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Fully-distributed Joint Clock Synchronization and Ranging in Wireless Sensor Networks Under Exponential Delays

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Abstract—In this paper, we study the global clock synchronization and ranging problem for wireless sensor networks in the presence of unknown exponential delays using the two-way message exchange mechanism. Based on the Alternating Direction Method of Multipliers (ADMM), we propose a fully-distributed synchronization and ranging algorithm which has low communication overhead and computation cost. Simulation results show that the proposed algorithm achieves better accuracy than consensus algorithm, and can always converge to the centralized optimal solution.

I. INTRODUCTION

Wireless Sensor Networks (WSNs) have widely been used for monitoring habitat environments, controlling industrial machines, object tracking and event detection [1], [2]. All these applications demand all the nodes running on a common time frame. However, every individual sensor in a WSN has its own clock. Different clocks will drift from each other over time due to imperfection in oscillator circuits. This necessitates global clock synchronization among nodes in the entire network.

A critical component of clock synchronization problem is the modeling of the random network delays that perturb the message-exchange process. It was observed in [3] that when the point-to-point Hypothesis Reference Connection topology is of interest, the cumulative link delay in WSNs is approximately represented as a single server M/M/1 queue, and the random delays should be modeled as exponential random variables, which is also supported by experimental measurements [4]–[7]. Hence the assumption of exponential delay distribution is worth investigating.

Traditionally, clock synchronization in WSNs relies on spanning tree or cluster-based structure [8], [9]. Under such structures, synchronization is achieved through level-by-level pairwise synchronization. The disadvantages of such protocols are that it requires large overhead to maintain the spanning tree or cluster structure, and synchronization error is accumulated quickly as distance from reference node increases.

Without special network structure, fully distributed synchronization based on averaged consensus algorithms have been proposed in [10], [11]. However, message delays are not considered in such algorithms, which causes large mean-square-error in converged clocks, and ranging information cannot be obtained. More recently, [12] pioneered the fully-distributed
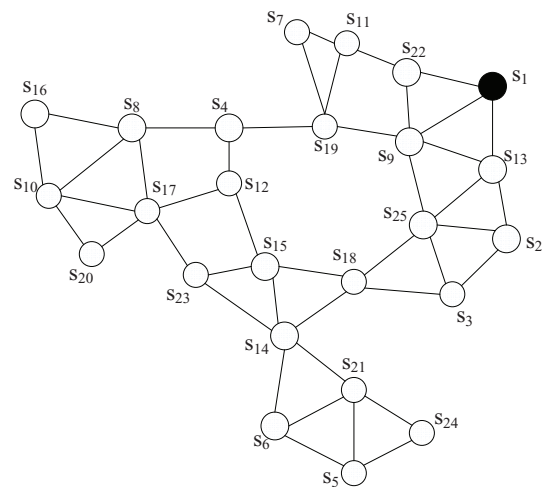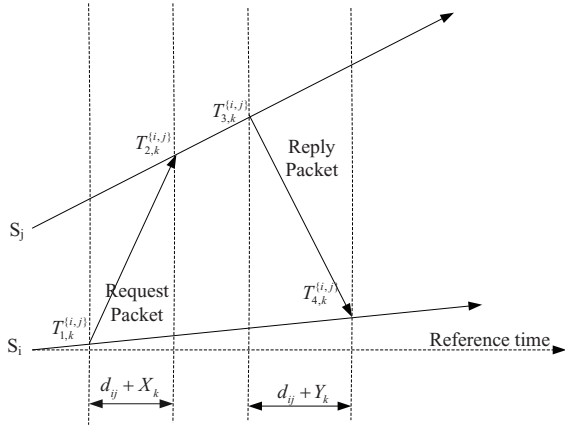

Figure 1: Topology of a network with 25 sensors $\{S_i\}_{i=1}^{25}$

clock offset estimation algorithm under exponential delays based on factor-graph, however, only clock offset is considered, resulting in potentially frequent re-synchronization.

In this paper, a network-wide joint estimator of clock offsets, clock skews and fixed delays is derived under exponential delay model. The joint maximum likelihood estimation problem is first cast into a linear programming (LP) problem, and then a distributed solution is derived based on ADMM [13]. Simulation results show that the proposed algorithm approaches the performance of the centralized LP solution.

