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| Title | Dynamic Coordinated Condition-Based Maintenance for Multiple Components With External Conditions |
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| Citation | IEEE Transactions on Power Delivery, 2015 |
| Issued Date | 2015 |
| URL | http://hdl.handle.net/10722/216950 |
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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRD.2015.2442291, IEEE Transactions on Power Delivery


# Dynamic Coordinated Condition-Based Maintenance for Multiple Components with External Conditions

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***Abstract*— This paper proposes a method to establish an optimal dynamic coordinated condition-based maintenance strategy that considers harsh external conditions, e.g., harsh weather conditions. Component deterioration is modeled as a Markov process based on physical characteristics, with the effects of harsh external conditions represented as probabilistic models. The proposed model involves interactions between different maintenance strategies on various components, as well as influences on the operation of the entire system. The optimal maintenance strategies are obtained by optimizing the proposed model with the cost-to-go, including the system reliability cost and the maintenance cost. This proposed model is solved using a backward induction algorithm associated with a search space reduction approach developed to reduce the simulation time. Two IEEE systems and one actual system validate the proposed model. Results show that this optimal maintenance strategy model that considers harsh external conditions provides insight for scheduling appropriate maintenance activities.**

***Index Terms*— Backward induction, condition-based maintenance, coordinated scheduling, harsh external conditions, Markov decision processes.**

## NOMENCLATURE

### A. Markov Chain Notation

| | |
|---|---|
| $N$ | Number of states for a component. |
| $N_c$ | Number of components. |
| $t$ | The *t*th time interval. |
| $i$ | The *i*th component. |
| $D_k$ | One state of a component, $k = \{1, 2, \cdots, N\}$. |
| $\Theta_D$ | Set of states, $\Theta_D = \{D_k, 1 \le k \le N\}$. |
| $MA_m$ | The *m*th activity, $m = \{1, 2, \cdots, N\}$. |
| $\boldsymbol{P}_{i,0}$ | An $N\times N$ transition matrix of component $i$ with deterioration processes. |
| $\boldsymbol{P}_{i,m}$ | An $N\times N$ transition matrix of component $i$ with maintenance activity $MA_m$. |
| $p_{jk}^{(i)}$ | Transition probability of component $i$ from state $D_k$ to state $D_j$. |
| $\boldsymbol{a}_{i,t}$ | An $(N+1)\times 1$ activity vector of component $i$ at the *t*th time interval. |
| $\boldsymbol{S}_{i,t}$ | An $N\times 1$ state vector of component $i$ at the *t*th time interval. |
| $\boldsymbol{\Omega}_t$ | An $N\times N_c$ state matrix for multiple components at the *t*th time interval. |
| $\boldsymbol{A}_t$ | An $(N+1)\times N_c$ activity matrix for multiple components at the *t*th time interval. |
| $P(\cdot)$ | Probability of going from one state to another for a given set of activities. |
| $\Theta_C$ | Set of all components. |

### B. Notation for the Model without Harsh External Conditions

| | |
|---|---|
| $C_{A,t}(\cdot)$ | Cost caused by activities at the *t*th time interval. |
| $C_{L,t}(\cdot)$ | Cost caused by the loss of load at the *t*th time interval. |
| $C_{S,t}(\cdot)$ | Successive cost at the *t*th time interval. |
| $v_t(\cdot)$ | Expected cost-to-go at the *t*th time interval. |
| $v_t^*(\cdot)$ | Minimum expected cost-to-go at the *t*th time interval. |
| $\Theta_N$ | Set of state matrices with all components in their normal operating states. |
| $\Theta_F$ | Set of state matrices with any components in failure states. |
| $\Theta_A$ | Set of activity matrices with all components in normal operating states. |

### C. Notation for the Model with Harsh External Conditions

| | |
|---|---|
| $M$ | Number of fault components. |
| $l$ | The *l*th fault component, $l = \{1, 2, \cdots, M\}$. |
| $l'$ | Serial number of the *l*th fault component in all |

This research was supported by National Natural Science Foundation of China (51277155), Research Grant Council of Hong Kong SAR (under grant GRF712411E, ECS739713 and GRF17202714), National Key Research Program of China (973 Program, 2012CB215102), Research Funding of Zhejiang Power Grid Corporation, China, and State Key Laboratory of Advanced Electromagnetic Engineering and Technology of China at HUST (2015KF006).

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components.

$\boldsymbol{B}$ An $M$×1 vector, where the value of each element in $\boldsymbol{B}$ can be '1' or '0'.

$P_t^{(E)}(\cdot)$ Probability that certain fault components can be repaired and others cannot be repaired.

$P_{l,t}^{(R)}$ Probability that the $l$th fault component can be repaired at the $t$th time interval.

$\boldsymbol{A}_t^{(E)}$ An $(N+1)\times N_c$ activity matrix for multiple components at the $t$th time interval with harsh external conditions.

$C_{L,t}^{(E)}(\cdot)$ Cost caused by the loss of load at the $t$th time interval considering harsh external conditions.

$C_{R,t}^{(E)}(\cdot)$ Repair cost at the $t$th time interval considering harsh external conditions.

$C_{S,t}^{(E)}(\cdot)$ Successive cost at the $t$th time interval considering harsh external conditions.

$c_{R,l}$ Repair cost for the $l$th fault component.

## I. Introduction

ELECTRIC power utilities always try to maximize profits while maintaining acceptable reliability levels, employing maintenance activities to mitigate deterioration of components since such deterioration can increase operating costs, aggravate potential fault losses, and reduce power system reliability levels. However, such maintenance activities often increase total operating costs. To achieve an appropriate trade-off between system reliability and the operating costs caused by deterioration, utilities must consider developing a long-term series of combined maintenance activities on various components in the power system.

