Title	Robust Energy and Reserve Dispatch Under Variable Renewable Generation
Author(s)	Wei, W; Liu, F; Mei, S; Hou, Y
Citation	IEEE Transactions on Smart Grid, 2015, v. 6, p. 369-380
Issued Date	2015
URL	http://hdl.handle.net/10722/216949
Rights	Creative Commons: Attribution 3.0 Hong Kong License

Robust Energy and Reserve Dispatch Under Variable Renewable Generation

Wei Wei, Feng Liu, Shengwei Mei, *Senior Member, IEEE*, and Yunhe Hou

***Abstract*—Global warming and environmental pollution concerns have promoted dramatic integrations of renewable energy sources all over the world. Associated with benefits of environmental conservation, essentially uncertain and variable characteristics of such energy resources significantly challenge the operation of power systems. In order to implement reliable and economical operations, a robust energy and reserve dispatch (RERD) model is proposed in this paper, in which the operating decisions are divided into pre-dispatch and re-dispatch. A robust feasibility constraint set is imposed on pre-dispatch variables, such that operation constraints can be recovered by adjusting re-dispatch after wind generation realizes. The model is extended to more general dispatch decision making problems involving uncertainties in the framework of adjustable robust optimization. By revealing the convexity of the robust feasibility constraint set, a comprehensive mixed integer linear programming based oracle is presented to verify the robust feasibility of pre-dispatch decisions. A cutting plane algorithm is established to solve associated optimization problems. The proposed model and method are applied to a five-bus system as well as a realistic provincial power grid in China. Numeric experiments demonstrate that the proposed methodology is effective and efficient.**

***Index Terms*—Adjustable robust optimization, energy and reserve dispatch, mixed integer linear programming, renewable power generation, uncertainty.**

NOMENCLATURE

A. Uncertainty Set

N_Q Number of loads.

N_W Number of wind farms.

p_m^w Possible generation of wind farm m, uncertain parameter.

p_m^{wl} Minimal output of wind farm m in the dispatch interval.

p_m^{wu} Maximal output of wind farm m in the dispatch interval.

p_m^{we} Forecast output of wind farm m in the dispatch interval.

p_m^{wh} Half of the possible output range of wind farm m in the dispatch interval.

W Vectorized uncertainty set.

W^D Discrete uncertainty set.

$\boldsymbol{w}$ Vector of uncertain parameters.

w_i The ith element of $\boldsymbol{w}$.

w_i^e Forecasted (nominal) value of w_i.

w_i^h Half of the range of w_i.

z_m^+ Binary variable indicating if the output of wind farm m reaches its upper bound in the dispatch interval.

z_m^- Binary variable indicating if the output of wind farm m reaches its lower bound in the dispatch interval.

z_i^+ Binary variable indicating if w_i reaches its upper bound.

z_i^- Binary variable indicating if w_i reaches its lower bound.

Γ^S Budget parameter representing geographical dispersion effects of wind farms in a vast area.

α Confident level when selecting p_m^{wh} and Γ^S.

σ_{m} The variance of generation forecast of wind farm m.

B. Robust Energy and Reserve Dispatch

(a_g, b_g) Generation cost coefficients of unit g.

c_g Reserve cost coefficient of unit g.

d_g^+ Up-regulation cost coefficient of unit g.

d_g^- Down-regulation cost coefficient of unit g.

F_l Power flow limits of transmission line l.

N_G Number of generating units.

P_g^l Minimum output power of unit g.

P_g^u Maximal output power of unit g.

p_g^f Output of unit g before wind generation is known.

p_g^c Output of unit g after actual wind generation is known.

p_q Demand of load q

Δp_g^+ Up-regulation power of unit g after actual wind generation is known.

Δp_g^- Down-regulation power of unit g after actual wind generation is known.

p_m^r Actual output of wind farm $\boldsymbol{m}$.

R_g^+ Ramp up limit of unit g.

R_g^- Ramp down limit of unit g.

r_g Reserve capacity of unit g.

π_{gl} Power flow distribution factor from unit g to line l.

π_{ml} Power flow distribution factor from wind farm m to line l.

Manuscript received November 05, 2013; revised February 10, 2014, April 04, 2014; accepted April 10, 2014. Date of publication July 23, 2014; date of current version December 17, 2014. This work was supported in part by the National Natural Science Foundation of China (No. 51007041), the Foundation for Innovative Research Groups of the National Natural Science Foundation of China (No. 51321005), and the Special grant from EPRI of China (XTB51201303968). paper no. TSG-00830-2013.

W. Wei, F. Liu, and S. Mei are with the State Key Laboratory of Power Systems, Department of Electrical Engineering, Tsinghua University, 100084 Beijing, China (e-mail: wei-wei04@mails.tsinghua.edu.cn).

Y. Hou is with the Department of Electrical and Electronic Engineering, University of Hong Kong, Hong Kong, China.

Digital Object Identifier 10.1109/TSG.2014.2317744

π_{ql}	Power flow distribution factor from load q to line l.
Δt	Time duration of the dispatch interval.

C. Abbreviations

ARO	Adjustable robust optimization.
BLP	Bilinear program.
CP	Cutting plane.
ERD	Extended robust dispatch.
ISO	Independent system operator.
LP	Linear program.
MILP	Mixed integer linear program.
OPF	Optimal power flow.
PHEV	Plug-in hybrid electric vehicle.
RERD	Robust energy and reserve dispatch.
RFR	Robust feasible region.
RO	Robust optimization.
RUC	Robust unit commitment.
SCED	Security-constrained economic dispatch.
SCUC	Security-constrained unit commitment.
SO	Stochastic optimization.
SR	Spinning reserve.
TED	Traditional economic dispatch.

I. INTRODUCTION

DUE to its remarkable advantages in reducing the dependence on fossil fuels and environment protection, renewable energy generation, especially wind power, has been rapidly developed throughout the world [1]. However, unlike conventional synchronous generation, renewable generation is variable with limited controllability and predictability and raises great difficulties on balancing generation and load [2]. Constructing reliable operation strategies is one of the most critical challenges for energy management of power systems integrating large-scale renewable generation.

Traditionally, the energy management problems for power systems can be boiled down to a deterministic optimization subject to necessary operation constraints [3], such as the security-constrained unit commitment (SCUC) [4] and the security-constrained economic dispatch (SCED) [5], provided that the load forecast is accurate. When high-penetration renewable energy is integrated into power systems, to enhance the reliability, more spinning reserve (SR) is engaged, which is often costly. Moreover, operational experiences from ISO confirmed that only adequate SR could not guarantee the operation reliability because it ignores possible transmission congestions after reserve is deployed. This phenomenon will be shown later in this paper. Therefore, a decision-making approach considering both economy and reliability is desired for power system operation. Recently, two trends of such methodologies, the stochastic optimization (SO) and the robust optimization (RO), have drawn much attention.

The SO approaches have been extensively studied during the past decade. Representative achievements include the stochastic SCUC [6]–[8], stochastic OPF [9], demand response scheduling [10], and PHEV charging [11]. The SO provides perfect paradigms for decision problems under uncertainties. However, when complex operation details are considered, identifying accurate probability distributions of uncertain factors and selecting minimal scenarios that sufficiently reflect the impact of uncertainties on the reliability of system operation is another challenging task.

More recently, the RO theory emerged as a powerful tool to manage data uncertainties in optimizations [12]–[14]. It is becoming popular among various decision-making problems in power systems, including planning the power transition of PHEV [15], scheduling demand response [16], transmission network expansion planning [17], robust unit commitment (RUC) [18]–[20], robust optimal power flow (OPF) [21], and energy and reserve scheduling under n-K security criterion [22]. These RO-based approaches have shown appealing features since it allows a distribution-free model of uncertainties, and provides solutions immune against all possible scenarios of uncertainties. In general, the worst case scenario oriented RO approach would be more conservative than the SO approach, because the cost at the worst case scenario may be very high.

According to the current research achievements, it is believed that the SO and the RO provide complementary approaches to deal with uncertainties in decision making. Each of them has its own advantages and drawbacks. For instance, if high-accuracy distribution functions of the underlying uncertainty are available, the SO is usually preferred; otherwise, the RO is a good alternative. An interesting problem is that if only a part of the information on the stochastic nature of uncertainty is available, then how to utilize it to build proper uncertainty sets in RO to reduce its conservativeness. It is revealed in [13] that the uncertainty set W can be built by properly selecting a "budget of uncertainty," then the robust optimal solution that immunizes against all outcomes of the data from W is also feasible for a chance-constrained SO. Another way to reduce the conservativeness of the RO is to divide the decision process into multiple stages, in which some decision variables can be adjusted with respect to the realized uncertainty [14]. Thus, we believe the RO is also promising in practice as its conservativeness can be reduced effectively.

In this paper, we generalize the traditional economic dispatch (TED) problem to a robust energy and reserve dispatch (RERD) problem within the framework of adjustable robust optimization (ARO). Compared with [22], this paper considers the renewable generation uncertainty, which is quite different from contingencies such as generator outage and line tripping. Compared with [18]–[20], the contribution of this paper is threefold:

1) Mathematical Formulation of RERD and Its Extensions: While previous work mainly focuses on unit commitment problems, this work generalizes TED model to RERD model, and further, a general extended robust dispatch (ERD) model, by introducing robust pre-dispatch and adaptive re-dispatch to guarantee operation reliability under variations of renewable generation. The model differs from previous work in two aspects: first, in the robust models of [18]–[20], the worst case cost at the second stage is considered in the objective function. Admittedly, immunizing against the worst case scenarios follows the standard modeling paradigm of RO, but a worst case scenario rarely happens in real operation. The actual realization of the uncertainty is more likely to be near to the nominal scenario in most cases. Motivated by this fact, we consider the re-dispatch (second stage) cost of the nominal scenario in the objective function, while guaranteeing feasibility (security) for all possible realizations of underlying uncertainties, including the worst case scenario. In this way, our model retains the main spirit of robustness in the RO methodology (remain feasible under uncertainties), and may be less conservative around the nominal scenario, at the cost of suffering a higher optimal value at the worst case scenario. Second, slack variables (representing real time load shedding, etc.) are used in [19] to keep the model always feasible. In this paper, the feasibility of RERD is considered to be a prerequisite and is guaranteed by unit commitment decisions. This modeling approach is suitable for power systems when real-time load shedding is not allowed. This is particularly true in many power grids, e.g., those in China.

