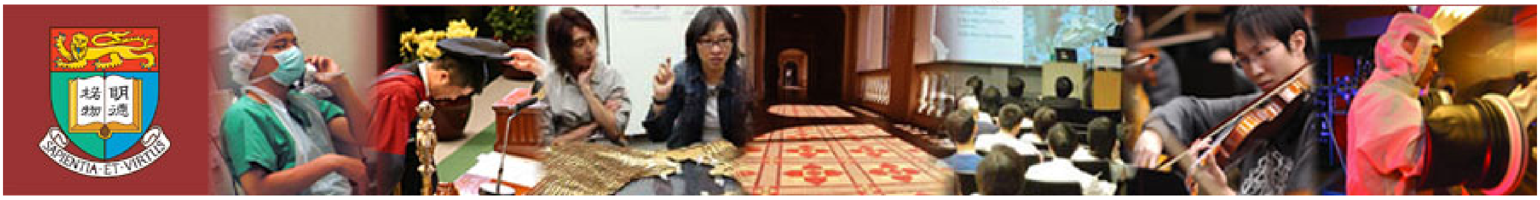

Title	Capacity analysis of hybrid wireless networks with long-range social contacts behavior
Author(s)	Hou, R; Yu, C; Li, J; Min, S; Wong Lui, KS
Citation	The 2015 IEEE Conference on Computer Communications (INFOCOM), Hong Kong, 26 April 26-1 May 2015. In Conference Proceedings, 2015, p. 2209-2217
Issued Date	2015
URL	http://hdl.handle.net/10722/218944
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Capacity Analysis of Hybrid Wireless Networks with Long-Range Social Contacts Behavior

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***Abstract*—Hybrid wireless networks are networks that are composed of both ad hoc transmissions and cellular transmissions. Many existing works have analyzed the capacity of hybrid wireless networks. By assuming the uniform traffic model that a source node would select a random node as the destination, the network capacity is a function of number of nodes and number of base stations. Nevertheless, the real network traffic pattern is related to the social behaviors of users. In this work, we study the capacity of hybrid wireless networks with the social traffic model under the L-maximum-hop routing policy. If two nodes are within L hops away, packets will be transmitted in the ad hoc mode; otherwise, packets are transmitted through the base stations. To our best knowledge, we are the first to study this problem and develop the capacity as a function of number of nodes, number of stations, traffic model parameters, and L.**

I. Introduction

The capacity of wireless networks has been received much attention in the recent years. It is well known that Gupta and Kumar derived the achievable throughput capacity of wireless ad hoc networks [1]. They give the per node network capacity as a function of number of nodes in a unit area. Their study suggests to increase network capacity by augmenting base stations to pure ad hoc network, and thus, the hybrid wireless network composed by ad hoc transmissions and cellular transmissions was proposed [2].

In the hybrid wireless networks, there are two ways for transmitting data packets from the source to the destination. In the ad hoc mode, the data packets are forwarded by the intermediate nodes with multiple hops. The whole procedure of multi-hop transmission consumes the wireless bandwidth resources. The data packets can also be transmitted in the cellular mode. In that case, the source node transmits the data to the nearest base station, and then the destination can fetch the data from its associated base station. The base stations are assumed to be connected via a wired network, and thus, data transmission between two base stations does not consume wireless bandwidth resources.

Different routing policies can be used in a hybrid wireless network. The two major ones are *same cell routing policy* [2] and *L-maximum-hop routing policy* [3]. Under the same cell routing policy, the data packets are transmitted in ad hoc mode if the source and the destination are in the same cell (covered by the same base station); otherwise, they are transmitted in cellular mode. In L-maximum-hop routing policy, the data packets are transmitted in ad hoc mode if the source and the destination are within L hops; otherwise, they are transmitted in cellular mode. This work applies the L-maximum-hop routing policy, since it can efficiently utilize the short-range ad hoc transmission opportunities.

Traffic model is also a major factor on capacity analysis. Most works assume the uniform traffic model. However, it cannot truly reflect the actual user behaviors. Recently, the social traffic model has been actively studied to model the traffic in wireless networks [12]. The long-range contacts traffic model is used in [12]. Each node has a set of long-range social contacts, and the traffic is generated probably between the source and a node selected from the long-range social contacts, which is similar as human behavior. We apply the social traffic model as that in [12]. While [12] studies the capacity of ad hoc networks, we perform capacity analysis of hybrid wireless networks.

To our best knowledge, our work is the first to study the capacity of hybrid wireless networks with social traffic model. We derive the network capacity as a function of number of nodes, number of base stations, traffic model parameters, and routing policy L. Our derived results demonstrate that traffic model and routing policy are both the important factors on the scaling law of network capacity. More importantly, our results facilitate us to find the optimal L to maximize the network capacity.

The rest of the paper is organized as follows: Section II introduces the related works on the capacity analysis of hybrid wireless networks. Section III presents the network model and traffic model. In Section IV, we study the hybrid wireless network capacity with the social traffic model. We compare our results with the existing results in Section V. Our comparison results demonstrate the correctness of our methodology to calculate the network capacity. Moreover, our results illustrate the efficiency of the routing policy derived by this paper to improve the network capacity. Section VI concludes our work.

II. Related Works

Ref. [2] proposes to analyze the hybrid wireless network capacity with the same cell routing policy. In the same cell routing policy, a source node transmits data packets to the destination in an ad hoc manner if they are in the same cell. The work in [2] shows that if the number of base stations m

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grows asymptotically slower than $\sqrt{n}$, the maximum network throughput capacity is $\Theta(\sqrt{\frac{n}{\log \frac{n}{m^2}}}W)$[1], where n is the number of nodes and W is the channel capacity. On the other hand, if m grows asymptotically faster than $\sqrt{n}$, the maximum throughput capacity is $\Theta(mW)$. The work in [4] assumes that m grows asymptotically with n, and sets the transmission range of each node such that any two nodes in the same cell can directly communicate with each other. If the source and the destination are in the same cell, the packets are transmitted in ad hoc manner; otherwise, the packets are forwarded by the base station. The authors derive the network throughput capacity as $\Theta(n\frac{W}{\log n})$. The work in [5] assumes that a source node transmits a portion of traffic to the destination using ad hoc mode, and transmits another portion through the base station, so that their solutions are independent of routing policy. Their study shows that if m grows asymptotically slower than $\sqrt{\frac{n}{\log n}}$, adding base station does not take benefit according to the scaling law of network throughput capacity. If m grows asymptotically faster than $\sqrt{\frac{n}{\log n}}$ and slower than $\frac{n}{\log n}$, the network capacity is $\Theta(mW)$. If m grows asymptotically faster than $\frac{n}{\log n}$, the network throughput capacity becomes $\Theta(\frac{nW}{\log n})$. The work in [3] applies the L-maximum-hop routing policy. If the source can reach to the destination within L hops, the packets are transmitted in ad hoc manner. Otherwise, the packets are forwarded by the base stations. It is shown in [3] that when $L = \Omega(\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n})$, the network capacity is $\Theta(\frac{nW_1}{L \log n}) + \Theta(mW_2)$, where W_1 and W_2 are the bandwidth allocated for ad hoc mode and cellular mode, respectively. When L grows asymptotically slower than $\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}$, the network capacity becomes $\Theta(L^2 \log nW_1)+\Theta(mW_2)$. Ref. [7] uses the same cell routing policy and allows the source node to transmit to a base station using multiple hops. The authors derive the network capacity as $\Theta(\sqrt{\frac{nm}{\log n}}W_1)+\Theta(\frac{m}{n}W_2)$. When m grows asymptotically slower than $\frac{n}{\log n}$, the maximum capacity is denoted by $\Theta(\sqrt{\frac{nm}{\log n}}W)$, which follows that if the number of base stations are increased by k times, we can have a gain of $\sqrt{k}$ on capacity. The work in [6] considers the two-dimensional strip network topology, and finds the optimal ratio of the width to the length for the strip network model, so as to improve the network capacity scaling behavior.

