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Communications Using Ubiquitous Antennas: Free-Space Propagation

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Abstract—The inefficiency of the cellular-network architecture has prevented the promising theoretic gains of communication technologies such as network MIMO, massive MIMO and distributed antennas from fully materializing in practice. The revolutionary cell-less *cloud radio access networks* (C-RANs) are under active development to overcome the drawbacks of cellular networks. In C-RANs, centralized cloud signal processing and minimum onsite hardware make it possible to deploy ubiquitous distributed antennas and coordinate them to form a gigantic array, called the *ubiquitous array* (UA). This paper focuses on designing techniques for UA communications and characterizing their performance. To this end, the UA is modeled as a continuous circular array enclosing target mobiles and free-space propagation is assumed, which allows the use of mathematical tools including Fourier series and Bessel functions in the analysis. First, exploiting the UA's large circular structure, a novel scheme for multiuser channel estimation is proposed to support *noiseless* channel estimation using only *single* pilot symbols. Channel estimation errors due to interference are proved to be Bessel functions of inter-user distances normalized by the wavelength. Besides increasing the distances, it is shown that the errors can be also suppressed by using pilot sequences and eliminated if the sequence length is longer than the number of mobiles. Next, for data communication, we first consider channel conjugate transmission that compensates the phase shift in propagation and thereby allows receive coherent combining. The multiuser interference powers are derived as Bessel functions of normalized inter-user distances. Last, we propose the design of multiuser precoders in the form of Fourier series whose coefficients excite different phase modes of the UA. Under the zero-forcing constraints, the precoder coefficients are proved to lie in the null space of a derived matrix with elements being Bessel functions of normalized inter-user distances.

I. INTRODUCTION

Mobile-network densification is the primary solution for the explosive growth of data traffic. A revolutionary network architecture called *cloud radio access networks* (C-RANs) provides an efficient platform for network densification, which promises low energy consumption and operational cost, enables more efficient resource utilization, and eliminates inter-cell interference [1]. In C-RANs, *base stations* (BSs) are implemented as software processes in a data center and on-site hardware reduces to low-complexity *remote radio heads* (RRHs) comprising antennas and RF components. The rapid advancements in fiber-fronthaul solutions for connecting RRHs to the data center allows the deployment of dense

RRHs in an urban area. Consequently, C-RANs will become effectively a virtual gigantic antenna array with ubiquitous elements, referred to as a *ubiquitous array* (UA). Communications using a UA is a paradigm shift in wireless communications and the focus of this paper.

MIMO communications relying on the use of arrays at both ends of a wireless channel to yield the *linear* growth of data rates with the numbers of antennas [2]. However, the performance of MIMO technologies in wireless networks is limited by inter-cell interference and severe propagation loss at cell edges. Tackling these challenges as well as maximizing MIMO gains have motivated researchers to develop large-scale MIMO technologies including network MIMO [3], massive MIMO [4] and distributed antennas [5].

However, the theoretic gains of these technologies fail to fully materialize in practice due to implementation issues inherent in the cellular architecture of existing networks. First, *network MIMO* that enables cooperation between BSs by data sharing via backhaul links, which coordinates the arrays of different BSs to create a gigantic virtual array for multi-cell joint transmissions [3]. Given constraints on backhaul data rates and BS-computation power, multi-cell cooperation is limited to be within relatively small BS clusters [6], [7], which exposes cluster-edge mobiles to strong inter-cluster interference [8]. This issue as well as others such as inaccurate channel feedback have resulted in marginal throughput gains for practical network MIMO [9]. Next, massive MIMO is the direct approach for maximizing MIMO gains by scaling up the array at each BS to have hundreds to thousands of elements [10]. Nevertheless, the throughput of networks provisioned with large-scale arrays is limited by a phenomenon called *pilot contamination*, namely that inter-cell interference arises from the use of non-orthogonal pilot sequences in different cells [4]. Last, a large-scale virtual array can be created by replacing collocated antennas at each BS with many antennas distributed over the cell region, which shortens propagation distances and thereby enhances network coverage [5]. The performance of such systems is limited by inter-cell interference.

UA communications can be considered as an extreme form of network MIMO, massive MIMO or distributed antennas where antennas are ubiquitous and fully coordinated. In UA networks (or C-RANs), cells varnish together with their resul-

tant performance bottlenecks. The performance limit of UA communications is unknown and investigated in this paper for free-space channels. Results for scattering channels are presented in a separate paper due to the space limitation. In this paper, a geometric system model is adopted where the UA is modeled as a continuous circular array in the horizontal plane and transmit data to single-antenna mobiles enclosed by the array. The antennas of the UA and mobiles are assumed to be omnidirectional. This model allows tractable analysis using mathematical tools including Fourier series and Bessel functions, yielding the following findings.