2) Theoretical Justification of the Robust Feasible Region of Pre-Dispatch: The definition of robust feasibility of a pre-dispatch is presented, and then the robust feasible region (RFR) for the pre-dispatch is defined. Moreover, it is proved that the RFR is a polyhedron and thus convex, providing a theoretical foundation for developing solution algorithms.

3) Cutting Plane Algorithm to Solve Associated Problems: Based on the reformulation-linearization technique, a mixed integer linear program (MILP) oracle is proposed to identify whether a given pre-dispatch is robustly feasible. Compared with the linearization method used in [17], due to the special structure of the robust feasibility checking problem, our reformulation is parameter-free. That is, a pre-specified big-M parameter is not needed any longer. Based on that oracle, a cutting plane algorithm is developed to solve the RERD and the ERD problems.

The rest of this paper is organized as follows. The mathematical formulation of the RERD and its extension, the ERD, are proposed in Section II. The convexity of robust feasibility constraints is proved and the solution algorithm is presented in Section III. Test results on a five-bus systems and realistic Guangdong power grid of China are presented in Section IV. Conclusions are given in Section V.

II. Formulation of RERD and ERD

In this section, TED is shortly reviewed first. Then, the model of RERD is discussed. Finally, the model is extended to a general formulation, the ERD. The concept of robust feasibility is addressed at the end of this section.

A. TED and Countermeasure for Renewable Generation

The TED problem can be formulated as the following deterministic optimization:

$$\min F = \sum_{g=1}^{N_G} \left(a_g p_g^2 + b_g p_g\right) \tag{1a}$$

$$\text{s.t.} \quad P_g^l \le p_g \le P_g^u \quad \forall g \tag{1b}$$

$$\sum_{g=1}^{N_G} p_g = \sum_{q=1}^{N_Q} p_q \tag{1c}$$

$$-F_l \le \sum_{g=1}^{N_G} \pi_{gl} p_g - \sum_{q=1}^{N_Q} \pi_{ql} p_q \le F_l, \quad \forall l \tag{1d}$$

where decision variable $\{p_g\}$ is the output of generators, other parameters are defined in Nomenclature. The objective function (1a) is to minimize the sum of generation cost. Constraints (1b)–(1d) are the generation capacity constraint, the power balance condition and DC power flow constraints, respectively. When renewable generation is integrated into the system, model (1) is simply modified through adding reserve capacity in constraint (1b) and renewable generation term in constraint (1c) and (1d) as follows:

$$\min F = \sum_{g=1}^{N_G} \left(a_g p_g^2 + b_g p_g\right) \tag{2a}$$

$$\text{s.t.} \quad P_g^l + r_g \le p_g \le P_g^u - r_g \quad \forall g \tag{2b}$$

$$\sum_{g=1}^{N_G} p_g + \sum_{m=1}^{N_W} p_m^{we} = \sum_{q=1}^{N_Q} p_q \tag{2c}$$

$$-F_l \le \sum_{g=1}^{N_G} \pi_{gl} p_g + \sum_{m=1}^{N_W} \pi_{ml} p_m^{we} - \sum_{q=1}^{N_Q} \pi_{ql} p_q \le F_l, \quad \forall l \tag{2d}$$

where p_m^{we} is the forecasted generation from wind farm m, r_g is the reserve capacity offered by unit g. In view that the actual wind generation is uncertain, generation-reserve capacity constraint (2b) is used to guarantee generation adequacy. Constraint (2c) and constraint (2d) are the balancing condition and DC power flow limit of each transmission line at the nominal scenario, respectively. When actual renewable generation is observed, TED (2) adopts a readjustment of output p_g within the range determined by the reserve capacity. The key point of constructing model (2) is to identify proper r_g for each unit by making a compromise between the reliability and economy. Previous research has been focused on using probabilistic methods to determine an optimal r_g [23], [24]. If additional information on the distribution of uncertainties is available, these methods may produce less conservative results. If not, industrial applications often utilize much intuitive criteria for large-scale power systems, such as a weighted average assignment

$$r_g = \frac{P_g^u}{\sum_{g=1}^{N_G} P_g^u} R_T \tag{2e}$$

where R_T is the total reserve capacity required to guarantee reliability, such as a fixed percentage of the demand, or the variation range of renewable generation. Constraint (2e) makes the reserve capacity "uniformly" distributed in the grid.

Since the predicted wind generation $\{p_m^{we}\}$ in constraint (2c) and (2d) is constant, TED (2) is still deterministic. However, once the output of generator is changed, the power flows also change, causing transmission congestion. In a word, the feasibility of re-dispatch in TED lacks theoretical guarantees. Thus, it is important for the operators to make more robust decisions to ensure the system reliability. This motivates us to extend the TED model to a robust version.

B. Formulations of RERD

The first key issue of the RERD is to appropriately model the variation of renewable generation. In view of the wind generation's dominant status among renewable energies, this paper takes wind generation as an example without loss of generality. In the RERD, the uncertainty of wind generation is characterized by a set that contains all its possible realizations, which is referred as the uncertainty set. It can be formulated in the similar manner as that in [18]. First, the output range of wind farm m is written as

$$p_m^{wl} \le p_m^w \le p_m^{wu} \quad \forall m \tag{3}$$

where the upper and lower bounds, p_m^{wl} and p_m^{wu}, can be obtained from the statistic information of the forecast or empirical data. For instance, if the predicted output is p_m^{we}, a simple method is to set $p_m^{wl} = (1-\theta)p_m^{we}$ and $p_m^{wu} = (1+\theta)p_m^{we}$, where θ is a constant. If more information on the accuracy of forecast is available, say, the variance of forecast error σ_m, the following Chebyshev's inequality will hold

$$\Pr\left(|p_m^w - p_m^{we}| \ge k\sigma_m\right) \le 1/k^2.$$

If we choose $k = \sqrt{1/(1-\alpha)}$, $p_m^{wl} = p_m^{we} - k\sigma_{\rm m}$ and $p_m^{wu} = p_m^{we} + k\sigma_{\rm m}$, inequality (3) will hold with a probability higher than α.

For the wind farms scattering in a vast area, due to the geographic dispersion effect, the probability that actual outputs of all wind farms reach their upper or lower bounds simultaneously is extremely small. Thus, a spatial constraint on outputs of all wind farms can be imposed as follows:

$$\sum_{m=1}^{N_W} |p_m^w - p_m^{we}| / p_m^{wh} \le \Gamma^S \tag{4}$$

where $p_m^{wh} = 0.5(p_m^{wu} - p_m^{wl}) = k\sigma_{\rm m}$. Inequality (4) restricts the total deviation of wind power injections from their predicted values over spatial scale. The parameters $\Gamma^{\rm S}$ is referred as the "budget of uncertainty" [13]. The tradeoff between the system reliability and operation cost can be implemented through an appropriate selection of Γ^S. The theory and method of budget selection have been well discussed in [13] and [18]. In this work, parameter Γ^S is calculated by equation $\Gamma^S = [\Phi^{-1}(\alpha)\sqrt{M}]$, where the function $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution; α is a given confidence level and the function $[x]$ denotes the nearest integer to the real number x. Specifically, if $x = n + 0.5$, where n is an integer, $[x] = n + 1$.

In the RERD, all constraints are linear, thus the worst case scenario must happen at one of the vertices of the polyhedral uncertainty set (this will be proved in Theorem 1 in the next subsection). It means that in the polyhedron characterized by constraints (3)–(4), we only need to consider its vertices set

$$W^D = \left\{ \begin{array}{l} p_m^w = p_m^{we} + z_m^+ p_m^{wh} \\ \quad -z_m^- p_m^{wh}, \quad \forall m \end{array} \middle| \begin{array}{l} z_m^+, z_m^- \in \{0,1\}, \quad \forall m \\ z_m^+ + z_m^- \le 1, \quad \forall m \\ \sum_{m=1}^{N_W} z_m^+ + z_m^- \le \Gamma^S \end{array} \right\}. \tag{5}$$

The task of RERD is to decide the output and reserve capacity of each generating unit, such that in a certain dispatch interval, no matter how wind generation fluctuates, operating constraints can be satisfied by deploying available reserve resources. The time scale of the RERD is often in less than an hour.

Note that the generation must meet the load on a moment-by-moment basis. So if wind generation deviates from its forecasted value, generators should also be re-dispatched to balance the electric power. In view of this, the RERD is divided into the pre-dispatch stage and the re-dispatch stage. The former uses forecasted wind generation while the latter use actual wind generation. At the same time, reserve capacity constraint couples the output of units in these two stages. Since actual wind generation is not known in advance, all scenarios that possibly happen in the re-dispatch stage should be considered in the pre-dispatch stage.

