



<b>Title</b>	<b>Cooperative output regulation of multi-agent systems coupled by dynamic edges</b>
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<b>Citation</b>	<b>The 19th World Congress of the International Federation of Automatic Control (IFAC), Cape Town, South Africa, 24-29 August 2014. In Conference Proceedings, 2014, p. 1813-1818</b>
<b>Issued Date</b>	<b>2014</b>
<b>URL</b>	<b><a href="http://hdl.handle.net/10722/217362">http://hdl.handle.net/10722/217362</a></b>
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# Cooperative output regulation of multi-agent systems coupled by dynamic edges <sup>★</sup>

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**Abstract:** This paper investigates a new class of linear multi-agent systems, in which nodes are coupled by dynamic edges in the sense that each edge has a dynamic system attached. The outputs of the edge dynamic systems combine to form the external input of the node dynamic system, which is called neighboring input; while the outputs of the node dynamic systems are inputs of the edge dynamic systems that therefore can not be directly controlled. Distributed controllers for nodes are presented to realize output synchronization and output cooperation. Output cooperation makes the outputs of nodes realize some cooperation that here is specified as making the neighboring input track a predefined trajectory. The controllers depending on local state and neighboring inputs are designed by combining the feedback passivity theory and the internal model principle. A simulation example on the cooperated current control of an electrical network illustrates the potential applications of the analytic results.

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## 1. INTRODUCTION

Multi-agent systems and the closely related subject of complex dynamic networks have emerged in the control system literature during the past decade, see for example, [Pogromsky, 2001], [Liu and Passino, 2006], [Yu et al., 2009], [Zhao et al., 2011], [Guan et al., 2012], [Siljak, 2012], [Liu et al., 2012], [Chen et al., 2013], because interactions and cooperations between units become the trend as the popularity of intelligent devices and the associated communication technology increases. Such a system is often described by a graph, where nodes represent the dynamic subsystems and edges the interactions between these subsystems. One significant feature of these systems is that they can achieve some collective behaviours, such as synchronization, swarming, formations and so on, with each node running a local/distributed feedback controller, rather than a centralized controller.

Among these collective behaviors, consensus and synchronization are the most extensively studied ones. The term consensus means that all the agents have their interesting variables converge to one common constant value. Since the seminal work [Jadbabaie et al., 2003], where the jointly connected condition was presented for consensus of undirected multi-agent systems with first-order integrators, many results have been reported extending to first-order or second-order multi-agent systems. Readers

can refer to the recent surveys [Cao et al., 2013], [Chen et al., 2013] and the earlier surveys [Olfati-Saber et al., 2007], [Ren et al., 2007] for details. The term synchronization means that all the agents have their interesting variables converge to one time-varying trajectory. In fact, synchronization has a long history of study in the field of physics, including phase synchronization, limit-cycle synchronization and chaos synchronization. All of them are for nonlinear systems. For nonlinear synchronization in networks of dynamic systems, refer to the survey [Arenas et al., 2008]. In recent years, several works with focuses on output feedback synchronization have been reported for linear multi-agent systems, including [Tuna, 2009], [Seo et al., 2009], [Scardovi and Sepulchre, 2009], and [Li et al., 2010].

Differently from consensus or synchronization requiring that agents are going to have an identical states, output synchronization that might happen in non-identical agents is more ubiquitous and interesting. Xiang et al. [2009] studied output synchronization in networks of identical agents by using the output regulation method. Output synchronization in networks of single-input and single-output non-identical agents was studied in [Kim et al., 2011]. Wang et al. [2010] presented an internal model controller for output synchronization of more general heterogeneous multi-agents systems. It is proved in [Wieland et al., 2011] that the internal model principle is a necessary and sufficient condition for non-trivial output synchronization. Grip et al. [2012] studied output synchronization problem of general right-invertible linear systems with no knowledge about their own state or output of nodes but there is knowledge of the relative outputs.

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<sup>★</sup> This work was supported by a grant from the National 863 Program of China (2011AA050204), the National Natural Science Foundation of China (61104149, 61374174), the Zhejiang Province Natural Science Fund (LY13F030001), and the Programm for New Century Excellent Talents in University (NCET-11-0459).