II. SYSTEM MODEL AND MAXIMUM LIKELIHOOD ESTIMATOR

Consider a strictly connected network with N sensor nodes $\{S_1, S_2, \cdots, S_N\}$. These sensors are randomly distributed in the field and can be self-organized into a network by establishing connections between neighbor nodes lying within each other's communication range. An example of 25 sensor nodes is shown in Figure 1, where each edge represents the ability to transmit and receive packets between the pair of nodes. The communication network topology is described by the link set $\mathcal{L} \triangleq \{(i,j) : \text{there is a link between nodes } S_i \text{ and } S_j\}$. Each

Figure 2: Two-way message exchange between nodes S_i and S_j

sensor S_i has a clock which gives clock reading $c_i(t)$ at real time t. The second-order model for the function $c_i(t)$ is

$$c_i(t) \triangleq \beta_i t + \theta_i, \qquad (1)$$

where β_i and θ_i represent the clock skew and offset of S_i, respectively.

In order to establish clock relationship between two neighboring nodes, two-way message exchange process [8], [14] is performed. Assume that nodes S_i and S_j are in the communication range of each other, i.e., $(i,j) \in \mathcal{L}$, the k-th round message exchange between S_i and S_j is shown in Figure 2. In the message exchange process, node S_i sends a synchronization message to node S_j with its sending time $T_{1,k}^{\{i,j\}}$, S_j records its time $T_{2,k}^{\{i,j\}}$ at the reception of that message and replies S_i at time $T_{3,k}^{\{i,j\}}$. The replied message contains both $T_2^{\{i,j\}}$ and $T_3^{\{i,j\}}$. Then S_i records the reception time of S_j's reply as $T_{4,k}^{\{i,j\}}$. The message exchange process is repeated for K rounds.

With the clock model (1), the above message exchange procedure can be described as

$$(T_{2,k}^{\{i,j\}} - \theta_j)/\beta_j = (T_{1,k}^{\{i,j\}} - \theta_i)/\beta_i + d_{ij} + X_k^{\{i,j\}} \qquad (2)$$

$$(T_{4,k}^{\{i,j\}} - \theta_i)/\beta_i = (T_{3,k}^{\{i,j\}} - \theta_j)/\beta_j + d_{ij} + Y_k^{\{i,j\}}, \qquad (3)$$

where d_{ij} stands for the fixed portion of message delay between S_i and S_j, which is considered unknown but symmetric in uplink and downlink transmission ($d_{ij} = d_{ji}$); $X_k^{\{i,j\}}$ and $Y_k^{\{i,j\}}$ are variable portions of the message delay in the uplink and downlink. In this paper, we focus on the case where the random delays are independent and identically distributed (i.i.d.) exponential variables with mean $1/\lambda$ ($\lambda > 0$).

Since $\{X_k^{\{i,j\}}, Y_k^{\{i,j\}}\}_{k=1}^K$ are i.i.d. exponential random variables, based on (2) and (3) and introducing $\alpha_i = 1/\beta_i$, $\gamma_i = \theta_i/\beta_i$, the network-wide likelihood function, when given all the unknowns, can be represented as

$$f\left(\left\{T_{1,k}^{\{i,j\}}, T_{2,k}^{\{i,j\}}, T_{3,k}^{\{i,j\}}, T_{4,k}^{\{i,j\}}\right\}_{(i,j)\in\mathcal{L}, k\in[1,K]}; \mathbf{x}, \mathbf{d}, \lambda\right)$$

$$= \lambda^{4K|\mathcal{L}|} \exp\left\{ -\lambda \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \sum_{k=1}^K \left[(T_{4,k}^{\{i,j\}} - T_{1,k}^{\{i,j\}})\alpha_i \right.\right.$$
$$\left.\left. + (T_{2,k}^{\{i,j\}} - T_{3,k}^{\{i,j\}})\alpha_j - 2d_{ij} \right] \right\}$$
$$\times \prod_{i=1}^N \prod_{j\in\mathcal{N}_i} \prod_{k=1}^K \mathcal{I}_+\left(-T_{1,k}^{\{i,j\}}\alpha_i + \gamma_i + T_{2,k}^{\{i,j\}}\alpha_j - \gamma_j - d_{ij} \right)$$
$$\times \prod_{i=1}^N \prod_{j\in\mathcal{N}_i} \prod_{k=1}^K \mathcal{I}_+\left(T_{4,k}^{\{i,j\}}\alpha_i - \gamma_i - T_{3,k}^{\{i,j\}}\alpha_j + \gamma_j - d_{ij} \right), \qquad (4)$$

where $\mathcal{N}_i$ denotes the set of neighbors of node S_i; $\mathbf{x} = [\alpha_2, \gamma_2, \cdots, \alpha_N, \gamma_N]^T$ and $\mathbf{d}$ is a vector containing d_{ij} as elements with $j \in \mathcal{N}_i$ and $j > i$, and the indexes are arranged in ascending order on i and then on j; $|\mathcal{L}|$ is the number of elements in link set $\mathcal{L}$; $\mathcal{I}_+(\cdot)$ is the indicator function of the nonnegative orthant. Here, without loss of generality, S_1 is selected as the reference node with known clock parameters α_1 and γ_1. Notice that due to the invariance property of maximum-likelihood estimator (MLE) [15], the MLE of $\{\beta_i\}_{i=1}^N$, $\{\theta_i\}_{i=1}^N$, and $\mathbf{d}$ is equivalent to that of $\mathbf{x}$ and $\mathbf{d}$, since they are related by an invertible one-to-one transformation.

