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Citation	The 2015 IEEE International Conference on Communications (ICC 2015), London, UK., 8-12 June 2015. In IEEE International Conference on Communications, 2015, p. 4745 - 4750
Issued Date	2015
URL	http://hdl.handle.net/10722/214823
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Tight Probabilistic MSE Constrained Multiuser MISO Transceiver Design under Channel Uncertainty

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Abstract—A novel optimization method is proposed to solve the probabilistic mean square error (MSE) constrained multiuser multiple-input single-output (MU-MISO) transceiver design problem. Since the probabilistic MSE constraints cannot be expressed in closed-form under Gaussian channel uncertainty, existing probabilistic transceiver design methods rely on probability inequality approximations, resulting in conservative MSE outage realizations. In this paper, based on local structure of the feasible set in the probabilistic MSE constrained transceiver design problem, a set squeezing procedure is proposed to realize tight MSE outage control. Simulation results show that the MSE outage can be realized tightly, which results in significantly reduced transmit power compared to the existing inequality based probabilistic transceiver design.

Index Terms—Probabilistic MSE constrained transceiver design, Tight probabilistic control, Channel uncertainty.

I. INTRODUCTION

Due to diverse nature of data (e.g., video call, VoIP, online game, etc) simultaneously transmitting through modern heterogeneous wireless networks [1], [2], different quality of services (QoS) are needed from different users. Since the mean square error (MSE) of data can be mapped to the bit error rate [3], using MSE as a QoS criterion is popular in transceiver design [5]–[8], and is also the focus of this paper.

However, the ideal case of exact MSE control is hindered by channel uncertainty [4], [6]. By modeling the channel uncertainties lie in a bounded region, MSE constrained robust transceiver design are proposed to tackle the worse-case error in [6], [7]. On the other hand, under Gaussian channel uncertainty, bounded robust optimization is not suitable, and probabilistic MSE constrained transceiver design provides a soft MSE control. Previous probabilistic transceiver design schemes only provide approximation solutions by using different probability inequalities, e.g., Markov inequality and duality based method for probabilistic MSE constrained transceiver design [8], Vysochanskii-Petunin inequality for probabilistic MSE constrained power allocation [9]. However, owing to the restricted feasible set in those safe approximations, the MSE requirement of these designs are over-satisfied, which leads to unnecessarily high transmit powers.

In this paper, a tight probabilistic MSE control is achieved in MU-MISO transceiver design under Gaussian channel uncertainty. Facing the challenge of intractable probabilistic constraints, a successive method is proposed to reconstruct the feasible set. In particular, we first locate a feasible subset based

on the moment information of the channel uncertainty. Then, a joint feasible subsets refinement and sequential optimization is proposed to analyze the unexplored feasible subsets. The proposed set squeezing procedure ensures the transmit power decrease monotonically and the realized outage probability approach the outage target. Simulation results show that the probabilistic MSE requirement are fulfilled tightly, and the tight MSE outage control in turns provides excellent performance on transmit power compared to existing approximation based probabilistic transceiver design.

The rest of this paper is organized as follows. In Section II, the probabilistic transceiver design problem is formulated and a feasible subset is located. Joint feasible subsets refinement and optimization is described in Section III. The computation details of the set squeezing procedure is presented in Section IV. Simulation results are presented in Section V, and conclusions are drawn in Section VI.

Notation: In this paper, $\mathbb{E}(\cdot)$, $(\cdot)^T$, and $(\cdot)^H$ denote statistical expectation, transposition and Hermitian, respectively. In addition, $\|\cdot\|_2$ and $\|\cdot\|_F$ refer to the norm of a vector and Frobenius norm of a matrix, respectively, while $\text{vec}(\cdot)$ stands for the vectorization from a matrix into a column vector. Symbol $\text{Diag}(\mathbf{x})$ denotes a diagonal matrix with vector \mathbf{x} on its diagonal, and \mathbf{I}_K is a $K \times K$ identity matrix.

II. PROBLEM FORMULATION AND FINDING A FEASIBLE SUBSET

The downlink MU-MISO system consists of one base station (BS) equipped with N transmit antennas and K single-antenna active users. Let \mathbf{G} be the $N \times K$ precoding matrix at BS, \mathbf{h}_k and $1/a_k$ (with $a_k > 0$ and the phase rotation factor are embedded in the precoder [6]) are the $N \times 1$ channel vector and the equalizer of the k^{th} user, respectively. The Gaussian noise n_k at the k^{th} user is distributed as $\mathcal{CN}(0, \delta_k^2)$. With transmitted $K \times 1$ data vector \mathbf{s} , the recovered data at the k^{th} user is

$$\hat{s}_k = \frac{1}{a_k}(\mathbf{h}_k^T \mathbf{G} \mathbf{s} + n_k). \quad (1)$$

Since $\mathbb{E}(\mathbf{s}_k \mathbf{s}_k^H) = \mathbf{I}_K$ and the transmitted data are independent of the noise, the MSE of the k^{th} user's data is

$$\begin{aligned} \text{MSE}_k(\underbrace{[\text{vec}(\mathbf{G})^T, 1/a_1, \dots, 1/a_K]^T}_{\mathbf{w}}, \mathbf{h}_k) \\ = \mathbb{E}_{\mathbf{s}_k, \mathbf{n}_k} ((\mathbf{e}_k^T \mathbf{s} - \hat{s}_k)(\mathbf{e}_k^T \mathbf{s} - \hat{s}_k)^H) \\ = \|1/a_k \mathbf{h}_k^T \mathbf{G} - \mathbf{e}_k^T\|_2^2 + (\delta_k/a_k)^2, \end{aligned} \quad (2)$$

where the $K \times 1$ vector $\mathbf{e}_k = [0, \dots, 1, 0, \dots, 0]^T$ with the 1 appears at the k^{th} position [6], [8].