In most studies on consensus, agents do not have interactions with each other before their controllers are performed. The received or measured neighboring information forms somewhat virtual edges between agents. Each edge can be thought as an algebraic map to get the relative error between two connected agents. However, there are many real large systems, in which the subsystems are inherently coupled to each other, such as power networks, ecological systems and so on. Previous studies on these large systems focus on stability analysis or decentralized controllers whose purpose is to overcome the coupled influences for stability [Davison, 1976, Moylan and Hill, 1978, Araki, 1978, Michel, 1983]. Such a kind of system was seldom described by a graph, partly because the interactions between the subsystems can not be represented by edges with an algebraic map. Therefore, many results on networked systems can not be applied on such systems.

This paper presents a new class of multi-agent systems which has not only dynamic nodes but also dynamic edges, being able to cover many large-scale systems. Outputs of edge dynamic systems combine to form external inputs of node dynamic system, which are called neighboring inputs, while outputs of node dynamic systems are inputs of edge dynamic systems. Two problems of output synchronization and output cooperation are studied here. Output cooperation means that the nodes have their output cooperate for some object that in this study is specified as making the neighboring inputs track some non-trivial trajectories. The proposed controller is distributed in the sense that the feedback information contains not only the local output of the agent itself, but also the neighboring input which represents some indirect information of neighboring agents. More recently, a similar but different work with dynamic edges has been reported in [Burger and Persis, 2013]. There agents interact with each other by the controllers placed on edges, that means, it is the edge dynamics rather than the node dynamics to be designed to achieve output synchronization.

The development is passivity-based. The edge dynamics is assumed to be strictly passive and the node dynamics to be feedback passive from the neighboring input to the local output of the node itself. We recognized that there are several works to exploit passivity for seeking consensus or synchronization of multi-agent systems. In [Chopra and Spong, 2006], output synchronization in networks of nonlinear systems with input-output passive was investigated. There a linear coupling controller for balanced directed graphs, a nonlinear coupling controller for undirected graphs, and extensions to time-delay communication are addressed. In [Arcak, 2007], a passivity-based design is proposed for a coordination problem of second-order multi-agent systems by making the feedback channel be passive. These results are not applicable here, because the exosystems being not Hurwitz have to be considered (due to internal model principle) in the closed-loop system that is no longer formed by a negative feedback interconnection of two passive systems. In the following developments, all the proofs are omitted due to the space limitation.

## 2. PROBLEM FORMULATION

Consider a multi-agent system of  $N$  nodes and  $M$  edges, where the node dynamics has the form of

$$\begin{cases} \dot{x}_i = A_i x_i + B_i u_i + D_i v_i \\ y_i = C_i x_i \end{cases}, \quad i = 1, 2, \dots, N, \quad (1)$$

where  $x_i \in \mathcal{R}^{n_i}$  is the state of node  $i$ ,  $u_i \in \mathcal{R}^{m_i}$  the input,  $y_i \in \mathcal{R}^p$  the output and  $v_i \in \mathcal{R}^p$  is the neighboring input to present the influences from other nodes.  $A_i, B_i, C_i$  and  $D_i$  are constant matrices with compatible dimensions.

Differently from the general coordination problem where  $v_i$  is a algebraic function of  $x_i$  and  $x_j$  with node  $j$  being a neighboring node of node  $i$ , what we consider here is that all the nodes are coupled by dynamic edges, that is, each edge, similar to the node, has a dynamic system model,

$$\begin{cases} \dot{z}_i = E_i z_i + F_i s_i \\ w_i = G_i z_i \end{cases}, \quad i = 1, 2, \dots, M, \quad (2)$$

where  $z_i \in \mathcal{R}^{n_{ei}}$ ,  $s_i \in \mathcal{R}^{m_{ei}}$  and  $w_i \in \mathcal{R}^p$  are the state, input and output of edge  $i$ , respectively.  $E_i, F_i$  and  $G_i$  are constant matrices with compatible dimensions. Without loss of generality,  $B_i, D_i$  and  $F_i$  are assumed to be of full column rank;  $C_i$  and  $G_i$  are assumed to be of full row rank.