- The UA enables *noiseless* estimation of multiuser channels using only single pilot symbols. Given free-space propagation, channels are determined by mobiles' locations. As a result, the estimation errors caused by interference are proved to be Bessel functions of inter-mobile distances normalized by the wavelength. Thus the channel-estimation accuracy improves with increasing inter-mobile distances.
- Pilot sequences are shown to improve channel-estimation accuracy. The optimal design of the sequences is found to be equivalent to the well known mathematical problem of Grassmannian line packing. As a result, channel estimation errors scale approximately as the inverse of the sequence length and vanish when the sequences are longer than the number of mobiles and become orthogonal.
- Assuming perfect channel estimation, techniques are designed and analyzed for data transmissions using the UA. First, consider channel-conjugate transmissions that compensate phase shifts in propagation to achieve coherent combining at mobiles. The signal-to-interference-and-noise ratios (SINRs) are derived in closed-form. In particular, the power of interference between any two mobiles is shown to be proportional to a squared Bessel function of their (wavelength) normalized separation distance.
- Next, it is proposed that circular precoders for multiuser transmission using the UA are designed in the form of Fourier series. Their coefficients are controlled to excite different *phase modes* of the circular array so as to null multiuser interference. To satisfy the zero-forcing constraints, the vector comprising the coefficients of each precoder is proved to lie in the null space of a derived matrix whose elements are Bessel functions of the normalized distances from the intended mobile to the interferences.

The remainder of the paper is organized as follows. The UA-system model is presented in Section II. Techniques for channel estimation and data transmission using the UA are proposed in Sections III and IV, respectively. Numerical results are presented in Section V followed by concluding remarks in Section ??.

II. SYSTEM MODEL

As illustrated in Fig. 1, an UA communication system comprises a continuous circular UA with a fixed radius r_0

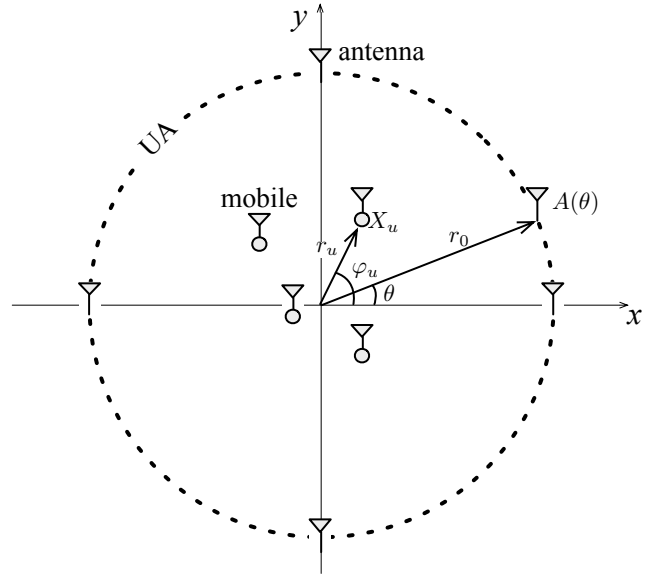


Fig. 1. UA communication system.

and centered at the origin and a set of single-antenna mobiles enclosed by the UA. All antennas are assumed to be isotropic. An arbitrary element of the UA, $A \in \mathbb{R}^2$, with the polar coordinates (r_0, θ) is represented by the function $A(\theta)$. Let $\mathcal{D} \subset \mathbb{R}^2$ denote the region enclosed by the UA. There are U mobiles at the locations X_1, X_2, \dots, X_U within \mathcal{D} . The polar coordinates of a particular location X_u are represented by (r_u, φ_u) . Uplink/downlink channel reciprocity is assumed, corresponding to a time-division duplexing system.

Assumption 1. Mobiles are assumed to be sufficiently far away from the UA such that $r_u/r_0 \ll 1$ for all $1 \leq u \leq U$.

This assumption allows the application of a common technique (see e.g., [11]) for simplifying the expression for the propagation distance between a mobile X and an UA element $A(\theta)$ as:

$$\begin{aligned} |X - A(\theta)| &= \sqrt{r_0^2 + r_X^2 - 2r_0r_X \cos(\varphi_X - \theta)} \\ &= r_0 - r_X \cos(\varphi_X - \theta) + O\left(\frac{r_X}{r_0}\right). \end{aligned} \quad (1)$$

Let $h_u(\theta)$ represents the gain of the channel between X_u and $A(\theta)$. Then

$$h_u(\theta) = \sqrt{\frac{gq\lambda^2}{(4\pi)^2 |X_u - A(\theta)|^2}} e^{-j\frac{2\pi}{\lambda} |X_u - A(\theta)|}$$

where g and q are the constant antenna gains for a mobile and a UA element, respectively, and λ denotes the wavelength. Based on (1),

$$h_u(\theta) = \frac{\eta}{r_0} e^{j\frac{2\pi}{\lambda} r_u \cos(\varphi_u - \theta)} + O\left(\frac{r_u}{r_0^2}\right) \quad (2)$$

where the constant $\eta = \lambda\sqrt{gq}e^{-j\frac{2\pi}{\lambda}r_0}/4\pi$.