For a given $\mathbf{x}$ and $\mathbf{d}$, differentiating the logarithm of (4) with respect to λ and setting the result to zero, we can obtain the conditional MLE $\hat{\lambda}$ for λ. Putting $\hat{\lambda}$ back into (4) and after some manipulations, the MLE of $\mathbf{x}$ and $\mathbf{d}$ is equivalent to the solution of the following LP problem

$$\min_{\mathbf{x},\mathbf{d}} \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \left[T_s^{\{i,j\}}\alpha_i + T_r^{\{i,j\}}\alpha_j - 2Kd_{ij} \right]$$
$$\text{s.t.} \begin{cases} T_{1,k}^{\{i,j\}}\alpha_i - \gamma_i - T_{2,k}^{\{i,j\}}\alpha_j + \gamma_j + d_{ij} \le 0 \\ -T_{4,k}^{\{i,j\}}\alpha_i + \gamma_i + T_{3,k}^{\{i,j\}}\alpha_j - \gamma_j + d_{ij} \le 0 \\ d_{ij} \ge 0, \qquad (i,j) \in \mathcal{L}, k \in \{1, \cdots, K\}, \end{cases} \qquad (5)$$

where $T_s^{\{i,j\}} = \sum_{k=1}^K (T_{4,k}^{\{i,j\}} - T_{1,k}^{\{i,j\}})$ and $T_r^{\{i,j\}} = \sum_{k=1}^K (T_{2,k}^{\{i,j\}} - T_{3,k}^{\{i,j\}})$. Since the constraints in (5) define a feasible domain $\mathbb{X}$ which relies on unknown parameters $\mathbf{x}$ and $\mathbf{d}$, there is no simple closed-form solution. However, it can be solved in a centralized way using different numerical methods, such as the simplex method or the interior-point method, and the centralized solution is guaranteed to be globally optimal. Unfortunately, such numerical methods are computationally expensive, especially for large scale WSNs. In addition to computational complexity, in centralized approach, all the local information should be sent to a central processing unit, and the estimation results need to be forwarded back to each individual node, thus putting heavy communication burden to the network. In the following section, fully-distributed synchronization algorithms will be proposed based on ADMM, which make use of the local information along with message passing among direct neighbors, thus is energy-efficient and scalable with network size.

III. DISTRIBUTED CLOCK SYNCHRONIZATION ALGORITHM

First, we transform the problem (5) into a compact form with the introduction of slack variables. Define $\mathbf{x}_i = [\alpha_i \ \gamma_i]^T$, $\mathbf{a}_i = [\sum_{j \in \mathcal{N}_i}(T_s^{\{i,j\}} + T_r^{\{j,i\}}) \ \ 0]^T$,

$$\mathbf{B}^{\{i,j\}} = \begin{bmatrix} T_{1,1}^{\{i,j\}} & -T_{4,1}^{\{i,j\}} \dots T_{1,K}^{\{i,j\}} & -T_{4,K}^{\{i,j\}} \\ -1 & 1 & -1 & 1 \end{bmatrix}^T,$$

and $\mathbf{E}^{\{i,j\}} = \begin{bmatrix} -T_{2,1}^{\{i,j\}} & T_{3,1}^{\{i,j\}} \dots -T_{2,K}^{\{i,j\}} & T_{3,K}^{\{i,j\}} \\ 1 & -1 & 1 & -1 \end{bmatrix}^T$,