Some works study the capacity of hybrid wireless mobile networks, in which nodes randomly move in the network area. The work in [10] assumes that users move randomly within a bounded distance around its home. The authors also let the network area scales with $f^2(n)$, and derive the per node capacity as the function of $f(n)$. On the other hand, the work in [8] considers the local node mobility in hybrid wireless network. The authors assume that each node moves only in its local cell. The authors in [9] develop an analytical model by assuming the infinite buffer to study the average packet delivery delay with base station. The results show that if the number of base stations grows asymptotically faster than $\sqrt{n}$, the average packet delay is independent of the number of nodes. Our work does not consider the mobility of nodes.

Few works study the effect of traffic model on network capacity. In [11], the probability that a source node communicates with a destination x distance units away is proportional with $x^{-\alpha}$. With the social traffic model, the authors analyzes the effect of α on network capacity. The traffic model in [12] considers the long-range social contacts. Each node has q long-range social contact nodes, from which the destination is randomly selected. The probability for each node contacting another node x distances away is also proportional with $x^{-\alpha}$. [12] derives the network capacity as a function of number of nodes, α, and q. Both works in [11] and [12] consider the wireless ad hoc networks. Besides traffic model, routing policy is another major factor on the capacity of hybrid wireless networks. Therefore, the problem becomes more complicated since the network capacity is affected by both the two factors simultaneously.

III. SYSTEM MODELS

A. Hybrid Wireless Network Model

We consider the hybrid wireless network consisting n nodes and m base stations. The hybrid wireless network is composed by two layers: ad hoc layer and cellular layer, which use different bandwidth resources so that there is no interference between them. In the ad hoc layer, n nodes are uniformly and independently distributed on the surface of a torus of unit area. As said in [3], the assumption of torus enables us to omit the area edge issue, but the results derived with this assumption are suitable for the unit square as well. In the cellular layer, m base stations are uniformly deployed in the network: dividing the area into m cells, and each cell has the $\frac{1}{m}$ area size. The base stations are connected by a wired network, such that there is no bandwidth limit.

The transmission range $r(n)$ for each node to communicate with another node is the same. Following [12], $r(n) = \Theta(\sqrt{\frac{\log n}{n}})$, so that the probability that a node has no neighbor tends to zero as n goes to infinity. In other words, the setting of $r(n)$ assures the connectivity of the network.

Following many existing works [1], we apply the protocol interference model. Assume that node i transmits to node j, the packet can be correctly received by j if the following conditions are satisfied: 1) The distance between i and j is no larger than $r(n)$; 2) no node inside the interference range of node j is transmitting concurrently, where the interference range is $(1 + \Delta)r(n)$, and $\Delta > 0$.

In the hybrid wireless networks, a source node can transmit the packet to the destination either in ad hoc mode or in cellular mode. If the source transmits packets to the destination in ad hoc mode, the data is forwarded by the intermediate nodes. If the packets are transmitted in cellular mode, the source node first transmits the packet to the nearest base station, and then, the data is sent to the base station from which the destination can fetch the data.

Denote by W_a and W_c the bandwidth resources allocated for ad hoc transmission and cellular transmission, respectively. Given a source and destination pair, the source node should

[1] $f(n) = O(g(n))$ for if and only if there exist constants N and C such that $|f(n)| \le C|g(n)|$ for all $n > N$; $f(n) = \Omega(g(n))$ means that $g(n) = O(f(n))$; $f(n) = \Theta(g(n))$ implies that $f(n) = O(g(n))$ and $g(n) = O(f(n))$; If $g(n) \ne 0$, $f(n) = o(g(n))$ if and only if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$.

determine in which mode the data is transmitted. Following [3], we use the *L-maximum-hop routing policy*. If the minimum hops between the source and the destination is within L hops, the data is transmitted using ad hoc resources; otherwise, the data is transmitted using cellular resources.

B. Preliminaries

In this section, we introduce the traffic model with social behavior. Following [12], this work applies the Kleinberg's traffic model, and each node s has long-range social contacts (LSC). Following [12], we assume that each source node has the same number of LSCs, denoted by q. The long-range contacts are selected independently, while closer nodes to the source are selected with higher probability. The probability for s to contact node i is denoted by $\frac{d_i^{-\alpha}}{\sum_{j=1}^{n} d_j^{-\alpha}}$, where d_i denotes the distance between s and i. The sum of the probabilities for s to contact each node in the network is 1. The probability for node s to have the set of LSCs $\{v_{i_1}, \ldots, v_{i_q}\}$ can be written as

$$P(\mathrm{LSC} = \{v_{i_1}, \ldots, v_{i_q}\}) = \frac{d_{i_1}^{-\alpha} \ldots d_{i_q}^{-\alpha}}{\sum_{1 \le j_1, \ldots, j_q \le n} d_{j_1}^{-\alpha} \ldots d_{j_q}^{-\alpha}} \quad (1)$$

By (1), the sum of all the probabilities of s selecting q LSCs is 1. Please refer to [12] for more details. We now calculate the probability that v_k is selected as a LSC:

$$\begin{aligned} &P(v_k \in \mathrm{LSC}) \\ &= \textstyle\sum_{1 \le i_1, \ldots, i_{q-1} \le n, i_j \ne k} P(\mathrm{LSC} = \{\mathrm{v_k}, \mathrm{v_{i_1}}, \ldots, \mathrm{v_{i_q}}\}) \\ &= \frac{\sum_{1 \le i_1, \ldots, i_{q-1} \le n, i_j \ne k} d_k^{-\alpha} d_{i_1}^{-\alpha} \ldots d_{i_{q-1}}^{-\alpha}}{\sum_{1 \le j_1, \ldots, j_q \le n} d_{j_1}^{-\alpha} \ldots d_{j_q}^{-\alpha}} \end{aligned} \quad (2)$$

In case that v_k is selected as a LSC of source node s, the probability that v_k is the destination of the flow originated from s is $\frac{1}{q}$. In a word, the probability for node v_k to be the destination corresponding to source node s is calculated as

$$P(\vartheta_s = v_k) = \frac{1}{q} P(v_k \in \mathrm{LSC}) \quad (3)$$

IV. Network Capacity Study

In this section, we study the hybrid wireless network capacity under the social traffic model. Generally speaking, the network capacity is contributed by two parts: the capacity in ad hoc layer and the capacity in cellular layer.

A. Main conclusions

Theorem 1: Consider a hybrid wireless network consisting n nodes and m base stations. Each source node has a social group which consists q long-range contacts selected independently. Denote by W_1 and W_2 the bandwidth resources allocated for ad hoc transmission and cellular transmission, respectively. The probability for each node contacting another node x distances away is proportional with $x^{-\alpha}$

Case I: $\lim_{n\to\infty} q = \infty$.