Channel estimation at the UA is assisted by pilot signals transmitted by mobiles. Time is divided in time slots with unit durations. The total pilot signal received at antenna $A(\theta)$ in an arbitrary slot, denoted as $q(\theta)$, is given as

$$q(\theta) = \sum_{u=1}^U h_u(\theta) s_u + z(\theta) \quad (3)$$

where s_u is a pilot symbol transmitted by mobile u and the noise $z(\theta)$ a sample of the white $\mathcal{CN}(0, \sigma^2)$ process $\{z(\theta) \mid 0 \leq \theta < 2\pi\}$. Substituting (2) gives

$$q(\theta) = \frac{\eta}{r_0} \sum_{u=1}^U e^{j \frac{2\pi}{\lambda} r_u \cos(\varphi_u - \theta)} s_u + z(\theta) + O\left(\frac{r_+}{r_0^2}\right). \quad (4)$$

where $r_+ = \max_u r_u$.

Next, consider downlink data transmission. The data symbol intended for mobile u , denoted as x_u , is precoded by a circular precoder $f_u(\theta)$. They satisfy the following power constraints:

$$\mathbb{E}[x_u] \leq P, \quad \frac{1}{2\pi} \int_0^{2\pi} f(\theta) d\theta \leq 1 \quad (5)$$

where P is the transmission power. With the channel gains in (2), the multiuser signal received at mobile u in an arbitrary slot, denoted as y_u , is given as

$$\begin{aligned} y_u &= \frac{1}{2\pi} \int_0^{2\pi} h_u(\theta) \sum_{k=1}^U f_k(\theta) x_k d\theta \\ &= \frac{\eta}{2\pi r_0} \int_0^{2\pi} e^{j \frac{2\pi}{\lambda} r_u \cos(\varphi_u - \theta)} \sum_{k=1}^U f_k(\theta) x_k d\theta + z_u + O\left(\frac{r_+}{r_0^2}\right) \end{aligned} \quad (6)$$

where $\{z_u\}$ are i.i.d. $\mathcal{CN}(0, \sigma^2)$ random variables representing channel noise.

III. UA COMMUNICATIONS: CHANNEL ESTIMATION

For free space propagation, channels are determined by mobile locations. Accurate estimation of the locations are feasible using the UA as a gigantic spatial filter. In contrast, conventional arrays only support estimation of directions of arrival. In the sub-section, the techniques for channel estimation using the UA are proposed and analyzed.

A. Channel estimation with single pilot symbols

Consider the scenario where mobiles simultaneously transmit single pilot symbols $\{s_u\}$ to facilitate channel estimation at the UA. Without loss of generality, assume that the pilot symbols are all ones: $s_u = 1 \forall u$. The proposed estimation method is based on representing the circular received signal $\{q(\theta) \mid \theta \in [0, 2\pi)\}$ in (3) using the Fourier series:

$$q(\theta) = \sum_{k=-\infty}^{\infty} Q_k e^{-jk\theta} + z(\theta)$$

where the Fourier coefficients $\{Q_k\}$ are defined as

$$Q_k = \frac{1}{2\pi} \int_0^{2\pi} \sum_{u=1}^U h_u(\theta) e^{jk\theta} d\theta. \quad (7)$$

The Fourier coefficients contain all information in the receive signal and can be thus used for channel estimation in place of signal. To this end, a few notations are introduced for convenience. Let \mathbf{Q} represent the infinite sequence $[\dots, Q_{-1}, Q_0, Q_1, \dots]^T$. Moreover, J_k denotes the Bessel function of the first kind with an integer order k that can be defined in its integral form as [12]

$$J_k(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{j(x \sin \theta - k\theta)} d\theta. \quad (8)$$

Lemma 1. The sequence \mathbf{Q} can be decomposed as

$$\mathbf{Q} = \frac{\eta}{r_0} \sum_{u=1}^U \mathbf{V}(X_u) + O\left(\frac{r_+}{r_0^2}\right), \quad \text{a.s.} \quad (9)$$

where $\mathbf{V}(X_u) = [\dots, V_{-1}(X_u), V_0(X_u), V_1(X_u), \dots]^T$ with the function $V_k(Y)$ defined for a given location $Y = (r_Y, \varphi_Y)$ as

$$V_k(Y) = j^k e^{jk\varphi_Y} J_k\left(\frac{2\pi r_Y}{\lambda}\right). \quad (10)$$

Due to the space limitation, only proof sketches are provided for all lemmas and theorems in this paper where the proof techniques are highlighted.

Sketch of proof: First, the law of large numbers is invoked to show the suppression of noise as follows:

$$\frac{1}{2\pi} \int_0^{2\pi} z(\theta) e^{jk\theta} d\theta = 0, \quad \text{a.s.} \quad (11)$$

Next, the technique in deriving the main result in the lemma statement is derived by decomposing $q(\theta)$ in (4) using the following Jacobi-Anger expansion [12]:

$$e^{j \frac{2\pi r_u}{\lambda} \cos(\varphi_u - \theta)} = \sum_{m=-\infty}^{\infty} j^m J_m\left(\frac{2\pi r_u}{\lambda}\right) e^{jm(\varphi_u - \theta)}$$

and combining the result with (2) and (7). \blacksquare

One can observe from Lemma 1 that the sequence \mathbf{Q} is the superposition of U component sequences $\{\mathbf{V}(X_u)\}$ determined by corresponding mobile locations. The proposed method for channel estimation is to estimate $\{X_u\}$ by extracting $\mathbf{V}(X_u)$ from \mathbf{Q} . For this purpose, it is desirable that two sequences $\mathbf{V}(X)$ and $\mathbf{V}(Y)$ with $X \neq Y$ can be differentiated if the distance $|X - Y|$ is sufficiently large. Interestingly, such a property indeed exists as shown in the following lemma.