(5) can be written into a compact form as

$$\min_{\mathbf{x},\mathbf{d},\mathbf{w}} \sum_{i=2}^{N} \mathbf{a}_i^T \mathbf{x}_i + \sum_{i=1}^{N}\sum_{j \in \mathcal{N}_i}(-2K)d_{ij} + \mathcal{I}_{\mathcal{D}}(\mathbf{d}) + \mathcal{I}_{\mathcal{W}}(\mathbf{w})$$

$$\text{s.t. } \mathbf{B}^{\{i,j\}}\mathbf{x}_i + \mathbf{E}^{\{i,j\}}\mathbf{x}_j + d_{ij}\mathbf{1}_{2K} + \mathbf{w}_{ij} = \mathbf{0}, \quad (i,j) \in \mathcal{L}, \quad (6)$$

where $\mathbf{1}_{2K}$ denotes the vector of $2K$ ones; $\mathbf{w}$ is a slack vector containing subvectors $\mathbf{w}_{ij} \in \mathbb{R}^{2K \times 1}$ with $j \in \mathcal{N}_i$ and the indexes are arranged in ascending order on i and then on j; $\mathcal{I}_{\mathcal{D}}(\mathbf{d})$ is the indicator function on the constraint set $\mathcal{D}$ (*i.e.*, $\mathcal{I}_{\mathcal{D}}(\mathbf{d}) = 0$ for $\mathbf{d} \in \mathcal{D}$, and $\mathcal{I}_{\mathcal{D}}(\mathbf{d}) = \infty$ for $\mathbf{d} \notin \mathcal{D}$) with $\mathcal{D} = \{\mathbf{d} | \mathbf{d} \geq \mathbf{0}\}$ and $\mathcal{I}_{\mathcal{W}}(\mathbf{w})$ is the indicator function on $\mathcal{W}$ with $\mathcal{W} = \{\mathbf{w} | \mathbf{w}_{ij} \geq \mathbf{0}, (i,j) \in \mathcal{L}\}$.

The challenge of solving the constrained optimization problem (6) in a distributed fashion comes from the coupling of constraints in (6) among different nodes. Fortunately, with the introduction of auxiliary replica variables as shown next, the original problem can be rewritten as an equivalent optimization problem with the structure more amenable for decomposition, which will allow us to split the original problem into subtasks that can be implemented in a distributed way with the classic ADMM while still guaranteeing convergence to the centralized solution.

In particular, we further transform the problem (6) by introducing an additional set of auxiliary "replica" variables $\{\mathbf{z}_q^{\{i,j\}}\}_{(i,j) \in \mathcal{L}, q \in \{1, \cdots, 4\}}$, and the auxiliary vector $\mathbf{z} = \left[\{ [(\mathbf{z}_1^{\{1,j\}})^T (\mathbf{z}_2^{\{1,j\}})^T (\mathbf{z}_3^{\{1,j\}})^T (\mathbf{z}_4^{\{1,j\}})^T] \}_{j \in \mathcal{N}_1}, \cdots, \{ [(\mathbf{z}_1^{\{N,j\}})^T (\mathbf{z}_2^{\{N,j\}})^T (\mathbf{z}_3^{\{N,j\}})^T (\mathbf{z}_4^{\{N,j\}})^T] \}_{j \in \mathcal{N}_N} \right]^T$, then we can obtain

$$\min_{\mathbf{x},\mathbf{d},\mathbf{w},\mathbf{z}} \sum_{i=2}^{N} \mathbf{a}_i^T \mathbf{x}_i + \sum_{i=1}^{N}\sum_{j \in \mathcal{N}_i}(-2K)d_{ij} + \mathcal{I}_{\mathcal{D}}(\mathbf{d}) + \mathcal{I}_{\mathcal{W}}(\mathbf{w})$$

$$\text{s.t. } \mathbf{B}^{\{i,j\}}\mathbf{x}_i = \mathbf{z}_1^{\{i,j\}}, \quad \mathbf{E}^{\{i,j\}}\mathbf{x}_j = \mathbf{z}_2^{\{i,j\}},$$

$$d_{ij}\mathbf{1}_{2K} = \mathbf{z}_3^{\{i,j\}}, \quad \mathbf{w}_{ij} = \mathbf{z}_4^{\{i,j\}},$$

$$\sum_{q=1}^{4} \mathbf{z}_q^{\{i,j\}} = \mathbf{0}, \qquad (i,j) \in \mathcal{L}. \quad (7)$$

