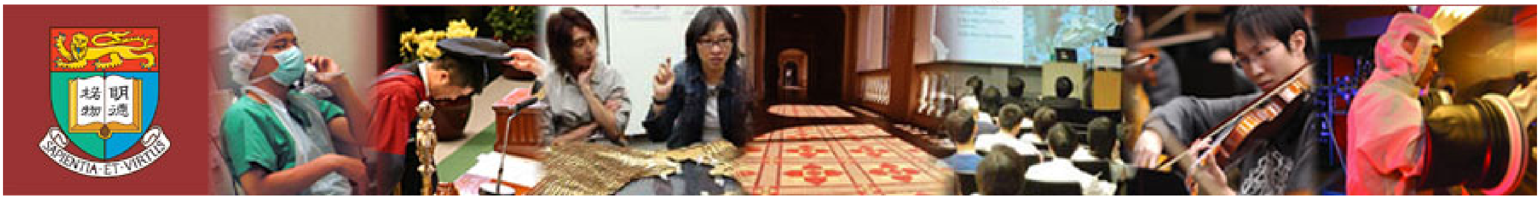

Calderón Preconditioned PMCHWT Equations for Analyzing Penetrable Objects in Layered Medium

Yongpin P. Chen, *Member, IEEE*, Lijun Jiang, *Senior Member, IEEE*, Sheng Sun, *Senior Member, IEEE*, Weng Cho Chew, *Fellow, IEEE*, and Jun Hu, *Senior Member, IEEE*

***Abstract*—We study Calderón preconditioners for analyzing electromagnetic scattering by penetrable objects in a layered medium. To account for the scattering effects of the multilayered background, the layered medium Green's function is adopted in the Poggio–Miller–Chang–Harrington–Wu–Tsai (PMCHWT) method. However, similar to the free-space case, the spectrum of the resulting equation is undesirable. This leads to a slow convergence of an iterative solver, especially when the geometry is densely meshed. To improve the convergence, a highly effective preconditioner is proposed. Different from its free-space counterpart, the preconditioning operator is constructed based on the Calderón identities for inhomogeneous medium. To reduce the relatively high construction cost of the preconditioning operator, several alternative simplified schemes are proposed and analyzed. Finally, the performances of different preconditioners are examined and compared carefully through different numerical examples. It is shown that the convergence of the PMCHWT system in a layered medium can be significantly improved by using the proposed Calderón preconditioners.**

***Index Terms*—Calderón preconditioner, layered medium Green's function, method of moments, penetrable objects, Poggio–Miller–Chang–Harrington–Wu–Tsai (PMCHWT) equation, surface integral equations.**

I. Introduction

SCATTERING of electromagnetic (EM) wave by embedded penetrable objects in a layered medium is a problem of long-standing interest. The method of moments (MoM) based on integral equations [1] is considered to be one of the most powerful methods in dealing with this kind of open-domain problems. However, most research works focus on electromagnetic scattering in the homogeneous environment (or free space), where the Green's function is simple and analytic. If the free-space Green's function is used to solve layered medium problems, the inhomogeneous background becomes part of the scatterer as well. Then, the unknown count increases considerably and the MoM becomes impractical. To overcome this difficulty, one introduces the layered medium Green's function [2], [3] that incorporates the scattering effect from the inhomogeneous layered medium. Therefore, the background effect is implicit in the integral equation and the object is the sole scatterer contributing to the unknowns.

Due to a different description of the scattering mechanism, there are basically two types of integral equations: the volume integral equation (VIE) and the surface integral equation (SIE) [4]. VIE requires volume discretization, and the number of unknowns is relatively large compared with SIE with only surface discretization. If the object is made of homogeneous material, SIE is usually more favorable in terms of memory requirement. However, the situation is subtle when considering the real computational cost. Taking the free-space Green's function, for instance, it is commonly accepted that the VIE operator has an excellent spectrum property and the iterative solution converges rapidly [5], [6]. The spectrum of the SIE operator, however, depends on formulations. Although the Müller formulation is reported to yield good spectrum, the accuracy is less satisfactory [7]–[9]. On the other hand, the PMCHWT method yields higher accuracy, but its operators suffer from a poor spectrum due to the electric field integral equation (EFIE) operator involved [10]–[12].

Recently, a highly effective preconditioner based on Calderón identities was developed to precondition the EFIE operator for analyzing EM scattering by perfect electrically conducting (PEC) objects in free space [13]. This technique fully exploits the self-regularizing property of the EFIE operator [14], namely, its square is a compact perturbation of the identity operator. Owing to this property, the spectrum of the EFIE operator, which clusters at zero and infinity, can be improved, and the operator is both bounded and well posed after preconditioning. Therefore, the resultant matrix is well conditioned and is free from the dense-discretization breakdown. Based on its success in EFIE for the PEC scattering problems [15]–[19], this preconditioner is further explored in the PMCHWT formulation for analyzing penetrable objects [20], [21].

However, most previous works mainly focus on scattering or radiation problems in the free space, where no scattering effect from the inhomogeneous background is considered. The corresponding Calderón identities [14], the foundation of this preconditioner, are derived based on the free-space Green's function.

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Y. P. Chen is with the School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, 611731, China, and also with the Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong, China (e-mail: yongpinchen@gmail.com).

L. Jiang and S. Sun are with the Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong, China (e-mail: jianglj@hku.hk; sunsheng@ieee.org).

W. C. Chew is with the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (e-mail: w-chew@uiuc.edu).

J. Hu is with the School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, 611731, China (e-mail: hujun@uestc.edu.cn).

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In a layered medium, such identities need to be reinvestigated. Currently, we have reported a work on the Calderón preconditioner for the EFIE in the analysis of PEC scattering in a layered medium [22].

In this paper, we further investigate the Calderón preconditioner for the PMCHWT analysis of scattering by penetrable objects in a layered medium. The rest of the paper is organized as follows. Section II first reviews the general PMCHWT formulation with the layered medium Green's function. A full Calderón preconditioner is constructed in Section III, where the spectral property of the preconditioned system is discussed. After that, in Section IV, several possible simplified schemes are proposed to reduce the cost of the preconditioner construction. Finally, in Section V, numerical examples are presented to validate the effectiveness of the preconditioners.

II. PMCHWT Formulation in Layered Medium

Let us consider a penetrable object with a closed boundary Γ embedded in a layered medium. The object is filled with a homogeneous material. The relative permittivity and permeability are denoted as ϵ and μ, respectively. Similarly, the material in layer i of the layered medium is characterized by ϵ_i and μ_i, where $i = 1, 2, \ldots N_L$. The outward unit normal of Γ is denoted as $\hat{n}$. This object is excited by an electromagnetic field generated from a source outside the scatterer. According to the surface equivalence principle, the induced surface electric current $\mathbf{J}$ and magnetic current $\mathbf{M}$ are obtained from the following PMCHWT equation [10]–[12]:

$$\begin{bmatrix} \hat{n} \times \left(\mathcal{L}_{\mathrm{E}}^{o} + \mathcal{L}_{\mathrm{E}}^{i}\right) & \hat{n} \times \left(\mathcal{K}_{\mathrm{E}}^{o} + \mathcal{K}_{\mathrm{E}}^{i}\right) \\ \hat{n} \times \left(\mathcal{K}_{\mathrm{H}}^{o} + \mathcal{K}_{\mathrm{H}}^{i}\right) & \hat{n} \times \left(\mathcal{L}_{\mathrm{H}}^{o} + \mathcal{L}_{\mathrm{H}}^{i}\right) \end{bmatrix} \cdot \begin{bmatrix} \mathbf{J} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} -\hat{n} \times \mathbf{E}_{\mathrm{inc}}^{o} \\ -\hat{n} \times \mathbf{H}_{\mathrm{inc}}^{o} \end{bmatrix}. \quad (1)$$

The detailed information of the operators and the corresponding integration kernels can be found in [23]–[25] and will not be repeated here.