The importance of maintenance scheduling for aging components over a given time horizon is already well recognized. Using the advantages of reliability centered maintenance (RCM), e.g., minimizing the frequency of overhauls and increasing the reliability of components, [1] and [2] proposed a practical framework by which the RCM procedure could be implemented in power distribution systems. Considering the influence of maintenance activities implemented on a component, [3] solved the problem of generator maintenance scheduling with network constraints by using Benders decomposition. Reference [4] studied the impact of a power market on the maintenance scheduling of a generating unit. However, maintenance scheduling should also depend on the different operating states of a component at different time intervals. A Markov model was introduced to represent the deterioration processes of an individual component [5] [6], while [7] enhanced the Markov model by using state diagrams. Due to the various aging rates of components in a power system, different maintenance activities on multiple components should be involved in maintenance scheduling. References [8] and [9] proposed models that included several components with different stochastic deterioration states. References [10] and [11] developed a two-stage maintenance management model that incorporated joint midterm and short-term maintenance. The proposed maintenance model considered both network constraints and maintenance constraints.

In practice, harsh external conditions also have a considerable impact on maintenance scheduling. Compliance application notices concerning protection system maintenance [12], provided by the North American Electric Reliability Corporation (NERC), verifies that maintenance activities should consider scheduling conflicts, reliability issues, extreme weather conditions, and other unforeseen occurrences. The NERC report on the 2011 southwest cold weather event [13] illustrates how extreme weather conditions can influence maintenance scheduling. *IEEE Standard 516-2009* [14] dictates that live work, e.g., maintenance activities on electrical devices, should not be performed under adverse weather conditions such as lightning activity, storms, heavy snow, and high humidity. *IEEE Standard 3007.2-2010* [15] emphasizes that maintenance activities on electrical devices such as generators and switches should not be performed under unfavorable weather conditions.

The motivation of this paper is to schedule the maintenance activities to improve the performance of the system over the entire time horizon, given the stochastic information of the harsh external conditions, and the requirements of industrial standards [14] [15]. The proposed maintenance activities enhance the system reliability to avoid severe damage caused by repair delays due to harsh external conditions. One contribution of this paper is the proposed probabilistic model associated with a Markov-based model, which establishes an optimized dynamic coordinated maintenance strategy that considers the influence of harsh external conditions. Given the deterioration of components, operating constraints, and potential harsh external conditions, the established strategy guarantees a high reliability level of all components under harsh external conditions. The other contribution is a backward induction with search space reduction method to improve computational efficiency while still maintaining good accuracy. Case studies on IEEE standard systems and an existing system validate the proposed model and the solution, and the results show that the proposed maintenance strategy, obtained by using the established model under harsh external conditions, has very practical applications.

The paper is organized as follows. Section II describes the deterioration process of components using a Markov model. Section III introduces the proposed model, which considers harsh external conditions. Section IV shows the solution for the proposed model, and case studies are presented in Section V. The work is concluded in Section VI.

## II. Markov Model for Individual and Multiple Components

This section presents a Markov-based deterioration model for components and the basic optimization model, which are prerequisites for the proposed model in Section III.

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### A. Transition Matrices of an Individual Component

#### 1) Transition matrices without maintenance activities

At any time interval, an operating component may be in a failure state or in a deteriorated state with a certain probability. This paper employs a Markov model to represent the stochastic state transitions of components for power system equipment such as generating units, lines, and transformers.

A component may include several normal operating states and a failure state. For the state $D_k$, we assume that a larger value of $k$ is associated with a worse state, while $D_N$ represents the failure state. The transition diagram of component $i$ without any maintenance activities is illustrated in Fig. 1(a). Mathematically, $\boldsymbol{P_{i,0}}$ can be written as

$$\boldsymbol{P_{i,0}} = [p_{jk}^{(i)}]_{N\times N} \tag{1}$$

In practice, each component's condition can be divided into different levels according to industrial standards, with each level considered a state. To determine a specific component's current state, a system monitoring and condition tracking infrastructure monitors the states of all components. Note that since this paper focuses on optimization models and solutions, we do not discuss condition monitoring technologies for the current model.

#### 2) Transition matrices with maintenance activities

Maintenance activities are used to mitigate the negative effects that deterioration processes have on components. For a component with $N$ states, there are $N$ activities, i.e., $MA_1$, $MA_2$,…,$MA_N$, which can be performed on the component. For example, state $D_q$ ($1\le q \le N$-1) can be driven to state $D_{q-m+1}$ through maintenance activity $MA_m$ when $m \le q$. When $m > q$, state $D_q$ ($1\le q \le N$) can be driven to state $D_1$, as shown in Fig. 1(b). Furthermore, $MA_N$ denotes a 'repair' activity that can be conducted on a component in failure, i.e., driving $D_N$ to $D_1$. In realistic systems, non-repairable outages may instead cause the replacement of failed components. Under the proposed model, replacement of a failed component can be considered a "repair activity" with a different cost. Moreover, the process whereby the component's state returns to a good state after the replacement is also satisfied through the established Markov process.

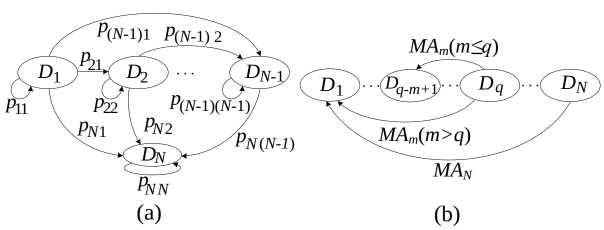


Fig. 1. (a) Transitions between different states without maintenance activity. (b) Transitions between different states with maintenance activity.

The transition matrix for component $i$ with activity $MA_m$ ($1\le$ m $\le N$) can be expressed as

$$\boldsymbol{P}_{i,m} = [p_{jk}^{(i)}]_{N\times N} = \begin{cases} 1, & j = [k-m]_+ + 1, k = 1,\cdots,N \\ 0, & else \end{cases} \tag{2}$$

where $[\bullet]_+$ is defined as $\max(\bullet,0)$. Based on the state transitions, the component can either retain its present state or return to a better state after a maintenance activity. In other words, maintenance activities reduce the future failure rates for components.

### B. Transition Probabilities of an Individual Component with Maintenance Activities

Using transition matrix (2), we can model the transition probabilities for any states in adjacent time intervals as a Markov chain. Given that only one activity can be selected in a given time interval, $\boldsymbol{a_{i,t}}$ consists of $N$+1 elements where only one element is '1' and all the remaining elements are '0'. The first element of $\boldsymbol{a_{i,t}}$ indicates the $i$th component without any activity. Meanwhile, the second element to the ($N$+1)th element of $\boldsymbol{a_{i,t}}$ represent $N$ activities. The $i$th component in state $D_k$ at the $t$th time interval is represented by $\boldsymbol{S}_{i,t}$, in which the $k$th element is equal to '1' and '0' elsewhere.