Since the channel cannot be perfectly known in practice, the channel is modelled as $\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{x}_k$, where $\hat{\mathbf{h}}_k$ is the estimated channel and \mathbf{x}_k is the Gaussian channel uncertainty distributed as $\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_k)$. Due to the channel uncertainty, the MSE_k is also a random variable. Therefore, the transceiver design aims at minimizing transmit power at the BS under probabilistic MSE constraints for different users is formulated as

$$\begin{aligned} \min_{\mathbf{G}, \{a_k\}_{k=1}^K} & \|\mathbf{G}\|_F \\ \text{s.t.} & \Pr\{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k\} \geq 1 - p_k, \quad \forall k \in \mathcal{K}, \end{aligned} \quad (4)$$

where $\mathcal{K} = \{1, \dots, K\}$, ε_k and p_k are the MSE target and the outage probability at the k^{th} user, respectively.

Owing to the unknown \mathbf{G} and a_k , and the nonlinear relationship between MSE_k and \mathbf{h}_k as shown in (3), the probabilistic MSE constraints in (4) cannot be expressed in closed-form, and subsequently the feasible set of problem (4) \mathcal{W}_0 is not directly available. A usual way to tackle the problem is to find a tractable upper bound function of $\Pr\{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \geq \varepsilon_k\}$. In this paper, we take the supremum of $\Pr\{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \geq \varepsilon_k\}$ under moment constraints as the upper bound. Then a feasible subset of problem (4) can be obtained from the feasible set of the following problem

$$\begin{aligned} \min_{\mathbf{G}, \{a_k\}_{k=1}^K} & \|\mathbf{G}\|_F \\ \text{s.t.} & \sup_{\substack{\mathbb{E}(\mathbf{x}_k) = \mathbf{0} \\ \mathbb{E}(\mathbf{x}_k \mathbf{x}_k^H) = \mathbf{\Sigma}_k}} \Pr\{\text{MSE}_k(\mathbf{w}, \hat{\mathbf{h}}_k + \mathbf{x}_k) \geq \varepsilon_k\} \leq p_k, \quad \forall k \in \mathcal{K}. \end{aligned} \quad (5)$$

According to [8], the problem (5) is equivalent to the following convex problem

$$\begin{aligned} \min_{\mathbf{G}, \{a_k, c_k, \tilde{\beta}_k, \tilde{\mathbf{Z}}_k\}_{k=1}^K} & \|\mathbf{G}\|_F \\ \text{s.t.} & \text{Tr}(\tilde{\mathbf{Z}}_k \mathbf{\Sigma}_k) \leq p_k \tilde{\beta}_k, \quad \tilde{\beta}_k > 0, \quad \tilde{\mathbf{Z}}_k \succeq 0, \quad \begin{bmatrix} a_k & 1 \\ 1 & c_k \end{bmatrix} \succeq 0, \quad \forall k \in \mathcal{K} \\ & \begin{bmatrix} \tilde{\mathbf{Z}}_k + \text{Diag}(\mathbf{0}, a_k \varepsilon_k - c_k \delta_k^2 - \tilde{\beta}_k) & \tilde{\mathbf{Q}}_k^H \\ \tilde{\mathbf{Q}}_k & a_k \mathbf{I}_K \end{bmatrix} \succeq 0, \quad \forall k \in \mathcal{K}, \end{aligned} \quad (6)$$

where $\tilde{\mathbf{\Sigma}}_k := \begin{bmatrix} \mathbf{\Sigma}_k & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}$, $\tilde{\mathbf{Q}}_k := [\mathbf{G}^T, \mathbf{G}^T \hat{\mathbf{h}}_k - a_k \mathbf{e}_k]$. Therefore, any feasible solution $(\mathbf{G}, \{1/a_k\}_{k=1}^K)$ in (6) is a feasible solution of (4). Note that the semidefinite programming (SDP) problem (6) can be solved by standard numerical optimization tool [14].