When $L = \Theta\left(\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}\right)$, the maximum network capacity is $\Theta\left(\frac{n^{\frac{2}{3}}}{\log^{\frac{1}{3}} n} W_1\right) + \Theta\left(mW_2\right)$.

Case II: $\lim_{n\to\infty} q < \infty$.

1) $0 \le \alpha < 2$. When $L = \Theta\left(\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)}\right)^{\frac{1}{3-\alpha}}\right)$, where $a(n) = \Theta(\frac{\log n}{n})$, we have the maximum network capacity $\Theta\left(\frac{n^{\frac{6-\alpha}{6-2\alpha}}}{(\log n)^{\frac{2-\alpha}{6-2\alpha}} (n-q+1)^{\frac{1}{3-\alpha}}} W_1\right) + \Theta\left(mW_2\right)$.
2) $2 \le \alpha < 3$. When $L \ge 1$ and $L = \Theta\left(1\right)$, the maximum network capacity is $\Theta\left(\frac{n}{\log n} W_1\right) + \Theta\left(mW_2\right)$.
3) $\alpha \ge 3$. The network capacity is $\Theta\left(\frac{n}{\log n} W_1\right) + \Theta\left(mW_2\right)$, which is independent of L.

□

When q goes to infinity as $n \to \infty$, the traffic model is the same as the uniform model. That is, the probability for a source node to select a node as the destination is denoted by $\frac{1}{n}$. Thus, the optimal L is independent of q. Our derived result is the same as that in [3] which considers the uniform traffic model and the same routing policy. When each node has finite number of long range contacts, $\lim_{n\to\infty} q < \infty$, we give the optimal L under the different traffic models. In the case of $0 \le \alpha < 2$, the optimal L depends on n, α and q. The optimal scaling law of L is lower as α increases. In the case of $2 < \alpha < 3$, the optimal scaling law of L should be set to a constant. When $\alpha > 3$, the destination is probably located close to the source node, such that the number of ad hoc flows and the average hop count are both independent of L. Therefore, the ad hoc network capacity is independent of L. In other words, in the case of $\alpha > 3$, the routing policy has no impact of the network capacity. We also find that the maximum ad hoc network capacity with $2 < \alpha < 3$ is the same as that with $\alpha > 3$.

In case that m does not grow fast, the maximum network capacity is mainly contributed by the ad hoc network capacity. Our study suggests that an appropriate transmission hop threshold should be set to limit the long range ad hoc transmission, so as to improve the network capacity.

B. Baseline of Network Capacity Derivation

The capacity in the cellular layer denotes the throughput contributed by cellular transmission. Assume that there are at most Δ_c cells interfering with a given cell. The bandwidth allocated for each cell is lower bounded by $\frac{W_c}{\Delta_c}$. On the other hand, the bandwidth for each cell is upper bounded by W_c. Δ_c is independent of n and m [3], and thus, the throughput capacity for each cell is $\Theta(W_c)$. Since there are totally m cells, the throughput capacity contributed by the cellular layer is denoted by

$$\Lambda_c = \Theta(mW_c). \quad (4)$$

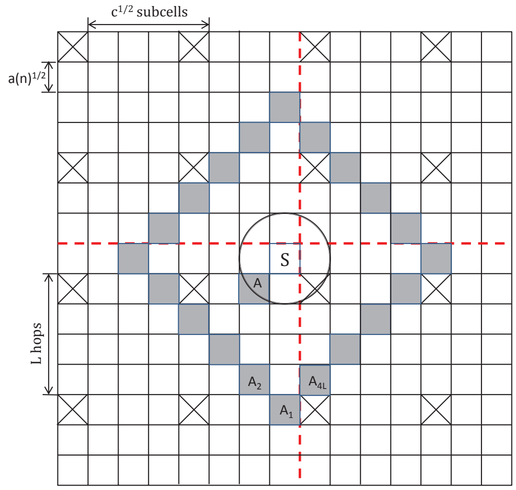

Fig. 1. A illustration for network division.

In the following, we focus on analyzing the throughput capacity contributed by ad hoc layer. Before we present the details, we describe the baseline of our derivation process.

We divide the network area into multiple subcells, and each subcell has the size of $a(n)$, as illustrated in Fig. 1 which is borrowed from [12]. We also let $a(n) = \Theta(\frac{\log n}{n})$, such that a node in a subcell can transmit to any node in the neighboring subcells. Note that a cell may consist several subcells. Fig. 1 illustrates four cells divided by dash lines. Denote by P_{ad} the probability for each source node to transmit to the destination in an ad hoc manner. Based on P_{ad}, we calculate the number of flows transmitted in the ad hoc layer, denoted by N_{ad}. Afterwards, we calculate the expected hop count for each ad hoc flow, denoted by $E[h]$, and then, we calculate the total number of hops for all the ad hoc flows, $H = N_{\text{ad}} \cdot E[h]$. Since nodes are randomly deployed in the network area, following [13], we calculate the average number of flows going through a certain subcell, denoted by $E[Z] = H \cdot a(n)$. As referred to [7], it is shown that there is at most $c = O((2+\Delta)^2)$ number of interfering subcells of any given subcell. In Fig. 1, only one subcell in each c subcells can be active at the same time. Since c is independent of n and m, the bandwidth allocated for each subcell is asymptotically proportional with W_a. We thus calculate the bandwidth allocated for each hop in a given subcell as $\Lambda^0_{\text{ad}} = \Theta(\frac{W_a}{E[Z]})$. Finally, we discuss the network throughput contributed by ad hoc layer as $\Lambda_{\text{ad}} = N_{\text{ad}} \cdot \Lambda^0_{\text{ad}}$.

C. Calculating Total Number of Ad hoc Flows

As illustrated in Fig. 1, when the destination is located at the gray areas, the number of hops from the source to the destination is L. We can observe that are $4x$ subcells in which the destination is x hops away from the source node. Note that we assume the network area is a surface torus of unit area, and so we do not consider the edge issue. If the destination is located in the area surrounded by the grey subcells, the flow is transmitted in ad hoc layer. Denote by $P(X = x)$ the probability that the destination is x away from the source node. We thus have the following

$$P(X = x) = \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(\vartheta_s = v_k) \tag{5}$$

A_l represents a subcell l hops away from the source node. The probability for a destination located in A_l is calculated by $\sum_{v_k \in A_l} P(\vartheta_s = v_k)$, that is, the sum of the probabilities that each node in A_l is the destination. Since the nodes are randomly deployed in the network area, the probability for any node located in A_l is $a(n)$, and the number of nodes contained in A_l is thus $n \cdot a(n)$. We thus have

$$\begin{aligned} P(X = x) &= \textstyle\sum_{l=1}^{4x} \sum_{v_k \in A_l} P(\vartheta_s = v_k) \\ &= \textstyle\sum_{l=1}^{4x} n \cdot a(n) P(\vartheta_s = v_k) \end{aligned} \tag{6}$$

In Fig. 1, if the destination is located in subcell A, we consider that it takes two hops from the source to the destination, and actually the destination may be in the transmission range of the source node. This assumption would not affect the analysis on the scaling behavior, as referred to [12]. Actually, Our results obtained based on the assumption agree with those obtained without this assumption, as shown later.

We are going to consider two cases: $\lim_{n\to\infty} q = \infty$ and $\lim_{n\to\infty} q < \infty$.