The RERD problem is described below. The pre-dispatch variables $\{p_g^f, r_g\}$ are the contemporary output and reserve capacity of generators, respectively; the uncertain parameter $\{p_m^w\}$ is the possible output of wind farms; the re-dispatch variables $\{\Delta p_g^+, \Delta p_g^-\}$ are the incremental output of generators after $\{p_m^w\}$ is observed. The RERD problem can be formulated as an ARO as follows:

$$\min_{p_g^f,\, r_g} \quad F = \sum_{g=1}^{N_G} \left(a_g \left(p_g^f\right)^2 + b_g p_g^f + c_g r_g \right) \tag{6a}$$

$$\text{s.t.} \quad p_g^f + r_g \le P_g^u \quad \forall g \tag{6b}$$

$$P_g^l \le p_g^f - r_g \quad \forall g \tag{6c}$$

$$\sum_{g=1}^{N_G} p_g^f + \sum_{m=1}^{N_W} p_m^{we} = \sum_{q=1}^{N_Q} p_q \tag{6d}$$

$$-F_l \le \sum_{g=1}^{N_G} \pi_{gl} p_g^f + \sum_{m=1}^{N_W} \pi_{ml} p_m^{we} - \sum_{q=1}^{N_Q} \pi_{ql} p_q \le F_l, \quad \forall l \tag{6e}$$

$$0 \le r_g \le \min\left\{R_g^- \Delta t,\ R_g^+ \Delta t\right\} \quad \forall g \tag{6f}$$

$$\forall \{p_m^w\} \in W^D, \exists \{\Delta p_g^+, \Delta p_g^-\} \text{ such that}$$

$$0 \le \Delta p_g^+ \le r_g,\ 0 \le \Delta p_g^- \le r_g \quad \forall g \tag{6g}$$

$$p_g^c = p_g^f + \Delta p_g^+ - \Delta p_g^- \quad \forall g \tag{6h}$$

$$\sum_{g=1}^{N_G} p_g^c + \sum_{m=1}^{N_W} p_m^w = \sum_{q=1}^{N_Q} p_q \tag{6i}$$

$$-F_l \le \sum_{g=1}^{N_G} \pi_{gl} p_g^c + \sum_{m=1}^{N_W} \pi_{ml} p_m^w - \sum_{q=1}^{N_Q} \pi_{ql} p_q \le F_l, \quad \forall l \tag{6j}$$

where the uncertainty set W^D is defined in (5). The parameters are defined in Nomenclature. The objective function (6a) is to minimize the sum of generation cost and reserve cost. The quadratic term can be linearized by using the piecewise linear approximation technique given in [25] if a nonlinear solver is not preferred. The pre-dispatch constraints consist of (6b)–(6f), among which (6b) and (6c) are the generation capacity constraints considering reserve capacity; (6d) and (6e) are the power balance constraint and DC power flow constraint corresponding to the normal (forecasted) scenario; inequality (6f) ensures that unit g has enough ramp capability to offer its assigned reserve capacity r_g in the dispatch interval. Due to the presence of uncertainty $\{p_m^w\}$ restricted in the set W^D, the existence of re-dispatch action $\{\Delta p_g^+, \Delta p_g^-\}$ must be guaranteed for all possible realizations of wind generation. Therefore, re-dispatch constraints (6g)–(6j) are imposed, among which (6g) restricts the incremental output of unit g within its reserve capacity during the re-dispatch stage; constraint (6h) represents the output of unit g after re-dispatch; constraints (6i) and (6j) are the power balance condition and the DC power flow limit on each of the transmission lines. Constraints (6g)–(6j) ensures there is at least one re-dispatch action $\{\Delta p_g^+, \Delta p_g^-\}$ that satisfies operating constraints for every possible wind generation scenario $\{p_m^w\} \in W^D$.

Provided the feasibility of re-dispatch, the optimal re-dispatch model with respect to the observed wind generation $\{p_m^r\}$ is as follows:

$$\begin{aligned}
\min_{\Delta p_g^+, \Delta p_g^-, p_g^c} \quad & \sum_{g=1}^{N_G} \left(d_g^+ \Delta p_g^+ + d_g^- \Delta p_g^-\right) \\
\text{s.t.} \quad & 0 \le \Delta p_g^+ \le r_g,\ 0 \le \Delta p_g^- \le r_g \quad \forall g \\
& p_g^c = p_g^f + \Delta p_g^+ - \Delta p_g^- \quad \forall g \\
& \sum_{g=1}^{N_G} p_g^c + \sum_{m=1}^{N_W} p_m^r = \sum_{q=1}^{N_Q} p_q \\
& -F_l \le \sum_{g=1}^{N_G} \pi_{gl} p_g^c + \sum_{m=1}^{N_W} \pi_{ml} p_m^r - \sum_{q=1}^{N_Q} \pi_{ql} p_q \le F_l, \\
& \forall l.
\end{aligned} \tag{6k}$$

Linear program (LP) (6k) is to minimize the actual re-dispatch cost (the sum of up- and down-regulation costs), and recover operation constraints for observed wind generation $\{p_m^r\}$ on the basis of pre-dispatch strategy $\{p_g^f, r_g\}$ offered by the RERD (6a)–(6j). LP (6k) is always feasible as its constraints have been considered in (6g)–(6j) for all possible $\{p_m^r\}$.

Comparing the TED model (2) with the RERD model (6), the essential difference is that the reserve capacity $\{r_g\}$ in the RERD is also decision variable. Moreover, re-dispatch constraints for all possible wind generation scenarios have been considered in pre-dispatch stage. As a result, when $\{p_m^r\}$ is observed or can be predicted accurately, the output of each generator will be adaptively adjusted. Above features distinguish the RERD from the TED, and endow RERD with robustness under uncertainties, as well as the flexibility to cope with uncertainties of renewable generations.

C. Model Extensions

The mathematical model of the RERD can be written in a compact form of an ARO model

$$\min \quad \boldsymbol{c}^T \boldsymbol{x} + \boldsymbol{d}^T \boldsymbol{y}^0 \tag{7a}$$

$$\boldsymbol{s.t.}\ \boldsymbol{A x} + \boldsymbol{B y}^0 \le \boldsymbol{b} - \boldsymbol{C w}^0, \quad \boldsymbol{x} \in X, \tag{7b}$$

$$\forall \boldsymbol{w} \in W, \exists \boldsymbol{y}: \quad \boldsymbol{B y} \le \boldsymbol{b} - \boldsymbol{C w} - \boldsymbol{A x} \tag{7c}$$

where $\boldsymbol{x}$ is the pre-dispatch decision vector; $\boldsymbol{y}$ the re-dispatch decision vector; $\boldsymbol{y}^0$ the re-dispatch decision vector associated with the nominal scenario ($\boldsymbol{w} = \boldsymbol{w}^0$). Constraint (7b) is the feasible set of pre-dispatch under the nominal scenario. Constraint (7c) is the feasible set of re-dispatch for a fixed $\boldsymbol{w}$. Pre-dispatch $\boldsymbol{x}$ should ensure that constraint (7c) is feasible for all $\boldsymbol{w} \in W$, where the set W is defined as

$$W = \left\{ \boldsymbol{w} \,\middle|\, \begin{array}{c} w_i = w_i^e + \left(z_i^+ - z_i^-\right) w_i^h, \quad \forall i \\ (\boldsymbol{z}^+, \boldsymbol{z}^-) \in Z \end{array} \right\}$$

$$Z = \left\{ (\boldsymbol{z}^+, \boldsymbol{z}^-) \,\middle|\, \begin{array}{c} z_i^+, z_i^- \in \{0,1\}, \quad \forall i \\ z_i^+ + z_i^- \le 1, \quad \forall i \\ \sum_i z_i^+ + z_i^- \le \Gamma^S \end{array} \right\}. \tag{7d}$$

The following theorem indicates the equivalence of using the discrete set W defined in (7d) and using the continuous uncertainty set defined by $\mathbf{Conv}(W)$, where $\mathbf{Conv}(\cdot)$ denote the convex hull of a set.

Theorem 1: Suppose that a pre-dispatch $\boldsymbol{x}$ is feasible in (7) with a discrete uncertainty set W defined in (7d), it is also feasible for all $\boldsymbol{w} \in \mathbf{Conv}(W)$.

Proof: Denote all elements in W by $\boldsymbol{w}^1, \boldsymbol{w}^2, \ldots, \boldsymbol{w}^N$, where N is the number of elements in W. Because $\boldsymbol{x}$ is feasible in constraint (7c) for W; thus, there is corresponding re-dispatch $\boldsymbol{y}^1, \boldsymbol{y}^2, \ldots, \boldsymbol{y}^N$, such that

$$\begin{aligned}
&\boldsymbol{B y}^1 \le \boldsymbol{b} - \boldsymbol{A x} - \boldsymbol{C w}^1 \\
&\boldsymbol{B y}^2 \le \boldsymbol{b} - \boldsymbol{A x} - \boldsymbol{C w}^2 \\
&\vdots \\
&\boldsymbol{B y}^N \le \boldsymbol{b} - \boldsymbol{A x} - \boldsymbol{C w}^N.
\end{aligned}$$

The following inequality holds for $\forall\{\lambda^1, \lambda^2, \ldots, \lambda^{\mathrm{N}}\}$ belongs to an N-simplex (defined as $\{\lambda \in R^N | \lambda \geqslant \mathbf{0}, \mathbf{1}^{\mathrm{T}}\lambda = 1\}$)

$$\sum_{i=1}^{N} \lambda_i \boldsymbol{B y}^i \le \sum_{i=1}^{N} \boldsymbol{\lambda}_i (\boldsymbol{b} - \boldsymbol{A x} - \boldsymbol{C w}^i).$$

That is

$$\boldsymbol{B} \sum_{i=1}^{N} \lambda_i \boldsymbol{y}^i \le \boldsymbol{b} - \boldsymbol{A x} - \boldsymbol{C} \sum_{i=1}^{N} \lambda_i \boldsymbol{w}^i.$$

This implies $\sum_{i=1}^{N} \lambda_i \boldsymbol{y}^i$ is the re-dispatch strategy with respect to $\sum_{i=1}^{N} \lambda_i \boldsymbol{w}^i \in \mathbf{Conv}(W)$; thus, $\boldsymbol{x}$ is also feasible for arbitrary elements in $\mathbf{Conv}(W)$. This completes the proof. ■■

Theorem 1 indicates that if we intend to design a robust pre-dispatch $\boldsymbol{x}$ for model (7) with a polyhedral uncertainty set, we only need to consider all its vertices.