In this problem, the scattering effect from the layered medium shall be included in the Green's function. However, this effect is usually weaker than the direct interaction between the source and the field. Hence, the layered medium effect is named the "secondary term" in the Green's function, in contrast with the "primary term" from the direct interaction. When the source and observation points are in the same layer, the interaction consists of the primary and secondary terms, or the Green's function is a summation of the aforementioned two terms. When the source and observation points are in different layers, only the transmitted term exists, and the Green's function comprises only the secondary term. It should be noted that the primary term always has a closed-form expression. However, the secondary term generally has no such simple solution. It can only be expressed by a series of infinite, slowly convergent, and highly oscillatory Sommerfeld integrals [23].

By noting the magnitude difference in $\mathcal{L}_{\mathrm{E}}$ and $\mathcal{L}_{\mathrm{H}}$ [22], the following PMCHWT system is suggested to balance the diagonal blocks:

$$\mathcal{P} \begin{bmatrix} \eta_m \mathbf{J} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} -\hat{n} \times \mathbf{E}_{\mathrm{inc}}^{o} \\ -\eta_m \hat{n} \times \mathbf{H}_{\mathrm{inc}}^{o} \end{bmatrix} \quad (2)$$

where

$$\mathcal{P} = \begin{bmatrix} \hat{n} \times \frac{\left(\mathcal{L}_{\mathrm{E}}^{o} + \mathcal{L}_{\mathrm{E}}^{i}\right)}{\eta_m} & \hat{n} \times \left(\mathcal{K}_{\mathrm{E}}^{o} + \mathcal{K}_{\mathrm{E}}^{i}\right) \\ \hat{n} \times \left(\mathcal{K}_{\mathrm{H}}^{o} + \mathcal{K}_{\mathrm{H}}^{i}\right) & \hat{n} \times \left(\mathcal{L}_{\mathrm{H}}^{o} + \mathcal{L}_{\mathrm{H}}^{i}\right) \cdot \eta_m \end{bmatrix} \quad (3)$$

and η_m is the characteristic impedance in region m.

In the MoM implementation, a triangular mesh is generated over the surface of the object. The unknown current $\eta_m \mathbf{J}$ and $\mathbf{M}$ are expanded by the Rao–Wilton–Glisson (RWG) basis function $\mathbf{f}_{\mathrm{RWG}}$ [26]. The integral equation can then be converted to a matrix equation by testing both sides with the rotated RWGs $\hat{n} \times \mathbf{f}_{\mathrm{RWG}}$. This resulting matrix is, unfortunately as in the free space, ill conditioned. This is so because the diagonal operators are unbounded and ill posed. For most applications, an effective preconditioner is indispensable to improve the spectrum of the system in order to accelerate the convergence of the iterative solution.

III. Calderón Preconditioner for PMCHWT Equation in Layered Medium

The Calderón identities in an inhomogeneous medium, which forms the theoretical basis of the desired preconditioner, can be derived as [22]

$$(\hat{n} \times \mathcal{L}_{\mathrm{H}})(\hat{n} \times \mathcal{L}_{\mathrm{E}}) = (\hat{n} \times \mathcal{K}_{\mathrm{H}})^2 - \frac{1}{4} \quad (4)$$

$$(\hat{n} \times \mathcal{K}_{\mathrm{H}})(\hat{n} \times \mathcal{L}_{\mathrm{H}}) - (\hat{n} \times \mathcal{L}_{\mathrm{H}})(\hat{n} \times \mathcal{K}_{\mathrm{E}}) = 0 \quad (5)$$

$$-(\hat{n} \times \mathcal{L}_{\mathrm{E}})(\hat{n} \times \mathcal{K}_{\mathrm{H}}) + (\hat{n} \times \mathcal{K}_{\mathrm{E}})(\hat{n} \times \mathcal{L}_{\mathrm{E}}) = 0 \quad (6)$$

$$(\hat{n} \times \mathcal{L}_{\mathrm{E}})(\hat{n} \times \mathcal{L}_{\mathrm{H}}) = (\hat{n} \times \mathcal{K}_{\mathrm{E}})^2 - \frac{1}{4}. \quad (7)$$

It is shown in (4) and (7) that a "dual" $\hat{n} \times \mathcal{L}_{\mathrm{H,E}}$ operator can be utilized to improve the spectrum of the $\hat{n} \times \mathcal{L}_{\mathrm{E,H}}$ operator in the diagonal blocks of (3), where the "dual squared" operator is a compact perturbation of the identity.

Motivated by the identities, we can construct the preconditioning operator to (2) as

$$\mathcal{Q} = \begin{bmatrix} \hat{n} \times \left(\mathcal{L}_{\mathrm{H}}^{o} + \mathcal{L}_{\mathrm{H}}^{i}\right) \cdot \eta_m & -\hat{n} \times \left(\mathcal{K}_{\mathrm{H}}^{o} + \mathcal{K}_{\mathrm{H}}^{i}\right) \\ -\hat{n} \times \left(\mathcal{K}_{\mathrm{E}}^{o} + \mathcal{K}_{\mathrm{E}}^{i}\right) & \hat{n} \times \frac{\left(\mathcal{L}_{\mathrm{E}}^{o} + \mathcal{L}_{\mathrm{E}}^{i}\right)}{\eta_m} \end{bmatrix}. \quad (8)$$

The preconditioned PMCHWT equation is, hence,

$$\mathcal{M} \begin{bmatrix} \eta_m \mathbf{J} \\ \mathbf{M} \end{bmatrix} = \mathcal{Q} \begin{bmatrix} -\hat{n} \times \mathbf{E}_{\mathrm{inc}}^{o} \\ -\eta_m \hat{n} \times \mathbf{H}_{\mathrm{inc}}^{o} \end{bmatrix} \quad (9)$$

where

$$\mathcal{M} = \mathcal{Q}\mathcal{P}. \quad (10)$$

The discretization of this preconditioned operator follows the multiplicative Calderón preconditioners in free space [13]. More specifically, the inner matrix operator $\mathcal{P}$ is discretized by using the RWG basis functions $\mathbf{f}_{\mathrm{RWG}}$ as the expansion function in the domain space, and the rotated RWGs $\hat{n} \times \mathbf{f}_{\mathrm{RWG}}$ as the testing function in the range space. The outer matrix operator $\mathcal{Q}$ is discretized by the Buffa–Christiansen (BC) basis functions $\mathbf{f}_{\mathrm{BC}}$ [27] and the rotated BCs $\hat{n} \times \mathbf{f}_{\mathrm{BC}}$ for expansion and testing, respectively. After discretization, the matrix system becomes

$$\mathcal{M}_{\mathrm{dis}} = \overline{\mathbf{Q}}_{\mathrm{BC}} \cdot \begin{bmatrix} \overline{\mathbf{G}}_m^{-1} & 0 \\ 0 & \overline{\mathbf{G}}_m^{-1} \end{bmatrix} \cdot \overline{\mathbf{P}}_{\mathrm{RWG}} \quad (11)$$

where the Gram matrix linking the domain of $\mathcal{Q}$ and the range of $\mathcal{P}$ is defined as

$$\left[\overline{\mathbf{G}}_m\right]_{ji} = \int d\mathbf{r}' \hat{n} \times \mathbf{f}_{\mathrm{RWG}j} \cdot \mathbf{f}_{\mathrm{BC}i}. \tag{12}$$

A. Spectrum Analysis of the Preconditioned Operator

The preconditioned operator in (10) is a 2×2 system

$$\mathcal{M} = \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{bmatrix}. \tag{13}$$