III. JOINT FEASIBLE SUBSETS REFINEMENT AND OPTIMIZATION

Since the upper bound function in (5) is only a conservative bound, the obtained feasible set is a conservative feasible

subset of (4). In this section, the local structure of a given feasible solution is utilized systematically to explore other feasible subset of (4). Since the support of the Gaussian random channel \mathbf{h}_k is \mathbb{C}^N , with any given feasible transceiver solution $\bar{\mathbf{w}}$ of (4), i.e., $\Pr\{\text{MSE}_k(\bar{\mathbf{w}}, \mathbf{h}_k) \leq \varepsilon_k\} \geq 1 - p_k$, a support subset of the random channel \mathbf{h}_k is

$$\mathcal{H}_k(\bar{\mathbf{w}}) := \{\mathbf{h}_k | \text{MSE}_k(\bar{\mathbf{w}}, \mathbf{h}_k) \leq \varepsilon_k\}. \quad (7)$$

Then, a feasible subset of problem (4) can be generated as follows,

$$\mathcal{W}(\bar{\mathbf{w}}) := \{\mathbf{w} | \{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k, \quad \forall \mathbf{h}_k \in \mathcal{H}_k(\bar{\mathbf{w}})\}_{k=1}^K\}. \quad (8)$$

The reason for $\mathcal{W}(\bar{\mathbf{w}})$ being a feasible subset of \mathcal{W}_0 is shown below.

Property 1. $\bar{\mathbf{w}} \in \mathcal{W}(\bar{\mathbf{w}}) \subseteq \mathcal{W}_0$

Proof. According to the definition $\mathcal{H}_k(\bar{\mathbf{w}}) := \{\mathbf{h}_k | \text{MSE}_k(\bar{\mathbf{w}}, \mathbf{h}_k) \leq \varepsilon_k\}$, the constraint $\text{MSE}_k(\bar{\mathbf{w}}, \mathbf{h}_k) \leq \varepsilon_k$ is automatically satisfied for all $\mathbf{h}_k \in \mathcal{H}_k(\bar{\mathbf{w}})$. Combining with the definition in (8) $\mathcal{W}(\bar{\mathbf{w}}) := \{\mathbf{w} | \{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k, \quad \forall \mathbf{h}_k \in \mathcal{H}_k(\bar{\mathbf{w}})\}_{k=1}^K\}$, then we directly have $\bar{\mathbf{w}} \in \mathcal{W}(\bar{\mathbf{w}})$. Furthermore, any \mathbf{w} in $\mathcal{W}(\bar{\mathbf{w}})$ satisfies the following condition

$$\begin{aligned} \Pr\{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k\} \\ = \int_{\mathbf{h}_k \in \mathcal{H}_k(\bar{\mathbf{w}})} \mathbf{1}_{\mathcal{F}_k}(\mathbf{h}_k) \cdot f(\mathbf{h}_k) d\mathbf{h}_k + \int_{\mathbf{h}_k \in \{\mathbb{C}^N \setminus \mathcal{H}_k(\bar{\mathbf{w}})\}} \mathbf{1}_{\mathcal{F}_k}(\mathbf{h}_k) \cdot f(\mathbf{h}_k) d\mathbf{h}_k \quad (9) \\ = \underbrace{\int_{\mathbf{h}_k \in \mathcal{H}_k(\bar{\mathbf{w}})} \mathbf{1} \cdot f(\mathbf{h}_k) d\mathbf{h}_k}_{\geq 1 - p_k} + \underbrace{\int_{\mathbf{h}_k \in \{\mathbb{C}^N \setminus \mathcal{H}_k(\bar{\mathbf{w}})\}} \mathbf{1}_{\mathcal{F}_k}(\mathbf{h}_k) \cdot f(\mathbf{h}_k) d\mathbf{h}_k}_{\geq 0} \quad (10) \\ \geq 1 - p_k, \quad \forall k \in \mathcal{K}, \end{aligned} \quad (11)$$

where $\mathbf{1}_{\mathcal{F}_k}(\mathbf{h}_k)$ is an indicator function of the set $\mathcal{F}_k := \{\mathbf{h}_k | \text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k\}$. Therefore, any \mathbf{w} in $\mathcal{W}(\bar{\mathbf{w}})$ is a feasible solution of (4), i.e., $\mathcal{W}(\bar{\mathbf{w}}) \subseteq \mathcal{W}_0$. \square

Therefore, each feasible solution $\bar{\mathbf{w}}$ of (4) can generate a feasible subset $\mathcal{W}(\bar{\mathbf{w}})$ which contains $\bar{\mathbf{w}}$ itself. Although optimization over $\mathcal{W}(\bar{\mathbf{w}})$ may find better solution than $\bar{\mathbf{w}}$, a larger feasible subset than $\mathcal{W}(\bar{\mathbf{w}})$ can be obtained as follows.