Lemma 2. Given two locations $X, Y \in [0, r_0] \times [0, 2\pi)$, the projection distance between the two sequences $\mathbf{V}(X)$ and $\mathbf{V}(Y)$ is given as

$$\mathbf{V}(X)^\dagger \mathbf{V}(Y) = J_0\left(\frac{2\pi}{\lambda} |X - Y|\right).$$

Sketch of proof: The main technique is to use a property of Bessel functions from [13, (6.61)]. Specifically, two infinite sequences of Bessel functions, $\{J_k(x)\}_{k=-\infty}^{\infty}$ and $\{J_{k+n}(x)\}_{k=-\infty}^{\infty}$, can be combined to give a single Bessel function as follows:

$$J_n(|X - Y|) e^{jn\omega} = \sum_{k=-\infty}^{\infty} J_k(|X|) J_{n+k}(|Y|) e^{jk\angle(X, Y)}$$

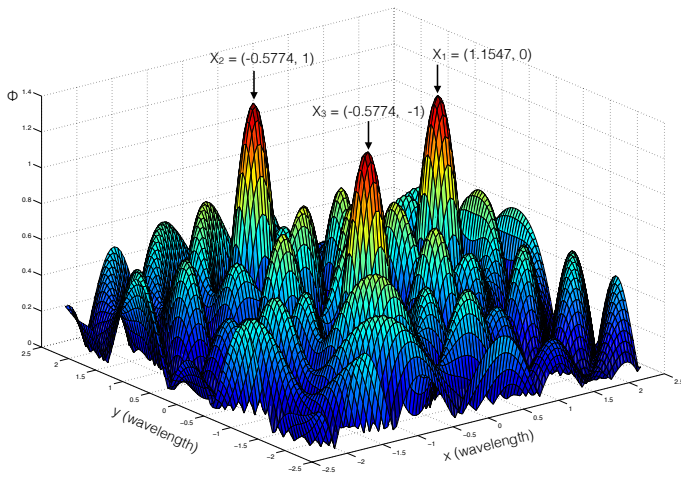


Fig. 2. default

where the angle ω is defined using $\sin \omega = (|X|/|X - Y|) \sin \angle(X, Y)$. ■

Note that $J_0(x)$ is a monotone decreasing function of x for $x \geq 0$. Thus the result in Lemma 2 suggests that the vectors $\mathbf{V}(X)$ and $\mathbf{V}(Y)$ become almost orthogonal if the distance between X and Y is sufficiently large.

The result in Lemma 2 suggests that the function $\Phi: \mathcal{D} \rightarrow \mathbb{R}^+$ defined below, called the *mobile location profile*, has its peaks at the mobile locations $Y = X_1, X_2, \dots, X_U$ if they are sufficiently separated:

$$\Phi(Y) = \frac{r_0^2}{\eta^2} |\mathbf{V}^\dagger(Y) \mathbf{Q}|^2. \quad (12)$$

This fact can be stated rigorously in the follow theorem obtained using Lemmas 1 and 2.

Theorem 1. The mobile location profile $\Phi(Y)$ is noiseless and given as

$$\Phi(Y) = \left| \sum_{u=1}^U J_0 \left(\frac{2\pi}{\lambda} |X_u - Y| \right) \right|^2 + O \left(\frac{r_+^2}{r_0^2} \right). \quad (13)$$

Based on Theorem 1, it is proposed that the mobile locations (or equivalently the multiuser channels) are estimated by detecting peak locations of the mobile location profile $\Phi(Y)$ given in the theorem. The estimation is accurate if the mobile inter-distances are sufficiently large as explained shortly. Consider an arbitrary mobile location X_u and a disk \mathcal{A} centered at X_u and with a radius d . Assume that other locations are sufficiently far away from X_u : $|X_k - X_u| \gg d \forall k \neq u$. Theorem 1 reveals that the portion of the profile in the region \mathcal{A} has a similar shape as a Bessel function centered at X_u . Specifically,

$$\Phi(Y) \approx J_0 \left(\frac{2\pi}{\lambda} |Y - X_u| \right), \quad Y \in \mathcal{A}. \quad (14)$$

Since J_0 is a monotone decreasing function, X_u can be accurately estimated by detecting the peak of $\Phi(Y)$ in \mathcal{A} .

On the other hand, mobile locations being too close cause large estimation errors due to interference resulting from the

tails of Bessel functions. The condition on accurate location estimation can be quantified using the following bound on the Bessel-function envelop [14]:

$$J_0 \left(\frac{2\pi}{\lambda} d \right) \leq \nu \left(\frac{2\pi}{\lambda} d \right)^{-\frac{1}{3}}. \quad (15)$$

For d larger than $d_{0.1} \approx 77\lambda$, the Bessel function is smaller than 0.1. Thus, $d_{0.1}$ gives a rule of thumb for minimum mobile separation to achieve accurate channel estimation. For the carrier frequency of 1 Gz and 10 Gz, $d_{0.1}$ is 23 m and 2.3 m, respectively.