According to the definition of $\boldsymbol{S}_{i,t}$ and $\boldsymbol{a_{i,t}}$, the conditional probability to go from $\boldsymbol{S}_{i,t}$ to $\boldsymbol{S}_{i,(t+1)}$ under $\boldsymbol{a}_{i,t}$ can be represented as

$$P(\boldsymbol{S}_{i,(t+1)} \mid \boldsymbol{S}_{i,t}, \boldsymbol{a}_{i,t}) = (\boldsymbol{S}_{i,(t+1)})^T \cdot \boldsymbol{P}_{i,m} \cdot \boldsymbol{S}_{i,t} \tag{3}$$

where $\boldsymbol{P}_{i,m}$ is defined by (1) and (2).

### C. Transition Probabilities of Multiple Components with Maintenance Activities

The state and maintenance activities of a system with multiple components can be described by the state matrix $\boldsymbol{\Omega}_t$ and the activity matrix $\boldsymbol{A}_t$, respectively. These two matrices consist of the state vector $\boldsymbol{S}_{i,t}$ and the activity vector $\boldsymbol{a_{i,t}}$, containing all components in the system. These vectors are defined as

$$\boldsymbol{\Omega}_t = [\boldsymbol{S}_{1,t}, \cdots, \boldsymbol{S}_{i,t}, \cdots, \boldsymbol{S}_{N_c,t}] \tag{4}$$

$$\boldsymbol{A}_t = [\boldsymbol{a}_{1,t}, \cdots, \boldsymbol{a}_{i,t}, \cdots, \boldsymbol{a}_{N_c,t}] \tag{5}$$

Note that the deterioration processes for adjacent components are not independent, given similar operating conditions and external conditions. Thus, changing the transition probabilities for a component can directly alter the dependency of the deterioration process of other components. In addition, different components are also dependent on each other due to operating constraints. The conditional probability of $\boldsymbol{\Omega}_{t+1}$ under states $\boldsymbol{\Omega}_t$ and $\boldsymbol{A}_t$ can be described as

$$P(\boldsymbol{\Omega}_{t+1} \mid \boldsymbol{\Omega}_t, \boldsymbol{A}_t) = \prod_{i\in\Theta_c} P(\boldsymbol{S}_{i,(t+1)} \mid \boldsymbol{S}_{i,t}, \boldsymbol{a}_{i,t}) \tag{6}$$

### D. Mathematical model without Harsh External Conditions

In this section, we establish a recursive model to optimize maintenance strategies without considering harsh external conditions. The expected cost-to-go at the $t$th time interval consists of three parts:

- the cost due to maintenance activities, $C_{A,t}$,
- the cost caused by the loss of load, $C_{L,t}$, and
- the successive cost, $C_{S,t}$.

For these three costs, $C_{A,t}$ is only related to the maintenance activities, while $C_{L,t}$ is related to the maintenance activities and the load level of the system. We can calculate $C_{L,t}$ by determining the optimal power flow with the objective of minimizing the loss of load, representing the system reliability.

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By varying the weights of $C_{A,t}$ and $C_{L,t}$, system operators can emphasize whether system reliability or maintenance cost is the most important aspect. Note that $C_{A,t}$ and $C_{L,t}$ can be expressed as functions of the maintenance activity matrix $\boldsymbol{A}_t$, and can be written as $C_{A,t}(\boldsymbol{A}_t)$ and $C_{L,t}(\boldsymbol{A}_t)$, while the successive cost $C_{S,t}$ is determined by the maintenance activities performed on all components and the state of the system and can be written as $C_{S,t}(\boldsymbol{\Omega}_t,\boldsymbol{A}_t)$. Therefore, the expected cost-to-go $v_t(\boldsymbol{\Omega}_t,\boldsymbol{A}_t)$ at the $t$th time interval can be written as

$$v_t(\boldsymbol{\Omega}_t,\boldsymbol{A}_t)=C_{A,t}(\boldsymbol{A}_t)+C_{L,t}(\boldsymbol{A}_t)+C_{S,t}(\boldsymbol{\Omega}_t,\boldsymbol{A}_t) \tag{7}$$

$$C_{S,t}(\boldsymbol{\Omega}_t,\boldsymbol{A}_t)=\sum_{\boldsymbol{\Omega}_{t+1}\in\Theta_N\cup\Theta_F}\left[v^*_{t+1}(\boldsymbol{\Omega}_{t+1})\cdot P(\boldsymbol{\Omega}_{t+1}\mid\boldsymbol{\Omega}_t,\boldsymbol{A}_t)\right] \tag{8}$$

The objective is to minimize the expected cost-to-go for the given state $\boldsymbol{\Omega}_t$ by finding an activity matrix $\boldsymbol{A}_t$ for the components, that is

$$v^*_t(\boldsymbol{\Omega}_t)=\min\left\{v_t(\boldsymbol{\Omega}_t,\boldsymbol{A}_t),\ \boldsymbol{A}_t\in\Theta_A\right\},\quad \boldsymbol{\Omega}_t\in\Theta_N \tag{9}$$

Given realistic states at each time interval, the above recursive model provides optimal maintenance activity scheduling without considering harsh external conditions.

## III. A Dynamic Markov Model with Harsh External Conditions

### A. Potential Harsh External Conditions

Recent years have seen exceptionally harsh weather conditions occurring in many different areas of the world. Table I highlights some of these events [16] [17]. Such events, while rare, can result in very high economic losses [18], and the influences of these extreme weather conditions on power systems have brought the concept of a resilient grid to the attention of many governments. *A Policy Framework for the 21st Century Grid* [19], released by the U.S. government in June 2011, emphasizes the importance of resilient grids to counter the effects of the increased frequency and intensity of severe weather [20]. As such, appropriate maintenance scheduling that takes into consideration severe weather and climate events is an important step in constructing a resilient grid. Given the difficulty in determining the occurrence of harsh extreme conditions, many research studies [21], [22] have focused on models and methodologies, using historical data provided by weather observations.