Case I: $\lim_{n\to\infty} q = \infty$.

As [12] shows that when $\lim_{n\to\infty} q = \infty$, we have $P(\vartheta_s = v_k) = \frac{1}{n}$. In this case, the traffic model is the same as the uniform traffic model in the existing works. We calculate

$$\begin{aligned} P_{\text{ad}} &= \textstyle\sum_{x=1}^{L} \sum_{l=1}^{4x} \sum_{v_k \in A_l} P(\vartheta_s = v_k) \\ &= \textstyle\sum_{x=1}^{L} \sum_{l=1}^{4x} n \cdot a(n) \cdot \frac{1}{n} \\ &= 2L(1+L)a(n) \end{aligned} \tag{7}$$

We thus have

$$N_{\text{ad}} = nP_{\text{ad}} = 2nL(1+L)a(n). \tag{8}$$

Case II: $\lim_{n\to\infty} q < \infty$.

We have the following lemma, and the detailed derivation can be found in Appendix.

Lemma 1: The probability for each source node to transmit the flow using ad hoc mode is

$$P_{\text{ad}} \equiv \begin{cases} \Theta\left(\frac{n}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}\right) & 0 \le \alpha < 2 \\ \Theta\left(\frac{n}{n-q+1} \frac{\ln L}{\ln a^{-\frac{1}{2}}(n)}\right) & \alpha = 2 \\ \Theta\left(\frac{n}{n-q+1}\right) & \alpha > 2 \end{cases} \tag{9}$$

By Lemma 1, we calculate the total number of ad hoc flows as

$$N_{\text{ad}} \equiv \begin{cases} \Theta\left(\frac{n^2}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}\right) & 0 \le \alpha < 2 \\ \Theta\left(\frac{n^2}{n-q+1} \frac{\ln L}{\ln a^{-\frac{1}{2}}(n)}\right) & \alpha = 2 \\ \Theta\left(\frac{n^2}{n-q+1}\right) & \alpha > 2 \end{cases} \tag{10}$$

(10) shows that the number of ad hoc flows grows faster as α increases from zero to two. As α is larger, more destinations are located close to the source, and thus, the probability that the flows are transmitted in the ad hoc layer is higher. When $\alpha > 2$, L has no impact on the scaling law of the number of ad hoc flow.

D. Calculating Number of Ad hoc Flows in Each Subcell

In the following, we describe how to calculate the total number of flows going through a given subcell. Denote by H the total hops of the ad hoc flows. Let h_i be the hops of an ad hoc flow i. We have

$$E[H] = E[\sum_{i=1}^{N_{\text{ad}}} h_i] = \sum_{i=1}^{N_{\text{ad}}} E[h_i] \tag{11}$$

Let $Z_i^j = 1$ represent that flow i goes through subcell j; otherwise, $Z_i^j = 0$. Following [7], we have $E[Z_i^j] = a(n)$, where $a(n)$ is the size of each subcell. The average number of flows going through a certain subcell can be calculated as follows [13].

$$\begin{aligned} E[Z] &= E_H[E[Z|H]] \\ &= E_H[HE[Z_i^j]] \\ &= E[H] \cdot Z[Z_i^j] \\ &= N_{\text{ad}} \cdot E[h_i] \cdot a(n) \end{aligned} \tag{12}$$

By (12), in order to calculate $E[Z]$, the main point is to calculate $E[h_i]$.

$$\begin{aligned} E[h_i] &= \textstyle\sum_{x=1}^{L} xP(X = x|\text{ad hoc flow}) \\ &= \textstyle\sum_{x=1}^{L} x\frac{P(X=x,\text{ad hoc flow})}{P(\text{ad hoc flow})} \\ &= \textstyle\sum_{x=1}^{L} x\frac{P(X=x)}{P_{\text{ad}}} \end{aligned} \tag{13}$$

We also have

$$\sum_{x=1}^{L} xP(X = x) = \sum_{x=1}^{L} x \sum_{l=1}^{4x} \sum_{v_k \in s_l} P(\vartheta_d = v_k). \tag{14}$$

Now, we describe how to calculate $E[h_i]$. Similarly, we need to consider two cases: $\lim_{n\to\infty} q = \infty$ and $\lim_{n\to\infty} q < \infty$.

Case I: $\lim_{n\to\infty} q = \infty$.

Since $P(\vartheta_d = v_k) = \frac{1}{n}$, we have

$$\sum_{x=1}^{L} xP(X = x) = \frac{2L(1+L)(2L+1)a(n)}{3}. \tag{15}$$

Therefore, $E[h_i] = \frac{2L+1}{3}$.

Case II: $\lim_{n\to\infty} q < \infty$.

We have the following lemma, and the derivation details can be found in Appendix.

Lemma 2: The expected value for the hops of each ad hoc flows is

$$E[h_i] \equiv \begin{cases} \Theta\left(\frac{L^{3-\alpha}}{L^{2-\alpha}}\right) = \Theta(L) & 0 \le \alpha < 2 \\ \Theta\left(\frac{L}{\ln L}\right) & \alpha = 2 \\ \Theta\left(L^{3-\alpha}\right) & 2 < \alpha < 3 \\ \Theta\left(\ln L\right) & \alpha = 3 \\ \Theta\left(1\right) & \alpha > 3 \end{cases} \tag{16}$$

(16) shows that when $0 \le \alpha \le 3$, the average hop count of ad hoc flows increases with L, and the growth rate reduces as α increases. When α is larger, more destinations are located close to the source node, and thus, the impact factor of L on the scaling law of $E[h_i]$ is less. When $\alpha > 3$, we find that the scaling law of $E[h_i]$ is independent of L. For the same reason, most of the flows are short range, so that L would not affect significantly on $E[h_i]$.

E. Ad hoc Network Capacity Analysis

We then calculate the throughput of each ad hoc flow

$$\Lambda_{\text{ad}}^0 = \Theta\left(\frac{W_a}{E[Z]}\right). \tag{17}$$

In the case that $\lim_{n\to\infty} q = \infty$, we have

$$\begin{aligned} \Lambda_{\text{ad}}^0 &= \Theta\left(\frac{W_a}{E[Z]}\right) \\ &= \Theta\left(\frac{W_a}{N_{\text{ad}} \cdot E[h_i] \cdot a(n)}\right) \\ &= \Theta\left(\frac{W_a}{nL^3 a^2(n)}\right) \end{aligned} \tag{18}$$

In case that $L = \Omega\left(\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}\right)$, we have

$$\begin{aligned} \Lambda_{\text{ad}} &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0 \\ &= N_{\text{ad}} \cdot \Theta\left(\frac{W_a}{N_{\text{ad}} \cdot E[h_i] \cdot a(n)}\right) \\ &= \Theta\left(\frac{nW_a}{L \log n}\right) \quad (a(n) = \Theta(\tfrac{\log n}{n})) \end{aligned} \tag{19}$$

In the following, we remove W_a when there is no confusion. When $L = O\left(\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}\right)$, we have $\Lambda_{\text{ad}}^0 = \Omega(1)$. That is, per node ad hoc capacity grows asymptotically faster than a constant. On the other hand, the maximum capacity for each ad hoc flow must be less than W_a, and thus we have $\Lambda_{\text{ad}}^0 = O(1)$. Finally, we have $\Lambda_{\text{ad}}^0 = \Theta(1)$ under this situation. The network ad hoc capacity is calculated as

$$\begin{aligned}\Lambda_{\text{ad}} &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0 \\ &= N_{\text{ad}} \cdot \Theta(1) \\ &= \Theta\left(L^2 \log n\right) \quad (a(n) = \Theta(\tfrac{\log n}{n}))\end{aligned} \tag{20}$$