From the coupling effect between the support subset $\mathcal{H}_k(\bar{\mathbf{w}})$ and feasible subset $\mathcal{W}(\bar{\mathbf{w}})$ in (8), it can be seen that reducing the number of elements in the support subset $\mathcal{H}_k(\bar{\mathbf{w}})$ may enlarge the feasible subset $\mathcal{W}(\bar{\mathbf{w}})$. Therefore, we consider a squeezed support subset $\mathcal{H}_k(\bar{\mathbf{w}})$ as

$$\mathcal{H}_k(\bar{\mathbf{w}}, q_k) := \{\mathbf{h}_k | \text{MSE}_k(\bar{\mathbf{w}}, \mathbf{h}_k) \leq q_k\}, \quad (12)$$

where $q_k \leq \varepsilon_k$, and we have

$$\mathcal{H}_k(\bar{\mathbf{w}}, q_k) \subseteq \mathcal{H}_k(\bar{\mathbf{w}}). \quad (13)$$

Then a set generated from $\mathcal{H}_k(\bar{\mathbf{w}}, q_k)$ is constructed as

$$\mathcal{W}(\bar{\mathbf{w}}, \mathbf{q}) := \{\mathbf{w} | \{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k, \quad \forall \mathbf{h}_k \in \mathcal{H}_k(\bar{\mathbf{w}}, q_k)\}_{k=1}^K\}, \quad (14)$$

where $\mathbf{q} = [q_1, q_2, \dots, q_K]^T$. In order to make $\mathcal{W}(\bar{\mathbf{w}}, \mathbf{q})$ a feasible subset of \mathcal{W}_0 , the parameters $\{q_k\}_{k=1}^K$ should be

chosen such that for any $\mathbf{w} \in \mathcal{W}(\bar{\mathbf{w}}, \mathbf{q})$, it must satisfy the constraints in (4). With similar derivations to (10), it can be established that $\Pr\{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k\} = \Pr\{\mathbf{h}_k \in \mathcal{H}_k(\bar{\mathbf{w}}, q_k)\} + c$, where c is always nonnegative. Since increasing q_k would decrease $\Pr\{\mathbf{h}_k \in \mathcal{H}_k(\bar{\mathbf{w}}, q_k)\}$, in order to guarantee $\Pr\{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k\} \geq 1 - p_k$, the maximum q_k is chosen to satisfy $\Pr\{\mathbf{h}_k \in \mathcal{H}_k(\bar{\mathbf{w}}, q_k)\} = 1 - p_k$.

To reveal the inter-relationship between $\mathcal{W}(\bar{\mathbf{w}})$ and $\mathcal{W}(\bar{\mathbf{w}}, \mathbf{q})$, we consider

$$\begin{aligned} & \mathcal{W}(\bar{\mathbf{w}}, \mathbf{q}) \cap \mathcal{W}(\bar{\mathbf{w}}) \\ &= \left\{ \mathbf{w} \mid \{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k, \forall \mathbf{h}_k \in \mathcal{H}_k(\bar{\mathbf{w}}, q_k)\}_{k=1}^K, \right. \\ & \quad \left. \{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k, \forall \mathbf{h}_k \in \mathcal{H}_k(\bar{\mathbf{w}})\}_{k=1}^K \right\} \quad (15) \end{aligned}$$

$$= \left\{ \mathbf{w} \mid \{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k, \forall \mathbf{h}_k \in \mathcal{H}_k(\bar{\mathbf{w}})\}_{k=1}^K \right\} \quad (16)$$

$$= \mathcal{W}(\bar{\mathbf{w}}), \quad (17)$$

where the second equality comes from the inclusive relationship in (13) and the final equality comes from the definition in (8). Therefore, an important property of those constructed feasible subsets is

$$\bar{\mathbf{w}} \in \mathcal{W}(\bar{\mathbf{w}}) \subseteq \mathcal{W}(\bar{\mathbf{w}}, \mathbf{q}) \subseteq \mathcal{W}_0. \quad (18)$$

That is, the squeezed support sets $\{\mathcal{H}_k(\bar{\mathbf{w}}, q_k)\}_{k=1}^K$ in (12) enlarge the corresponding feasible subset $\mathcal{W}(\bar{\mathbf{w}}, \mathbf{q})$ in (14). Therefore, the complementary phenomenon between support subsets and feasible subset reveals the duality property locally.

Owing to $\bar{\mathbf{w}} \in \mathcal{W}(\bar{\mathbf{w}}, \mathbf{q})$, better feasible solution than $\bar{\mathbf{w}}$ can be found via $\min \{\|\mathbf{G}\|_F \mid \mathbf{w} \in \mathcal{W}(\bar{\mathbf{w}}, \mathbf{q})\}$. With the obtained new solution, we can construct another feasible subset of \mathcal{W}_0 and perform another optimization, and so on. That makes iterative improvement of the objective function becomes possible. The proposed set squeezing procedure begins with finding a feasible solution $\mathbf{w}^{[0]} = [\text{vec}(\mathbf{G})^T, 1/a_1, \dots, 1/a_K]^T$ from (6) or any other safe approximation, followed by iterations between the following two steps until convergence.

- **P-step:** Finding $q_k^{[i]} \leq \varepsilon_k$ such that $\Pr\{\text{MSE}_k(\mathbf{w}^{[i]}, \mathbf{h}_k) \leq q_k^{[i]}\} = 1 - p_k$.
- **O-step:** Solving the i^{th} subproblem $\min \{\|\mathbf{G}\|_F \mid \{\text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k, \forall \mathbf{h}_k \in \mathcal{H}_k(\mathbf{w}^{[i]}, q_k^{[i]})\}_{k=1}^K\}$, denoting the solution as $\mathbf{w}^{[i+1]}$. Increment i by one.