TABLE I
RECENT HARSH WEATHER

| Time | Location | Event |
|---|---|---|
| Aug 2005 | United States | Hurricane Katrina |
| Jan 2008 | China | Snow Storm |
| Feb 2011 | United States | Snow Storm |
| Oct 2012 | United States and Canada | Hurricane Sandy |
| Feb 2014 | Slovenia and Australia | Heat wave |

### B. Mathematical Model with Harsh External Conditions

This paper considers harsh external conditions that directly affect repair activities on failed components. If repair activities are delayed due to harsh external conditions, the system can suffer severe damage. Therefore, the maintenance scheduling algorithm should schedule and coordinate maintenance activities at different time intervals and on different components to ensure high system reliability during harsh external conditions, taking into consideration the deterioration processes of different components, the different maintenance costs of different components, and system operating constraints. To take into account harsh external conditions, we quantify the influence of harsh external conditions on repair delays as an expected cost in the proposed recursive model, while the uncertainties of repair delays caused by harsh external conditions are modeled as probabilities in the expected cost. These probabilities are determined using historical data.

*1) Probabilistic model of repair delays caused by harsh external conditions*

Given $M$ ($M < N_c$) fault components in a certain future time interval, there will be $2^M$ combinations of repair scenarios when considering potential repair delays caused by harsh external conditions. This is written as

$$C^0_M+C^1_M+C^2_M+\cdots+C^M_M=2^M \tag{10}$$

where $C^r_M, r\in\{0,1,\cdots,M\}$ denotes that repair activities on $r$ components will be delayed due to harsh external conditions.

Given the probabilistic characteristics of harsh external conditions, the general formula of the probability of one scenario where the repair activities on certain components are delayed is represented as

$$P^{(E)}_t(\boldsymbol{B})=\prod_{l=1}^{M}\left[\boldsymbol{B}(l)\cdot P^{(R)}_{l,t}+(1-\boldsymbol{B}(l))\cdot(1-P^{(R)}_{l,t})\right] \tag{11}$$

where $\boldsymbol{B}(l)=0$ when the repair on the $l$th fault component is delayed, otherwise $\boldsymbol{B}(l)=1$. Usually, harsh weather conditions affect power systems over an entire area, which means that adjacent facilities in need of repairs are highly vulnerable to the same harsh weather event. In this case, we just need to set the unrepaired probabilities of these adjacent facilities to appropriately high values.

*2) Activity vectors corresponding to repair delays*

Different repair delays denote different activity vectors. When the repair on the $l$th fault component is not delayed at the $t$th time interval, the element in the activity vector, which represents the repair activity, should be '1' and '0' elsewhere. That is

$$\boldsymbol{A}^{(E)}_t(N+1,l')=1 \tag{12}$$

$$\boldsymbol{A}^{(E)}_t(n,l')=0,\quad n=\{1,2,\cdots,N\} \tag{13}$$

where $l'$ is the serial number of the $l$th fault component in all components. When the repair on the $l$th fault component is delayed at the $t$th time interval, the element in the activity vector, which represents no activity, should be '1' and '0' elsewhere. That is

$$\boldsymbol{A}^{(E)}_t(1,l')=1 \tag{14}$$

$$\boldsymbol{A}^{(E)}_t(n,l')=0,\quad n=\{2,3,\cdots,N+1\} \tag{15}$$

We assume that there are no maintenance activities occurring on other normal operating components when any component is in a failure state. Therefore, the activity vectors for the $i$th normal operating component can be represented as

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$$\boldsymbol{A}_t^{(E)}(1,i)=1,\quad i\neq l' \tag{16}$$

$$\boldsymbol{A}_t^{(E)}(n,i)=0,\quad i\neq l',\ n=\{2,3,\cdots,N+1\} \tag{17}$$

*3) Expected cost*

Based on the established probabilities for the potential repair delay scenarios and the corresponding activity vector, the expected cost when certain components are in a failure state can be established as

$$C_{R,t}^{(E)}(\boldsymbol{B})=\sum_{l=1}^{M}\left[B(l)\cdot c_{R,l}\right] \tag{18}$$

$$C_{S,t}^{(E)}(\boldsymbol{\Omega}_t,\boldsymbol{A}_t^{(E)})=\sum_{\boldsymbol{\Omega}_{t+1}\in\Theta_N\cup\Theta_F}\left[v_{t+1}^{*}(\boldsymbol{\Omega}_{t+1})\cdot P(\boldsymbol{\Omega}_{t+1}\mid\boldsymbol{\Omega}_t,\boldsymbol{A}_t^{(E)})\right] \tag{19}$$

$$v_t^{*}(\boldsymbol{\Omega}_t)=C_{L,t}^{(E)}(\boldsymbol{\Omega}_t)+\sum_{B(1)=0}^{1}\cdots\sum_{B(M)=0}^{1}\left[P_t^{(E)}(\boldsymbol{B})\times\left(C_{R,t}^{(E)}(\boldsymbol{B})+C_{S,t}^{(E)}\left(\boldsymbol{\Omega}_t,\boldsymbol{A}_t^{(E)}\right)\right)\right] \tag{20}$$

where (18) denotes the repair cost of a possible repaired scenario, (19) is the successive cost with a possible repaired scenario where $\sum\cdots\sum[\bullet]$ indicates all possible repair delay scenarios, and (20) calculates the expected cost with potential delays of repair activities on failure components. In practice, system operators determine repair activities based on realistic external conditions. Because harsh external conditions are assumed to affect repair activities, (18)-(20) in conjunction with (7)-(9) are employed to achieve the optimal scheduling of maintenance activities for non-failure states. Since the proposed model is a recursive model that includes the costs of various maintenance activities on different components, the recursive characteristic is capable of handling maintenance alternatives with dissimilar time requirements, and the included costs of maintenance activities is capable of handling maintenance alternatives with dissimilar costs.

## IV. Solving the Model Using a Backward Induction Algorithm

Backward induction [23] associated with a search space reduction method is employed to solve the proposed dynamic model in this paper.

### A. Backward Induction

Backward induction is a deduction process operating backwards from the end of a problem to determine a sequence of optimal activities. At each time interval, the expected cost-to-go of one state with a certain activity can be calculated based on Bellman's equations. For each state at each time interval, the action with the minimal expected cost-to-go is the optimal strategy. This process continues backwards until all time intervals are covered.