Thus the augmented Lagrangian function of (7) is expressed as

$$\mathcal{F}(\mathbf{x},\mathbf{d},\mathbf{w},\mathbf{z},\boldsymbol{\mu}) = \sum_{i=2}^{N} \mathbf{a}_i^T \mathbf{x}_i + \sum_{i=1}^{N}\sum_{j \in \mathcal{N}_i}(-2K)d_{ij} + \mathcal{I}_{\mathcal{D}}(\mathbf{d}) + \mathcal{I}_{\mathcal{W}}(\mathbf{w})$$

$$+\sum_{i=1}^{N}\sum_{j \in \mathcal{N}_i}\left[(\boldsymbol{\mu}_1^{\{i,j\}})^T(\mathbf{B}^{\{i,j\}}\mathbf{x}_i - \mathbf{z}_1^{\{i,j\}}) + (\boldsymbol{\mu}_2^{\{i,j\}})^T(\mathbf{E}^{\{i,j\}}\mathbf{x}_j - \mathbf{z}_2^{\{i,j\}}) \right.$$

$$\left. +(\boldsymbol{\mu}_3^{\{i,j\}})^T(d_{ij}\mathbf{1}_{2K} - \mathbf{z}_3^{\{i,j\}}) + (\boldsymbol{\mu}_4^{\{i,j\}})^T(\mathbf{w}_{ij} - \mathbf{z}_4^{\{i,j\}}) \right]$$

$$+\sum_{i=1}^{N}\sum_{j \in \mathcal{N}_i}\frac{\rho_{ij}}{2}\left[\|\mathbf{B}^{\{i,j\}}\mathbf{x}_i - \mathbf{z}_1^{\{i,j\}}\|_2^2 + \|\mathbf{E}^{\{i,j\}}\mathbf{x}_j - \mathbf{z}_2^{\{i,j\}}\|_2^2 \right.$$

$$\left. +\|d_{ij}\mathbf{1}_{2K} - \mathbf{z}_3^{\{i,j\}}\|_2^2 + \|\mathbf{w}_{ij} - \mathbf{z}_4^{\{i,j\}}\|_2^2 \right], \quad (8)$$

where $\mathbf{z} \in \mathcal{Z}$, the set $\boldsymbol{\mu} \triangleq \{\boldsymbol{\mu}_q^{\{i,j\}}\}_{(i,j) \in \mathcal{L}, q \in \{1, \cdots, 4\}}$ are the Lagrange multipliers, and the constants $\rho_{ij} > 0$ denote penalty coefficients. With the introduction of auxiliary variables $\{\mathbf{z}_q^{\{i,j\}}\}_{(i,j) \in \mathcal{L}, q \in \{1, \cdots, 4\}}$, it is obvious that $\mathbf{x}_i$ and $\mathbf{x}_j$ are decoupled in (8), and the classic ADMM can be applied.

Based on the steps of classic ADMM, the augmented lagrangian function (8) can be minimized by sequential optimization over the primal variables $\{\mathbf{x}, \mathbf{d}, \mathbf{w}, \mathbf{z}\}$ followed by the gradient ascent method for the dual variables $\boldsymbol{\mu}$ update. With m defined as the iteration number, the steps are as follows:

■ Update the variables $\mathbf{x}, \mathbf{d}$, and $\mathbf{w}$ by solving the problem

$$[\mathbf{x}(m+1), \mathbf{d}(m+1), \mathbf{w}(m+1)] = \arg\min_{\mathbf{x},\mathbf{d},\mathbf{w}} \mathcal{F}(\mathbf{x}, \mathbf{d}, \mathbf{w}, \mathbf{z}(m), \boldsymbol{\mu}(m)). \quad (9)$$

■ Update the auxiliary variables $\mathbf{z}$ by solving the problem

$$\mathbf{z}(m+1) = \arg\min_{\mathbf{z}} \mathcal{F}(\mathbf{x}(m+1), \mathbf{d}(m+1), \mathbf{w}(m+1), \mathbf{z}, \boldsymbol{\mu}(m)). \quad (10)$$

■ Update the Lagrangian multipliers $\boldsymbol{\mu}$ via

$$\boldsymbol{\mu}_q^{\{i,j\}}(m+1) = \boldsymbol{\mu}_q^{\{i,j\}}(m) + \rho_{ij}(\boldsymbol{\phi}_q^{\{i,j\}}(m+1) - \mathbf{z}_q^{\{i,j\}}(m+1)),$$

$$(i,j) \in \mathcal{L}, q \in \{1, \cdots, 4\}, \quad (11)$$

where the intermediate variables $\boldsymbol{\phi}_1^{\{i,j\}}(m) = \mathbf{B}^{\{i,j\}}\mathbf{x}_i(m)$, $\boldsymbol{\phi}_2^{\{i,j\}}(m) = \mathbf{E}^{\{i,j\}}\mathbf{x}_j(m)$, $\boldsymbol{\phi}_3^{\{i,j\}}(m) = d_{ij}(m)\mathbf{1}_{2K}$, and $\boldsymbol{\phi}_4^{\{i,j\}}(m) = \mathbf{w}_{ij}(m)$, and the reason for choosing the penalty parameter ρ_{ij} as the step size is to guarantee that the solution is dual feasible after each iterate of primal variables and dual update.