Lemma 1. *If $\mathbf{w}^{[i]}$ generated from the $(i-1)^{\text{th}}$ O-step does not activate the k^{th} inequality constraint in the original problem (4), then $\mathbf{w}^{[i]}$ does not activate the k^{th} inequality constraint of the i^{th} O-step subproblem.*

Proof. If the $(i-1)^{\text{th}}$ O-step solution $\mathbf{w}^{[i]}$ does not activate the k^{th} constraint in (4), i.e., $\Pr\{\text{MSE}_k(\mathbf{w}^{[i]}, \mathbf{h}_k) \leq \varepsilon_k\} > 1 - p_k$, the parameter $q_k^{[i]} < \varepsilon_k$ is needed to make $\Pr\{\text{MSE}_k(\mathbf{w}^{[i]}, \mathbf{h}_k) \leq q_k^{[i]}\} = 1 - p_k$ at P-step. Together with the definition $\mathcal{H}_k(\mathbf{w}^{[i]}, q_k^{[i]}) = \{\mathbf{h}_k \mid \text{MSE}_k(\mathbf{w}^{[i]}, \mathbf{h}_k) \leq q_k^{[i]}\}$, we have $\text{MSE}_k(\mathbf{w}^{[i]}, \mathbf{h}_k) \leq q_k^{[i]} < \varepsilon_k$ is satisfied for all $\mathbf{h}_k \in \mathcal{H}_k(\mathbf{w}^{[i]}, q_k^{[i]})$. Therefore, $\mathbf{w}^{[i]}$ does not activate the k^{th} constraint $\text{MSE}_k(\mathbf{w}^{[i]}, \mathbf{h}_k) \leq \varepsilon_k$ in the i^{th} O-step subproblem. \square

By using *Lemma 1*, the convergence property of the set squeezing procedure is presented as follows.

Proposition 1. *If the optimal solution of O-step subproblem is obtained, the set squeezing procedure converges and the limit solution activates all constraints in problem (4).*

Proof. First, since $\mathbf{w}^{[i]} \in \mathcal{W}(\mathbf{w}^{[i]}, \mathbf{q})$ is established in (18), the optimal solution of O-step subproblem guarantees $\|\mathbf{G}^{[i+1]}\|_F \leq \|\mathbf{G}^{[i]}\|_F$. With the monotonic decreasing property of $\|\mathbf{G}\|_F$, and the transmit power is bounded below by zero, the convergence of set squeezing procedure is guaranteed.

Second, if $\mathbf{w}^{[i]}$ does not activate the k^{th} constraint in the original problem (4), according to *Lemma 1*, $\mathbf{w}^{[i]}$ does not activate the k^{th} constraint $\|\mathbf{h}_k^T \mathbf{G}^{[i]} / a_k^{[i]} - \mathbf{e}_k^T\|_2^2 + (\delta_k / a_k^{[i]})^2 \leq \varepsilon_k$ in i^{th} O-step subproblem. This implies directly scaling down the k^{th} column of $\mathbf{G}^{[i]}$, which becomes $\mathbf{G}^{[i+1]}$, until $\|\mathbf{h}_k^T \mathbf{G}^{[i+1]} / a_k^{[i+1]} - \mathbf{e}_k^T\|_2^2 + (\delta_k / a_k^{[i+1]})^2 = \varepsilon_k$ would reduce transmit power strictly, hence $\|\mathbf{G}^{[i+1]}\|_F < \|\mathbf{G}^{[i]}\|_F$ becomes possible. Furthermore, scaling down the k^{th} column of $\mathbf{G}^{[i]}$ reduces other users' MSEs, and other MSE constraints would remain valid. Therefore, the set squeezing procedure with optimal solution in successive O-steps would not stop, as long as any of the user's constraint in (4) is not active. That is, the limit solution activates all constraints in (4). \square

The proposed set squeezing procedure can be generalized to any quadratically perturbed chance-constrained programming with continuous uncertainty distributions [10].

IV. COMPUTATION DETAILS OF THE SET SQUEEZING PROCEDURE FOR MU-MISO TRANSCIVER DESIGN

The details of P-step and O-step are derived in this section.

A. P-step

For a given feasible solution $(\mathbf{G}^{[i]}, \{1/a_k^{[i]}\}_{k=1}^K)$, the P-step is to find the quantile $q_k^{[i]}$ such that

$$\Pr \left(\|\mathbf{h}_k^T \mathbf{G}^{[i]} / a_k^{[i]} - \mathbf{e}_k^T\|_2^2 + (\delta_k / a_k^{[i]})^2 \leq q_k^{[i]} \right) = 1 - p_k, \quad (19)$$

which can be solved by probability evaluation with bisection candidate $q_k^{[i]} \in [(\delta_k / a_k^{[i]})^2, \varepsilon_k]$. Since $\mathbf{h}_k \sim \mathcal{CN}(\hat{\mathbf{h}}_k, \Sigma_k)$, the normalization and singular value decomposition

$$(\mathbf{G}^{[i]})^T / a_k^{[i]} (\Sigma_k / 2)^{\frac{1}{2}} = \mathbf{U}_k [\text{Diag}([\sigma_1, \dots, \sigma_K]), \mathbf{0}_{K \times (N-K)}] \mathbf{V}_k^H, \quad (20)$$