The  $N \times M$  incidence matrix  $H$  describes the coupled relationship among the nodes, which is defined as

$$h_{ij} = \begin{cases} +1 & \text{node } i \text{ is the positive end of edge } j, \\ -1 & \text{node } i \text{ is the negative end of edge } j, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

The orientation of an edge does not influence the results developed in the following. It only reflects the action of an edge on its connecting nodes. The neighboring input  $v_i$  and the edge input  $s_i$  are assumed to satisfy

$$\begin{cases} v_i = -\sum_{j=1}^M h_{ij} w_j, & i = 1, 2, \dots, N, \\ s_i = \sum_{j=1}^N h_{ji} y_j, & i = 1, 2, \dots, M \end{cases}. \quad (4)$$

The goal is to design  $u_i$  for each node  $i$ ,  $i = 1, \dots, N$ , which depends on the information of  $x_i$  and  $v_i$ , so as to generate some cooperative behaviors. In this paper, the following two kinds of cooperative problems are considered,

*Definition 1.* (Output Synchronization). Given a multi-agent system consisting of dynamic systems (1) and (2) with algebraic relationships (3) and (4), design a distributed control law depending on local state  $x_i$  and neighboring input  $v_i$  such that all the node outputs of the closed-loop system asymptotically converge to one nontrivial common trajectory.

*Definition 2.* (Output Cooperation). Given a multi-agent system consisting of dynamic systems (1) and (2) with algebraic relationships (3) and (4), design a distributed control law depending on the local output  $y_i$  and neighboring input  $v_i$  such that all the nodes have their neighboring inputs (determined by the node outputs) of the closed-loop system asymptotically converge to predefined trajectories  $\bar{v}_i$ s satisfying some conditions.

We firstly provide an assumption for the edge dynamic system.

A1) The edge dynamic  $(E_i, F_i, G_i)$  is strictly passive in the sense defined in [Hill and Moylan, 1976].

Assumption A1) implies that matrix  $E_i$  is Hurwitz, hence in the output synchronization state,  $s_i = 0$  and subsequently  $v_i = 0$ . This means that the dynamic edges play

no role in the steady output synchronization state, which therefore leads to that output synchronization considered here is a generalization of that considered by Wieland et al. in Wieland et al. [2011], where it is shown that the necessary and sufficient for a nontrivial output synchronization in networks of linear dynamic systems is that all the nodes have their controllers contain the same internal model. Accordingly, such a internal model is also necessary in this paper. We assume that all the nodes have a priori knowledge of this internal model, which is denoted by  $S_\eta \in \mathcal{R}^q$  and assumed to satisfy

- A2) Matrix  $S_\eta$  has no eigenvalue of negative real part, but there is a symmetric positive definite matrix  $P_\eta$  such that  $P_\eta S_\eta + S_\eta^T P_\eta \leq 0$ .

### 3. OUTPUT SYNCHRONIZATION

Since all the nodes have a priori knowledge about the desired model for their outputs, every node can independently construct an exosystem of the desired model and then force its output to asymptotically track the output of the constructed exosystem, which in general can be

$$\dot{\eta}_i = S_\eta \eta_i, \quad y_{\eta_i} = Q_\eta \eta_i \quad (5)$$

where  $\eta_i \in \mathcal{R}^q$  is the exosystem state,  $y_{\eta_i} \in \mathcal{R}^p$  the exosystem output, and  $Q_\eta$  the output matrix. The tracking error is defined by

$$e_i = y_i - y_{\eta_i} = C_i x_i - Q_\eta \eta_i. \quad (6)$$

Noticing all the exosystems have the same state matrix  $S_\eta$ , the internal model controller to make  $e_i \rightarrow 0$  has the form of

$$\begin{cases} \dot{\zeta}_i = G_{1i} \zeta_i + G_{2i}(y_i - Q_\eta \eta_i) \\ u_i = K_{xi} x_i + K_{\zeta i} \zeta_i \end{cases} \quad (7)$$