B. Channel Estimation with Pilot Sequences

In this section, the results in the preceding section for single pilot symbols are extended to the case with pilot sequences. Let the pilot sequence for the u th mobile be denoted as $\mathbf{s}_u = [s_u(1), \dots, s_u(L)]^T$ with length L and unit norm: $\mathbf{s}_u^\dagger \mathbf{s}_u = 1$. As a result of the pilot-sequence transmissions, the UA receives L infinite vectors $\mathbf{Q}(\ell), \mathbf{Q}(2), \dots, \mathbf{Q}(L)$ over L symbol durations where $\mathbf{Q}(\ell)$ is modified from its single-symbol counterpart in Lemma 1 as

$$\mathbf{Q}(\ell) = \frac{\eta}{r_0} \sum_{u=1}^U s_u(\ell) \mathbf{V}(X_u) + O \left(\frac{r_+}{r_0^2} \right), \quad \text{a.s.} \quad (16)$$

To estimate the mobile location X_u , $\{\mathbf{Q}(\ell)\}$ are coherently combined using the pilot sequence \mathbf{s}_u as $\sum_{\ell=1}^L s_u^*(\ell) \mathbf{Q}(\ell)$. The result denoted as \mathbf{Q}_u follows from (16) as

$$\mathbf{Q}_u = \frac{\eta}{r_0} \sum_{k=1}^U \mathbf{s}_u^\dagger \mathbf{s}_k(\ell) \mathbf{V}(X_u) + O \left(\frac{r_+}{r_0^2} \right), \quad \text{a.s.} \quad (17)$$

A corresponding mobile location profile $\Phi_u(Y)$ can be defined similarly as (12):

$$\Phi_u(Y) = \frac{r_0^2}{\eta^2} |\mathbf{V}^\dagger(Y) \mathbf{Q}_u|^2. \quad (18)$$

The profile $\Phi_u(Y)$ can be decomposed into the desired and interference terms as follows:

$$\Phi_u(Y) = \left| J_0 \left(\frac{2\pi |X_u - Y|}{\lambda} \right) + \sum_{k \neq u} \mathbf{s}_u^\dagger \mathbf{s}_k J_0 \left(\frac{2\pi |X_k - Y|}{\lambda} \right) + O \left(\frac{1}{r_0^2} \right) \right|^2$$

The estimation accuracy can be improved by designing the pilot sequences to suppress the interference. To this end, the estimation error can be bounded as:

$$\left| \Phi_u(Y) - J_0^2 \left(\frac{2\pi |X_u - Y|}{\lambda} \right) \right| \leq \sum_{k \neq u} |\mathbf{s}_u^\dagger \mathbf{s}_k|^2 J_0^2 \left(\frac{2\pi |X_k - Y|}{\lambda} \right) + O \left(\frac{r_+^2}{r_0^2} \right). \quad (19)$$

Minimizing the upper bound is equivalent to the minimization of the pairwise projection distance of the pilot sequences $\{\mathbf{s}_u\}$:

$$\min \sum_{u \neq k} |\mathbf{s}_{u_1}^\dagger \mathbf{s}_{u_2}|^2, \quad u \neq k. \quad (20)$$

Under the unit-norm constraint on the sequences, this optimization problem is the well-known mathematical problem of Grassmannian line packing [15]. The maximum pairwise distance for the sequences $\{\tilde{s}_u\}$ resulting from the optimal packing can be lower bounded as [15]

$$\max_{u \neq k} |\tilde{s}_u^\dagger \tilde{s}_k|^2 \geq \max\left(\frac{U-L}{L(U-1)}, 0\right). \quad (21)$$

Combining this bound and (19) gives the main result of this section as follows.

Theorem 2. The error for estimating the u th mobile's channel, or equivalently the location X_u , can be upper bounded as

$$\left| \Phi_u(Y) - J_0^2\left(\frac{2\pi|X_u - Y|}{\lambda}\right) \right| \leq \max\left(\frac{U-L}{L(U-1)}, 0\right) \times \sum_{k \neq u} J_0^2\left(\frac{2\pi}{\lambda}|X_k - Y|\right) + O\left(\frac{r_{\pm}^2}{r_0^2}\right).$$

Since $J_0(x) \leq 1$, a simpler but looser bound is

$$\left| \Phi_u(Y) - J_0^2\left(\frac{2\pi|X_u - Y|}{\lambda}\right) \right| \leq \max\left(\frac{U-L}{L}, 0\right) + O\left(\frac{r_{\pm}^2}{r_0^2}\right).$$

Comparing Theorems 1 and 2, the length of pilot sequences contributes the reduction factor $\max\left(\frac{U-L}{L(U-1)}, 0\right)$ of the channel estimation error. In particular, if $L \geq U$, the factor is zero given orthogonal pilot sequences, resulting in:

$$\left| \Phi_u(Y) - J_0^2\left(\frac{2\pi|X_u - Y|}{\lambda}\right) \right| \leq O\left(\frac{r_{\pm}^2}{r_0^2}\right), \quad L \geq U.$$

IV. UA COMMUNICATIONS: DATA TRANSMISSIONS

In this section, two transmission techniques are designed for the UA based on the conventional methods of channel conjugate and multiuser zero-forcing transmissions. For simplicity, it is assumed that the UA has perfect knowledge of the mobile locations.