For the updating of the variables $\{\mathbf{x}, \mathbf{d}, \mathbf{w}\}$, from the augmented Lagrangian function (8), it is noticed that the expression is separable in terms of $\mathbf{x}$, $\mathbf{d}$, and $\mathbf{w}$. Furthermore, the components of $\mathbf{x}$, $\mathbf{d}$, and $\mathbf{w}$ in (8) are in form of summation, therefore, different components are also separable. Based on this observation, the updating of different variables are given as follow.

Updating of clock parameter variables $\mathbf{x}_i$

Based on (9), setting the gradient of the augmented Lagrangian function $\mathcal{F}$ with respect to $\mathbf{x}_i$ equal to zero and solving for $\mathbf{x}_i$, we obtain

$$\mathbf{x}_i(m+1) = \left(\sum_{j \in \mathcal{N}_i}\left[\rho_{ij}(\mathbf{B}^{\{i,j\}})^T\mathbf{B}^{\{i,j\}} + \rho_{ji}(\mathbf{E}^{\{j,i\}})^T\mathbf{E}^{\{j,i\}} \right] \right)^{-1}$$

$$\left\{ -\mathbf{a}_i - \sum_{j \in \mathcal{N}_i}\left[(\mathbf{B}^{\{i,j\}})^T\boldsymbol{\mu}_1^{\{i,j\}}(m) + (\mathbf{E}^{\{j,i\}})^T\boldsymbol{\mu}_2^{\{j,i\}}(m) \right] \right.$$

$$+\sum_{j\in\mathcal{N}_i}\left[\rho_{ij}(\mathbf{B}^{\{i,j\}})^T\mathbf{z}_1^{\{i,j\}}(m)+\rho_{ji}(\mathbf{E}^{\{j,i\}})^T\mathbf{z}_2^{\{j,i\}}(m)\right]\Bigg\}.$$
(12)

Updating of fixed delay variables d_{ij}

Based on (9), it is noticed that $\mathcal{F}$ is quadratic with respect to d_{ij} without constraint, thus the optimal d_{ij} can be obtained by firstly setting the gradient of $\mathcal{F}$ (without constraint) with respect to d_{ij} equal to zero, and applying the constraint $d_{ij}\geq 0$ afterward. This gives

$$d_{ij}(m+1)=\max\left\{0,\frac{1}{2K(\rho_{ij}+\rho_{ji})}\left(4K-[(\boldsymbol{\mu}_3^{\{i,j\}}(m))^T\right.\right.$$
$$\left.\left.+(\boldsymbol{\mu}_3^{\{j,i\}}(m))^T]\mathbf{1}_{2K}+\mathbf{1}_{2K}^T[\rho_{ij}\mathbf{z}_3^{\{i,j\}}(m)+\rho_{ji}\mathbf{z}_3^{\{j,i\}}(m)]\right)\right\}.$$
(13)

Updating of slack variables $\mathbf{w}_{ij}$

Following the same logic as in the updating of d_{ij}, we have

$$\mathbf{w}_{ij}(m+1)=\max\left\{\mathbf{0},-\boldsymbol{\mu}_4^{\{i,j\}}(m)/\rho_{ij}+\mathbf{z}_4^{\{i,j\}}(m)\right\}.$$
(14)

Updating of auxiliary variables $\mathbf{z}_q^{\{i,j\}}$

For the updating of $\mathbf{z}$, it is noticed that once $\mathbf{x}$, $\mathbf{d}$, and $\mathbf{w}$ are given, the augmented Lagrangian function (8) is separable in terms of $\mathbf{z}_q^{\{i,j\}}$. Therefore, the problem (10) is equivalent to $2|\mathcal{L}|$ sub-LP problems in the following form:

$$\min_{\mathbf{z}_q^{\{i,j\}}}\left\{-\sum_{q=1}^4(\boldsymbol{\mu}_q^{\{i,j\}}(m))^T\mathbf{z}_q^{\{i,j\}}+\frac{\rho_{ij}}{2}\sum_{q=1}^4\|\phi_q^{\{i,j\}}(m+1)-\mathbf{z}_q^{\{i,j\}}\|_2^2\right\}$$
$$\text{s.t. }\sum_{q=1}^4\mathbf{z}_q^{\{i,j\}}=\mathbf{0}\qquad(i,j)\in\mathcal{L}.$$
(15)