### B. Search Space Reduction Method

As the number of states and time intervals increases, the size of the search space grows dramatically, leading to a computationally intensive task. In this paper, we employ a search space reduction method to reduce the search space. The key point of the proposed search space reduction method is to neglect state transitions with tiny probabilities.

In realistic systems, the deterioration probability of each component is usually small, especially considering advances in manufacturing technologies and materials. Given the small deterioration probability of each component, the probability that many components deteriorate simultaneously in one time interval is even smaller. When calculating the expected cost-to-go of each state, state transitions with minor probabilities are neglected to reduce the huge search space.

### C. Solving the Model

The steps to solve the proposed model are as follows.

Step 1) Using the deterioration process of each component, generate the sets $\Theta_N$ and $\Theta_F$ .

Step 2) Repeat for $t$=$T$, $T$-1,…,1.

--Step 2.1) For $\boldsymbol{\Omega}_t\in\Theta_F$ , calculate the probability and the corresponding active matrix using (10)-(17).

--Step 2.2) Calculate the repair cost and the successive cost using (18) and (19) in conjunction with the search space reduction method. Then, get the expected cost for $\boldsymbol{\Omega}_t\in\Theta_F$ using (20).

--Step 2.3) For $\boldsymbol{\Omega}_t\in\Theta_N$ with activity $\boldsymbol{A}_t$ , use (7) and (8) to calculate the corresponding expected cost-to-go. Again, the search space reduction method is employed.

--Step 2.4) For all possible activities, calculate all expected costs and find the optimal strategy for $\boldsymbol{\Omega}_t\in\Theta_N$ using (9).

--Step 2.5) Set $t=t-1$ and return to Step 2.1. For the $T$th time interval, the successive cost of each state is not considered.

## V. Case Studies

In this section, we employ three test systems to validate the proposed model. Case I verifies the proposed model and shows the influences of harsh external conditions on the maintenance scheduling, Case II shows the feasibility of the proposed search space reduction method, and Case III presents simulation results for an actual power system in China. For the sake of exposition, we focus on the maintenance of transformers in these three cases. However, it is worth pointing out that the proposed model also determines appropriate maintenance scheduling for other components, e.g., transmission and distribution lines. All parameters come from [2], [3], [8], [9], [11] and Zhejiang Electric Power Grid of China.

### A. Data Description

The following data is required for the model

- Costs of different activities on different components.
- Costs of loss of load when components are out of service due to failure.
- Transition probabilities between different states, e.g., deterioration states and failure states.
- Probability of repair delays caused by harsh external conditions.

In addition to the above data, the following parameters, which again are used for the sake of exposition, are employed for each case study. The proposed model will work just as effectively with other parameters.

- There are three states for each component: the good state ($D_1$), the deteriorated state ($D_2$), and the failure state ($D_3$). Note that the proposed model and algorithms can analyze scenarios that are more comprehensive.
- Three activities, consisting of the repair activity ($M_3$) and two maintenance activities, i.e., the minor maintenance activity ($M_1$) and the major maintenance activity ($M_2$).
- 52 weeks for the maintenance scheduling using a one-week time interval.

*B. Case I: Verification of the Proposed Model*

For this case, we used an IEEE 30-bus system as the test system. Table II shows the maintenance activity costs and repair costs for each of the transformers in the system, where $T_1$, $T_2$, $T_3$, and $T_4$ represent the transformers at bus 6-10, bus 6-9, bus 27-28, and bus 4-12, respectively. Unrepaired probabilities are shown in Table III, and unrepaired probabilities of different transformers at the same time interval are assumed to be consistent. The load curve over 52 weeks is shown in Fig. 2(a). The load losses with regard to different offline transformers are presented in Fig. 2(b). For all transformers, the probabilities from $D_1$ to $D_2$ and from $D_1$ to $D_3$ are 0.055 and 0.005, respectively. The probability from $D_2$ to $D_3$ is 0.015.

TABLE II
COSTS ($10^3$ \$) OF TRANSFORMERS

| | $T_1$ | $T_2$ | $T_3$ | $T_4$ |
|---|---|---|---|---|
| $M_1$ | 0.8 | 1.0 | 1.0 | 1.0 |
| $M_2$ | 6.5 | 6.4 | 6.0 | 6.5 |
| Repair | 14 | 15 | 16 | 15.5 |

TABLE III
UNREPAIRED PROBABILITIES OVER ALL TIME INTERVALS

| Weeks | Unrepaired Probability |
|---|---|
| 1-3 | 0.12 |
| 4 - 24, 38 - 41 | 0.05 |
| 25 - 32, 42 - 47 | 0.10 |
| 33 - 37, 48 - 52 | 0.15 |

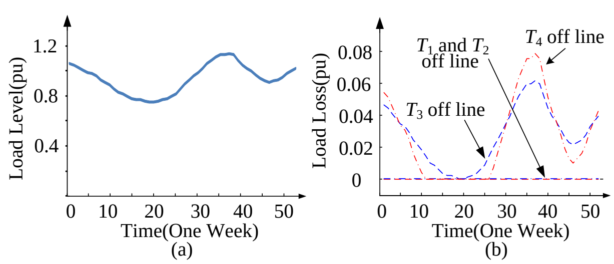

Fig. 2. (a) Load curve. (b) Load losses with different offline transformers.

*1) Verifying the model using Monte Carlo simulations*

This subcase employs the Monte Carlo method to verify the proposed model. Take an initial state with all transformers in the state $D_1$ as an example. The expected cost, based on 5000 Monte Carlo simulations, is \$20218.4, which is close to the expected cost of \$ 20164.4 calculated by the proposed model. For different initial state scenarios, the relative errors of the expected costs based on Monte Carlo simulations and the expected costs calculated with the proposed model are presented in Fig. 3(a), while Fig. 3(b) shows the probability density function (PDF) of the relative errors. The tiny errors indicated by these figures show that the proposed model and its solution are correct.