where  $\zeta_i \in \mathcal{R}^c$  is the controller state, matrix pair  $(G_{i1}, G_{i2})$  incorporates a  $p$ -copy internal model of matrix  $S_\eta$  [Huang, 2004], and  $K_{xi}$  and  $K_{\zeta i}$  are feedback gains. With controller (7), the dynamics of node system (1) becomes

$$\dot{\hat{x}}_i = \hat{A}_i \hat{x}_i + \hat{D}_i v_i + \hat{D}_{\eta i} \eta_i, \quad y_i = \hat{C}_i \hat{x}_i \quad (8)$$

where  $\hat{x}_i = [x_i^T, \zeta_i^T]^T$ ,

$$\hat{A}_i = \begin{bmatrix} A_i + B_i K_{xi} & B_i K_{\zeta i} \\ G_{2i} C_i & G_{1i} \end{bmatrix}, \quad \hat{D}_i = \begin{bmatrix} D_i \\ 0 \end{bmatrix},$$

$$\hat{D}_{\eta i} = \begin{bmatrix} 0 \\ -G_{2i} Q_\eta \end{bmatrix}, \quad \hat{C}_i = [C_i \ 0].$$

In a single-node system, it is enough to make  $\hat{A}_i$  be Hurwitz for output regulation, which requires each node satisfies the following assumptions,

- A3) Matrix pair  $(A_i, B_i)$  is stabilizable.  
A4) For any eigenvalue  $\lambda$  of  $S_\eta$ ,

$$\text{rank} \begin{pmatrix} A_i - \lambda I & B_i \\ C_i & 0 \end{pmatrix} = n_i + p.$$

Here for cooperations between nodes, we take an extra requirement on the controller design.

- A5) Controller (7) is such that  $\hat{A}_i$  is Hurwitz and the closed-loop system  $(\hat{A}_i, \hat{D}_i, \hat{C}_i)$  is passive.

Since  $\hat{A}_i$  is Hurwitz and  $S_\eta$  has no stable eigenvalue, there is a unique solution  $\Pi_i$  satisfying,

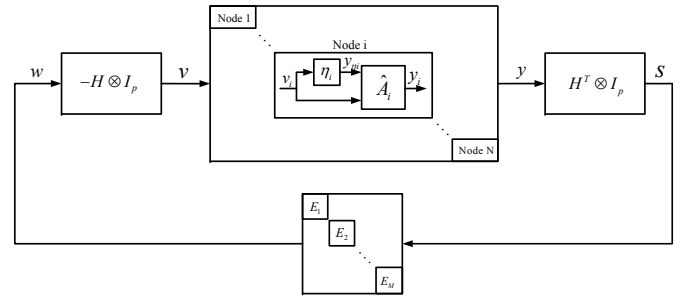


Fig. 1. A block diagram representation for the interconnection of multi-agent systems (1)~(4) with controller (11).

$$\Pi_i S_\eta = \hat{A}_i \Pi_i + \hat{D}_{\eta i}, \quad (9)$$

and, since matrix pair  $(G_{i1}, G_{i2})$  incorporates a  $p$ -copy internal model of matrix  $S_\eta$ ,

$$\hat{C}_i \Pi_i = Q_\eta. \quad (10)$$

which implies that  $e_i \rightarrow 0$  under controller (7) when  $v_i = 0$ . In the presence of  $v_i$ , making  $e_i \rightarrow 0$  is a decentralized servomechanism problem [Davison, 1976], because  $\eta_i$  is independent of  $\hat{x}_i$  and  $z_j$ . For this goal, the following result is given,

*Theorem 1.* Given a multi-agent system consisting of (1)~(4) and exosystem (5). If assumptions A1)~A5) hold, then  $e_i$  will exponentially converge to zero for all  $i = 1, \dots, N$ , under controller (7).

The above theorem means that when the exosystem  $\eta_i$ , whose output node  $i$  will track, has the same dynamic model for all the nodes, then the decentralized internal model controller can work well for the network coupled by dynamic edges if some passive properties are satisfied.