In ARO (7), it requires that pre-dispatch decision $\boldsymbol{x}$ should be made without accurate information of $\boldsymbol{w}$, but for any possible realization of $\boldsymbol{w}$, there should be at least one re-dispatch decision $\boldsymbol{y}$ that satisfies constraint (7c). Clearly, the admissible re-dispatch set (7c) is parameterized by $\boldsymbol{x}$ and $\boldsymbol{w}$ and can be written as follows:

$$Y(\boldsymbol{x}, \boldsymbol{w}) = \{\boldsymbol{y} \mid \boldsymbol{B}\boldsymbol{y} \le \boldsymbol{b} - \boldsymbol{C}\boldsymbol{w} - \boldsymbol{A}\boldsymbol{x}\}. \tag{8}$$

The optimal actual re-dispatch action $\boldsymbol{y}^r$ associated with the realized uncertainty $\boldsymbol{w}^r$ can be determined from the following LP:

$$\begin{aligned} \boldsymbol{y}^r &= \arg\min_{y} \boldsymbol{d}^T \boldsymbol{y} \\ \text{s.t.} \quad & \boldsymbol{y} \in Y(\boldsymbol{x}, \boldsymbol{w}^r). \end{aligned} \tag{9}$$

Since ARO (7) considers every vertex of the set W, according to Theorem 1, LP (9) is always feasible provided $\boldsymbol{x}$ is a feasible solution of (7). This important feature epitomizes the concept of *robust feasibility* that is defined as follows.

Definition 1 (Robust Feasibility): A pre-dispatch $\boldsymbol{x}$ is said to be robustly feasible in model (7) if and only if $\forall \boldsymbol{w} \in W, Y(\boldsymbol{x}, \boldsymbol{w}) \neq \emptyset$.

Moreover, the robust feasible region (RFR) for $\boldsymbol{x}$ can be further defined as follows.

Definition 2 (RFR): The RFR of pre-dispatch $\boldsymbol{x}$ is defined as

$$X_R = \{\boldsymbol{x} \in X \mid \forall \boldsymbol{w} \in W,\ Y(\boldsymbol{x}, \boldsymbol{w}) \neq \emptyset\}. \tag{10}$$

With the help of RFR, model (7) can be written as

$$\begin{aligned} \min \quad & \boldsymbol{c}^T \boldsymbol{x} + \boldsymbol{d}^T \boldsymbol{y}^0 \\ & \text{s.t. } \boldsymbol{x} \in X \cap X_R. \end{aligned} \tag{11}$$

Model (11) is called an extended robust dispatch (ERD) problem. Note that because $\boldsymbol{w}^0 \in W$, so constraint (7b) has already been included in X_R. Under the following notations and conventions: $\boldsymbol{x} = \{p_g^f, r_g\}$ $\boldsymbol{y} = \{\Delta p_g^+, \Delta p_g^-\}, \boldsymbol{w}^0 = \{p_m^{we}\}, \boldsymbol{w} = \{p_m^w\}, \boldsymbol{y}^0 = \boldsymbol{0}, X = R^{2N_G}$, (7a) represents (linearized) objective function (6a), (7b) represents constraints (6b)–(6f), (7c) represents constraints (6g)-(6j), ERD (7) (or its equivalent form (11)) realizes as an RERD problem. If a unit commitment problem under uncertain renewable generation is considered, the pre-dispatch decisions will be the commitment schedule, and the re-dispatch decisions will be the generation output. In such situation, ERD (11) renders a special case of RUC that is slightly different from the RUC models proposed in [18]–[20], because the second stage cost at the nominal scenario is considered in the former's objective function.

In general, the ERD is a class of semi-infinite programs. The algorithm to solve (11) will be given in the coming section. There is something more need to be clarified regarding the modeling framework of the ERD:

1) Regarding the variable $\boldsymbol{y}^0$. The decision variable $\boldsymbol{y}^0$ is re-dispatch with respect to the nominal scenario, so ERD (11) is not identical as the ARO model proposed in [12], because the latter does not consider the second stage re-dispatch cost, which will influence the pre-dispatch decision $\boldsymbol{x}$. In practice, only $\boldsymbol{x}$ is deployed in the pre-dispatch stage, while the actual re-dispatch $\boldsymbol{y}^r$ will be determined through LP (9) and deployed after uncertainty is realized as $\boldsymbol{w}^r$.

2) Regarding the objective function and conservativeness. Since the actual optimal re-dispatch cost $\boldsymbol{d}^{\mathrm{T}}\boldsymbol{y}^r$ hinges on the revealed uncertainty $\boldsymbol{w}^r$, which is not known in advance, how to deal with the re-dispatch cost in the objective function depends on the decision maker's attitude. For instance, the weighted average of re-dispatch costs with respect to a number of scenarios is used in the SO models, while the worst case re-dispatch cost is used in the RO models of [18]–[20]. Here we alternatively use the re-dispatch cost $\boldsymbol{d}^{\mathrm{T}}\boldsymbol{y}^0$ at the nominal scenario in the objective function (7a) based on two considerations. On the one hand, the probability that a worst case scenario happens in reality is extremely low, the actual realization of uncertainty $\boldsymbol{w}^r$ is more likely to be "near" to the nominal scenario $\boldsymbol{w}^0$. Thus our model will be less conservative around the nominal scenario in general. On the other hand, if a worst case scenario really happens, the total cost of our model will probably be higher than those of the worst case oriented RO models in [18]–[20], but the feasibility of constraint (7c) still can be guaranteed by deploying a proper re-dispatch $\boldsymbol{y}$ with the pre-dispatch $\boldsymbol{x}$ unchanged. This is the critically important task in the case of contingencies, and retains the essential spirit of RO methods. To strike a proper balance between the reliability and economy, an alternative choice is to use the combination of both SO and RO model [26].

3) Regarding the none-emptiness of X_R. Different from the model in [19] adding slack variables (representing real time load shedding, etc.) in the second stage to keep the model always feasible, in the RERD, load shedding is not allowed. This is particularly true in many power grids, such as those in China, in which real-time load shedding is identified as an operating failure. In the RERD, the none-emptiness of X_R is a prerequisite and is guaranteed by unit commitment decisions. If ERD (11) is applied to the unit commitment problem, its feasibility should be guaranteed by generation expansion planning decisions. In some extreme cases, the load shedding in RERD is inevitable. However, the probabilities of such cases are very low. In such situations, unit commitment should be recalculated to arrange more reserve capacity, or start fast-response units as additional available re-dispatch resource of the RERD to circumvent infeasibility. It should be noticed that load shedding is also non-preferred in the RUC model in [19]. This is implemented through adding penalty terms with a large enough parameter M in the objective function, so that the slack variables in the optimal solution will forced to zero. That is, load shedding is not used unless it is inevitable.

4) What happens if $w^r \notin W$. When the realization of uncertainty w^r is not included in the uncertainty set W in RERD (6) and ERD (11), the feasibility of re-dispatch cannot be guaranteed in theory, but it may be still feasible in practice. In other

words, that uncertainty is realized within the uncertainty set is only a sufficient (not necessary) condition that the feasibility of operation constraints can be recovered in re-dispatch. Therefore, for risk-aversive decision makers, it is important to include all possible scenarios that may occur in the uncertainty set W.

III. Convexity of X_R and Solution Algorithm

To solve the proposed RERD (6) and ERD (11), the convexity of X_R is proved first. Then it is revealed that the robust feasibility checking problem can be formulated as a game between the uncertainties associated with wind power and the system operator. A reformulation-linearization oracle is then proposed to transform the game into a parameter-free MILP. Based on the above results, a cutting plane based algorithm is established to solve the whole problem.

A. Convexity of X_R

Since the set X_R defined in (10) does not have a closed form in general, ERD (11) cannot be solved directly. An alternative way is to generate cutting plane approximation for X_R. However, if X_R is non-convex, cutting planes may remove feasible points from X_R. Fortunately, the following theorem indicates that X_R is a polyhedron, so is convex.

Theorem 2: Suppose that both $\boldsymbol{x}$ and $\boldsymbol{y}$ are continuous, W is described by (7d) and $X_R \neq \emptyset$, then X_R is a polyhedron.

Proof: Consider the set

$$\Theta = \underset{w \in W}{\cap} \{(\boldsymbol{x}, \boldsymbol{y}) \mid \boldsymbol{Ax} + \boldsymbol{By} \leq \boldsymbol{b} - \boldsymbol{Cw}\}.$$

So Θ is the intersection of a finite number of polyhedral sets; hence, Θ is also a polyhedron. Moreover, set X_R is the projection of Θ onto the subspace that $\boldsymbol{x}$ lies in, so X_R is also a polyhedron because the projection operation is a linear mapping. This completes the proof. ■ ■

Theorem 1 reveals that the set X_R is a polyhedron without providing an analytical expression. The cutting plane (CP) method [27], [28] can be used to generate a sequence of linear inequalities to approximate X_R. Consequently, the problem turns to be: how to verify whether $\boldsymbol{x} \in X_R$ or not. If not, how to generate feasibility cuts. From Definition 1, this primarily refers to validate the none-emptiness of $Y(\boldsymbol{x}, \boldsymbol{w})$ for all $\boldsymbol{w} \in W$.

