438-452, 2019.
15. Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. *Journal of medical systems*, vol 43(2), pp. 38, 2019.
16. Abdel-Basset, M., Atef, A., & Smarandache, F. A hybrid Neutrosophic multiple criteria group decision making approach for project selection. *Cognitive Systems Research*, vol 57, pp. 216-227, 2019.
17. Abdel-Basset, Mohamed, Mumtaz Ali, and Asma Atef. "Resource levelling problem in construction projects under neutrosophic environment." *The Journal of Supercomputing*, pp.1-25, 2019.

18. Saqlain M, Sana M, Jafar N, Saeed. M, Said. B, Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Sets, *Neutrosophic Sets and Systems (NSS)*, vol 32, pp. 317-329, 2020.
19. S. Pramanik, P. P. Dey and B. C. Giri, "TOPSIS for single valued neutrosophic soft expert set based multiattribute decision making problems," *Neutrosophic Sets and Systems*, vol 10, pp. 88-95, 2015.
20. Saqlain. M., Jafar. N. and Riffat. A., "Smart phone selection by consumers' in Pakistan: FMCGDM fuzzy multiple criteria group decision making approach," *Gomal University Journal of Research*, vol 34(1), pp. 27-31, 2018.
21. Saqlain. M, Jafar.N. M, and Muniba. K, "Change in The Layers of Earth in Term of Fractional Derivative: A Study," *Gomal University Journal of Research*, vol 34(2), pp. 1-13, 2018.
22. Saqlain M, Jafar N, Hamid R, Shahzad A. "Prediction of Cricket World Cup 2019 by TOPSIS Technique of MCDM-A Mathematical Analysis," *International Journal of Scientific & Engineering Research*, vol 10(2), pp. 789-792, 2019.
23. Saqlain M, Saeed M, Ahmad M. R, Smarandache. F. "Generalization of TOPSIS for Neutrosophic Hypersoft set using Accuracy Function and its Application," *Neutrosophic Sets and Systems (NSS)*, vol 27, pp. 131-137, 2019.
24. Riaz.M., Saeed.M. Saqlain.M. and Jafar.N,"Impact of Water Hardness in Instinctive Laundry System based on Fuzzy Logic Controller," *Punjab University Journal of Mathematics*, vol 51(4), pp. 73-84, 2018.
25. Riaz. M., Saqlain. M. and Saeed. M., "Application of Generalized Fuzzy TOPSIS in Decision Making for Neutrosophic Soft set to Predict the Champion of FIFA 2018: A Mathematical Analysis," *Punjab University Journal of Mathematics*, vol 51(8), pp.111-126, 2019.
26. T. Bera and N. K. Mahapatra, Introduction to neutrosophic soft groups, *Neutrosophic Sets and Systems*, vol 13, pp. 118-127, 2016. doi.org/10.5281/zenodo.570845.
27. P. Biswas, S. Pramanik, and B. C. Giri. "A new methodology for neutrosophic multi-attribute decision making with unknown weight information," *Neutrosophic Sets and Systems*, vol 3, pp. 42-52, 2014.
28. K. Mondal, and S. Pramanik. Neutrosophic decision making model of school choice. *Neutrosophic Sets and Systems*, vol 7, pp. 62-68, 2015.
29. Smarandache, F., Pramanik, S., "New Neutrosophic Sets via Neutrosophic Topological Spaces," *In Neutrosophic Operational Research; Eds.; Pons Editions: Brussels, Belgium*, vol I, pp. 189–209, 2017.
30. Saqlain, M. Sana, M., Jafar. M. N., Saeed, M., Smarandache, F. "Aggregate Operators of Neutrosophic Hypersoft Set," *Neutrosophic Sets and Systems*, vol. 32, pp. 294-306, 2020.
31. Saqlain, M., Jafar, M. N., Riaz, M. "A New Approach of Neutrosophic Soft Set with Generalized Fuzzy TOPSIS in Application of Smart Phone Selection," *Neutrosophic Sets and Systems*, vol. 32, pp. 307-316, 2020. DOI: 10.5281/zenodo.3723161
32. Saqlain, M., Jafar, M. N., Moin, S., Saeed, M. and Broumi, S. "Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Sets," *Neutrosophic Sets and Systems*, vol. 32, pp. 317-329, 2020.
33. Saqlain.M , A. Hamza, and S. Farooq, "Linear and Non-Linear Octagonal Neutrosophic Numbers: Its Representation, α -Cut and Applications," *International Journal of Neutrosophic Science*, vol. 3, no. 1, pp. 29–43, 2020.

34. Saqlain, M., A. Hamza, and M. Saeed, R. M. Zulqarnain, Aggregate, Arithmetic and Geometric Operators of Octagonal Neutrosophic Numbers and Its Application in Multi-Criteria Decision-Making Problems, Springer Book Series 2020.
35. Abdel-Basset, M., Gamal, A., Chakraborty, R. K., & Ryan, M. A new hybrid multi-criteria decision-making approach for location selection of sustainable offshore wind energy stations: A case study. *Journal of Cleaner Production*, 280, 124462.
36. Abdel-Basset, M., Manogaran, G. and Mohamed, M., 2019. A neutrosophic theory-based security approach for fog and mobile-edge computing. *Computer Networks*, 157, pp.122-132.
37. Abdel-Basset, M., Mohamed, R., & Elhoseny, M. (2020). A model for the effective COVID-19 identification in uncertainty environment using primary symptoms and CT scans. *Health Informatics Journal*, 1460458220952918.
38. Abdel-Basset, M., Gamal, A., Chakraborty, R. K., & Ryan, M. J. (2020). Evaluation of sustainable hydrogen production options using an advanced hybrid MCDM approach: A case study. *International Journal of Hydrogen Energy*.
39. Abdel-Basset, R. Mohamed, F. Smarandache and M. Elhoseny, "A new decision-making model based on plithogenic set for supplier selection," *Computers, Materials & Continua*, vol. 66, no.3, pp. 2751–2769, 2021.