Applying the KKT optimality conditions [16] to (15), we obtain

$$\begin{cases}-\boldsymbol{\mu}_q^{\{ij\}}(m)+\rho_{ij}[\mathbf{z}_q^{\{ij\}}-\phi_q^{\{ij\}}(m+1)]+\boldsymbol{\xi}^{\{i,j\}}(m+1)=\mathbf{0}&(16)\\\sum_{q=1}^4\mathbf{z}_q^{\{i,j\}}=\mathbf{0}\qquad\forall\,(i,j)\in\mathcal{L},q\in\{1,\cdots,4\},&(17)\end{cases}$$

where $\boldsymbol{\xi}^{\{i,j\}}\in\mathbb{R}^{2K\times 1}$ is the Lagrange multiplier associated with the equality constraint $\sum_{q=1}^4\mathbf{z}_q^{\{i,j\}}=\mathbf{0}$. Thus from (16), we can obtain $\mathbf{z}_q^{\{i,j\}}$ as

$$\mathbf{z}_q^{\{i,j\}}(m+1)=\phi_q^{\{i,j\}}(m+1)+\frac{1}{\rho_{ij}}[\boldsymbol{\mu}_q^{\{i,j\}}(m)-\boldsymbol{\xi}^{\{i,j\}}(m+1)].$$
(18)

By adding the update formula (16) for $q=1,\cdots,4$ together and substituting (17) to it, we obtain the multiplier update formula as

$$\boldsymbol{\xi}^{\{i,j\}}(m+1)=\frac{1}{4}\sum_{l=1}^4\boldsymbol{\mu}_l^{\{i,j\}}(m)+\frac{\rho_{ij}}{4}\sum_{l=1}^4\phi_l^{\{i,j\}}(m+1).\quad(19)$$

Finally substituting (19) into (18), we obtain the final updating formula for $\mathbf{z}_q^{\{i,j\}}$ as

$$\mathbf{z}_q^{\{i,j\}}(m+1)=\phi_q^{\{i,j\}}(m+1)-\frac{1}{4}\sum_{l=1}^4\phi_l^{\{i,j\}}(m+1)$$

$$+\frac{1}{\rho_{ij}}[\boldsymbol{\mu}_q^{\{i,j\}}(m)-\frac{1}{4}\sum_{l=1}^4\boldsymbol{\mu}_l^{\{i,j\}}(m)].\quad(20)$$

The proposed distributed synchronization and ranging algorithm is summarized in Algorithm 1. Notice that the computation of this algorithm is localized and each node only communicates with its neighbors. Thus the proposed algorithm is scalable with network size in terms of communication overhead and computation cost.

Algorithm 1 Distributed synchronization and ranging algorithm

1: **Initialization:**
2: Set the initial clock parameter variable $\mathbf{x}_i(0)$ for $i=2,\cdots,N$.
3: Set initial $d_{ij}(0)$, $\mathbf{w}_{ij}(0)$, ρ_{ij} and $\boldsymbol{\mu}_q^{\{i,j\}}(0)$ for $q\in\{1,\cdots,4\}$ and $(i,j)\in\mathcal{L}$.
4: **Iteration until convergence:**
5: **for** the mth iteration **do**
6:     Sensors S_i with $i=1,\cdots,N$ **in parallel**
7:     Update $\mathbf{x}_i$, $\{d_{ij}\}_{j\in\mathcal{N}_i}$, and $\{\mathbf{w}_{ij}\}_{j\in\mathcal{N}_i}$ by (12), (13) and (14), respectively;
8:     Broadcast the updated $\mathbf{x}_i$ to neighbors;
9:     Update $\{\mathbf{z}_q^{\{i,j\}}\}_{j\in\mathcal{N}_i,q\in\{1,\cdots,4\}}$, $\{\boldsymbol{\mu}_q^{\{i,j\}}\}_{j\in\mathcal{N}_i,q\in\{1,\cdots,4\}}$ by (20) and (11) respectively, with received $\mathbf{x}_j$ from neighbor S_j;
10:     Transmit $\mathbf{z}_2^{\{i,j\}}$, $\mathbf{z}_3^{\{i,j\}}$, $\boldsymbol{\mu}_2^{\{i,j\}}$ and $\boldsymbol{\mu}_3^{\{i,j\}}$ to its neighbor S_j.
11: **end parallel**
12: **end for**