1) Diagonal Operators: The upper left operator is

$$\begin{aligned} \mathcal{A} = &\left[\hat{n} \times \left(\mathcal{L}_{\mathrm{H}}^{o} + \mathcal{L}_{\mathrm{H}}^{i}\right) \cdot \eta_m\right] \cdot \left[\hat{n} \times \frac{\left(\mathcal{L}_{\mathrm{E}}^{o} + \mathcal{L}_{\mathrm{E}}^{i}\right)}{\eta_m}\right] \\ &+ \left[-\hat{n} \times \left(\mathcal{K}_{\mathrm{H}}^{o} + \mathcal{K}_{\mathrm{H}}^{i}\right)\right] \cdot \left[\hat{n} \times \left(\mathcal{K}_{\mathrm{H}}^{o} + \mathcal{K}_{\mathrm{H}}^{i}\right)\right] \\ = &-\frac{1}{2} + \left(\hat{n} \times \mathcal{L}_{\mathrm{H}}^{o}\right)\left(\hat{n} \times \mathcal{L}_{\mathrm{E}}^{i}\right) + \left(\hat{n} \times \mathcal{L}_{\mathrm{H}}^{i}\right)\left(\hat{n} \times \mathcal{L}_{\mathrm{E}}^{o}\right) \\ &- \left(\hat{n} \times \mathcal{K}_{\mathrm{H}}^{o}\right)\left(\hat{n} \times \mathcal{K}_{\mathrm{H}}^{i}\right) - \left(\hat{n} \times \mathcal{K}_{\mathrm{H}}^{i}\right)\left(\hat{n} \times \mathcal{K}_{\mathrm{H}}^{o}\right). \end{aligned} \tag{14}$$

In the derivation, (4) is adopted to simplify the expression. Noting that the secondary terms of the operators are supposed to be compact perturbations [22], it is clear that the last two terms involving $\mathcal{K}$ in the last equality are compact. The other two terms involving $\mathcal{L}$ are, though not a direct consequence of (4), well behaved [28], [21]. Namely, the operator $\mathcal{A}$ is well behaved. The situation for the lower right operator $\mathcal{D}$ is similar, and (7) is needed in the derivation.

2) Off-Diagonal Operators: The upper right operator is

$$\begin{aligned} \mathcal{B} = &\left[\hat{n} \times \left(\mathcal{L}_{\mathrm{H}}^{o} + \mathcal{L}_{\mathrm{H}}^{i}\right)\right] \cdot \left[\hat{n} \times \left(\mathcal{K}_{\mathrm{E}}^{o} + \mathcal{K}_{\mathrm{E}}^{i}\right)\right] \cdot \eta_m \\ &+ \left[-\hat{n} \times \left(\mathcal{K}_{\mathrm{H}}^{o} + \mathcal{K}_{\mathrm{H}}^{i}\right)\right] \cdot \left[\hat{n} \times \left(\mathcal{L}_{\mathrm{H}}^{o} + \mathcal{L}_{\mathrm{H}}^{i}\right)\right] \cdot \eta_m \\ = &\left(\hat{n} \times \mathcal{L}_{\mathrm{H}}^{o}\right)\left(\hat{n} \times \mathcal{K}_{\mathrm{E}}^{i}\right)\eta_m + \left(\hat{n} \times \mathcal{L}_{\mathrm{H}}^{i}\right)\left(\hat{n} \times \mathcal{K}_{\mathrm{E}}^{o}\right)\eta_m \\ &- \left(\hat{n} \times \mathcal{K}_{\mathrm{H}}^{o}\right)\left(\hat{n} \times \mathcal{L}_{\mathrm{H}}^{i}\right)\eta_m - \left(\hat{n} \times \mathcal{K}_{\mathrm{H}}^{i}\right)\left(\hat{n} \times \mathcal{L}_{\mathrm{H}}^{o}\right)\eta_m. \end{aligned} \tag{15}$$

In the derivation, (5) is invoked to simplify the expression. The four terms in the last equality are supposed to be compact [6], [20], and the upper right operator $\mathcal{B}$ is thus compact. The situation is similar for the lower left operator $\mathcal{C}$, where (6) is needed in the derivation.

To numerically validate the properties of these operators, we consider the example shown in Fig. 1. A dielectric sphere with $r = 1$ m is located at the top layer of a three-layer medium. The configuration as well as the material parameters of the layered medium are shown in the figure. The sphere is filled with a material of $\epsilon = 4$ and $\mu = 1$. A y-polarized plane wave of $f = 100$ MHz is incident on the object along the negative z direction. The eight operators in (14) and (15) are discretized, and their eigenvalue distributions are recorded in Figs. 2 and 3. The corresponding condition numbers are also listed in the captions of the figures. It is shown that the eigenvalues of the six operators involving $\mathcal{K}$ cluster around zero, and the condition numbers are relatively high, validating the previous statement. However, for the first two operators involving double $\mathcal{L}$, the eigenvalues accumulate at two finite and nonzero values. The much smaller condition numbers further reinstate the good property of these two operators. It should be noted that even though the predicted

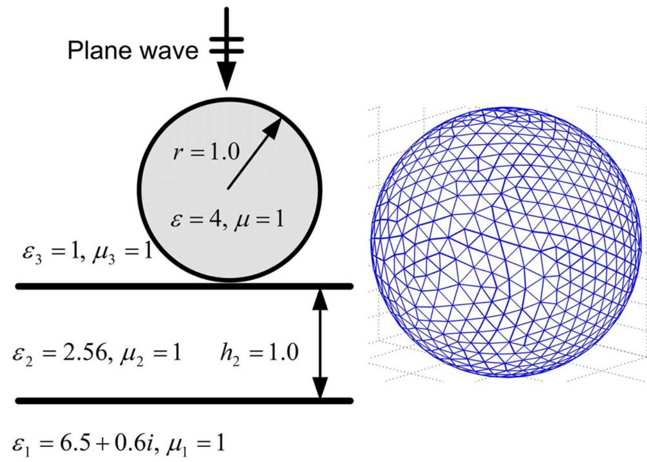

Fig. 1. A dielectric sphere with unit radius and $\epsilon = 4$, $\mu = 1$ is located above a three-layer medium (unit: m). It is under the illumination of a $f = 100$ MHz plane wave. The parameters of the layer medium are shown in the figure. (a) 2D profile of the configuration. (b) 3D model of the object.

accumulation points are $1/4$ according to the Calderón identities, the actual accumulation points may move due to the material contrast between the scatterer and the surrounding medium. Such observation was first made and analyzed in [21] for the free-space case. It was also indicated in [21] that if the material contrast is very high, the preconditioner may break down. According to our preliminary tests, the situation is similar in a layered medium. However, the detailed effects on the accumulation points from, for instance, the contrast between the object and the layered background, the contrast between different layers, or the relative positions of the object in the layered medium, are interesting topics requiring further studies.

In summary, the preconditioned operator $\mathcal{M}$ is well behaved, which validates the effective use of the Calderón preconditioner for the PMCHWT equation in a layered medium.

B. Discussion

The preconditioning operator in (8) is designed based on the Calderón identities in (4)–(7). Compared with the Calderón preconditioned PMCHWT equation in the free space [20], [21], additional discussions are needed.

First, the diagonal blocks of (8) are constructed due to the dual product suggested by (4) and (7). It is helpful to interchange the diagonal blocks of $\mathcal{P}$ to form $\mathcal{Q}$, though this is not necessary in the free space. For example, a direct use of the PMCHWT operator $\mathcal{P}$ is suggested as the preconditioner in [21], namely

$$\mathcal{Q} = \mathcal{P} = \begin{bmatrix} \hat{n} \times \frac{\left(\mathcal{L}_{\mathrm{E}}^{o} + \mathcal{L}_{\mathrm{E}}^{i}\right)}{\eta_m} & \hat{n} \times \left(\mathcal{K}_{\mathrm{E}}^{o} + \mathcal{K}_{\mathrm{E}}^{i}\right) \\ \hat{n} \times \left(\mathcal{K}_{\mathrm{H}}^{o} + \mathcal{K}_{\mathrm{H}}^{i}\right) & \hat{n} \times \left(\mathcal{L}_{\mathrm{H}}^{o} + \mathcal{L}_{\mathrm{H}}^{i}\right) \cdot \eta_m \end{bmatrix}. \tag{16}$$

This construction is motivated by the argument that a PEC body is a high conductivity limit of a dielectric one. Hence, there shall be similarity in the Calderón preconditioners for PMCHWT and EFIE, while the direct use of the operator has been previously verified in the EFIE analysis [13]. This can be done in the free space, since $\mathcal{L}_{\mathrm{H}}$ and $\mathcal{L}_{\mathrm{E}}$ differ only with a scaling constant [22], and the Calderón identity is usually expressed by a "direct square". In a layered medium, however, the two operators