where the singular values $\{\sigma_j\}_{j=1}^K$ are arranged in descending order. Therefore, the statistical representation of the random variable in (19) is

$$\|(\mathbf{G}^{[i]})^T / a_k^{[i]} \mathbf{h}_k - \mathbf{e}_k\|_2^2 \sim \sum_{j=1}^K \sigma_j^2 \chi_{(|\eta_j|^2, 2)}^2, \quad (21)$$

which is a weighted sum of independent noncentral chi-squared variables $\chi_{(|\eta_j|^2, 2)}^2$ with two degrees of freedom, and η_j is the j^{th} element of the vector $[\mathbf{I}_K, \mathbf{0}_{K \times (N-K)}] \mathbf{V}_k^H (\Sigma_k / 2)^{-\frac{1}{2}} \hat{\mathbf{h}}_k - \text{Diag}([1/\sigma_1, \dots, 1/\sigma_K]) \mathbf{U}_k^H \mathbf{e}_k$.

Therefore, the cumulant-generating function (CGF) of $\|\mathbf{h}_k^T \mathbf{G}^{[i]} / a_k^{[i]} - \mathbf{e}_k^T\|_2^2$ is

$$\kappa(t) = \sum_{j=1}^K \left(\frac{|\eta_j|^2 \sigma_j^2 t}{1 - 2\sigma_j^2 t} - \ln(1 - 2\sigma_j^2 t) \right), \quad (22)$$

with its domain $(-\infty, 1/(2\sigma_1^2))$.

With the CGF in (22), the left side of (19) can be evaluated using the second-order saddlepoint approximation [11, p. 53]

$$\Pr\left(\|\mathbf{h}_k^T \mathbf{G}^{[i]} / a_k^{[i]} - \mathbf{e}_k^T\|_2^2 \leq q_k^{[i]} - (\delta_k / a_k^{[i]})^2\right) \simeq \Phi(u) + \phi(u) \cdot \left\{ \frac{1}{u} - \frac{1}{v} - v^{-1} \left(\frac{O_4}{8} - \frac{5}{24} (O_3)^2 \right) + v^{-3} + \frac{O_3}{2v^2} - u^{-3} \right\}, \quad (23)$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ are the cumulate distribution function and probability density function of standard normal distribution, $u = \text{sign}(t_0) \sqrt{2(t_0 \cdot (q_k^{[i]} - (\delta_k / a_k^{[i]})^2) - \kappa(t_0))}$, $v = t_0 \sqrt{\kappa''(t_0)}$, $O_n = \kappa^{(n)}(t_0) / \{\kappa''(t_0)\}^{n/2}$ with $n = \{3, 4\}$, and the saddlepoint t_0 is calculated through

$$\kappa'(t_0) = q_k^{[i]} - (\delta_k / a_k^{[i]})^2 \quad (24)$$

by bisection in the domain $t_0 \in (-\infty, 1/(2\sigma_1^2))$. Note that the uniqueness of the saddlepoint is guaranteed by the fact that $\kappa''(t) > 0$ in its domain, i.e., $\kappa'(t)$ is monotonically increasing. Since the relative error by using (23) can be calculated according to the analysis in [12], pre-distorting the relative error in the outage target ensures tight outage probability control even under saddlepoint approximation error.

B. O-step

With the quantile $q_k^{[i]}$ obtained in the P-step, the corresponding subproblem in O-step is

$$\begin{aligned} \min_{\mathbf{G}, \{a_k, \lambda_k\}_{k=1}^K} & \|\mathbf{G}\|_F \\ \text{s.t.} & \text{MSE}_k(\mathbf{w}, \mathbf{h}_k) \leq \varepsilon_k, \forall \mathbf{h}_k: \text{MSE}_k(\mathbf{w}^{[i]}, \mathbf{h}_k) \leq q_k^{[i]}, \forall k \in \mathcal{K}. \end{aligned} \quad (25)$$

After applying the S-lemma [13, p.23], (25) is equivalent to the problem

$$\begin{aligned} \min_{\mathbf{G}, \{a_k, \lambda_k\}_{k=1}^K} & \|\mathbf{G}\|_F \\ \text{s.t.} & \lambda_k \mathbf{A}_k^{[i]} + \text{Diag}([\mathbf{0}, a_k \varepsilon_k - \delta_k^2 / a_k]) - \frac{1}{a_k} \mathbf{Q}_k^H \mathbf{Q}_k \succeq \mathbf{0}, \forall k \in \mathcal{K} \\ & \lambda_k \geq 0, \quad a_k > 0, \quad \forall k \in \mathcal{K}, \end{aligned} \quad (26)$$

where $\mathbf{Q}_k := [\mathbf{G}^T, -a_k \mathbf{e}_k]$, $\mathbf{A}_k^{[i]} = (\mathbf{Q}_k^{[i]})^H \mathbf{Q}_k^{[i]} - \text{Diag}([\mathbf{0}, q_k^{[i]} - (\delta_k / a_k^{[i]})^2])$ with $\mathbf{Q}_k^{[i]} = [(\mathbf{G}^{[i]})^T / a_k^{[i]}, -\mathbf{e}_k]$. Furthermore, with Schur complement, (26) is transformed into