B. Robust Feasibility Formulation and Feasibility Cuts

First of all, to detect if $Y(\boldsymbol{x}, \boldsymbol{w})$ is empty for a fixed $\boldsymbol{w} \in W$, two positive slack vectors, $\boldsymbol{s}^+$ and $\boldsymbol{s}^-$, which can be viewed as emergency measures to prevent the system from failure, are augmented, deriving the following LP:

$$\begin{aligned} r(\boldsymbol{x}^*, \boldsymbol{w}) = & \min_{y, s^+, s^-} \mathbf{1}^T \boldsymbol{s}^+ + \mathbf{1}^T \boldsymbol{s}^- \\ \text{s.t.} \quad & \boldsymbol{By} + \boldsymbol{Is}^+ - \boldsymbol{Is}^- \leq \boldsymbol{b} - \boldsymbol{Cw} - \boldsymbol{Ax}^* \end{aligned} \tag{12}$$

where the optimal solution $r(\boldsymbol{x}^*, \boldsymbol{w})$ is the sum of slack variables for a fixed $\boldsymbol{w}$. $r(\boldsymbol{x}^*, \boldsymbol{w}) = 0$ means $Y(\boldsymbol{x}^*, \boldsymbol{w}) \neq \emptyset$ and emergency measures are not necessary. $r(\boldsymbol{x}^*, \boldsymbol{w}) > 0$ implies $Y(\boldsymbol{x}^*, \boldsymbol{w}) = \emptyset$ and emergency measures must be deployed. To check the robustness of $\boldsymbol{x}^*$, we need to investigate the worst influence of $\boldsymbol{w}$ on the feasibility of LP (12). It is natural to model $\boldsymbol{w}$ as a player to represent the nature who wants to maximize $r(\boldsymbol{x}^*, \boldsymbol{w})$ on W, and the system operator as the opposite player who deploys the best defensive strategy with respect to the worst case attack $\boldsymbol{w}$ to avoid of using emergency measures. As a result, the robust feasibility problem can be viewed as a game

$$\begin{aligned} R(\boldsymbol{x}^*) = & \max_{w \in W} r(\boldsymbol{x}^*, \boldsymbol{w}) = \max_{w \in W} \min_{y, s^+, s^-} \mathbf{1}^T \boldsymbol{s}^+ + \mathbf{1}^T \boldsymbol{s}^- \\ \text{s.t.} \quad & \boldsymbol{By} + \boldsymbol{Is}^+ - \boldsymbol{Is}^- \leq \boldsymbol{b} - \boldsymbol{Cw} - \boldsymbol{Ax}^*. \end{aligned} \tag{13}$$

This type of game was studied in [29]. Here we use it to investigate the worst impact of the uncertainty on the feasibility of power system operation. Similar to LP (12), $Y(\boldsymbol{x}, \boldsymbol{w}) \neq \emptyset$ holds if and only if $R(\boldsymbol{x}^*) = 0$. To convert game (13) into a standard optimization problem, the dual formulation of LP (12) is used [29]

$$\begin{aligned} r(\boldsymbol{x}^*, \boldsymbol{w}) = & \max_{u} \boldsymbol{u}^T (\boldsymbol{b} - \boldsymbol{Cw} - \boldsymbol{Ax}^*) \\ \text{s.t.} \quad & \boldsymbol{u}^T \boldsymbol{B} \leq \mathbf{0}^T, -\mathbf{1}^T \leq \boldsymbol{u}^T \leq \mathbf{1}^T, \boldsymbol{u}^T \leq \mathbf{0}^T \end{aligned} \tag{14}$$

where $\boldsymbol{u}$ is the dual variable. Because $\boldsymbol{u}$ is bounded, LP (14) is always feasible and has a finite optimum. Substituting LP (14) into game (13) results in the following bilinear program (BLP)

$$\begin{aligned} R(\boldsymbol{x}^*) = & \max_{w \in W, u} \boldsymbol{u}^T (\boldsymbol{b} - \boldsymbol{Ax}^*) - \boldsymbol{u}^T \boldsymbol{Cw} \\ \text{s.t.} \quad & \boldsymbol{u}^T \boldsymbol{B} \leq \mathbf{0}^T, \quad -\mathbf{1}^T \leq \boldsymbol{u}^T \leq \mathbf{0}^T. \end{aligned} \tag{15}$$

As long as $R(\boldsymbol{x}^*)$ is computed, we can determine the robustness of $\boldsymbol{x}^*$. If $R(\boldsymbol{x}^*) = 0$, the pre-dispatch decision $\boldsymbol{x}^*$ is robustly feasible and there would be certain valid re-dispatch $\boldsymbol{y}$ to recover operation constraints for any $\boldsymbol{w} \in W$. If $R(\boldsymbol{x}^*) > 0, \boldsymbol{x}^*$ is not robustly feasible and should be changed to guarantee the system security under uncertainties. It is worth mentioning that $R(\boldsymbol{x}^*)$ can be regarded as an index for the level of risk, or the quantity of emergency measures should be taken under the worst case scenarios to prevent the system from collapse. However, it's usually difficult to acquire the global optimum of BLP (15) since it is non-convex. To circumvent this difficulty, BLP (15) is transformed into an MILP following the paradigm of reformulation-linearization technique [30].

In the reformulation phase, considering $\boldsymbol{w}$ defined by (7d) and defining auxiliary continuous variable $v_{ij}^+ = u_i z_j^+$ and $v_{ij}^- = u_i z_j^-$, BLP (15) is equivalent to

$$\begin{aligned} R(\boldsymbol{x}^*) = & \max \boldsymbol{u}^T (\boldsymbol{b} - \boldsymbol{Ax}^*) \\ & - \sum_i \sum_j c_{ij} \left(u_i w_j^e + v_{ij}^+ w_j^h - v_{ij}^- w_j^h \right) \\ \text{s.t.} \quad & \boldsymbol{u} \in U, \quad \{\boldsymbol{z}^+, \boldsymbol{z}^-\} \in Z, \\ & v_{ij}^+ = u_i z_j^+, \quad v_{ij}^- = u_i z_j^-, \quad \forall i, j \end{aligned} \tag{16}$$

where $U = \{\boldsymbol{u} \mid \boldsymbol{u}^T \boldsymbol{B} \leq 0, -\mathbf{1} \leq \boldsymbol{u} \leq \mathbf{0}\}$, Z is defined in (7d).

In the linearization phase, bilinear terms v_{ij}^+ and v_{ij}^- are replaced by their convex and concave envelopes. This is called the McCormick envelopes [31]. For example, because $-1 \leq$

$u_i \leq 0, z_j^+ \in \{0,1\}$, performing this operation on $v_{ij}^+ = u_i z_j^+$ results in the following four inequalities:

$$v_{ij}^+ \leq 0,\ u_i \leq v_{ij}^+,\ -z_j^+ \leq v_{ij}^+,\ v_{ij}^+ \leq -z_j^+ + u_i + 1. \quad (17)$$

Although the McCormick envelopes give convex relaxations and underestimations for general BLPs, the relaxation performed in (17) is also tight. It can be seen from (17) that if $z_j^+ = 1, v_{ij}^+ = u_i$; else if $z_j^+ = 0, v_{ij}^+ = 0$. The same procedure is applied to $v_{ij}^- = u_i z_j^-$. Finally, BLP (15) is transformed into the following equivalent MILP:

$$\begin{aligned} R(\boldsymbol{x}^*) = &\max \boldsymbol{u}^T(\boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}^*) \\ &- \sum_i \sum_j c_{ij} \left(u_i w_j^e + v_{ij}^+ w_j^h - v_{ij}^- w_j^h \right) \\ \text{s.t.}\quad & \boldsymbol{u} \in U,\ \{\boldsymbol{z}^+, \boldsymbol{z}^-\} \in Z, \\ & v_{ij}^+ \leq 0,\ u_i \leq v_{ij}^+, \\ & -z_j^+ \leq v_{ij}^+,\ v_{ij}^+ \leq -z_j^+ + u_i + 1, \quad \forall i, \forall j \\ & v_{ij}^- \leq 0,\ u_i \leq v_{ij}^- \\ & -z_j^- \leq v_{ij}^-,\ v_{ij}^- \leq -z_j^- + u_i + 1, \quad \forall i, \forall j \end{aligned} \quad (18)$$

An important feature of the proposed method is that the MILP (18) is parameter-free. More precisely, a pre-specified big-M parameter is not needed compared with some big-M based reformulations. Sometimes selecting an appropriate big-M parameter is not trivial [32], either because the bounds of dual variables are unknown or some decision variables are unbounded. In this paper, due to the special structure of LP (12), bound constraints of dual variables are explicitly derived in LP (14). This is important for constructing an MILP that is parameter-free. Noting that to eliminate each bilinear term in (16), one continuous variable and four linear constraints are introduced. Since matrix $\boldsymbol{C}$ is usually very sparse, the number of additional variables and constraints are not large.

If the optimal solution of MILP (18) gives $R(\boldsymbol{x}^*) > 0$, implying that $\boldsymbol{x}^*$ is not robustly feasible, a linear inequality to remove non-robust feasible points around $\boldsymbol{x}^*$ is needed. Another interpretation is that one can adjust $\boldsymbol{x}^*$ along the direction that makes $R(\boldsymbol{x}^*)$ decrease, until $R(\boldsymbol{x}^*) \leq 0$ is reached. The sub-gradient of $R(\boldsymbol{x})$ at $\boldsymbol{x}^*$ is

$$\boldsymbol{s}_g = -\boldsymbol{u}^{*T} \boldsymbol{A}.$$

Then the feasibility cut can be simply constructed by

$$R(\boldsymbol{x}^*) + \boldsymbol{s}_g(\boldsymbol{x} - \boldsymbol{x}^*) \leq 0. \quad (19)$$

Or in a standard form

$$\boldsymbol{s}_g \boldsymbol{x} \leq s_g \boldsymbol{x}^* - R(\boldsymbol{x}^*). \quad (20)$$

C. Overall Algorithm for Solving ERD

To solve RERD (6) and ERD (11), X_R is first relaxed and approximated by successively generating a sequence of feasibility cuts. The overall algorithm is presented below.