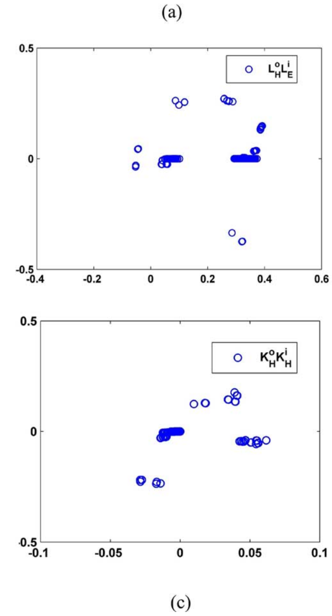

Fig. 2. Eigenvalue distribution and the condition numbers. (a) $(\hat{n}\times\mathcal{L}_{\rm H}^{o})(\hat{n}\times\mathcal{L}_{\rm E}^{i})$ (13.0752); (b) $(\hat{n}\times\mathcal{L}_{\rm H}^{i})(\hat{n}\times\mathcal{L}_{\rm E}^{o})$ (11.2573); (c) $(\hat{n}\times\mathcal{K}_{\rm H}^{o})(\hat{n}\times\mathcal{K}_{\rm H}^{i})$ (2.0519×10^{11}); (d) $(\hat{n}\times\mathcal{K}_{\rm H}^{i})(\hat{n}\times\mathcal{K}_{\rm H}^{o})$ (1.3458×10^{12}).

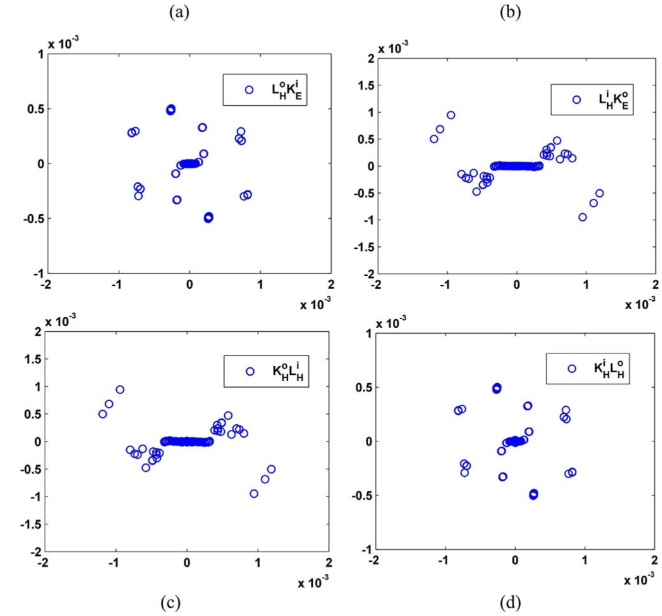

Fig. 3. Eigenvalue distribution and the condition numbers. (a) $(\hat{n}\times\mathcal{L}_{\rm H}^{o})(\hat{n}\times\mathcal{K}_{\rm E}^{i})$ (4.4419×10^{7}); (b) $(\hat{n}\times\mathcal{L}_{\rm H}^{i})(\hat{n}\times\mathcal{K}_{\rm E}^{o})$ (2.1479×10^{8}); (c) $(\hat{n}\times\mathcal{K}_{\rm H}^{o})(\hat{n}\times\mathcal{L}_{\rm H}^{i})$ (3.4452×10^{5}); (d) $(\hat{n}\times\mathcal{K}_{\rm H}^{i})(\hat{n}\times\mathcal{L}_{\rm H}^{o})$ (4.1063×10^{5}).

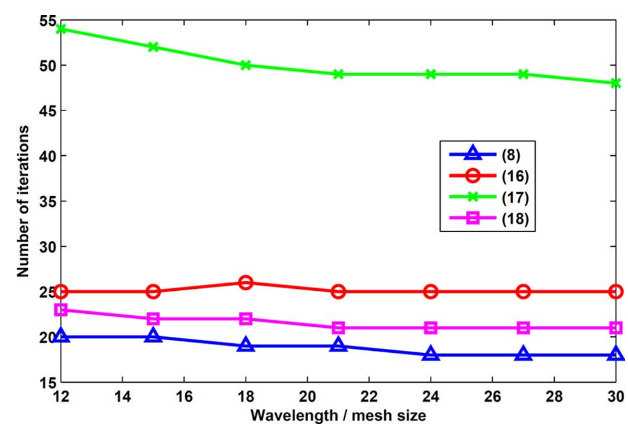

Fig. 4. Performance comparison of the operators in (8) and (16)–(18). Number of iterations versus discretization density (λ_0/δ) in the analysis of scattering from the dielectric sphere.

generally are not commutable. Hence, according to (4) and (7), the diagonal blocks of $\mathcal{P}$ should interchange in forming the $\mathcal{Q}$ to obtain a "dual square" when performing $\mathcal{M}=\mathcal{QP}$. However, it should be noted that such suggestion is optional for optimal purpose rather than mandatory. This is because of the weaker contribution from the secondary term. Due to this fact, the direct square $[(\hat{n}\times\mathcal{L}_{\rm E})^2$ or $(\hat{n}\times\mathcal{L}_{\rm H})^2]$ from (16) may still work in a layered medium but with a certain performance deterioration.

Second, the construction of the off-diagonal blocks in (8) is also owing to an apparent duality. The negative sign is suggested in order to invoke the other two identities (5), (6) as well when performing $\mathcal{M}=\mathcal{PQ}$. To our experience, such practice may lead to a better convergence. In fact, the choice of the off-diagonal blocks is not so crucial, and we can simply choose either $\mathcal{K}_{\rm E}$ or $\mathcal{K}_{\rm H}$. For example, we can set (the off-diagonal parts of) $\mathcal{Q}$ as either

$$\mathcal{Q}=\begin{bmatrix}\hat{n}\times\left(\mathcal{L}_{\rm H}^{o}+\mathcal{L}_{\rm H}^{i}\right)\cdot\eta_m & \hat{n}\times\left(\mathcal{K}_{\rm H}^{o}+\mathcal{K}_{\rm H}^{i}\right)\\ \hat{n}\times\left(\mathcal{K}_{\rm E}^{o}+\mathcal{K}_{\rm E}^{i}\right) & \hat{n}\times\frac{\left(\mathcal{L}_{\rm E}^{o}+\mathcal{L}_{\rm E}^{i}\right)}{\eta_m}\end{bmatrix} \tag{17}$$

or

$$\mathcal{Q}=\begin{bmatrix}\hat{n}\times\left(\mathcal{L}_{\rm H}^{o}+\mathcal{L}_{\rm H}^{i}\right)\cdot\eta_m & \hat{n}\times\left(\mathcal{K}_{\rm E}^{o}+\mathcal{K}_{\rm E}^{i}\right)\\ \hat{n}\times\left(\mathcal{K}_{\rm H}^{o}+\mathcal{K}_{\rm H}^{i}\right) & \hat{n}\times\frac{\left(\mathcal{L}_{\rm E}^{o}+\mathcal{L}_{\rm E}^{i}\right)}{\eta_m}\end{bmatrix}. \tag{18}$$

Apparently, these two preconditioners also end up with a well-behaved system after a simple analysis of the spectrum of $\mathcal{M}$. However, these two preconditioners may not be optimal according to our tests. The reason could be as follows. In using (17) or (18), the relations in (5), (6) are never invoked to cancel relevant terms when performing $\mathcal{M}=\mathcal{QP}$. On the other hand, the Q in (8) seems to benefit from such cancelation, as is shown in (15). Therefore, the off-diagonal blocks in (8) is also believed to be optimal.

To further verify this argument numerically, we compare the performance of the four proposed preconditioners in the example shown in Fig. 1 again. Fig. 4 shows the number of iterations versus the mesh density, where a targeted relative residual error of 10^{-6} is set in the generalized minimal residual (GMRES) solver [29]. It is shown that all the four preconditioners in (8), (16), (17), and (18) perform well and are stable with respect to the mesh configuration. The operator in (8) is the best and, hence, the optimal choice. Although the number of iterations of (17) is relatively high, it is still much (at least one order) lower than the one in the original PMCHWT formulation, as shall also be shown in Section V.