*2) Selection of optimal maintenance activities*

The optimal maintenance activities are selected based on observed states. Fig. 4 shows that the optimal maintenance activities are adjusted dynamically based on the observed states of the transformers at each time interval. For example, if $T_2$ is in state $D_2$ and $T_1$, $T_3$, and $T_4$ are in $D_1$ at the 26$^{th}$ time interval, the optimal strategy, based on the proposed model with harsh external conditions, should be a minor maintenance activity, i.e., $M_1$, on $T_4$. At the 30$^{th}$ time interval, if $T_1$ and $T_3$ deteriorate and other transformers retain their previous states, i.e., $T_1$, $T_2$, and $T_3$ are in state $D_2$ and $T_4$ is in $D_1$, the optimal strategy should be a major maintenance activity, i.e., $M_2$, on $T_1$.

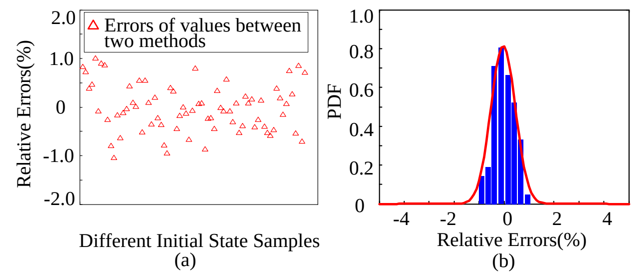

Fig. 3. (a) Errors of expected costs based on the proposed model and Monte Carlo simulations. (b) PDF of the relative errors.

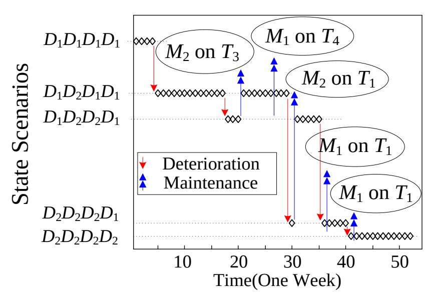

Fig. 4. Sample paths of states with regard to the costs of two scenarios.

*3) Influences of harsh external conditions on maintenance*

Harsh external conditions have an influence on the expected costs. The proposed optimal maintenance strategies have smaller expected costs than the strategies based on the model that does not consider harsh external conditions. The reason for this improvement is that future effects due to the non-repair of components caused by harsh external conditions are taken into account for the proposed maintenance scheduling. For example, given $T_1$, $T_2$, and $T_3$ in $D_2$ and $T_4$ in $D_1$ at the first time interval, the expected cost of the proposed optimal maintenance strategy with harsh external conditions is \$18582.8, while the expected cost of the strategy, based on the model without harsh external conditions, is \$20010.4. For all states at the first time interval, 6.13% - 9.25% of expected costs can be reduced with considering harsh external conditions.

Since expected costs are influenced by harsh external

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conditions, the maintenance strategies are correspondingly influenced by harsh external conditions. Fig. 5(a) and (b) show two sample paths of states with and without harsh external conditions, respectively. At the 28th time interval, the states of some transformers deteriorate. For the proposed maintenance scheduling in this paper, $M_2$ is performed on $T_4$. In this case, $T_4$ returns to a good state in the next time interval and has a small probability of failing in the future. However, the model performs no maintenance activities if harsh external conditions are not considered. In this case, $T_4$ has a larger probability of being in failure in the future. If $T_4$ fails at the 34th time interval and cannot be fixed immediately due to harsh external conditions, this will seriously influence system operations.

To provide an overview of the influences of harsh external conditions on maintenance scheduling, Fig. 6 shows the distribution of inconsistent maintenance strategies with and without harsh external conditions, while Figs. 7(a), (b), (c), and (d) show the distributions of inconsistent maintenance strategies on $T_1$, $T_2$, $T_3$, and $T_4$ with and without harsh external conditions. Fig. 6 indicates that most inconsistent strategies are performed on $T_1$, $T_2$, and $T_4$ rather than on $T_3$. The reason is that putting $T_3$ offline causes larger load losses in most time intervals. As shown in Fig. 7, there are more inconsistent maintenance activities on $T_1$ and $T_2$ around peak periods with and without considering harsh external conditions. There are two main reasons for this. The first reason is that taking $T_1$ and $T_2$ offline does not cause any load loss during peak periods. The second reason is that maintenance strategies are performed around peak periods, reducing multiple faults during peak periods, to avoid potential repair delays caused by harsh external conditions. In addition, we find that inconsistent maintenance activity on $T_4$ mainly occurs around the 11th to 28th time intervals. One reason is no loss of load during these periods, and another reason is to enhance the reliability during peak periods, avoiding potential repair delays.

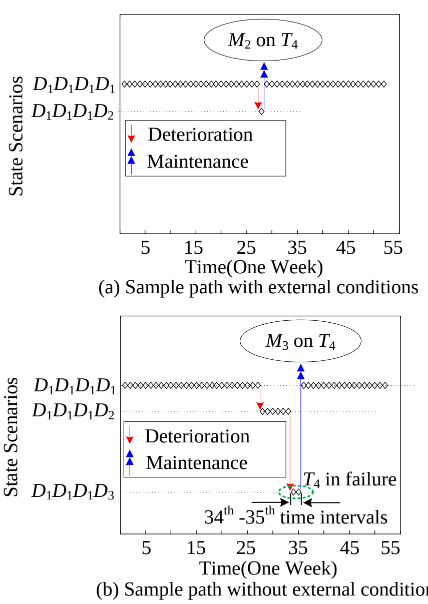


Fig. 5. (a) Sample path of states with external conditions. (b) Sample path of states without external conditions.

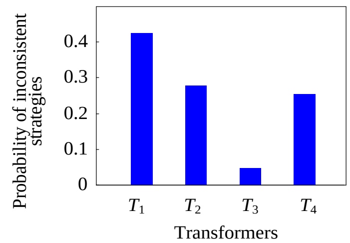


Fig. 6. Distribution of inconsistent strategies on $T_1$, $T_2$, $T_3$, and $T_4$.