Algorithm 1: Cutting plane algorithm for ERD

Step 1 (*Initialization*): Set parameters: $k = 0, \boldsymbol{x}^0 = \boldsymbol{0}, \boldsymbol{s}_g^0 = \boldsymbol{0}, R^0 = 0$;

Step 2 (*Pre-dispatch*): Solve the relaxed ERD problem

$$\begin{aligned} \min\quad & \boldsymbol{c}^T \boldsymbol{x} + \boldsymbol{d}^T \boldsymbol{y}^0 \\ \text{s.t.}\quad & \boldsymbol{x} \in X,\ \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y}^0 \leq \boldsymbol{b} - \boldsymbol{C}\boldsymbol{w}^0 \\ & \boldsymbol{s}_g^l \boldsymbol{x} \leq -R^l + \boldsymbol{s}_g^l \boldsymbol{x}^l \quad \forall 0 \leq l \leq k \end{aligned} \quad (21)$$

$k = k + 1$. Let $\boldsymbol{x}^k$ be the temporal optimal solution.

Step 3 (*Robust feasibility checking*): Solve MILP (18). Let $\boldsymbol{u}^k$ be the current optimal solution, and R^k the current optimal value. If $R^k = 0, \boldsymbol{x}^k$ is the final optimal solution of ERD and go to step 4; else generate the sub-gradient

$$\boldsymbol{s}_g^k = -(\boldsymbol{u}^k)^T \boldsymbol{A} \quad (22)$$

and a feasibility cut

$$\boldsymbol{s}_g^k \boldsymbol{x} \leq -R^k + \boldsymbol{s}_g^k \boldsymbol{x}^k \quad (23)$$

Augment (23) into the relaxed ERD problem (21) as a constraint. Go to step 2.

Step 4 (*Re-dispatch*): Wait until uncertainty $\boldsymbol{w}$ is observed or can be forecasted with high accuracy, solve LP (9) with $\boldsymbol{w}^r$ and deploy $\boldsymbol{y}^r$.

Remarks: 1) Theorem 2 requires that the pre-dispatch decision $\boldsymbol{x}$ should be continuous. However, if there is discrete decision variable in pre-dispatch $\boldsymbol{x}$, Algorithm 1 also applies, because most spirit of Theorem 2 remains for the convex hull of X_R. However, if there are discrete variables in the re-dispatch $\boldsymbol{y}$, Algorithm 1 is no longer valid since the game (13) of robust feasibility identification and BLP (15) is no longer equivalent because strong duality condition does not hold for problem (12) and (14).

2) In case that the objective function of (11) is not linear but remains convex, the global optimum of the relaxed problem (21) in Algorithm 1 can be computed by using a mixed integer convex program algorithm. Other steps remain the same. Particularly, if a convex quadratic cost function is used, the *cplexmiqp* solver in CPLEX is capable to solve problem (21).

3) Problem (7) also can be solved by using a column-and-constraint generation (C&CG) method proposed in [33]. In this method, a critical scenario is identified in each iteration, and then new primal variables and constraints are generated and added in the master problem. It seems that incorporating C&CG cuts in the first 1–2 iterations may enhance the overall computational performance of Algorithm 1.

IV. Case Studies

To validate the effectiveness and efficiency of the proposed model and algorithm, numeric experiments on a simple five-bus system and the realistic Guangdong power grid of China are carried out. CPLEX 12.2 is used to solve related LP and MILP

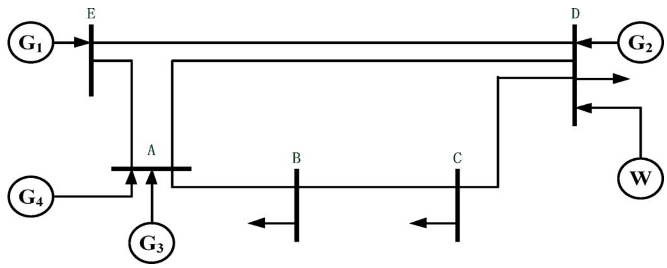

Fig. 1. Topology of the five-bus system.

TABLE I
PARAMETERS OF GENERATORS

Unit NO.	P_{min}/P_{max} MW	Offering Price ￥/MWh	Reserve Price ￥/MWh	Ramp MW/h
G_1	[180, 400]	200	300	100
G_2	[100, 300]	300	450	60
G_3	[150, 600]	360	540	150
G_4	[120, 500]	250	400	120

TABLE II
PARAMETERS OF TRANSMISSION LINES

Line NO.	From Node	To Node	Reactance p.u.	Transmission Limit MW
L_1	A	B	0.0281	600
L_2	A	D	0.0304	300
L_3	A	E	0.0064	200
L_4	B	C	0.0108	300
L_5	C	D	0.0297	420
L_6	D	E	0.0297	300

TABLE III
DISPATCH DECISIONS GIVEN BY TED

	G_1	G_2	G_3	G_4
Output / MW	395.6	195.4	156.7	452.3
Reserve / MW	4.44	3.33	6.67	5.56

problems. All case studies are conducted on a PC with Intel(R) Core(TM) 2Duo 2.2 GHz CPU and 4 GB memory.

A. A Rudimental 5-Bus System

The topology of the five-bus system is shown in Fig. 1. Four thermal units are connected to the grid at Bus A, D, and E. The load demand at Bus B, C, and D are 550 MW, 450 MW, and 350 MW, respectively, and assumed to be deterministic. The parameters of the generators and transmission lines are given in Tables I and II. A wind farm is connected to the grid at Bus D. The wind generation forecast is 150 MW, and the lower and upper bounds of wind farm output are assumed to be 130 MW and 170 MW, respectively. However, because the probability distribution of wind generation is unknown, the SO approach is not applicable. For comparison, both the TED and the RERD are tested.

Case 1 (TED): The TED involves two basic steps: the total reserve capacity determination and the economic dispatch decision making.

Total reserve capacity determination: To tackle possible fluctuations of wind generation variations in the dispatch interval, the total reserve capacity is selected as $R_T = 20$ MW and proportionally assigned to each unit according to (2e), which is shown in Table III. It should be pointed out that in practical engineering applications, there are several other kinds of uncertainties should be considered when arranging R_T, such as generator outages. However they are omitted in this case study for clarity.

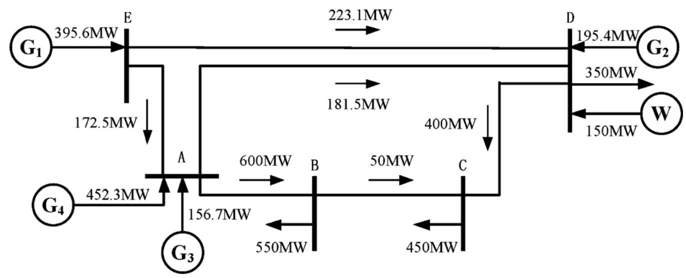

Fig. 2. Power flow in transmission lines for TED.

TABLE IV
DISPATCH DECISIONS GIVEN BY RERD UNDER ±20 MW UNCERTAINTY

	G_1	G_2	G_3	G_4
Output / MW	380	214.6	150	455.4
Reserve / MW	20	0	0	0

ED decision making: Solve TED (2) with the 150-MW predicted wind generation (nominal scenario) and assigned reserve capacity. The output of each unit is given in Table III.

In this test, the penetration rate of wind power is more than 11%. To ensure reliability, 20-MW reserve capacity is used. The generation cost is ￥ 307 225, and the reserve cost is ￥ 8656, contributing 2.74% of the total cost ￥ 315 881.

The active power flow on each transmission line is shown in Fig. 2. It is found that the power flow in Line AB has already reached its upper limit. If the wind farm produces less power than it is expected to, other synchronous generators will accordingly increase their generation for balancing the demand. From a mathematical point of view, the optimal re-dispatch problem (6k) may be infeasible for some wind generation scenarios under dispatch decisions given in Table III. Physically, the power flow on Line AB may exceed its limit when frequency regulation is deployed. If Line AB is tripped due to the overflow protection, cascading tripping of Line BC and Line CD will occur, resulting in blackout at load center B and C. This indicates that, only adequate reserve capacity could not reliably cope with the variability of wind generation in this case.

Case 2 (RERD With ±20 MW Uncertainty): First, the RERD (6a)–(6j) is solved by using Algorithm 1. The pre-dispatch strategies are obtained after 1 computation iteration within 0.2 s and shown in Table IV.

The generation cost in this case is ￥ 308 230, slightly higher than that of Case 1. The total reserve capacity is still 20 MW. However, all 20 MW reserve capacity is allocated on G_1 whose reserve price is the cheapest. As a result, the reserve cost is only ￥ 6000, about 30% less than that of Case 1. The total cost is also reduced by 0.52%.