In summary, it is our belief that the preconditioning operator in (8) is the optimal choice in a layered medium. This preconditioner is constructed based on the careful study of the Calderón identities in an inhomogeneous medium. It should be noted that none of the preconditioners in free space proposed in [20] and [21] directly leads to the one in (8). However, after a careful derivation and comparison, one can show that when reducing to the free-space case, the preconditioning operator in (8) turns out to be identical with Version Two in [20], which is also identical to the one in (18).

IV. Alternative Simplified Schemes

Compared with other preconditioner techniques like the diagonal preconditioner, the construction cost of the Calderón preconditioner is relatively high, since a full matrix involving all the operators needs to be assembled. If we assume the number of unknowns is $2N$ in the PMCHWT equation, the memory requirement for storing the preconditioning matrix is $4N^2$, which is the same as the original system. In addition, one should be reminded that, in order to fill each matrix element, the Sommerfeld integrals are required, and the CPU time is much longer than its free-space counterpart. At the iteration stage, the computational complexity of the matrix-vector product (MVP) is $8N^2$ in the preconditioned system, which also doubles the number in the original system.

From the spectral analysis in (14) and (15), one finds that the off-diagonal operators of $\mathcal{Q}$ in (8) is not necessary to form a well-behaved $\mathcal{M}$. To reduce the computational cost in the construction of the preconditioning matrix, several alternative simplified schemes are presented in this section.

A. Simplified Scheme One

A diagonal matrix operator

$$\mathcal{Q} = \begin{bmatrix} \hat{n} \times \left(\mathcal{L}_{\mathrm{H}}^{o} + \mathcal{L}_{\mathrm{H}}^{i}\right) \cdot \eta_m & 0 \\ 0 & \hat{n} \times \frac{\left(\mathcal{L}_{\mathrm{E}}^{o} + \mathcal{L}_{\mathrm{E}}^{i}\right)}{\eta_m} \end{bmatrix} \tag{19}$$

can also serve as the preconditioner. In this case, the diagonal operator $\mathcal{A}$ (upper left block of $\mathcal{M}$) is also well behaved

$$\begin{aligned} \mathcal{A} &= \left[\hat{n} \times \left(\mathcal{L}_{\mathrm{H}}^{o} + \mathcal{L}_{\mathrm{H}}^{i}\right) \cdot \eta_m\right] \cdot \left[\hat{n} \times \frac{\left(\mathcal{L}_{\mathrm{E}}^{o} + \mathcal{L}_{\mathrm{E}}^{i}\right)}{\eta_m}\right] \\ &= -\frac{1}{2} + \left(\hat{n} \times \mathcal{L}_{\mathrm{H}}^{o}\right)\left(\hat{n} \times \mathcal{L}_{\mathrm{E}}^{i}\right) + \left(\hat{n} \times \mathcal{L}_{\mathrm{H}}^{i}\right)\left(\hat{n} \times \mathcal{L}_{\mathrm{E}}^{o}\right) \\ &\quad + \left(\hat{n} \times \mathcal{K}_{\mathrm{H}}^{o}\right)^2 + \left(\hat{n} \times \mathcal{K}_{\mathrm{H}}^{i}\right)^2 \end{aligned} \tag{20}$$

and the off-diagonal operator $\mathcal{B}$ (upper right block of $\mathcal{M}$), is compact

$$\begin{aligned} \mathcal{B} &= \left[\hat{n} \times \left(\mathcal{L}_{\mathrm{H}}^{o} + \mathcal{L}_{\mathrm{H}}^{i}\right)\right] \cdot \left[\hat{n} \times \left(\mathcal{K}_{\mathrm{E}}^{o} + \mathcal{K}_{\mathrm{E}}^{i}\right)\right] \cdot \eta_m \\ &= \left(\hat{n} \times \mathcal{L}_{\mathrm{H}}^{o}\right)\left(\hat{n} \times \mathcal{K}_{\mathrm{E}}^{o}\right)\eta_m + \left(\hat{n} \times \mathcal{L}_{\mathrm{H}}^{o}\right)\left(\hat{n} \times \mathcal{K}_{\mathrm{E}}^{i}\right)\eta_m \\ &\quad + \left(\hat{n} \times \mathcal{L}_{\mathrm{H}}^{i}\right)\left(\hat{n} \times \mathcal{K}_{\mathrm{E}}^{o}\right)\eta_m + \left(\hat{n} \times \mathcal{L}_{\mathrm{H}}^{i}\right)\left(\hat{n} \times \mathcal{K}_{\mathrm{E}}^{i}\right)\eta_m. \end{aligned} \tag{21}$$

In this scheme, the memory requirement of the preconditioning matrix is reduced by a half to $2N^2$ and, hence, the CPU time in the matrix assembly is also reduced by a half. The computational complexity for MVP becomes $6N^2$.

B. Simplified Scheme Two

The inner part $\mathcal{L}_{\mathrm{E,H}}^{i}$ in (19) is proposed to be skipped in the preconditioner, hence

$$\mathcal{Q} = \begin{bmatrix} \hat{n} \times \mathcal{L}_{\mathrm{H}}^{o} \cdot \eta_m & 0 \\ 0 & \hat{n} \times \frac{\mathcal{L}_{\mathrm{E}}^{o}}{\eta_m} \end{bmatrix}. \tag{22}$$

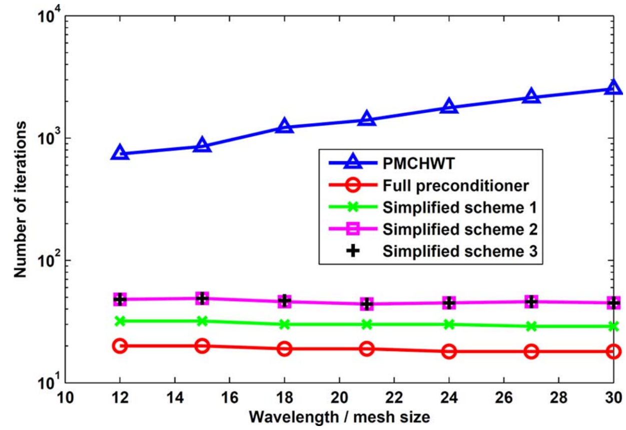

Fig. 5. Performance comparison of the PMCHWT formulation and the preconditioned systems. Number of iterations versus discretization density (λ_0/δ) in the analysis of scattering from the dielectric sphere.

This scheme is an incremental simplification to the previous one. Only the CPU cost for assembly reduces slightly. The memory requirement and the CPU cost for the MVP remain unchanged. This is actually identical to Version One of [20] when reduced to the free-space case, except that a small loss was also introduced to avoid internal resonance in [20].

C. Simplified Scheme Three

Noticing that the secondary term of the layered medium Green's function is a perturbation contribution, we can further abandon the secondary term of $\mathcal{L}_{\mathrm{E,H}}^{o}$ in the preconditioner. Although the memory requirement and the CPU cost for the MVP is the same as the previous one, the labor for Sommerfeld integrals can be totally avoided in the preconditioning matrix, and hence, its assembly can be much accelerated:

$$\mathcal{Q} = \begin{bmatrix} \hat{n} \times \mathcal{L}_{\mathrm{H}}^{o,\mathrm{P}} \cdot \eta_m & 0 \\ 0 & \hat{n} \times \frac{\mathcal{L}_{\mathrm{E}}^{o,\mathrm{P}}}{\eta_m} \end{bmatrix} \tag{23}$$

where $\mathcal{L}_{\mathrm{E,H}}^{\mathrm{P}}$ denotes the primary term.

V. Numerical Results

Several numerical examples are presented to validate the effectiveness of the Calderón preconditioners proposed in the previous sections. The GMRES algorithm [29] is adopted as the iterative solver, and the targeted relative residual error is 10^{-6} in all cases.