If these  $\eta_i$  are synchronous each other, then  $y_i - y_j \rightarrow 0$  can be obtained by controller (7). But in general,  $\eta_i \neq \eta_j$  due to different initial conditions. Meanwhile, the synchronization errors, either  $y_i - y_j$  or  $\eta_i - \eta_j$ , are not directly available for synchronization seeking of  $\eta_i$ . Here neighboring input  $v_i$  is the only available information that indirectly reflects the synchronization error. Our idea is to adjust the exosystem dynamics (5) by feeding  $v_i$  in order to synchronize  $\eta_i$ . With this alteration, the following dynamic controller is presented,

$$\begin{cases} \dot{\eta}_i = S_\eta \eta_i + \epsilon B_\eta v_i \\ \dot{\zeta}_i = G_{1i} \zeta_i + G_{2i}(y_i - Q_\eta \eta_i), \\ u_i = K_{xi} x_i + K_{\zeta i} \zeta_i \end{cases} \quad (11)$$

where  $B_\eta \in \mathcal{R}^{q \times p}$  is the input matrix of exosystem,  $\epsilon$  is a positive scalar expressing the strength of adjusting the dynamics of output reference, and  $(G_{i1}, G_{i2})$ ,  $K_{xi}$ ,  $K_{\zeta i}$  and  $K_{\eta i}$  are the same as those in (7). The block diagram of the resulted closed-loop system is shown in Fig. 1.

Now the assumption on the network topology is given

- A6) The network is connected, namely, the rank of incidence matrix  $H$  is  $N - 1$ .

Define a matrix  $T \in \mathcal{R}^{(N-1) \times N}$  satisfying  $T\mathbf{1} = 0$  and  $TT^T = I_{N-1}$ , where  $\mathbf{1}$  means a vector with all the elements being 1. Define an induced matrix  $\bar{H} = TH$ . It easily to verify that  $\bar{H}$  is of full row rank.

*Theorem 2.* Given a multi-agent system consisting of (1)~(4). If Assumptions A1)~A6) hold and  $B_\eta$  is designed such that  $(S_\eta, B_\eta, Q_\eta)$  is passive, namely,  $B_\eta = P_\eta^{-1}Q_\eta^T$ , then there is a scalar  $\epsilon^* > 0$  such that for all  $0 < \epsilon < \epsilon^*$ , controller (11) will solve the output synchronization problem. And moreover, the steady output trajectory is given by

$$y_s(t) = Q_\eta e^{St} \frac{1}{N} \sum_{i=1}^N \eta_i(0). \quad (12)$$

In order to guarantee stability, an interconnection of two passive systems only requires  $\epsilon$  more than zero to ensure that the feedback to be negative and permits an arbitrary value of  $\epsilon$ . But as shown in Fig. 1, here the signal channel from reference output  $y_{\eta_i}$  to  $s_j$  is not direct, but indirectly through output  $y_i$  of dynamic system  $\hat{x}_i$ . Critically, it is not passive from  $y_{\eta_i}$  to  $y_i$  due to  $\hat{C}_i \hat{D}_{\eta_i} = 0$ . This in turn results an adjusting strength having an upper bound.

#### 4. OUTPUT COOPERATION

This section extends the result developed above to the output cooperation problem, to make the influences between nodes,  $v_i$ , track some predefined trajectories  $\bar{v}_i(t)$  by adjusting output reference  $\eta_i$ . Notice that the output synchronization is a special case of the output cooperation with  $\bar{v}_i(t) = 0$  for all  $i = 1, \dots, N$ , and that it is necessary for the same dynamic model to be embedded in the controller for seeking output synchronization. Here we similarly assume that  $\bar{v}_i$  complies with the same dynamics,

$$\dot{v}_i = S_\eta v_i, \quad \bar{v}_i = Q_v v_i, \quad i = 1, \dots, N. \quad (13)$$

where  $v_i \in \mathcal{R}^q$  is the state of neighboring input reference system.