The active power flow on each transmission line is shown in Fig. 3. It can be seen that each transmission line has certain secure margin to its limit. Moreover, since all SR capacity is assigned to G_1, the increased power for re-dispatch action are mainly imposed to Line DE instead of the heavily loaded Line AB to compensate the variations of wind generation. Thus, the power flow in all transmission lines will not violate the security limits. In other words, optimal re-dispatch problem (6k) will be

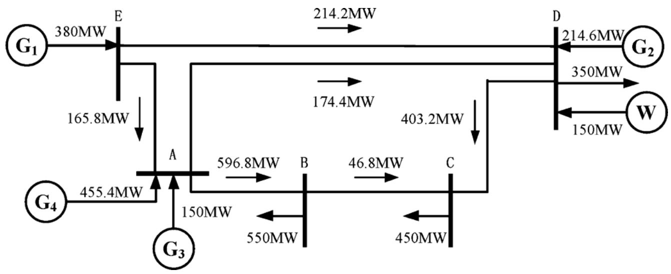

Fig. 3. Power flow of RERD under ±20 MW uncertainty.

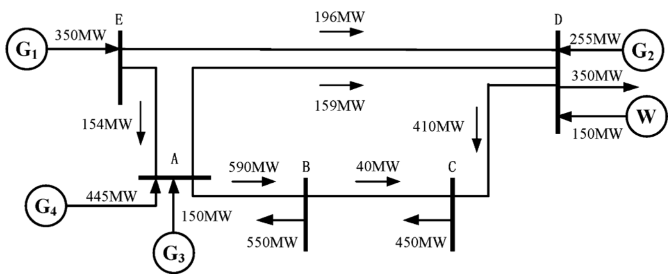

Fig. 4. Power flow for RERD under ±105 MW uncertainty.

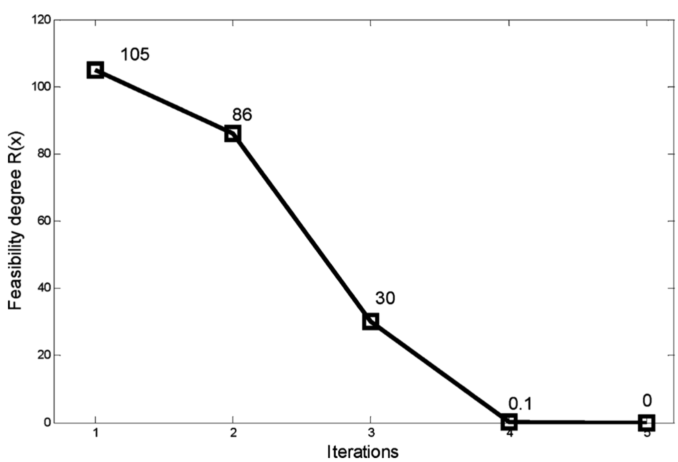

Fig. 5. $R(\boldsymbol{x})$ as a feasibility degree in each iteration.

TABLE V
DISPATCH DECISIONS GIVEN BY RERD UNDER ±105 MW UNCERTAINTY

	G_1	G_2	G_3	G_4
Output / MW	350	255	150	445
Reserve / MW	50	44.6	0	10.4

always feasible, the reliability and robustness is achieved with the least cost with the proposed RERD method.

Case 3 (RERD With ±105 MW Uncertainty): To verify the performance of RERD under a higher level of uncertainty, the forecast interval of wind generation is increased to [45 MW, 255 MW] in this case. The computation of Algorithm 1 converges after 5 iterations within 1.2 second. The dispatch strategies and the corresponding DC power flow are shown in Table V and Fig. 4, respectively. Details about $R(x)$ as a feasibility degree in each iteration are shown in Fig. 5.

Fig. 4 shows that G_2 located at load center D generates more electricity and the security margin of Line AB is further improved. Since the total requirement for reserve capacity is up to 105 MW, the generation and the reserve cost are increased to ¥ 311 750 and ¥ 39 230, respectively. The total cost is ¥ 350 980.

TABLE VI
DISPATCH DECISIONS GIVEN BY RERD INCORPORATING MULTIPLE WIND FARMS AND ±105 MW UNCERTAINTY

	6 Wind Farms (Γ^S=4)		9 Wind Farms (Γ^S=5)	
	Power MW	Reserve MW	Power MW	Reserve MW
G_1	350	50	350	50
G_2	244.6	20	244.6	8.33
G_3	150	0	150	0
G_4	455.4	0	455.4	0
Cost	Generation	Reserve	Generation	Reserve
/ ¥	311,230	24,000	311,230	187,48.5

For comparison, the TED is also performed. The obtained generation cost is ¥ 311 454, slightly lower than that of the RERD. However, the reserve cost is ¥ 45 442 and the total cost is ¥ 356 896, both higher than that of the RERD. Moreover, similar to Case 1, the delivered power on Line AB already reaches the limit, indicating that the dispatch strategy of TED is not robust to wind generation variation.

If the uncertain level (interval) of wind generation is further increased, $R(x)$ will be impossible to reach 0 and $X_R = \emptyset$, implying that this situation has exceeded the capability of the power grid to reliably accommodate the wind generation. System operators have to reduce the uncertainties of wind generation by either increasing the accuracy of prediction or drop off some wind generation, or enhance system dispatch-ability by either adjusting the unit commitment decision to increase SR capacity, or starting fast-response units to provide additional re-dispatch resources.

Case 4 (RERD With Multiple Wind Farms): In this case, the impact of the geographical dispersion of wind farms on dispatch strategies is studied. The original wind farm in the base case is divided into six and nine smaller identical wind farms. They are connected into the grid at bus A, D and E. By setting the confidence level $\alpha = 95\%$, the budget parameter can be chosen as $\Gamma^S = 4$ and $\Gamma^S = 5$, respectively. The results for the two situations are shown in Table VI.

Table VI shows that the wider the wind farms spread, the smaller the additional reserve capacity is required. Compared with Case 3, the reserve cost is reduced by 38.8% (52.2%) when 6 (9) smaller wind farms are connected and $\Gamma^S = 4$ (5) is considered. This means that, by fully utilizing the geographical dispersion of wind farms, the cost for mitigating the variations of wind generation can be significantly reduced.

B. *Guangdong Power System of China*

The real Guangdong power grid of China is studied in this case. The whole system possess 174 thermal units with 58 744 MW total installed capacity, 453 loads, 1880 buses, 2452 transmission lines and six large-scale wind farms in planning. The active power on the 500-kV main transmission network is modeled using constrained DC power flow. Hourly-ahead forecast data of wind generation in a normal winter day, predicted nodal loads, and network parameters, $\alpha = 95\%$ are used in this test. The predicted output of each of wind farms is 1000 MW in Guangzhou, 1000 MW in Shaoguan, 1500 MW in Shenzhen, 1000 MW in Dongguan, 1000 MW in Shantou, and 500 MW in

TABLE VII
COST OF RERD FOR GUANGDONG POWER GRID

Γ^S	Cost		
	Generation /¥	Reserve /¥	Total/¥
0	1.2385×10^7	0	1.2385×10^7
2	1.2401×10^7	1.5607×10^5	1.2557×10^7
4	$\mathbf{1.2417\times10^7}$	$\mathbf{3.1235\times10^5}$	$\mathbf{1.2729\times10^7}$
6	1.2432×10^7	4.6852×10^5	1.2901×10^7

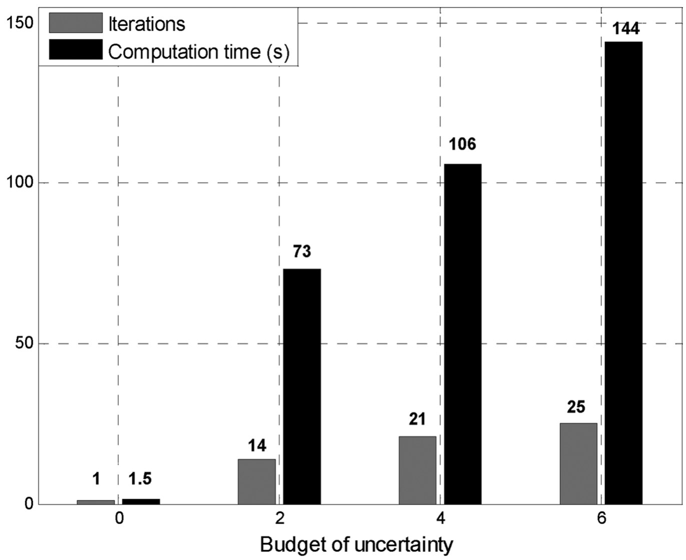

Fig. 6. Computation time for Guangdong power grid under different Γ^S.

Zhanjiang, respectively. It is assumed that there is a 15% forecast error for wind generation and the load forecast is accurate (this is not an actual limitation because the uncertainty of load can also be modeled in the RERD). Thus, as for the TED, 900 MW reserve capacity is required and shared by thermal units. The generation cost and reserve cost of TED is ¥ 1.2484×10^7 and ¥ 5.283×10^5, respectively. The total cost is ¥ 1.3012×10^7.

As for the RERD, similar to Case 4 of the previous subsection, the appropriate spatial budget is $\Gamma^S = 4$. In order to further investigate the impact of Γ^S on the generation and reserve costs, Γ^S is changed from 0 to its maximum, 6, and then the dispatch strategies are computed and illustrated in Table VII. It is observed that both the generation and reserve cost increase with Γ^S increasing. However, the total costs are always lower than that of TED. In fact, $\Gamma^S = 4$ is large enough to describe the variation of wind generation due to the dispersion effect.

The number of iteration and computation time for different Γ^S is presented in Fig. 6. It can be seen that, with the uncertain level of wind generation increasing, the computation time increases accordingly. It is found that most of the computation time is consumed in checking the robust feasibility because it is a large-scale MILP. However, the computational efficiency remains acceptable, even for online applications of this large-scale power system.