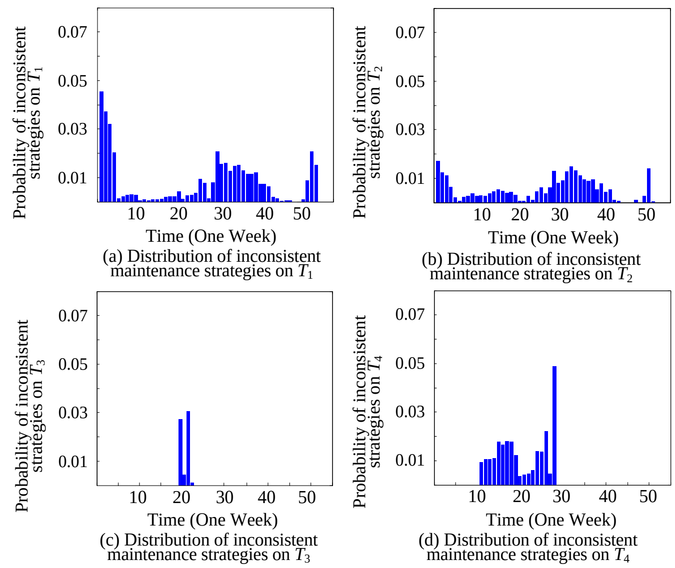


Fig. 7. Distribution of inconsistent maintenance strategies on $T_1$, $T_2$, $T_3$, and $T_4$ over the whole 52-week time interval.

## C. Case II: *Verification of the Search Space Reduction Method*

This case is performed on the IEEE 57-bus system that included ten transformers to show the efficiency of the proposed space reduction method. As shown in Fig. 8(a), 84% of simulation time can be reduced by the search space reduction method. Meanwhile, Fig. 8(b) indicates that the relative errors in the expected costs with and without the search space reduction method satisfy a Gaussian distribution with a zero mean and variance of 0.9, proving that the proposed method with search space reduction maintains good accuracy performance.

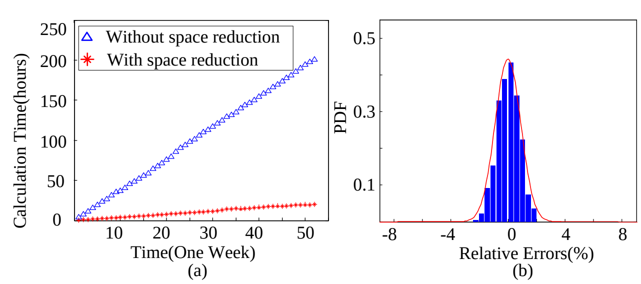


Fig. 8. (a) Simulation time with and without the search space reduction method. (b) PDF of the relative errors of the expected costs.

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### *D.Case III: Results Simulating the Zhejiang Electric Power Grid*

In this section, we implement dynamic optimal maintenance scheduling for an existing system belonging to the Zhejiang Electric Power Grid of China. The installed capacity of the grid is about 70 GW. For brevity, this paper focuses on a sub-network of Zhejiang Electric Power Grid containing six transformers. All data for this case come from Zhejiang Electric Power Grid, and consists of averaged values based on historical data over ten years. Table IV lists the costs of different maintenance and repair activities. Unrepaired probabilities are the same as in Table III. For all transformers, the probabilities for moving from $D_1$ to $D_2$ and from $D_1$ to $D_3$ are 0.055 and 0.005, respectively. The probability of moving from $D_2$ to $D_3$ is 0.015. Fig. 9 shows the loss of load when different transformers have failed. With the established maintenance scheduling, system operators first observe the states of all components at each time interval with monitoring systems, then choose the optimal activities that should be performed corresponding to the observed states.

TABLE IV
COSTS ($10^4$ RMB) OF TRANSFORMERS

| | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ | $T_6$ |
|---|---|---|---|---|---|---|
| $M_1$ | 16 | 18 | 16 | 15 | 16 | 20 |
| $M_2$ | 95 | 100 | 90 | 85 | 90 | 105 |
| Repair | 125 | 120 | 105 | 110 | 100 | 115 |

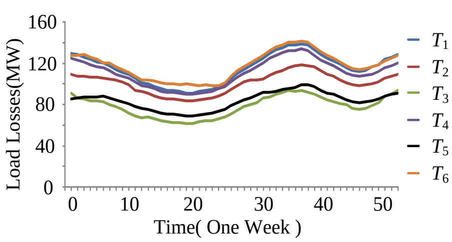

Fig. 9. Loss of load for each transformer failure.

Fig. 10(a) shows a comparison between expected costs caused by the proposed maintenance activities (ECPA) and the expected costs caused by given maintenance activities with regular time intervals (ECGA). Expected costs include maintenance activity costs, loss of load costs, and successive costs. With proposed maintenance activities and the given maintenance activities with regular time intervals, we can use (7), (8), and (10)-(21) to calculate the ECPA and the ECGA, respectively. Fig. 10(b) shows the improvement on expected costs and system reliability when using the ECPA. When the initial states of all transformers are in $D_2$, ECPA is 23% lower. When the initial states of all transformers are all in a good state, i.e., $D_1$, 65% of the expected cost can be reduced. For these two states, we also see that the reliability can also be enhanced. For other states, improvements on expected costs and the reliability (loss of loads) are presented in Fig. 10(b). The results show that the proposed strategy achieves a better balance between the reliability level and maintenance scheduling.

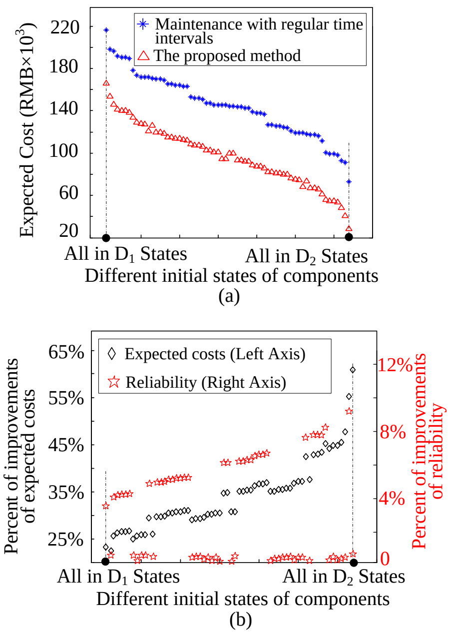

Fig. 10. (a) Expected costs for proposed maintenance activities and given maintenance activities with regular time intervals. (b) Improvements in expected costs and reliability.