A. Channel Conjugate UA Transmission

For channel conjugate transmission, the precoder applies a phase shift to each antenna for compensating the shift in propagation. Specifically, the precoder $f_u(\theta)$ is given as

$$f_u^{(cc)}(\theta) = \frac{h_u^*(\theta)}{|h_u(\theta)|}, \quad 0 \leq \theta < 2\pi \quad (22)$$

where the channel coefficient $h(\theta)$ is given in (2). The normalization $|f_u^{(cc)}(\theta)|^2 = 1$ facilitates the UA implementation under the constraint on the per-element transmission power e.g., the phase array. By substituting the precoder into (6), the signal received at mobile u is obtained as

$$y_u = x_u + \sum_{k \neq u} \frac{x_k}{2\pi} \int_0^{2\pi} e^{j\frac{2\pi}{\lambda}(r_u \cos(\varphi_u - \theta) - r_k \cos(\varphi_k - \theta))} d\theta + z_u + O\left(\frac{r_{\pm}}{r_0}\right). \quad (23)$$

Using some trigonometry identifies and the integral form of Bessel functions, a formula useful for the derivation in both this and next sections are obtained as follows.

Lemma 3. Given $X_u, X_k \in \mathbb{R}^2$ and an integer m ,

$$\frac{1}{2\pi} \int_0^{2\pi} e^{j\frac{2\pi}{\lambda}(r_u \cos(\varphi_u - \theta) - r_k \cos(\varphi_k - \theta)) + jm\theta} d\theta = e^{jm\beta_{u,k}} J_m\left(\frac{2\pi}{\lambda}|X_u - X_k|\right) \quad (24)$$

where the angle β is defined by

$$\tan \beta_{u,k} = \frac{r_u \cos(\varphi_u) - r_k \cos(\varphi_k)}{r_u \sin(\varphi_u) - r_k \sin(\varphi_k)}. \quad (25)$$

Using the result in Lemma 3 with $m = 0$, the received signal in (23) reduces to

$$y_u = x_u + \sum_{k \neq u} e^{jm\beta_{u,k}} J_0\left(\frac{2\pi}{\lambda}|X_k - X_u|\right) x_k + z_u + O\left(\frac{r_{\pm}}{r_0}\right).$$

This leads to the main result of this section as shown in the following theorem.

Theorem 3. For channel conjugate transmission, the received SINR for mobile u is given by

$$\text{SINR}_u = \frac{1}{\sum_{k \neq u} J_0^2\left(\frac{2\pi}{\lambda}|X_u - X_k|\right) + \sigma^2/P + O\left(\frac{r_{\pm}^2}{r_0^2}\right)}$$

where $u = 1, 2, \dots, U$.

Using the bound on the Bessel-function envelop in (15), for a high SNR, the signal-to-interference ratio (SIR) at mobile u scales with the distance to the nearest interferer, namely $\min_{k \neq u} |X_k - X_u|$ and the wavelength λ as

$$\text{SIR}_u \geq \frac{1}{\nu^2(U-1)} \left(\frac{2\pi}{\lambda} \min_{k \neq u} |X_k - X_u|\right)^{\frac{2}{3}}.$$

B. Zero-Forcing UA Transmission

In this section, precoders are designed under the zero-forcing constraints to null multiuser interference. Exploiting the circular structure of the UA, the precoder for each mobile, say $f_u^{(zf)}(\theta)$ for mobile u , can be expressed in terms of a Fourier series:

$$f_u^{(zf)}(\theta) = \frac{h_u^*(\theta)}{|h_u(\theta)|} \sum_{m=-\infty}^{\infty} c_{u,m} e^{-jm\theta} \quad (26)$$

where the factor in front of the summation is observed to be the channel-conjugate precoder for compensating the channel phase shift and the summation is the Fourier series. The coefficients $\{c_{u,m}\}$ are the *precoder coefficients* designed in the sequel under the power constraint:

$$\sum_{m=-\infty}^{\infty} c_{u,m}^2 \leq 1, \quad \forall u. \quad (27)$$

To this end, the received signal at mobile u is obtained in terms of the precoder coefficients as follows. By substituting the precoder in (26) into (6),

$$\begin{aligned}
 y_u &= \left(\frac{|h_k(\theta)|}{2\pi} \int_0^{2\pi} \sum_{m=-\infty}^{\infty} c_{u,m} e^{-jm\theta} d\theta \right) x_u + \\
 &\quad \sum_{k \neq u} \left(\frac{1}{2\pi} \int_0^{2\pi} \frac{h_k^*(\theta) h_u(\theta)}{|h_k(\theta)|} \sum_{m=-\infty}^{\infty} c_{k,m} e^{-jm\theta} d\theta \right) x_k + z_u \\
 &= \frac{\eta c_{u,0} x_u}{r_0} + \frac{\eta}{r_0} \sum_{k \neq u} \sum_{m=-\infty}^{\infty} c_{k,m} \mathcal{J}_{u,k,m} x_k + z_u + O\left(\frac{r_+}{r_0^2}\right) \quad (28)
 \end{aligned}$$

where the first term and the summation represents the signal and interference, respectively, and

$$\mathcal{J}_{u,k,m} = \frac{1}{2\pi} \int_0^{2\pi} e^{j \frac{2\pi}{\lambda} (r_u \cos(\varphi_u - \theta) - r_k \cos(\varphi_k - \theta)) - jm\theta} d\theta.$$