First, the scattering of the dielectric sphere shown in Fig. 1 is again studied. The numbers of iterations required to achieve the targeted relative residual error, with respect to the discretization density (free-space wavelength λ_0/mesh size δ), are shown in Fig. 5. It is shown that the number of iterations in the PMCHWT method increases rapidly as the mesh becomes denser, from 740 at the lower end ($\lambda_0/\delta = 12$) to 2525 at the higher end ($\lambda_0/\delta = 30$). On the other hand, the Calderón preconditioned systems have much lower and stable numbers. The full preconditioner of (8), with the largest complexity in construction, has the lowest convergence as expected. At the higher end, the number of iterations is 18. The performance of the simplified scheme 1 deteriorates slightly since it relieves the construction

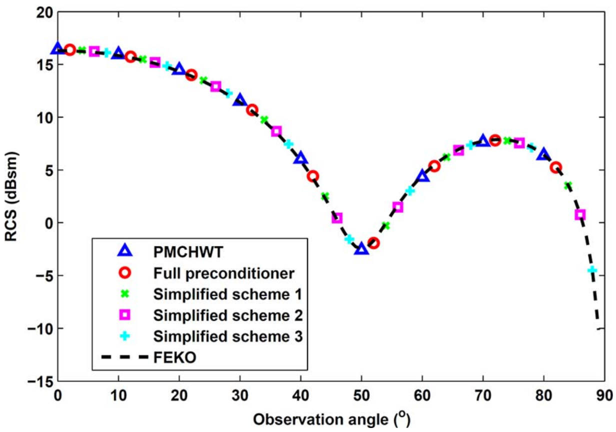

Fig. 6. Accuracy test of RCS calculated from different schemes in the paper, validated by FEKO.

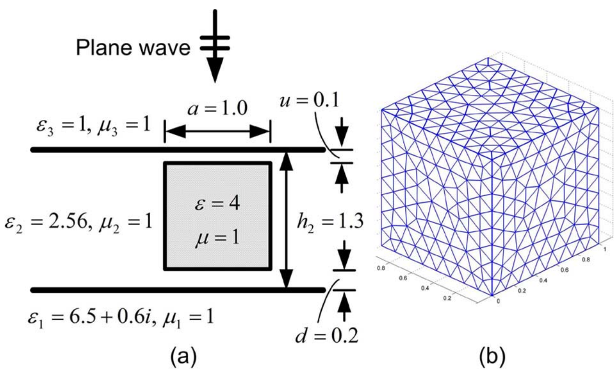

Fig. 7. Dielectric cube with unit side length and $\epsilon = 4$, $\mu = 1$ is embedded in a layered medium (unit: m). It is under the illumination of an $f = 100$ MHz plane wave. The parameters of the layer medium are shown in the figure. (a) 2D profile of the configuration; (b) 3D model of the object.

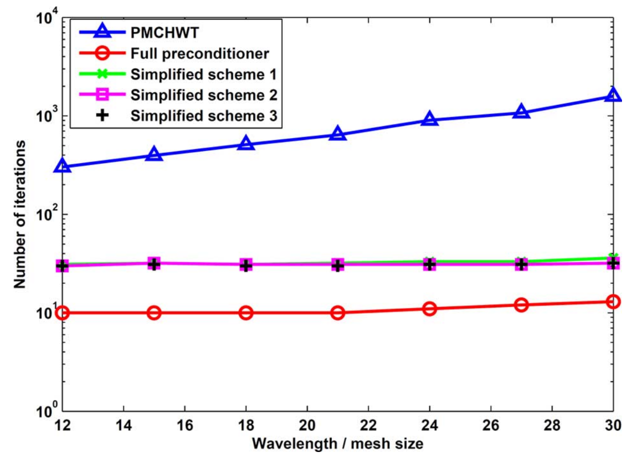

Fig. 8. Performance comparison of the PMCHWT formulation and the preconditioned systems. Number of iterations versus discretization density (λ_0/δ) in the analysis of scattering from the dielectric cube.

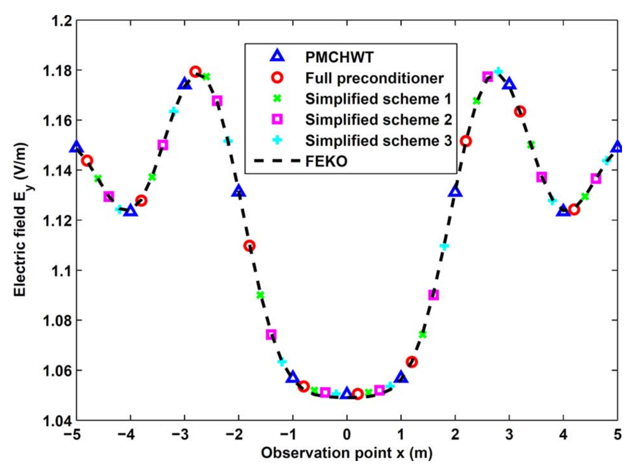

Fig. 9. Accuracy test of the near field distribution calculated from different schemes in the paper, validated by FEKO.

load, with the number of 29 at the higher end. The simplified schemes 2 and 3 are almost the same and further deteriorate, with the number of 45. The price paid is due to the fact that they require less computations in their construction, especially for scheme 3. Overall, we can see that the numbers of iterations of the preconditioners are in the same order and are much lower than that of the PMCHWT system. To test the accuracy, the far field or radar cross section (RCS) is further recorded for comparison in Fig. 6. The scanning is along θ at $\phi = 90^\circ$. Since there is no analytic solution available in a layered medium, the results are validated by the commercial software FEKO [30]. It is found that the accuracy of all these methods are satisfactory when compared with the reference data generated by FEKO with LU decomposition. Note that in order to show different curves clearly in the same figure, only the reference data is plotted with normal sampling rate in dash line. All the other results are plotted in symbols with a coarser sampling rate and with different starting points.

Next, the scattering of a dielectric cube with unit side length $(a = 1 \text{ m})$ and the material of $\epsilon = 4$, $\mu = 1$ is tested. As is shown in Fig. 7, the cube is embedded in the middle layer of a layered medium and is illuminated by a y-polarized plane wave of $f = 100$ MHz with normal incidence. The numbers of iterations versus discretization density are shown in Fig. 8. A similar phenomenon is observed in this case. Again, all the preconditioned equations converge rapidly to the targeted error bound, and are independent of the discretization density. The full preconditioner performs the best, and the simplified schemes are similar and slightly worse than the full one. The numbers of iterations in these preconditioned systems are all below 40. For the original PMCHWT system, however, the convergence is much slower and unstable, varying from 303 to 1586. To further show the accuracy, the near-field distribution along the observation line $(-5 \le x \le 5, y = 0, z = 1.5)$ m is collected in Fig. 9. Even though the dynamic range of the curve is small, the accuracy is reasonably good when compared with the one by FEKO.

In order to test objects with sharp corners, a star structure with $\epsilon = 4$, $\mu = 1$ is further investigated. As is shown in Fig. 10, the structure is put in the top layer of the layered medium, and again under the illumination of a plane wave with $f = 100$ MHz. The convergence history is shown in Fig. 11, where the original PMCHWT system can only converge to 0.0034 after 10 000 steps. However, the convergence rates of the preconditioned systems are much improved. The full preconditioner takes only 279 steps to converge to the targeted error criterion. The simplified scheme 1 needs 651 steps. Schemes 2 and 3 are similar and need 1003 and 967 steps, respectively. Scheme 3 is even slightly better than scheme 2 in this example. To show the accuracy of

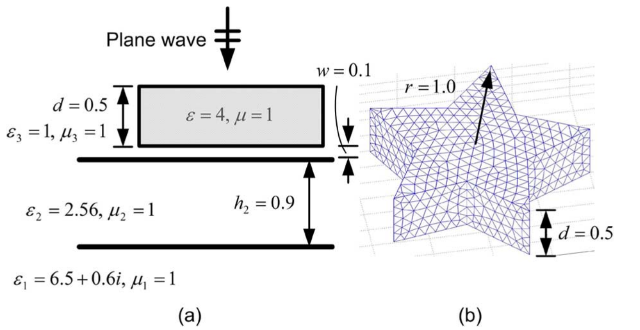

Fig. 10. Dielectric star structure with $\epsilon = 4$, $\mu = 1$ is located above a layered medium (unit: m). It is under the illumination of an $f = 100$ MHz plane wave. The parameters of the layer medium are shown in the figure. (a) 2D profile of the configuration. (b) 3D model of the object.