Our idea is to take the reference output  $Q_\eta \eta_i$  as a virtual control input of edge dynamics to regulate  $v_i \rightarrow \bar{v}_i$ . To this end, the error  $v_i - \bar{v}_i$  is fed into the dynamics of  $\eta_i$  which should be changed to incorporate a  $p$ -copy model of  $S_\eta$ , and the following controller is proposed,

$$\begin{cases} \dot{\bar{\eta}}_i = G_S \bar{\eta}_i + \epsilon G_B (v_i - Q_v v_i) \\ \dot{\zeta}_i = G_{1i} \zeta_i + G_{2i} (y_i - G_Q \bar{\eta}_i), \\ u_i = K_{xi} x_i + K_{\zeta i} \zeta_i \end{cases} \quad (14)$$

where  $\bar{\eta}_i \in \mathcal{R}^{pq}$ . Controllable matrix pair  $(G_S, G_B)$  incorporates a  $p$ -copy model of  $S_\eta$ . A possible choice for  $G_S$ ,  $G_B$  and  $G_Q$  is

$$G_S = I_p \otimes S_\eta, G_B = \begin{bmatrix} B_\eta^1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & B_\eta^p \end{bmatrix}, G_Q = \begin{bmatrix} Q_\eta^1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & Q_\eta^p \end{bmatrix} \quad (15)$$

where  $B_\eta^i$  denotes the  $i$ th column of  $B_\eta$  and  $Q_\eta^i$  the  $i$ th row of  $Q_\eta$ . It can be verified that

$$(I_p \otimes P_\eta) G_S + G_S^T (I_p \otimes P_\eta) \leq 0, \quad (I_p \otimes P_\eta) G_B = G_Q^T. \quad (16)$$

Compared to output synchronization, the analysis for output cooperation is much more complex, although the former is a special case of the latter. Output synchronization is directly realized by controlling the output of the node itself; output cooperation is a result of the output of the edge which however, is manipulated by the outputs of two nodes. In fact, the output cooperation goal can not

be arbitrary, on which the following *matching condition* assumption is made,

A7) The trajectories of neighboring input reference systems are in the manifold  $\sum_{i=1}^N v_i = 0$ .

Now, we are ready to present our main result for output cooperation,

*Theorem 3.* Given a multi-agent system consisting of (1)~(4). If Assumptions A1)~A7) hold, then there is a scalar  $\epsilon^* > 0$  such that for all  $0 < \epsilon < \epsilon^*$ , controller (14) with (15) and (16) will solve the output cooperation problem. And moreover, the sum of nodal outputs  $y_s = \sum_{i=1}^N y_i$  satisfies

$$y_s(t) \rightarrow G_Q e^{G_S t} \sum_{i=1}^N \bar{\eta}_i(0). \quad (17)$$

It can be seen that different initial conditions of the output reference systems will cause different steady outputs  $y_i$ , although steady neighboring inputs  $v_i$  hold invariant for all the nodes. This in turn means that for a given output cooperation target  $\bar{v}_i$ s, there are infinite feasible solutions for nodal outputs.

#### 5. MASTER-SLAVE OUTPUT COOPERATION

In order for a well-posed output cooperation problem, the matching condition of Assumption A7) is required. To relax this assumption, one possible way is to keep some nodes free, running without requirements on their neighboring inputs. Such nodes are called master nodes, in the sense that they do not receive any command, so that their output reference is not changed. The controller for master node is given by (7). The remaining nodes are called slave nodes whose controller is designed as (14), to adjust their output reference to fit in with command  $\bar{v}_i$ .

Without loss of generality, we assume that the first  $l$  nodes are slave nodes and the remaining  $N - l$  nodes are master nodes, and present the following result,

*Theorem 4.* Given a multi-agent system consisting of (1)~(4). If the nodes have the controllers described by

$$\begin{cases} \dot{\bar{\eta}}_i = G_S \bar{\eta}_i + \epsilon G_B (v_i - Q_v v_i) \\ \dot{\zeta}_i = G_{1i} \zeta_i + G_{2i} (y_i - G_Q \bar{\eta}_i), \\ u_i = K_{xi} x_i + K_{\zeta i} \zeta_i \end{cases}, i = 1, \dots, l, \quad (18a)$$

$$\begin{cases} \dot{\zeta}_i = G_{1i} \zeta_i + G_{2i} (y_i - Q_\eta \eta_i), \\ u_i = K_{xi} x_i + K_{\zeta i} \zeta_i \end{cases}, i = l + 1, \dots, N, \quad (18b)$$

where  $0 \leq l \leq N - 1$ , and both  $v_i$  and  $\eta_i$  are the state of exosystems satisfying

$$\begin{cases} \dot{v}_i = S_\eta v_i, & i = 1, 2, \dots, l \\ \dot{\eta}_j = S_\eta \eta_j, & j = l + 1, \dots, N \end{cases} \quad (19)$$