Using Lemma 3,

$$\mathcal{J}_{u,k,m} = e^{jm\beta_{u,k}} J_m \left(\frac{2\pi}{\lambda} |X_u - X_k| \right). \quad (29)$$

The interference term in the signal y_u can be nulled by enforcing the following zero-forcing constraints:

$$\sum_{m=-\infty}^{\infty} c_{k,m} \mathcal{J}_{u,k,m} = 0, \quad \forall k \neq u. \quad (30)$$

Though there exist infinite precoder coefficients, it is undesirable and unnecessary to design an unnecessarily large set of coefficients that results in high computation complexity. This can be avoided by considering only those elements with significant values from the infinite set $\{\mathcal{J}_{u,k,m}\}$. For this purpose, a useful property of Bessel functions is as follows: for $x \gg 1$ and $|n| \geq x$, $J_n(x) \approx 0$ [16]. Assuming that mobile are separated by distances much larger than a single wavelength, this property reveals that the set of significant elements in $\{\mathcal{J}_{u,k,m}\}$ correspond to $-M \leq m \leq M$ with

$$M = \max \left(\max_{u,k} \left\lfloor \frac{2\pi |X_u - X_k|}{\lambda} \right\rfloor, \left\lfloor \frac{U}{2} \right\rfloor \right). \quad (31)$$

By considering only these elements, the zero-forcing constraints in (30) can be approximated in the matrix form:

$$\mathcal{J}_u \mathbf{c}_u = \mathbf{0}, \quad (32)$$

where the matrix \mathcal{J}_u is defined as

$$\mathcal{J}_u = \begin{bmatrix} \mathcal{J}_{u,1,-M} & \cdots & \mathcal{J}_{u,1,0} & \cdots & \mathcal{J}_{u,1,M} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathcal{J}_{u,u-1,-M} & \cdots & \mathcal{J}_{u,u-1,0} & \cdots & \mathcal{J}_{u,u-1,M} \\ \mathcal{J}_{u,u+1,-M} & \cdots & \mathcal{J}_{u,u+1,0} & \cdots & \mathcal{J}_{u,u+1,M} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathcal{J}_{u,U,-M} & \cdots & \mathcal{J}_{u,U,0} & \cdots & \mathcal{J}_{u,U,M} \end{bmatrix}$$

and the vector \mathbf{c}_u as $\mathbf{c}_u = [c_{u,-M} \cdots c_{u,0} \cdots c_{u,M}]^T$.

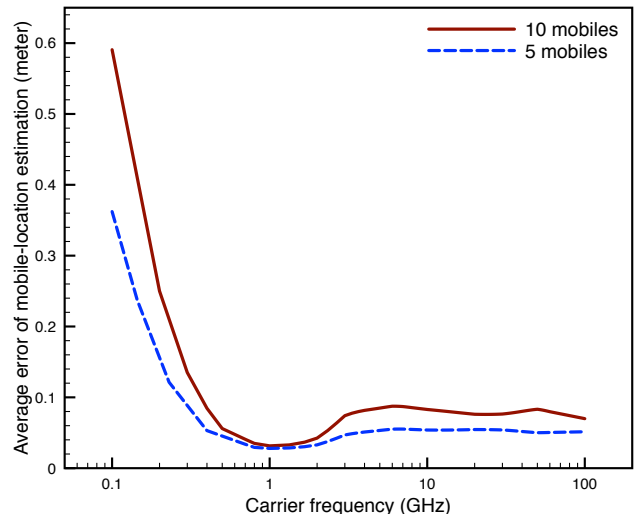


Fig. 3. Average error on estimated mobile locations versus carrier frequency for channel estimation using single pilot symbols.

Theorem 4. To satisfy the approximate zero-forcing constraints in (32), it is sufficient to choose precoder coefficients for mobile u as follows.

$$\mathbf{c}_u \in \text{span} \left(\mathbf{I} - \mathcal{J}_u^\dagger (\mathcal{J}_u \mathcal{J}_u^\dagger)^{-1} \mathcal{J}_u \right). \quad (33)$$

With interference nulled, the SINR for mobile u follows from (28) as

$$\begin{aligned}
 \text{SNR}_u &= \frac{\eta^2 P |c_{u,0}|^2}{\sigma^2 + O(r_+^2/r_0^2)} \\
 &= \frac{\eta^2 P |\mathbf{e}_0^\dagger \mathbf{c}_u|^2}{\sigma^2 + O(r_+^2/r_0^2)} \quad (34)
 \end{aligned}$$

where $\mathbf{e}_0 = [0, \dots, 0, 1, 0, \dots, 0]^T$. Thus, to maximize SNR_u , the precoder-coefficient vector \mathbf{c}_u should be chosen as the projection of \mathbf{e}_0 onto $\text{span} \left(\mathbf{I} - \mathcal{J}_u^\dagger (\mathcal{J}_u \mathcal{J}_u^\dagger)^{-1} \mathcal{J}_u \right)$ that contains \mathbf{c}_u according to Theorem 4. This gives the following corollary.