V. CONCLUSION

Inspired by the major technical challenges for economic dispatch of power systems with high penetration of renewable energy generation, this paper extends the TED method to a robust version, which is referred to as the RERD method and the ERD method, by introducing the concepts of pre-dispatch and re-dispatch. The existence of valid re-dispatch strategies depends on the robust feasibility of pre-dispatch decisions. By revealing the convexity of the robust feasible region, a MILP based cutting plane algorithm is proposed for solving associated robust optimization problems.

We apply the RERD to a five-bus system as well as a realistic large-scale power gird in China. Experiment results demonstrate that the RERD can reduce both the reserve cost and the total cost, while improve the operation reliability of the power system in the presence of uncertainties with unknown probability distributions. Besides, by incorporating the geographic dispersion of wind generation, the RERD remarkably reduces the amount of reserve capacity for hedging against unpredictable variations of renewable power generation. It is also shown that the RERD is computationally efficient and promising for online applications even in large-scale power systems.

In summary, the RERD and the ERD methods proposed in this paper show appealing advantages of easy implementation and explicit theoretical guarantees on the feasibility of real time dispatch. Such a method is consistent with the decision-making and operating preference of ISO, so is promising in practical power system applications.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers whose very insightful comments and sincere suggestions lead to a remarkable improvement on the quality of this paper.

REFERENCES

[1] WWEA, "World Wind Energy Report 2011," July 2012 [Online]. Available: http://www.wwindea.org/webimages/WorldWindEnergyReport2011.pdf

[2] L. Xie, P. M. S. Carvalho, L. A. F. M. Ferreira, J. H. Liu, B. H. Krogh, N. Popli, and M. D. Ilić, "Wind integration in power systems: Operational challenges and possible solutions," *Proc. IEEE*, vol. 99, no. 1, pp. 214–232, Jan. 2011.

[3] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*, 2nd ed. New York, NY, USA: Wiley, 1996.

[4] S. Wang, M. Shahidehpour, D. Kirschen, S. Mokhtari, and G. Irisarri, "Short-term generation scheduling with transmission constraints using augmented Lagrangian relaxation," *IEEE Trans. Power. Syst.*, vol. 10, no. 3, pp. 1294–1301, Aug. 1995.

[5] C. W. Sanders and C. A. Monroe, "An algorithm for real-time security constrained economic dispatch," *IEEE Trans. Power. Syst.*, vol. PS-2, no. 4, pp. 1068–1074, Nov. 1987.

[6] L. Wu, M. Shahidehpour, and T. Li, "Stochastic security-constrained unit commitment," *IEEE Trans. Power Syst.*, vol. 22, no. 2, pp. 800–811, May 2007.

[7] J. Wang, M. Shahidehpour, and Z. Li, "Security-constrained unit commitment with volatile wind power generation," *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1319–1327, Aug. 2008.

[8] P. Xiong and P. Jirutitijaroen, "A stochastic optimization formulation of unit commitment with reliability constraints," *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 2200–2208, Dec. 2013.

[9] L. M. Kimball, K. A. Clements, and P. W. Davis, "An implementation of the stochastic OPF problem," *Electr. Power Compon. Syst.*, vol. 31, no. 12, pp. 1193–1204, 2003.

[10] M. Parvania and M. Fotuhi-Firuzabad, "Demand response scheduling by stochastic SCUC," *IEEE Trans. Smart Grid*, vol. 1, no. 1, pp. 89–98, Jun. 2010.

[11] M. E. Khodayar, L. Wu, and M. Shahidehpour, "Hourly coordination of electric vehicle operation and volatile wind power generation in SCUC," *IEEE Trans. Smart Grid*, vol. 3, no. 3, pp. 1271–1279, Sep. 2012.

[12] A. Ben-Tal, L. Ghaoui, and A. S. Nemirovski, *Robust Optimization*. Princeton, NJ, USA: Princeton Univ. Press, 2009.

[13] D. Bertsimas and M. Sim, "The price of robustness," *Oper. Res.*, vol. 52, no. 1, pp. 35–53, Jan. 2004.

[14] A. Ben-Tal, A. Goryashko, E. Guslitzer, and A. Nemirovski, "Adjustable robust solutions of uncertain linear programs," *Math. Program.*, vol. 99, no. 2, pp. 351–376, Mar. 2004.

[15] A. Hajimiragha, C. Canizares, M. Fowler, S. Moazeni, and A. Elkamel, "A robust optimization approach for planning the transition to plug-in hybrid electric vehicles," *IEEE Trans. Power. Syst.*, vol. 26, no. 4, pp. 2264–2274, Nov. 2011.

[16] Z. Chen, L. Wu, and Y. Fu, "Real-time price-based demand response management for residential appliances via stochastic optimization and robust optimization," *IEEE Trans. Smart Grid*, vol. 3, no. 4, pp. 1822–1831, Dec. 2012.

[17] R. A. Jabr, "Robust transmission network expansion planning with uncertain renewable generation and loads," *IEEE Trans. Power. Syst.*, vol. 28, no. 4, pp. 4558–4567, Nov. 2013.

[18] R. Jiang, J. Wang, and Y. Guan, "Robust unit commitment with wind power and pumped storage hydro," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 800–810, May 2012.

[19] D. Bertsimas, E. Litvinov, X. A. Sun, J. Y. Zhao, and T. X. Zheng, "Adaptive robust optimization for the security constrained unit commitment problem," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 52–63, Feb. 2013.

[20] L. Zhao and B. Zeng, "Robust unit commitment problem with demand response and wind energy," in *Proc: IEEE Power Energy Soc, General Meeting*, San Diego, CA, USA, July 22–26, 2012.

[21] R. A. Jabr, "Adjustable robust OPF with renewable energy sources," *IEEE Trans. Power. Syst.*, vol. 28, no. 4, pp. 4742–4751, Nov. 2013.

[22] A. Street, A. Moreira, and J. M. Arroyo, "Energy and reserve scheduling under a joint generation and transmission security criterion: An adjustable robust optimization approach," *IEEE Trans. Power. Syst.*, vol. 29, no. 1, pp. 3–14, Jan. 2014.

[23] R. Doherty and M. O'Malley, "A new approach to quantify reserve demand in systems with significant installed wind capacity," *IEEE Trans. Power. Syst.*, vol. 20, no. 2, pp. 587–595, May 2005.

[24] M. Black and G. Strbac, "Value of bulk energy storage for managing wind power fluctuations," *IEEE Trans. Energy Convers.*, vol. 22, no. 1, pp. 197–205, Mar. 2007.

[25] A. Frangioni, C. Gentile, and F. Lacalandra, "Tighter approximated MILP formulations for unit commitment problems," *IEEE Trans. Power Syst.*, vol. 24, no. 1, pp. 105–113, Feb. 2009.

[26] C. Zhao and Y. Guan, "Unified stochastic and robust unit commitment," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 3353–3361, Aug. 2013.

[27] D. Bertsimas and J. N. Tsitsiklis*, Introduction to Linear Optimization*, 3rd ed. Belmont, MA, USA: Athena Scientific, 1997.

[28] J. W. Blankenship and J. E. Falk, "Infinitely constrained optimization problems," *J. Optimiz. Theory App.*, vol. 19, no. 2, pp. 261–281, Jun. 1976.

[29] J. E. Falk, "A linear max-min problem," *Math. Program.*, vol. 5, no. 1, pp. 169–188, 1973.

[30] H. D. Sherali and A. Alameddine, "A new reformulation-linearization technique for bilinear programming problems," *J. Glob. Optim.*, vol. 2, no. 4, pp. 379–410, 1992.

[31] G. P. McCormick, "Computability of global solutions to factorable nonconvex programs: Part I—Convex underestimating problems," *Math. Program.*, vol. 10, no. 1, pp. 147–175, 1976.

[32] S. Siddiqui and S. A. Gabriel, "An SOS1-based approach for solving MPECs with a natural gas market application," *Netw. Spat. Econ.*, vol. 13, no. 2, pp. 205–227, Jun. 2013.

[33] B. Zeng and L. Zhao, "Solving two-stage robust optimization problems using a column-and-constraint generation method," *Oper. Res. Lett.*, vol. 41, no. 5, pp. 457–461, Sep. 2013.

Wei Wei received the B.Sc. and Ph.D. degrees in electrical engineering from Tsinghua University, Beijing, China, in 2008 and 2013, respectively.

He is currently a Research Assistant at Tsinghua University. His research interests include robust optimization, bilevel optimization, game theory, and their applications in the smart grid networks.

Feng Liu received the B.Sc. and Ph.D. degrees in electrical engineering from Tsinghua University, Beijing, China, in 1999 and 2004, respectively.

He is currently an Associate Professor at Tsinghua University. His research interests include power system stability analysis and control, and smart grid operation technologies.

Shengwei Mei (SM'06) received the B.Sc. degree in mathematics from Xinjiang University, Urumqi, China, the M.Sc. degree in operations research from Tsinghua University, Beijing, China, and the Ph.D. degree in automatic control from Chinese Academy of Sciences, Beijing, China, in 1984, 1989, and 1996, respectively.

He is currently a Professor at Tsinghua University. His research interests include power system analysis and control, game theory, and its application in power systems.

Yunhe Hou (M'06) received the B.E. and Ph.D. degrees from Huazhong University of Science and Technology, Wuhan, China, in 1999 and 2005, respectively.

He was a Postdoctoral Research Fellow at Tsinghua University, Beijing, China, from 2005 to 2007. He was a Visiting Scholar at Iowa State University, Ames, IA, USA, and a Research Scientist at University College Dublin, Dublin, Ireland, from 2008 to 2009. He was also a Visiting Scientist with Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA, USA, in 2010. He is currently with the University of Hong Kong as a Research Assistant Professor. His research interests include electricity market theory, power system resource planning, and self-healing smart grid construction.