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, numerical simulations will be presented to assess the performance of the proposed distributed clock synchronization algorithm. The measure of parameter estimation fidelity at iteration m are Root Average Mean Squared Error (RAMSE) of clock offsets and clock skews over the whole network: $\text{RAMSE}(\nu(m))=\sqrt{\sum_{i=2}^N(\hat{\nu}_i(m)-\nu_i(m))^2/(N-1)}$, where $\nu\in\{\theta,\beta\}$, and the RAMSE of fixed delays: $\text{RAMSE}(d(m))=\sqrt{\sum_{i=1}^N\sum_{j\in\mathcal{N}_i,j>i}(\hat{d}_{ij}(m)-d_{ij}(m))^2/|\mathcal{L}|}$. We also compare to the consensus algorithm, which seeks to converge to the average value of all the nodes' clock parameters θ_i and β_i, and the RAMSE for consensus algorithm is defined as: $\text{RAMSE}(\nu(m))_{con}=\sqrt{\sum_{i=1}^N(\hat{\nu}_i(m)-\frac{1}{N}\sum_{i=1}^N\nu_i(m))^2/N}$.

Network of 25 nodes are randomly deployed in an area $5m\times 5m$ with communication radius 1.5m (an example is shown in Figure 1). 1000 independent networks are generated for averaging the network RAMSE in the figures. In each simulation run, clock skews, clock offsets and fixed delays are uniformly selected from ranges [0.99, 1.01], [-10, 10]ms and [1, 10]ms, respectively. The parameter of exponential delay is $\lambda=1$, and the number of rounds of two-way message exchange is $K=5$. For our proposed algorithm, the initial
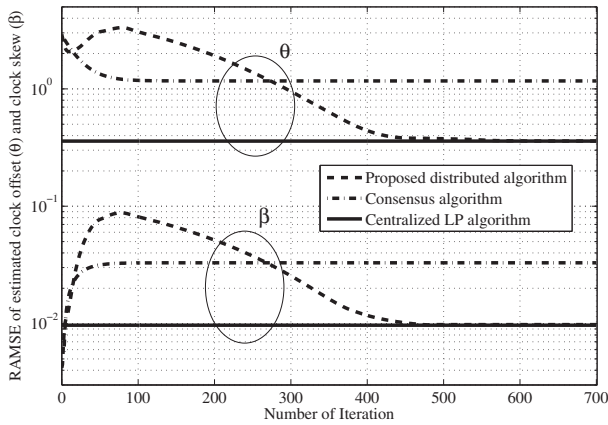
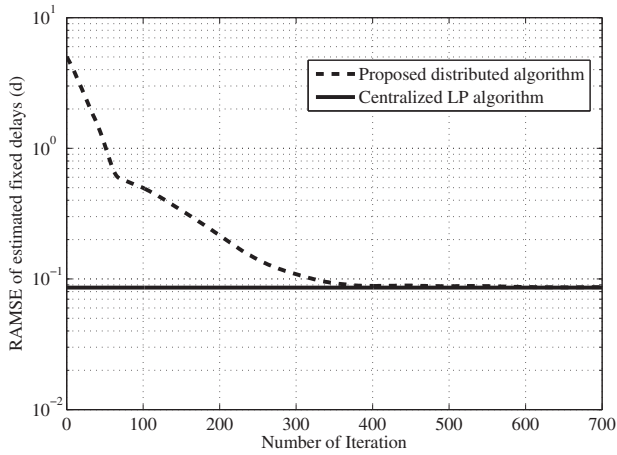Figure 3: RAMSE of clock offsets and skews estimation

Figure 4: RAMSE of fixed delays estimation

values of clock parameter variables are set as $\mathbf{x}_i(0) = [1\ 0]^T$ for all $i \in \{2, \cdots, N\}$, and the initial values of the fixed delay variables $d_{ij}(0)$, slack variables $\mathbf{w}_{ij}(0)$, and the Lagrangian multipliers $\boldsymbol{\zeta}^{\{i,j\}}(0)$ are all set to zero.

The performance of the consensus algorithm [10], the proposed algorithm, and the centralized LP algorithm using CVX are compared in Figure 3 for clock offsets and clock skews estimation. From Figure 3, it is obvious that the performance of the consensus algorithm converges to a value far away from that of the centralized LP algorithm. On the other hand, the proposed algorithm converges to an RAMSE coinciding with that of the centralized optimal solution. Furthermore, for the fixed delays estimation, it is noticed in Figure 4 that as the number of iterations increase, the RAMSE of the proposed algorithm gradually decreases and finally converges to the centralized optimal solution.

V. Conclusions

Global clock synchronization and ranging for WSNs in the presence of unknown exponential delays was investigated un-der the two-way message exchange mechanism. A distributed synchronization and ranging algorithm was proposed based on ADMM. The proposed algorithm requires communications only between neighboring sensors and the communication overhead was shown to be low. Simulation results showed that our proposed algorithm achieves better performance than the consensus algorithm, and always approaches the centralized LP solution.

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