If assumptions A1)~A6) are satisfied, then there is a positive scalar  $\epsilon^*$  such that for all  $0 < \epsilon < \epsilon^*$ ,

$$v_i \rightarrow Q_v v_i, \quad y_j \rightarrow Q_\eta \eta_j \quad (20)$$

for all  $i = 1, \dots, l$  and  $j = l + 1, \dots, N$ .

It can be seen that the master node plays an output tracking role to make its output  $y_i$  track the given reference  $Q_\eta \eta_i$ ; while the slave node plays an output cooperation role to make its neighboring input track the given reference  $Q_v v_i$ . Theorem 4 tells that if the two kinds of nodes simultaneously exist in the network, they can definitely realize

their targets without the so-called matching condition. Recalling Theorem 1, the number of slave node can reach  $l = 0$ .

## 6. EXAMPLE ON ELECTRICAL NETWORK

A simple electrical network consisting of two sources, two loads and one transmission line is selected as an application example to illustrate the analytic results. We consider the sources with their current being control input. Fig. 2 shows the electrical network. Node 1 and 2 are sources, and node 3 is the ground. Every edge contains a resistance and an inductance. The transmission line connects the outputs of two sources, which means that both sources jointly provide currents for loads.

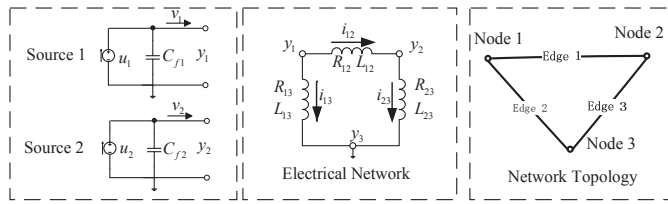


Fig. 2. The electrical network of three nodes

The incidence matrix is

$$H = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}. \quad (21)$$

For the edges, the following dynamic functions can be established,

$$L_{12}\dot{i}_{12} = -R_{12}i_{12} + y_1 - y_2, \quad (22a)$$

$$L_{13}\dot{i}_{13} = -R_{13}i_{13} + y_1 - y_3, \quad (22b)$$

$$L_{23}\dot{i}_{23} = -R_{23}i_{23} + y_2 - y_3. \quad (22c)$$

And for the sources,

$$C_{f1}\dot{u}_{f1} = u_1 - v_1, \quad y_1 = u_{f1} \quad (23a)$$

$$C_{f2}\dot{u}_{f2} = u_2 - v_2, \quad y_2 = u_{f2}. \quad (23b)$$

The desired output is a 50hz sinewave, that is,  $S_\eta = \begin{bmatrix} 0 & -w \\ w & 0 \end{bmatrix}$ , with  $w = 100\pi$ . Here we consider the master-slave output cooperation problem, as node 3 is the ground that can not be controlled. Both node 1 and node 2 are required to make their neighboring inputs track the desired currents,  $\bar{v}_1 = 10 \sin(\omega t + \pi/6)$  and  $\bar{v}_2 = 10 \cos(\omega t)$ , respectively. They can be regarded as to be produced by the following dynamic systems,

$$\dot{\nu}_i = S_\eta \nu_i, \quad \bar{\nu}_i = Q_v \nu_i \quad \text{with} \quad \nu_1(0) = \begin{bmatrix} 5 \\ -5\sqrt{3} \end{bmatrix},$$

$$\nu_2(0) = \begin{bmatrix} 10 \\ 0 \end{bmatrix}, \quad Q_v = [1, 0].$$

Node 3 is regarded as a master node with perfect voltage tracking performance, satisfying  $y_3 \equiv \eta_3$  with

$$\dot{\eta}_3 = S_\eta \eta_3, \quad \eta_3(0) = 0. \quad (24)$$

Such a configuration of output cooperation corresponds to the scenario where two sources work in the current control mode.