When L grows asymptotically faster than $\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}$, by (8), the number of ad hoc flows grows asymptotically with L^2. However, by (18), the per node capacity reduces asymptotically with L^3. Therefore, the network capacity reduces asymptotically with L, as shown in (19). This means that L should be better grow asymptotically with $\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}$ to improve the network capacity scaling behavior. On the other hand, when L grows asymptotically slower than $\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}$, we mentioned that the per node capacity is independent of L. Thus, the network capacity grows asymptotically with L^2, as shown in (20). Therefore, the ad hoc network capacity is maximized when $L = \Theta\left(\frac{n^{\frac{1}{3}}}{\log^{\frac{2}{3}} n}\right)$, which is

$$\Lambda_{\text{ad}}^* = \Theta\left(\frac{n^{\frac{2}{3}}}{(\log n)^{\frac{1}{3}}}\right) \tag{21}$$

We now consider the network capacity in the case that $\lim_{n\to\infty} q < \infty$. By (10), (16), (12), and (17), we calculate

$$\Lambda_{\text{ad}}^0 = \begin{cases} \Theta\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n) L^{3-\alpha}}\right) & 0 \le \alpha < 2 \\ \Theta\left(\frac{(n-q+1)\ln a^{-\frac{1}{2}}(n)}{n^2 L a(n)}\right) & \alpha = 2 \\ \Theta\left(\frac{n-q+1}{n^2 a(n) L^{3-\alpha}}\right) & 2 < \alpha < 3 \\ \Theta\left(\frac{n-q+1}{n^2 a(n)\ln L}\right) & \alpha = 3 \\ \Theta\left(\frac{n-q+1}{n^2 a(n)}\right) & \alpha > 3 \end{cases} \tag{22}$$

We have $\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n) L^{3-\alpha}} = o(\frac{(n-q+1)\ln a^{-\frac{1}{2}}(n)}{n^2 L a(n)})$ and $\frac{n-q+1}{n^2 a(n) L^{3-\alpha}} = o(\frac{n-q+1}{n^2 a(n)\ln L})$. For clarity, we discuss the network capacity scaling behavior when $0 \le \alpha < 2$, $2 < \alpha < 3$, and $\alpha > 3$.

(a). When $0 \le \alpha < 2$, we should discuss the per node ad hoc capacity when $L = \Omega\left((\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)})^{\frac{1}{3-\alpha}}\right)$ and $L = O\left((\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)})^{\frac{1}{3-\alpha}}\right)$, separately. In case that $L = O\left((\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)})^{\frac{1}{3-\alpha}}\right)$, we have $\Lambda_{\text{ad}}^0(n) = \Omega(1)$. Since the per node ad hoc capacity must be no greater than W_a, we have $\Lambda_{\text{ad}}^0(n) = \Theta(1)$. Therefore, we have

$$\Lambda_{\text{ad}}^0 = \begin{cases} \Theta\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n) L^{3-\alpha}}\right) & L = \Omega\left((\frac{n-q+1}{n a^{2-\frac{\alpha}{2}}(n)})^{\frac{1}{3-\alpha}}\right) \\ \Theta\left(1\right) & L = O\left((\frac{n-q+1}{n a^{2-\frac{\alpha}{2}}(n)})^{\frac{1}{3-\alpha}}\right) \end{cases} \tag{23}$$

When $L = \Omega\left((\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)})^{\frac{1}{3-\alpha}}\right)$, the network capacity is written as

$$\begin{aligned}\Lambda_{\text{ad}} &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0 \\ &= \Theta\left(\frac{n^2}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}\right) \cdot \Theta\left(\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n) L^{3-\alpha}}\right) \\ &= \Theta\left(\frac{1}{a(n)L}\right)\end{aligned} \tag{24}$$

When $L = O\left((\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)})^{\frac{1}{3-\alpha}}\right)$, the network capacity is written as

$$\begin{aligned}\Lambda_{\text{ad}} &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0 \\ &= \Theta\left(\frac{n^2}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}\right) \cdot \Theta\left(1\right) \\ &= \Theta\left(\frac{n^2}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}\right)\end{aligned} \tag{25}$$

By (24) and (25), we have the maximum ad hoc network capacity when $L = \Theta\left((\frac{n-q+1}{n^2 a^{2-\frac{\alpha}{2}}(n)})^{\frac{1}{3-\alpha}}\right)$, which is

$$\begin{aligned}\Lambda_{\text{ad}}^* &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^0 \\ &= \Theta\left(\frac{n^2}{n-q+1} a^{1-\frac{\alpha}{2}}(n) L^{2-\alpha}\right) \cdot \Theta\left(1\right) \\ &= \Theta\left(\frac{n^{\frac{6-\alpha}{6-2\alpha}}}{(\log n)^{\frac{2-\alpha}{6-2\alpha}} (n-q+1)^{\frac{1}{3-\alpha}}}\right) \quad (a(n) = \Theta(\tfrac{\log n}{n}))\end{aligned} \tag{26}$$

Assume that q is a constant. When $\alpha = 0$, we have $\Lambda_{\text{ad}}^* = \Theta\left(\frac{n^{\frac{2}{3}}}{(\log n)^{\frac{1}{3}}}\right)$ according to (26), and it is the same as (21) derived by assuming the uniform traffic model. When $\alpha = 0$, each node randomly selects q constant long-range social contacts from all the nodes in the network, and then randomly chooses the destination from the q LSCs. This implies that the destination is uniformly selected by the source node, and thus, the traffic model is basically the same as the uniform traffic model. Therefore, the capacity scaling behavior with $\alpha = 0$ and constant q is the same as that with the uniform traffic model.

(b). When $2 < \alpha < 3$, by (22), the capacity increases as L decreases. Since L must be larger than 1, we have the maximum capacity when $L = \Theta(1)$. By (22) and (10), the network capacity is written as

$$\begin{aligned} \Lambda_{\text{ad}}^{*} &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^{0} \\ &= \Theta\left(\frac{n^2}{n-q+1}\right) \cdot \Theta\left(\frac{n-q+1}{n^2 a(n) \cdot \Theta(1)}\right) \\ &= \Theta\left(\frac{n}{\log n}\right) \quad (a(n) = \Theta(\tfrac{\log n}{n})) \end{aligned} \tag{27}$$

(c). When $\alpha > 3$, by (22), the per node capacity is independent of L, and the network capacity is

$$\begin{aligned} \Lambda_{\text{ad}}^{*} &= N_{\text{ad}} \cdot \Lambda_{\text{ad}}^{0} \\ &= \Theta\left(\frac{n^2}{n-q+1}\right) \cdot \Theta\left(\frac{n-q+1}{n^2 a(n)}\right) \\ &= \Theta\left(\frac{n}{\log n}\right) \quad (a(n) = \Theta(\tfrac{\log n}{n})) \end{aligned} \tag{28}$$

When $\alpha > 3$, the destination is probably located locally around the source node, and therefore, the flow probably has small hop count and is transmitted in the ad hoc layer. That is why the network capacity is independent of the maximum hop count limit L. By (10) and (22), we can verify that the maximum ad hoc network capacity is $\Theta(\frac{n}{\log n})$ when $\alpha = 2$. Generally speaking, we have (29).