$$\begin{aligned} \min_{\mathbf{G}, \{a_k, \lambda_k\}_{k=1}^K} & \|\mathbf{G}\|_F \\ \text{s.t.} & \begin{bmatrix} \lambda_k \mathbf{A}_k^{[i]} + \text{Diag}([\mathbf{0}, a_k \varepsilon_k - \delta_k^2 / a_k]) & \mathbf{Q}_k^H \\ \mathbf{Q}_k & a_k \mathbf{I}_K \end{bmatrix} \succeq \mathbf{0}, \forall k \in \mathcal{K} \\ & \lambda_k \geq 0, \quad a_k > 0, \quad \forall k \in \mathcal{K}. \end{aligned} \quad (27)$$

Finally, by introducing slack variable c_k with $c_k \geq 1/a_k$, (27) is equivalent to [8]

$$\begin{aligned} \min_{\mathbf{G}, \{a_k, c_k, \lambda_k\}_{k=1}^K} & \|\mathbf{G}\|_F \\ \text{s.t.} & \begin{bmatrix} \lambda_k \mathbf{A}_k^{[i]} + \text{Diag}([\mathbf{0}, a_k \varepsilon_k - c_k \delta_k^2]) & \mathbf{Q}_k^H \\ \mathbf{Q}_k & a_k \mathbf{I}_K \end{bmatrix} \succeq \mathbf{0}, \forall k \in \mathcal{K} \\ & \lambda_k \geq 0, \quad \begin{bmatrix} a_k & 1 \\ 1 & c_k \end{bmatrix} \succeq \mathbf{0}, \quad \forall k \in \mathcal{K}, \end{aligned} \quad (28)$$

which is a convex SDP problem.

C. Summary

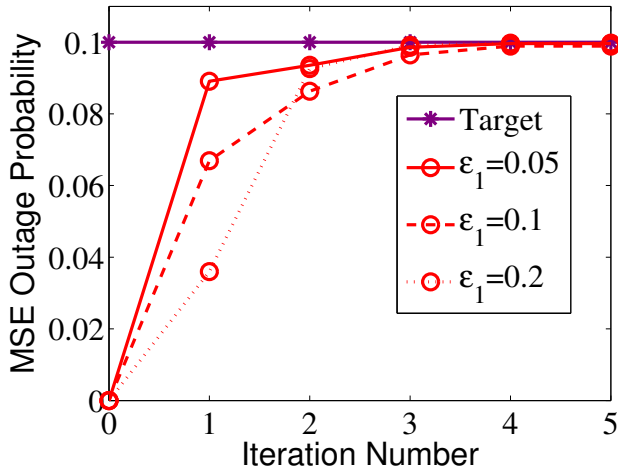
The proposed set squeezing procedure for the probabilistic beamforming problem starts with a feasible solution from (6), and follows iterations between (23) for P-step and (28) for O-step until the difference between successive transmit power is smaller than a pre-defined threshold. Since the optimal solution of O-step subproblem is obtained in (28), the limit solution activates all constraints of (4) according to the *Proposition 1*.

V. SIMULATION RESULTS AND DISCUSSIONS

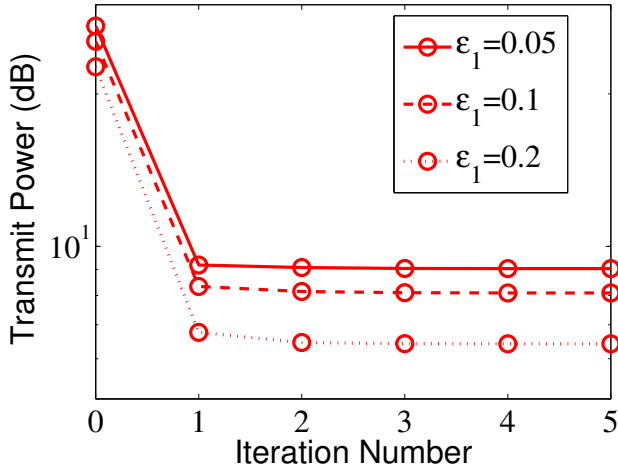
In this section, the performance of the set squeezing procedure is illustrated under different MSE requirements. The downlink channel for each user is modeled as $\mathbf{h}_k = \mathbf{R}_t^{\frac{1}{2}} \mathbf{h}_w$, where the elements of \mathbf{h}_w are standard complex Gaussian variables, and the channel correlation matrix is $[\mathbf{R}_t]_{ij} = \rho_t^{|i-j|}$ with correlation coefficient $\rho_t = 0.2$. The BS is equipped with four antennas and there are two active users (i.e., $N = 4, K = 2$), and the variance of the complex Gaussian noise at every antenna is $\delta_k^2 = 0.01$. The MSE requirement for the second user is fixed as $\varepsilon_2 = 0.2$ and $p_2 = 10\%$, while that for the first user is specified in the figures presented below, where each point is an average of 10^3 independent simulation runs. The relative power difference $\|\mathbf{G}^{[i]}\|_F^2 - \|\mathbf{G}^{[i+1]}\|_F^2 / \|\mathbf{G}^{[i]}\|_F^2 \leq 10^{-3}$ is used to terminate the set squeezing procedure. For the set squeezing procedure, the bisection accuracy in finding the quantile $q_k^{[i]}$ is 0.01%, and the bisection accuracy for the saddlepoint t_0 is 10^{-8} . To backoff the relative error of the saddlepoint method, all outage targets are predistorted $p_k / 1.015$ [12]. With the linear minimum mean square error channel estimator, the channel estimation error covariance matrix is $\Sigma_k = (\mathbf{R}_t^{-T} + P_t / \delta_n^2 \mathbf{I}_N)^{-1}$ [8]. In the following, the pilot-to-noise ratio is set as $P_t / \delta_n^2 = 10^2$ (i.e., 20dB).