Table 1. Parameters of the electrical network

$R_{12}$	$L_{12}$	$R_{13}$	$L_{13}$	$R_{23}$	$L_{23}$	$C_{f1}$	$C_{f2}$
$0.05\Omega$	$0.01\text{mH}$	$9\Omega$	$1\text{mH}$	$8\Omega$	$5\text{mH}$	$50\mu\text{F}$	$30\mu\text{F}$

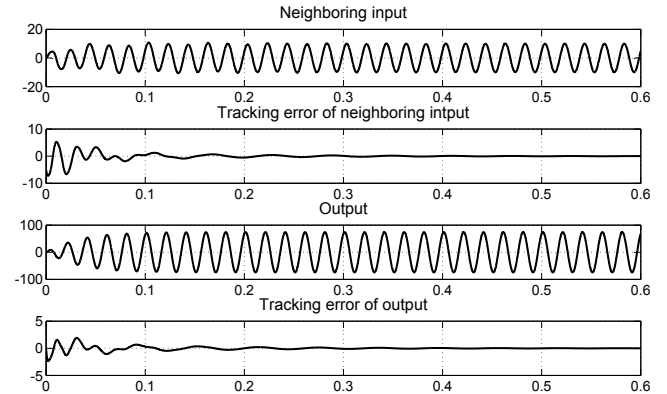


Fig. 3. Trajectories of output and neighboring output of node 1 with their tracking errors.

Take  $z_1 = i_{12}$ ,  $z_2 = i_{13}$ ,  $z_3 = i_{23}$  and  $x_i = u_{fi}$ ,  $i = 1, 2$ . Notice that

$$v_1 = i_{12} + i_{13}, \quad (25)$$

$$v_2 = -i_{12} + i_{23}.$$

The electrical network is just a multi-agent system consisting of (1)~(4), with assumption A1) and A6) being satisfied. The physical parameters are listed in Table 1. Assumption A2) can be also verified with  $P_\eta = I_2$ .

According to Theorem 4, the following controllers are designed for two source nodes,

$$\begin{cases} \dot{\bar{\eta}}_i = G_S \bar{\eta}_i + \epsilon G_B (v_i - \bar{v}_i) \\ \dot{\zeta}_i = G_{1i} \zeta_i + G_{2i} (y_i - G_Q \bar{\eta}_i), \quad i = 1, 2, \\ u_i = K_{xi} x_i + K_{\zeta i} \zeta_i \end{cases} \quad (26)$$

where

$$G_S = S_\eta, G_Q = [0 \ 1], G_B = P_\eta G_Q^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$G_{11} = S_\eta, G_{21} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, K_{x1} = -1, K_{\zeta 1} = [-500, -500],$$

$$G_{12} = \begin{bmatrix} 0 & 1 \\ -w^2 & 0 \end{bmatrix}, G_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, K_{x2} = -2, K_{\zeta 2} = [0, -500].$$

It can be verified that assumption A5) is satisfied for both source nodes under the controller gains defined above. All the conditions in Theorem 4 are satisfied, therefore, there is  $\epsilon$  for the above controller to make the electrical network realize the output cooperation.

Simulation results with  $\epsilon = 500$  obtained in the Matlab environment are given in Fig. 2, where due to space limitations only the trajectories related to node 1 are shown. The electrical network is built by making use of the SimPowerSystems Toolbox. It can be found that after the transition time, each source adjusts its output to make the neighboring input of itself converge to the desired one.

## 7. CONCLUSION

A new class of multi-agent systems was presented, where the nodes are indirectly coupled by dynamic systems, called dynamic edges. The node dynamics can be directly

controlled and is influenced by the neighboring input which is a weighted sum of outputs of edge dynamic systems; while the edge dynamics can not be directly controlled due to its input being the node outputs. Distributed controllers designed by feedback passivity theory and the internal model principle were presented for output synchronization and output cooperation, respectively. A simulation example of cooperated current control of a simple electrical network illustrates the efficacy of the analytic results.

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