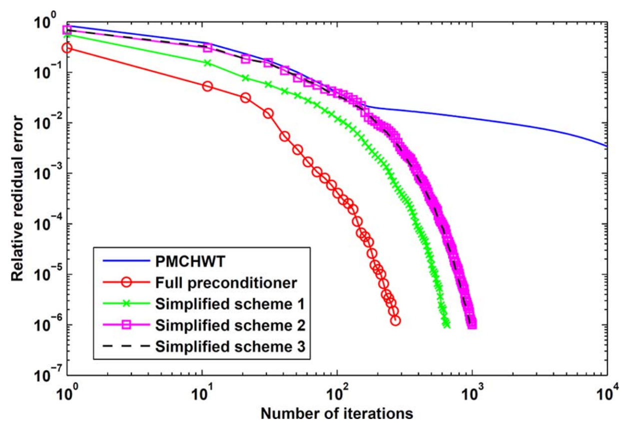

Fig. 11. Performance comparison of the PMCHWT formulation and the preconditioned systems. Iteration history recorded in GMRES to achieve a relative residual error of 10^{-6}, in the analysis of scattering from the dielectric star structure.

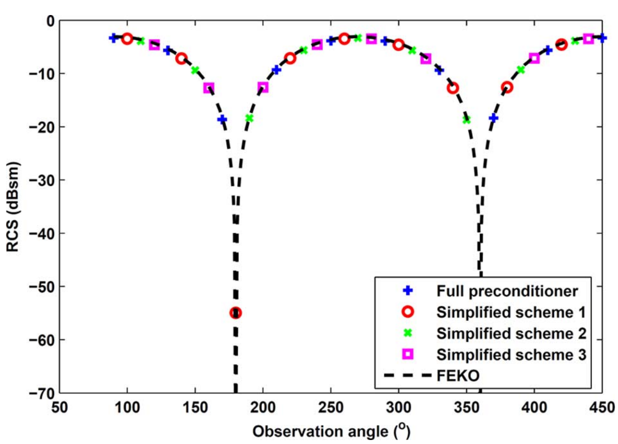

Fig. 12. Accuracy test of RCS calculated from different schemes in the paper, validated by FEKO.

our methods, the $\theta\theta$-polarized RCS is calculated along the scanning surface $\theta = 60^\circ$, shown in Fig. 12. It can be seen that the results agree well with the data calculated by FEKO. Since the PMCHWT method cannot converge to the targeted error, the result is not shown in the figure.

Finally, we consider an example where the object is straddling different layers and is excited by a Hertzian dipole. As is shown in Fig. 13, the dimension of the block is $a = b = 0.5$ m, $c = 1$ m, and the material is $\epsilon = 4$, $\mu = 1$. The working frequency

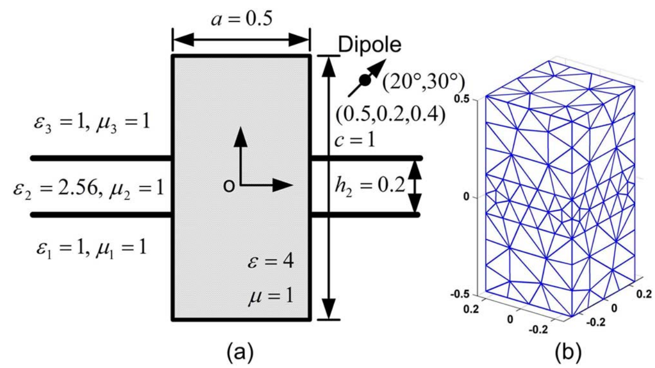

Fig. 13. Dielectric block with $\epsilon = 4$, $\mu = 1$ is straddling different layers of a three-layer medium (unit: m). It is excited by a Hertzian dipole with 150 MHz. The material parameters of each layer are shown in the figure. (a) 2D profile of the configuration. (b) 3D model of the object.

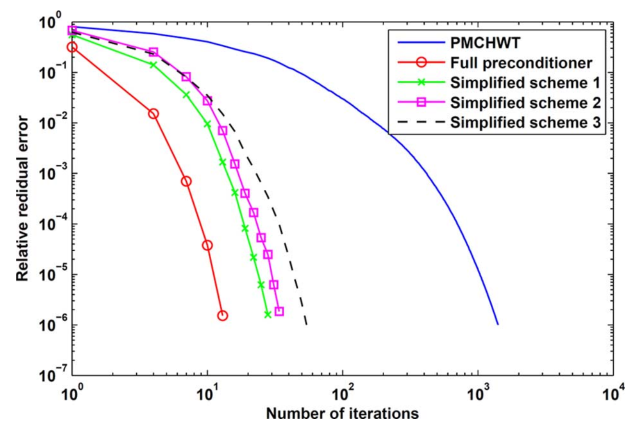

Fig. 14. Performance comparison of the PMCHWT formulation and the preconditioned systems. Iteration history recorded in GMRES to achieve a relative residual error of 10^{-6} in the analysis of scattering from the dielectric block.

is $f = 150$ MHz, and the source is located at $(x = 0.5, y = 0.2, z = 0.4)$ m, with the polarization of $(\theta = 20^\circ, \phi = 30^\circ)$. Again, it can be observed from Fig. 14 that all the preconditioners work well for this case. The number of iterations are 14, 30, 36, and 55 in these preconditioned systems, while the number is 1409 if the PMCHWT equation is solved without any preconditioner. The near-scattered field is calculated along the observation line $(-4 \leq x \leq 4, y = 0, z = 0.8)$ m and is shown in Fig. 15, where good agreement with the reference data can also be observed.

In all the four examples, the full preconditioner performs the best in terms of condition number improvement. The simplified schemes reduce the construction cost to some extent, at the expense of certain deterioration in the convergence. Therefore, it needs further investigation to determine the optimal choice in different applications. From these examples, the simplified scheme 3 is supposed to be a good compromise and may be the most useful in real engineering.

VI. Conclusion

Calderón preconditioned PMCHWT formulation is developed for the analysis of electromagnetic scattering by penetrable objects in a layered medium. Based on the Calderón identities in an inhomogeneous medium, a full preconditioner and several alternative simplified schemes are developed. The performance

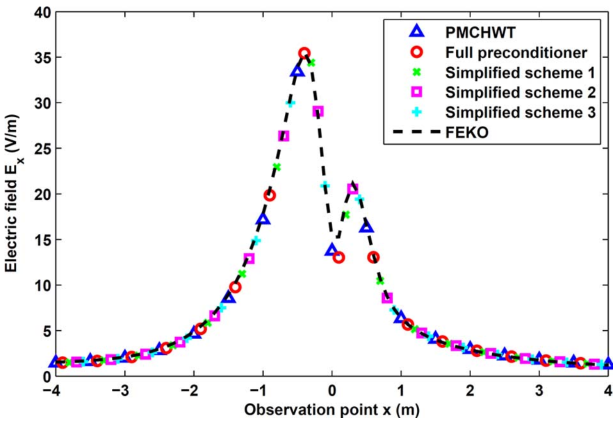

Fig. 15. Accuracy test of the near scattered field calculated from different schemes in the paper, validated by FEKO.

of these preconditioners are compared carefully through different scattering problems involving a layered medium. It is shown that the convergence of the iterative solutions of the PMCHWT equation can always be much improved by applying the preconditioners. It is also shown that the convergence of the preconditioned systems is independent of the discretization density, hence they are immune from the dense-discretization breakdown. The simplified scheme 3 is speculated to be a good compromise between the cost of preconditioning matrix assembly and the convergence improvement.

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Yongpin P. Chen (S'06–M'12) was born in Zhejiang, China, in 1981. He received the B.S. and the M.S. degrees in microwave engineering from the University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 2003 and 2006, respectively, and the Ph.D. degree in electrical engineering from the University of Hong Kong (HKU) in 2012.