The convergence performance of the set squeezing procedure is illustrated at Fig. 1 with $p_1 = 10\%$. Fig. 1(a) shows that the outage probabilities gradually approach the outage target, irrespective to the MSE requirement value ε_1 . Furthermore, it is noticed that the outage probabilities is very close to the 10% outage target at the second iteration, the remaining space to reduce the transmit power is small, and therefore the transmit powers in Fig. 1(b) decrease slowly after the second iteration.

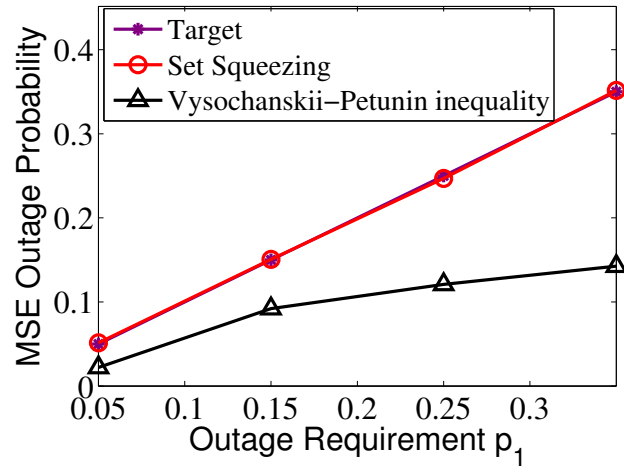
In Fig. 2, we compare the performance of the set squeezing procedure to that of the Vysotskii-Petunin (V-P) inequality based method [9] with $\varepsilon_1 = 0.1$ under different outage requirement p_1 . For fair comparison, the channel realizations are feasible for both methods. It is observed from Fig. 2(a) that the outage probability target is realized tightly by the set squeezing



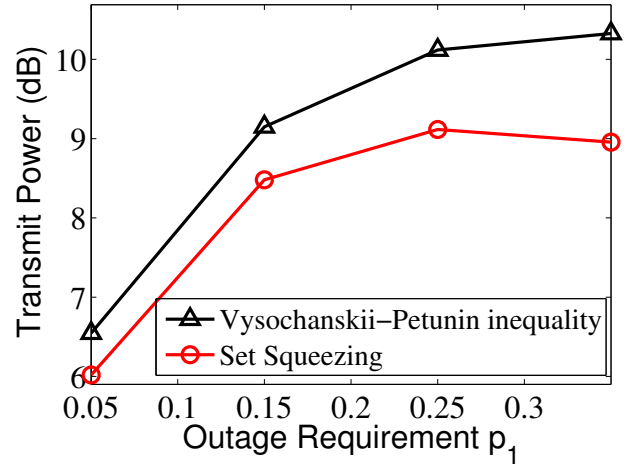
(a)



(b)

 Fig. 1. The convergence performance of the set squeezing procedure under $p_1 = 10\%$.


(a)



(b)

 Fig. 2. Achieved MSE outage probability and transmit power versus different outage targets under $\varepsilon_1 = 0.1$.

procedure over a wide range of outage target p_1 , while the V-P method reaches a much conservative outage probability. As a result of the tightly controlled outage performance from the set squeezing procedure, 0.5 to 1.5 dB transmit power gain is achieved compared to the V-P method as shown in Fig. 2(b). Note that the nonmonotonic transmit power behavior in Fig. 2(b) is due to the following two contrasting reasons. First, since only the good channel realizations with high channel gain are feasible at low outage scenario, the transmit power is small when p_1 is small. Second, as the outage requirement p_1 increases, the QoS requirement becomes less stringent and the transmit power should decrease.

VI. CONCLUSIONS

In this paper, a novel optimization method was proposed to achieve tight probabilistic MSE outage control in MU-MISO transceiver design under Gaussian channel uncertainty.

First, based on the moment information of channel uncertainty, a feasible solution of the probabilistic transceiver design problem was obtained. Then, with the proposed set squeezing procedure, the local structure of the obtained feasible solution is utilized systematically to explore other feasible subsets of the original problem, leading to tight outage control. Simulation results showed that, as a result of tight MSE outage control, significant transmit power was saved compared to the existing approximation based probabilistic transceiver design.

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