In case that $\lim_{n\to\infty} q = \infty$, if the number of base stations m asymptotically grows slower than $\Theta(\frac{n^{\frac{2}{3}}}{(\log n)^{\frac{1}{3}}})$, the network capacity is maximized when $W_a = W$ and $W_c = 0$, that is, $\Lambda^* = \Theta(\frac{n^{\frac{2}{3}}}{(\log n)^{\frac{1}{3}}} W)$; otherwise, we have the maximum network capacity when $W_c = W$ while $W_a = 0$, that is, $\Lambda^* = \Theta(mW)$.

We now consider the case that $\lim_{n\to\infty} q < \infty$. When $0 \leq \alpha < 2$, if m asymptotically grows slower than $\Theta(\frac{n^{\frac{4-\alpha}{6-2\alpha}}}{(\log n)^{\frac{2-\alpha}{6-2\alpha}} (n-q+1)^{\frac{1}{3-\alpha}}})$, we have the maximum network capacity $\Lambda^* = \Theta(\frac{n^{\frac{4-\alpha}{6-2\alpha}}}{(\log n)^{\frac{2-\alpha}{6-2\alpha}} (n-q+1)^{\frac{1}{3-\alpha}}} W)$; otherwise, the maximum network capacity is denoted by $\Lambda^* = \Theta(mW)$. When $\alpha \geq 2$, only if the number of base stations asymptotically grows faster than $\Theta(\frac{n}{\log n})$, augmenting base stations can play significant role in improving the scaling behavior of the network capacity. (29) also implies that limiting the long-range ad hoc flows can efficiently improve network capacity.

V. Performance Comparison

In this section, we first compare our results with the existing works. Our work applies the same routing policy, the L-maximum hop count, as [3], and so we first compare our results with those in [3]. In [3], the derived network capacities are totally the same as our results in the case that $\lim_{n\to\infty} q = \infty$, which are represented by (19) and (20).

Since our work applies the same traffic model as [12], we also compare our conclusions with those in [12]. In [12], all the flows are transmitted using ad hoc manner. In other words, L is set large enough so that any flow is within L hops. According [12], L grows asymptotically with $(\frac{n}{\log n})^{\frac{1}{2}}$. By (10), $N_{\text{ad}} = \Theta(n)$ for all possible α, which is true since all the flows are transmitted in ad hoc manner. By (18) and (22), we have (30).

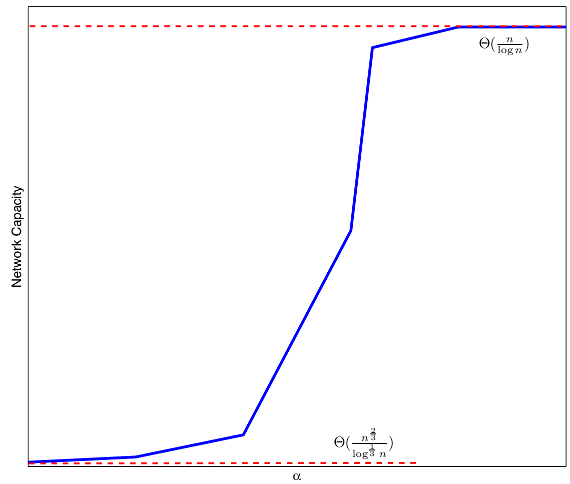

Fig. 2. Effect of α Network Capacity.

$$\Lambda_{\text{ad}}^{0} = \begin{cases} \Theta\left(\frac{1}{n a^{\frac{1}{2}}(n)}\right) & \lim_{n\to\infty} \to \infty \\ \Theta\left(\frac{n-q+1}{n^2 a^{\frac{1}{2}}(n)}\right) & q < \infty,\ 0 \leq \alpha < 2 \\ \Theta\left(\frac{n-q+1}{n^2 a^{\frac{\alpha-1}{2}}(n)}\right) & q < \infty, 2 < \alpha < 3 \\ \Theta\left(\frac{n-q+1}{n^2 a(n)}\right) & q < \infty,\ \alpha > 3 \end{cases} \tag{30}$$

(30) is totally the same as the results derived by [12]. The above analysis shows that our method to derive the hybrid network capacity with the social traffic model is reasonable. We consider a constant q, and Fig. 2 illustrates the maximum network capacity with different α when applying the optimal L. The figure is plotted by using a large constant n. We can observe that the traffic model plays significant role on network capacity, and therefore, studying network capacity with the specific traffic model is important to realize the service capacity provided by the network.

When $\alpha < 2$, the network capacity grows initially when L increases, and then reduces if L grows further. When L is too small, more traffic are transmitted over cellular layer, such that the ad hoc resources would not be fully utilized. When L is larger, more collision would be introduced in ad hoc layer, such that the network throughput would be reduced. Our analysis shows the importance of the optimal routing policy on the network capacity.

VI. Conclusion

The work study the capacity of hybrid wireless networks with social traffic model, and derive the network capacity as a function of number of nodes, number of base stations, traffic model parameters, and routing policy L. Moreover, we identify

$$\Lambda^* = \Lambda^*_{\rm ad} + \Lambda_{\rm c} = \begin{cases} \Theta\Big(\frac{n^{\frac{2}{3}}}{(\log n)^{\frac{1}{3}}} W_a\Big) + \Theta\Big(mW_c\Big) & \lim_{n\to\infty} q = \infty \\ \Theta\Big(\frac{n^{\frac{4-\alpha}{6-2\alpha}}}{(\log n)^{\frac{2-\alpha}{6-2\alpha}} (n-q+1)^{\frac{1}{3-\alpha}}} W_a\Big) + \Theta\Big(mW_c\Big) & \lim_{n\to\infty} q < \infty,\ 0 \le \alpha < 2 \\ \Theta\Big(\frac{n}{\log n} W_a\Big) + \Theta\Big(mW_c\Big) & \lim_{n\to\infty} q < \infty,\ \alpha \ge 2 \end{cases} \tag{29}$$

the optimal routing policy L to maximize the network capacity. In the future, we would consider that the source is allowed to transmit to the base station with multiple hops due to the transmitting power limitation. Moreover, analyzing the effect of the social mobility model of users on network capacity is still open.