## VI. CONCLUSION

Considering the influence of harsh external conditions, this paper establishes a sequentially coordinated condition-based maintenance scheduling with a Markov-based model. The established scheduling achieves better overall performance by coordinating the maintenance activities of multiple components over a time horizon. Highly efficient backward induction with the search space reduction approach is developed to solve the proposed model.

The major findings from the comprehensive case studies on standard and realistic systems are as follows. (1) The harsh external conditions are critical for the maintenance scheduling. When considering influences of harsh external conditions, 6.13 - 9.25% of the expected cost of the maintenance scheduling can be reduced. (2) Sequential coordination of the multiple components over a time horizon can significantly enhance the performance of the system. In realistic Zhejiang Power Grid, the cost may be reduced by over 20% compared with the current regular maintenance scheduling. (3) The proposed search space reduction approach can obtain accurate results within the acceptable computational time. Compared with the full space search method, about 10 times speedup with 0.8% absolute mean error has been achieved.

It is believed that with the development of the condition monitoring technology and the external conditions prediction, the proposed methods is promising for realistic systems.

## REFERENCES

[1] P. Dehghanian, M. Fotuhi-Firuzabad, F. Aminifar, and R. Billinton, “A comprehensive scheme for reliability centered maintenance in power distribution systems—Part I: methodology,” *IEEE Trans. Power Del.*, vol. 28, no. 2, pp. 761-770, Apr. 2013.

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[2] P. Dehghanian, M. Fotuhi-Firuzabad, F. Aminifar, and R. Billinton, "A comprehensive scheme for reliability-centered maintenance in power distribution systems—Part II: numerical analysis," *IEEE Trans. Power Del.*, vol. 28, no. 2, pp.771-778, Apr. 2013.

[3] E. L. Silva, M. T. Schilling, and M. C. Rafael, "Generation maintenance scheduling considering transmission constraints,'' *IEEE Trans. Power Syst.*, vol. 15, no. 2, pp. 838-843, May 2000.

[4] A. J. Conejo, R. Garcia-Bertrand, and M. Diaz-Salazar, "Generation maintenance scheduling in restructured power systems," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 984-992, May 2005.

[5] Y. Fu, S. M. Shahidehpour, and Z. Li, "Security-constrained optimal coordination of generation and transmission maintenance outage scheduling," *IEEE Trans. Power Syst.*, vol. 22, no. 3, pp.1302-1313, Aug. 2007.

[6] J. Endrenyi, G. J. Anders, and A. M. Silva, "Probabilistic evaluation of the effect of maintenance on reliability. an application to power systems," *IEEE Trans. Power Syst.*, vol. 13, no. 2, pp. 576-583, May 1998.

[7] T. M. Welte, "Using state diagrams for modeling maintenance of deteriorating systems," *IEEE Trans. Power Syst.*, vol. 24, no. 1, pp. 58-66, Feb. 2009.

[8] F. Yang and C. S. Chang, "Multiobjective evolutionary optimization of maintenance schedules and extents for composite power systems," *IEEE Trans. Power Syst.*, vol. 24, no. 4, pp. 1694-1702, Nov. 2009.

[9] J. H. Heo, M. K. Kim, G. P. Park, Y. T. Yoon, J. K. Park, S. S. Lee, and D. H. Kim, "A reliability-centered approach to an optimal maintenance strategy in transmission systems using a genetic algorithm," *IEEE Trans. Power Del.*, vol. 26, no. 4, pp. 2171-2179, Oct. 2011.

[10] A. Abiri-Jahromi, M. Parvania, F. Bouffard, and M. Fotuhi-Firuzabad, "A two-stage framework for power transformer asset maintenance management-Part I: models and formulations," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 1395-1403, May 2013.

[11] A. Abiri-Jahromi, M. Parvania, F. Bouffard, and M. Fotuhi-Firuzabad, "A two-stage framework for power transformer asset maintenance management- Part II: validation results," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 1404-1414, May 2013.

[12] NERC's report: Compliance application notice-protection system mainenance and testing evidence. [Online]. Available: http://www.nerc.com

[13] NERC's report: FERC/NERC staff report on the 2011 Southwest cold weather event. [Online]. Available: http://www.nerc.com

[14] IEEE Standard 516-2009: Guide for maintenance methods on energized power lines, Jun. 2009.

[15] IEEE Standard 3007.2-2010: Recommended practice for the maintenance of industrial and commercial power systems, Oct. 2010.

[16] IEEE Standard C57.94-1982: Recommended Practice for Installation, Application, Operation, and Maintenance of Dry-Type General Purpose Distribution and Power Transformers.

[17] IEEE Standard 3007.2-2010: Recommended Practice for the Maintenance of Industrial and Commercial Power Systems.

[18] U. S. Department of Energy's Report: Insurance as a risk management instrument for energy infrastructure security and resilience [Online]. Available: http://energy.gov/sites/prod/files/2013/03/f0/03282013_Final_Insurance_EnergyInfrastructure.pdf

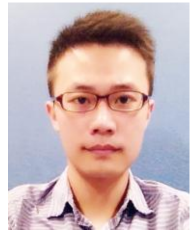
[19] Executive Office of the President's Report: A policy framework for the 21st century grid. [Online]. Available: https://www.whitehouse.gov/sites/default/files/microsites/ostp/nstc-smart-grid-june2011.pdf

[20] Executive Office of the President's Report: Economic benefits of increasing electric grid resilience to weather outages. [Online]. Available: http://energy.gov/sites/prod/files/2013/08/f2/Grid%20Resiliency%20Report_FINAL.pdf

[21] G. Li, P. Zhang, P.B. Luh, W. Li, Z. Bie, C. Serna, and Z. Zhao, "Risk analysis for distribution systems in the northeast U.S. under wind storms," *IEEE Trans. Power Syst.*, vol.29, no.2, pp.889-898, Mar. 2014.

[22] B. Ruszczak and M. Tomaszewski, "Extreme value analysis of wet snow loads on power lines," *IEEE Trans. Power Syst.*, vol. 30, no. 1, pp. 457-462, Jan. 2015.

[23] M. Puterman, *Markov Decision Process*. New York: Wiley, 1994, pp. 92-100.

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