From 2007 to 2008, he was a Research Assistant with the Electromagnetics and Optics Laboratory at HKU. Since 2012, he has been an Associate Professor with the School of Electronic Engineering, UESTC. Currently, he is also a Postdoctroal Fellow with the Department of Electrical and Electronic Engineering, HKU. His research interests include waves and fields in inhomogeneous media, computational electromagnetics, multi-physics modeling, and nano-optics.

Dr. Chen was the recipient of the General Assembly Young Scientists Award from the International Union of Radio Science (URSI), in 2014. He is a member of the Institute of Electrical and Electronics Engineers (IEEE), and a member of the Optical Society of America (OSA). He serves as a Reviewer for several journals, including the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, the *Journal of Computational Physics*, *Physical Review E*, the *Annals of Physics*, and *Progress in Electromagnetics Research* (PIER), etc.

Lijun Jiang (S'01–M'04–SM'13) received the B.S. degree in electrical engineering from the Beijing University of Aeronautics and Astronautics, China, in 1993, the M.S. degree from the Tsinghua University in 1996, and Ph.D. degree from the University of Illinois at Urbana-Champaign in 2004.

From 1996 to 1999, he was an Application Engineer with the Hewlett-Packard Company. Since 2004, he has been a Postdoctoral Researcher, a Research Staff Member, and a Senior Engineer at IBM T. J. Watson Research Center. Since the end of 2009, he has been an Associate Professor with the Department of Electrical and Electronic Engineering at the University of Hong Kong. His research interests focus on electromagnetics, computational electromagnetics, IC signal/power integrity, IC EMC/EMI, antennas, multi-physics modeling, etc.

Dr. Jiang received the HP STAR Award in 1998. In 2003, he received the IEEE MTT Graduate Fellowship Award and in 2004 the Y. T. Lo Outstanding Research Award. In 2008, he received the IBM Research Technical Achievement Award. He is Associate Editor of the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, Editor of *Progress in Electromagnetics Research*, Associate Guest Editor of *Proceedings of IEEE* special issues in 2011 and 2012, an IEEE AP-S Member, an IEEE MTT-S member, an ACES member, and a member of Chinese Computational Electromagnetics Society. He was the Semiconductor Research Cooperation (SRC) Industrial Liaison for several academic projects. He has been a TPC member of the IEEE Electrical Design for Advanced Packaging and Systems (EDAPS) since 2010; a TPC member of the 2013 IEEE International Conference on Microwave Technology and Computational Electromagnetics (ICMTCE); a scientific committee member of the 2010 Workshop on Simulation and Modeling of Emerging Electronics (SMEE); a special session organizer of the IEEE EDAPS, the International Review of Progress in Applied Computational Electromagnetics (ACES), and the Asia-Pacific Radio Science Conference (AP-RASC); a co-organizer of the HKU Computational Science and Engineering Workshops in 2010–2012; a TC-9 and TC-10 member of the IEEE EMC-S since 2011; a TPC Chair of the Seventh International Conference on Nanophotonics (ICNP); a TPC member of the Third Conference on Advances in Optoelectronics and Micro/Nano Optics (AOM); a Co-Chair of the International Workshop on Pulsed Electromagnetic Field, Delft, The Netherlands, 2013; the Chair of the Fourteenth IEEE Hong Kong AP/MTT Postgraduate Conference; and session chair of many international conferences. He also serves as the reviewer of the IEEE Transactions on several topics, and other primary electromagnetics and microwave related journals.

Sheng Sun (S'02–M'07–SM'12) received the B.Eng. degree in information engineering from the Xi'an Jiaotong University, China, in 2001 and the Ph.D. degree in electrical and electronic engineering from the Nanyang Technological University, Singapore, in 2006.

He was with the Institute of Microelectronics, Singapore, from 2005 to 2006, and with the Nanyang Technological University, Singapore, from 2006 to 2008. He was a Humboldt Research Fellow with the Institute of Microwave Techniques at the University of Ulm, Ulm, Germany, from 2008 to 2010. Since 2010, he has been a Research Assistant Professor with the Department of Electrical and Electronic Engineering at The University of Hong Kong. His research interests include electromagnetic theory and computational mathematics, multi-physics, numerical modeling of planar circuits and antennas, microwave passive and active devices, as well as the microwave and millimeter-wave communication systems. He has coauthored a book titled *Microwave Bandpass Filters for Wideband Communications* (Wiley, 2012) and authored and coauthored more than 100 journal and conference publications.

Dr. Sun received the Outstanding Reviewer Award for the IEEE MICROWAVE AND WIRELESS COMPONENTS LETTERS in 2010. He has been an Associate Editor for the IEICE TRANSACTIONS ON ELECTRONICS since 2010. He was the recipient of the General Assembly Young Scientists Award from the International Union of Radio Science (URSI) in 2014. He also received the Hildegard Maier Research Fellowship of the ALEXANDER VON HUMBOLDT FOUNDATION (Germany) in 2008 and is the recipient of the ISAP Young Scientist Travel Grant (Japan) in 2004.

Weng Cho Chew (S'79–M'80–SM'86–F'93) received the B.S. degree in 1976, both the M.S. and Engineer's degrees in 1978, and the Ph.D. degree in 1980, all in electrical engineering, from the Massachusetts Institute of Technology, Cambridge, MA, USA.

He worked on fast algorithms in computational electromagnetics in the last 20 years. From 1981 to 1985, he was a Manager with Schlumberger–Doll Research. In 1985, he joined the University of Illinois Urbana-Champaign (UIUC), IL, USA, where he was the Director of the Electromagnetics Laboratory from 1995 to 2007. He was also the Founder Professor at UIUC from 2000 to 2005, and the Y. T. Lo Endowed Chair Professor from 2005 to 2009. From 2007 to 2011, he served as the Dean of Engineering at The University of Hong Kong. In 2006, he was a Cheng Tsang Man Professor at Nanyang Technological University (NTU), Singapore. He has authored and coauthored three books, more than 350 journal papers, and more than 400 conference papers. His research interests are in wave and field and multiphysics.

Dr. Chew is a Fellow of the OSA, the IOP, the EM Academy, and the HKIE. He is the winner of an IEEE Graduate Teaching Award, a Schekulnof Best Paper Award, and an IEEE Distinguished Lecturer Award. In 2008, he received the CT Tai Distinguished Educator Award from IEEE AP-S. He has been the Editor-in-Chief of PIER journals since 2009, and he is an ISI highly cited author. Recently, he has been elected as a member of the U.S. National Academy of Engineering.

Jun Hu (M'06–SM'11) received the B.S., M.S., and Ph.D. degrees in electromagnetic field and microwave technique from the University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 1995, 1998, and 2000, respectively.

Currently, he is Full Professor at the School of Electronic Engineering, UESTC. During 2001, he was a Research Assistant with the Center of Wireless Communication in the City University of Hong Kong, Kowloon. During March to August in 2010, he was visiting scholar in the ElectroScience Laboratory of Department of Electrical and Computer Engineering of the Ohio State University. He was Visiting Professor of City University of Hong Kong during February to March in 2011. He is the author or coauthor of over 200 technical papers. His current research interests include integral equation methods in computational electromagnetics, computer-aided engineering, electromagnetic scattering, and radiation.

Dr. Hu serves as Chairman of the Student Activities Committee in the IEEE Cheng du Section, Chair of the IEEE AP/EMC Joint Chengdu Chapter from 2014 to 2015. He was listed in the 2013 *Who's Who in the World* by Marquis. He received 2011 Si Chuan Young Scientist Award, 2004 Best Young Scholar Paper Prize by Chinese Radio Propagation Society. His doctoral students were awarded Best Student Paper Awards many times, including in the 2010 and 2012 IEEE Chengdu Section, at the 2011 National Conference on Antennas, at the 2011 National Conference on Microwave, at the 2012 IEEE International Workshop on Electromagnetics: Applications and Students' Innovation Competition in Chengdu, China, at the Twelfth International Workshop on Finite Elements for Microwave Engineering—FEM 2014.