APPENDIX

Detailed Derivation of Equation (9)

By (2) and (3), we have

$$P(\vartheta_s = v_k) = \frac{\sum_{1\le i_1,\ldots,i_{q-1}\le n, i_j\ne k} d_k^{-\alpha} d_{i_1}^{-\alpha} \ldots d_{i_{q-1}}^{-\alpha}}{q \sum_{1\le j_1,\ldots,j_q\le n} d_{j_1}^{-\alpha} \ldots d_{j_q}^{-\alpha}} \tag{31}$$

Let $\tau = (\tau_1, \ldots, \tau_n)$ represent $(d_1^{-\alpha}, \ldots, d_n^{-\alpha})$, $\upsilon_{q,n}(\tau) = \sum_{1\le i_1\le i_2\le\ldots\le i_p\le n} \tau_{i_1}\ldots\tau_{i_p}$, and $\upsilon^k_{q,n-1}(\tau) = \upsilon_{q,n-1}(\tau_1,\ldots,\tau_{k-1},\tau_{k+1},\ldots,\tau_n)$. We then have

$$P(\vartheta_s = v_k) = \frac{d_k^{-\alpha} \upsilon^k_{q-1,n-1}(\tau)}{q\upsilon_{q,n}(\tau)} \tag{32}$$

(33)-(36) are borrowed from [12]. In (35), B_1 and B_2 are constants. d_k is the distance from a node in A_l to s, and so we have $B_1 x\sqrt{a(n)} \le d_k \le B_2 x \sqrt{a(n)}$, where x denotes the hop count from the node to s. In (36), $\gamma \le 1$ and $d_{\max}$ denotes the maximum distance between any two nodes in the network.

$$\begin{cases} P(\vartheta_s = v_k) \ge d_k^{-\alpha} \frac{\upsilon_{q-1,n}(\tau) - d_k^{-\alpha}\upsilon_{q-2,n}(\tau)}{q\upsilon_{q,n}(\tau)} \\ P(\vartheta_s = v_k) \le d_k^{-\alpha} \frac{\upsilon_{q-1,n}(\tau)}{q\upsilon_{q,n}(\tau)} \end{cases} \tag{33}$$

$$\frac{\upsilon_{1,n}(\tau)\upsilon_{q-1,n}(\tau)}{q\upsilon_{q,n}(\tau)} = \Theta(\frac{n}{n-q+1}) \tag{34}$$

$$\begin{cases} \sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-\alpha} \ge \sum_{l=1}^{4x}\sum_{v_k\in A_l} (B_1 x\sqrt{a(n)})^{-\alpha} \\ \sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-\alpha} \le \sum_{l=1}^{4x}\sum_{v_k\in A_l} (B_2 x\sqrt{a(n)})^{-\alpha} \end{cases} \tag{35}$$

$$\begin{aligned} \upsilon_{1,n}(\tau) &= \sum_{v_k} d_k^{-\alpha} \\ &\equiv \int_{\sqrt{a(n)}}^{\gamma d_{\max}} n x^{1-\alpha} dx \\ &\equiv \begin{cases} \Theta(n) & 0\le\alpha<2 \\ \Theta(n\ln a^{-\frac{1}{2}}(n)) & \alpha = 2 \\ \Theta(na^{1-\frac{\alpha}{2}}(n)) & \alpha > 2 \end{cases} \end{aligned} \tag{36}$$

Since the probability that each node v_k is located in the subcell A_l is $a(n)$, we now calculate

$$\begin{aligned} & \sum_{x=1}^{L}\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{\alpha} \\ \equiv\ & na^{1-\frac{\alpha}{2}}(n)\sum_{x=1}^{L} x^{1-\alpha} \\ \equiv\ & na^{1-\frac{\alpha}{2}}(n)\int_1^L x^{1-\alpha} dx \\ \equiv\ & \begin{cases} \Theta\Big(na^{1-\frac{\alpha}{2}}(n)L^{2-\alpha}\Big) & 0\le\alpha<2 \\ \Theta\Big(n\ln L\Big) & \alpha = 2 \\ \Theta\Big(na^{1-\frac{\alpha}{2}}(n)\Big) & \alpha > 2 \end{cases} \end{aligned} \tag{37}$$

$$\begin{aligned} P_{\rm ad} &= \sum_{x=1}^{L}\sum_{l=1}^{4x}\sum_{v_k\in A_l} P(\vartheta_s = v_k) \\ &\le \sum_{x=1}^{L}\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-\alpha}\frac{\upsilon_{q-1,n}(\tau)}{q\upsilon_{q,n}(\tau)}\ by(33) \\ &= \frac{\upsilon_{1,n}(\tau)\upsilon_{q-1,n}(\tau)}{q\upsilon_{q,n}(\tau)} \sum_{x=1}^{L}\sum_{l=1}^{4x}\sum_{v_k\in A_l} \frac{d_k^{-\alpha}}{\upsilon_{1,n}(\tau)} \\ &\equiv \Theta(\frac{n}{n-q+1})\sum_{x=1}^{L}\sum_{l=1}^{4x}\sum_{v_k\in A_l} \frac{d_k^{-\alpha}}{\upsilon_{1,n}(\tau)}\ by(34) \\ &\equiv \begin{cases} \Theta\Big(\frac{n}{n-q+1}a^{1-\frac{\alpha}{2}}(n)L^{2-\alpha}\Big) & 0\le\alpha<2 \\ \Theta\Big(\frac{n}{n-q+1}\frac{\ln L}{\ln a^{-\frac{1}{2}}(n)}\Big) & \alpha=2 \\ \Theta\Big(\frac{n}{n-q+1}\Big) & \alpha>2 \end{cases} \end{aligned} \tag{38}$$

With the similar method, we calculate

$$\begin{aligned} & \frac{\upsilon_{q-2,n}(\tau)}{q\upsilon_{q,n}(\tau)}\sum_{x=1}^{L}\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-2\alpha} \\ \equiv\ & \begin{cases} \Theta\Big(\frac{(q-1)n}{(n-q+1)(n-q+2)}a^{1-\alpha}(n)L^{2-2\alpha}\Big) & 0\le\alpha<1 \\ \Theta\Big(\frac{(q-1)n}{(n-q+1)(n-q+2)}\ln L\Big) & \alpha=1 \\ \Theta\Big(\frac{(q-1)n}{(n-q+1)(n-q+2)}a^{1-\alpha}(n)\Big) & 1<\alpha<2 \\ \Theta\Big(\frac{(q-1)n}{(n-q+1)(n-q+2)\ln^2 a^{-\frac{1}{2}}(n)}a^{-1}(n)\Big) & \alpha=2 \\ \Theta\Big(\frac{(q-1)n}{(n-q+1)(n-q+2)a(n)}\Big) & \alpha>2 \end{cases} \end{aligned} \tag{39}$$

We now consider the case of $0 \le \alpha < 1$. In order to compare the order of $\frac{n}{n-q+1}a^{1-\frac{\alpha}{2}}(n)L^{2-\alpha}$ with that of $\frac{(q-1)n}{(n-q+1)(n-q+2)}a^{1-\alpha}(n)L^{2-2\alpha}$, we calculate $\lim_{n\to\infty}\frac{\frac{n}{n-q+1}a^{1-\frac{\alpha}{2}}(n)L^{2-\alpha}}{\frac{(q-1)n}{(n-q+1)(n-q+2)}a^{1-\alpha}(n)L^{2-2\alpha}} = \infty$ based on $a(n) = \Theta(\frac{\log n}{n})$. Thus, we know that the order of $\frac{n}{n-q+1}a^{1-\frac{\alpha}{2}}(n)L^{2-\alpha}$ is higher than that of $\frac{(q-1)n}{(n-q+1)(n-q+2)}a^{1-\alpha}(n)L^{2-2\alpha}$. With the same method, we can verify that $\sum_{x=1}^{D}\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-\alpha}\frac{\upsilon_{q-1,n}(\tau)}{q\upsilon_{q,n}(\tau)} = \Omega(\sum_{x=1}^{D}\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-2\alpha}\frac{\upsilon_{q-2,n}(\tau)}{q\upsilon_{q,n}(\tau)})$. Therefore,

$$\begin{aligned} P_{\text{ad}} &= \sum_{x=1}^{L}\sum_{l=1}^{4x}\sum_{v_k\in A_l} P(\vartheta_s = v_k) \\ &\geq \sum_{x=1}^{L}\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-\alpha}\frac{\upsilon_{q-1,n}(\tau)-d_k^{-\alpha}\upsilon_{q-2,n}(\tau)}{q\upsilon_{q,n}(\tau)} \\ &\equiv \sum_{x=1}^{L}\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-\alpha}\frac{\upsilon_{q-1,n}(\tau)}{q\upsilon_{q,n}(\tau)} \end{aligned} \tag{40}$$

Combining (38) and (40), we have (9).