Corollary 1. For zero-forcing transmissions, the maximum received SNR for mobile u is given as

$$\text{SNR}_u = \frac{\eta^2 P \left| \mathbf{e}_0^\dagger \left(\mathbf{I} - \mathcal{J}_u^\dagger (\mathcal{J}_u \mathcal{J}_u^\dagger)^{-1} \mathcal{J}_u \right) \right|^2}{\sigma^2 + O(r_+^2/r_0^2)}. \quad (35)$$

V. SIMULATION RESULTS

For simulation, mobiles are uniformly distributed in a disk centered at the origin and with a radius of $r = 10$ meters. The UA radius r_0 is assumed to be much larger than r and its exactly affects only the *receive* SNR at a mobile, which is defined as $\eta^2/r_0^2\sigma^2$.

Consider channel estimation with single pilot symbols. The average error on the estimated mobile locations is plotted in Fig. 3 against the carrier frequency. In the frequency range below 1 GHz, the average error is observed to decrease rapidly as the frequency increases. This is due to the reduction of ripples in the mobile location profile Φ with the decreasing

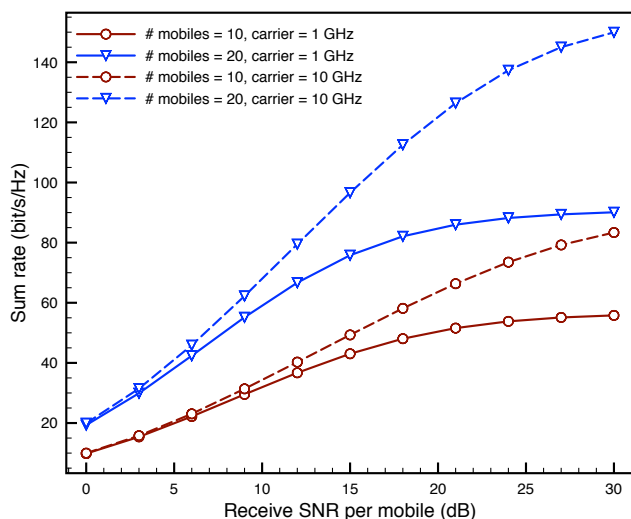


Fig. 4. Sum rate for channel-conjugate transmission versus receive SNR.

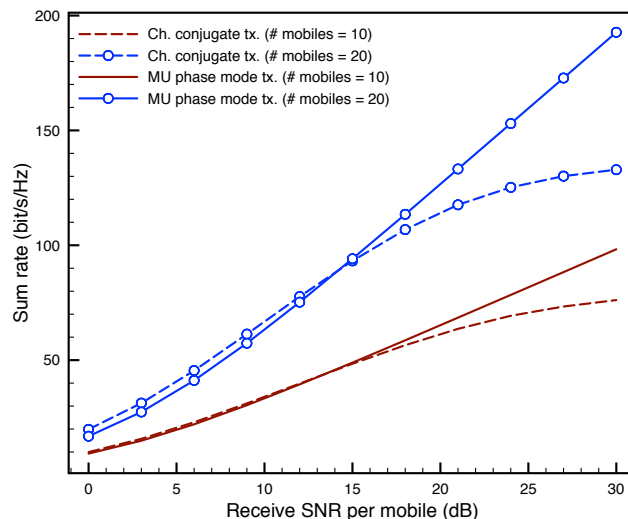


Fig. 5. Sum-rate comparison for channel-conjugate and MPM transmissions.

wavelength based on (15). However, for the frequency range above 2 GHz, the average error saturates about a small value caused by the fluctuating tails of Bessel functions in Φ . It can be observed from Fig. 3 that there exists an optimal carrier frequency at 1.5 GHz yielding the minimum average channel estimation error.

Fig. 4 shows the sum rate for channel-conjugate transmission for an increasing receive SNR. Different combinations of carrier frequencies $\{1, 10\}$ GHz, and numbers of mobiles, $\{5, 10\}$, are considered. One can see that the UA can support dense mobiles e.g., 20 mobiles in the circular region with a radius of 10 meters, and thereby achieves a high sum rate e.g., about 150 b/s/Hz for high SNRs. Furthermore, it is observed increasing the carrier frequency suppresses multiuser interference and hence alleviates the issue of rate saturation at high SNRs typical single-user transmission schemes.

The channel-conjugate and MPM transmissions are compared in Fig. 5 in terms of sum-rates. The scheme of MPM transmission is observed to substantially outperform the single-user scheme for high SNRs (above 20 dB) though their performance is almost identical for lower SNRs. In particular, for SNR of 30 dB, the sum rate for MPM transmission is about 60 b/s/Hz higher than that for channel-conjugate transmission.

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