Detailed Derivation of Equation (16)

With the same method as above, we have (41) and (42).

$$\begin{aligned} &\sum_{x=1}^{L} x\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-\alpha}\frac{\upsilon_{q-1,n}(\tau)}{q\upsilon_{q,n}(\tau)} \\ \equiv\ & \frac{n}{n-q+1}\sum_{x=1}^{L}\sum_{l=1}^{4x}\sum_{v_k\in A_l}\frac{d_k^{-\alpha}}{\upsilon_{1,n}(\tau)}\ by(34) \\ \equiv\ & \frac{n}{n-q+1}\frac{1}{\upsilon_{1,n}(\tau)}na^{1-\frac{\alpha}{2}}(n)\sum_{x=1}^{L}x^{2-\alpha}\ by(35) \\ \equiv\ & \frac{n}{n-q+1}\frac{1}{\upsilon_{1,n}(\tau)}na^{1-\frac{\alpha}{2}}(n)\int_1^L x^{2-\alpha}dx \\ \equiv\ & \begin{cases} \Theta\left(\frac{n}{n-q+1}a^{1-\frac{\alpha}{2}}(n)L^{3-\alpha}\right) & 0\leq\alpha<2 \\ \Theta\left(\frac{n}{n-q+1}\frac{L}{\ln a^{-\frac{1}{2}}(n)}\right) & \alpha=2 \\ \Theta\left(\frac{n}{n-q+1}L^{3-\alpha}\right) & 2<\alpha<3 \\ \Theta\left(\frac{n}{n-q+1}\ln L\right) & \alpha=3 \\ \Theta\left(\frac{n}{n-q+1}\right) & \alpha>3 \end{cases} \qquad by(36) \end{aligned} \tag{41}$$

$$\begin{aligned} &\sum_{x=1}^{L} x\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-\alpha}\frac{\upsilon_{q-1,n}(\tau)}{q\upsilon_{q,n}(\tau)} \\ \equiv\ & \begin{cases} \Theta\left(\frac{(q-1)n}{(n-q+1)(n-q+2)}a^{1-\alpha}(n)L^{3-2\alpha}\right) & 0\leq\alpha<\frac{3}{2} \\ \Theta\left(\frac{(q-1)n}{(n-q+1)(n-q+2)}a^{-\frac{1}{2}}(n)\ln L\right) & \alpha=\frac{3}{2} \\ \Theta\left(\frac{(q-1)n}{(n-q+1)(n-q+2)}a^{1-\alpha}(n)\right) & \frac{3}{2}<\alpha<2 \\ \Theta\left(\frac{(q-1)n}{(n-q+1)(n-q+2)\ln^2(a(n)^{-\frac{1}{2}})}a^{-1}(n)\right) & \alpha=2 \\ \Theta\left(\frac{(q-1)n}{(n-q+1)(n-q+2)a(n)}\right) & \alpha>2 \end{cases} \end{aligned} \tag{42}$$

By (13), (14), and (33) $E[h_i]$ is less than $\frac{1}{P_{\text{ad}}}\sum_{x=1}^{L} x\sum_{l=1}^{4x}\sum_{v_k\in A_l}\frac{d_k^{-\alpha}\upsilon_{q-1,n}(\tau)}{q\upsilon_{q,n}(\tau)}$, while larger than $\frac{1}{P_{\text{ad}}}\sum_{x=1}^{L} x\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-\alpha}\frac{\upsilon_{q-1,n}(\tau)-d_k^{-\alpha}\upsilon_{q-2,n}(\tau)}{q\upsilon_{q,n}(\tau)}$. Since $\sum_{x=1}^{L} x\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-\alpha}\frac{\upsilon_{q-1,n}(\tau)}{q\upsilon_{q,n}(\tau)}$ has the higher order than $\sum_{x=1}^{L} x\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-\alpha}\frac{d_k^{-\alpha}\upsilon_{q-2,n}(\tau)}{q\upsilon_{q,n}(\tau)}$, we have

$$E[h_i] \equiv \frac{1}{P_{\text{ad}}}\sum_{x=1}^{L} x\sum_{l=1}^{4x}\sum_{v_k\in A_l} d_k^{-\alpha}\frac{\upsilon_{q-1,n}(\tau)}{q\upsilon_{q,n}(\tau)} \tag{43}$$

By (41) and (9), we have (16).

ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundation of China (Grants Nos. 61231008, 61101143, 91338114, and 61271176), the important national science & technology specific projects 2013ZX03004007-003, and the 111 Project under Grant B08038.

Yu Chengs work was supported in part by the NSF under grants CNS-1053777 and CNS-1320736.

REFERENCES

[1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions and Information Theory*, vol. 46, no. 2, pp. 388-404, March 2000.

[2] B. Liu, Z. Liu and D. Towsley, "On the capacity of hybrid wireless networks," *IEEE Infocom*, April 2003.

[3] P. Li, C. Zhang, and Y. Fang, "Capacity and Delay of Hybrid Wireless Broadband Access Networks," *IEEE Journal of Selected Areas in Communications*, 27(2): 117-125, Feburary 2009.

[4] U. Kozat and L. Tassiulas, "Throughput Capacity of random ad hoc networks with Infrastructure support," *ACM Mobicom*, June 2003.

[5] A. Zemlianov and G. Veciana, "Capacity of ad hoc wireless networks with Infrastructure Support," *IEEE Journal of Selected Areas in Communications*, 23(3): 657-667, March 2005.

[6] B. Liu, P. Thiran and D. Towsley, "Capacity of a wireless ad hoc network with infrastructure," *ACM MobiHoc*, September 2007.

[7] D. M. Shila, Y. Cheng, and T. Anjali, "Throughput and delay analysis of hybrid wireless networks with multi-hop uplinks," *IEEE INFOCOM*, April 2011.

[8] D. M. Shila and Y. Cheng, "Ad hoc wireless networks meet the infrastructure: mobility, capacity, and delay," *IEEE INFOCOM*, April 2012.

[9] N. Banerjee, M. Corner, D. Towsley, and B. Levine, "Relays, base stations, and meshes: Enhancing mobile networks with infrastructure," *ACM MobiCom*, pp. 81C91, September 2008.

[10] W. Huang, Xinbing Wang, and Q. Zhang, "Capacity Scaling in Mobile Wireless Ad Hoc Network with Infrastructure Support," *IEEE ICDCS*, Genoa, Italy, 2010.

[11] J. Li, C.Blake, D. S. J. De Couto, H. Lee, and R. Morris, "Capacity of Ad Hoc Wireless Networks," *IEEE Mobicom*, July 2001.

[12] B. Azimdoost, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "Capacity of wireless networks with social behavior," *IEEE Transactions on Wireless Communications*, vol. 12, no. 1, January 2013.

[13] A. Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-delay trade-off in wireless networks," *IEEE INFOCOM*, April 2004.