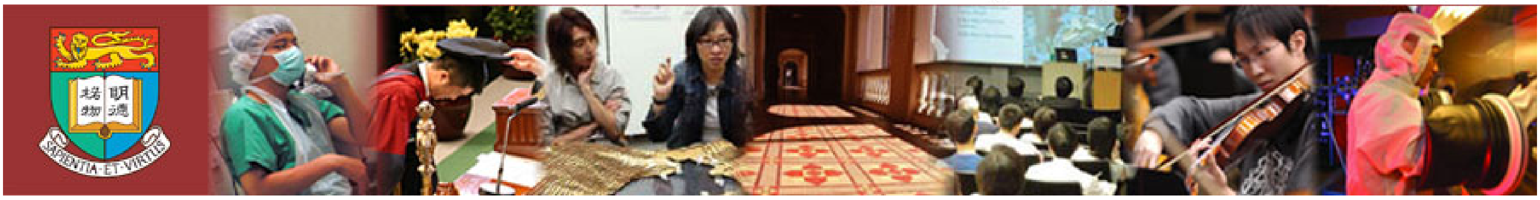

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SHORT COMMUNICATION

SELECTION BIAS IN BUILD-OPERATE-TRANSFER TRANSPORTATION PROJECT APPRAISALS

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ABSTRACT

Recent empirical studies have found widespread inaccuracies in traffic forecasts despite the fact that travel demand forecasting models have been significantly improved over the past few decades. We suspect that an intrinsic *selection bias* may exist in the competitive project appraisal process, in addition to the many other factors that contribute to inaccurate traffic forecasts. In this paper, we examine the potential for selection bias in the governmental process of build-operate-transfer (BOT) transportation project appraisals. Although the simultaneous consideration of multiple criteria is typically used in practice, traffic flow estimate is usually a key criterion in these appraisals. For the purposes of this paper, we focus on the selection bias associated with the highest flow estimate criterion. We develop two approaches to quantify the level and chance of inaccuracy caused by selection bias: the expected value approach and the probability approach. The expected value approach addresses the question "*to what* ***extent*** *is inaccuracy caused by selection bias?*" The probability approach addresses the question "*what is the* ***chance*** *of inaccuracy due to selection bias?*" The results of this analysis confirm the existence of selection bias when a government uses the highest traffic forecast estimate as the priority criterion for BOT project selection. In addition, we offer some insights into the relationship between the extent/chance of inaccuracy and other related factors. We do not argue that selection bias is the only reason for inaccurate traffic forecasts in BOT projects; however, it does appear that it could be an intrinsic factor worthy of further attention and investigation.

Keywords: BOT; selection bias; traffic forecasts; inaccuracy; project appraisals

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1 INTRODUCTION

The Build-Operate-Transfer (BOT) scheme is increasingly used as an innovative way to finance the construction of major public transportation infrastructure projects in many developing countries, and even in some developed countries. In a BOT project bidding process, each consultant (or consortium) develops a proposed plan that includes factors related to highway pricing and capacity (Yang and Meng, 2000; Subprasom and Chen, 2007). Bidding consortiums design their pricing and capacity plans to maximize the profit from the project and the government then evaluates the social welfare benefits of the proposed plans. Generally, both the investor's profits and the social welfare benefits of the project are dependent on the estimated forecast of the traffic demand of the project in the target year and the estimate of the construction and operation costs of the project. Therefore, the accuracy of the traffic forecasts directly affects the efficacy of the BOT decision-making process (e.g., the ranking of project tenderers and the selection of a successful tenderer).

Beyond the significance of the forecast accuracy in the BOT project bidding process, traffic (or travel demand) forecasting is also a fundamental step in the planning and management of transportation systems. The resulting estimates of traffic flows in the network can be used to evaluate the performance of existing systems, to assess the results of the proposed planning and management strategies (e.g., new road construction, road pricing, and traffic rationing), and to study the financial feasibility of candidate projects (Chen and Subprasom, 2007). The accuracy of a traffic forecast substantially affects the quality of the system performance assessment and the resulting decision.

It is well recognized that travel demand forecasting models have significantly improved in the past few decades. However, many recent empirical studies have demonstrated the inaccuracy of the traffic forecasts of various transportation projects. In other words, the traffic forecast estimate at the project design stage may be significantly different from the true value at the operating stage. These recent empirical and statistical studies include those of Flyvbjerg *et al*. (2005), who used data from 183 road projects and 27 rail projects in 14 nations; Bain (2009), who used data from 104 international and privately financed toll road projects; Li and Hensher (2010), who used data from 14 toll roads in the three largest Australian cities; Parthasarathi and Levinson (2010), who used data from 108 projects in Minnesota, USA; Roxas and Chalermpong (2010), who used data from 89 road and 40 bridge projects in Thailand and Philippines; and Nicolaisen and Næss (2015), who used data from 35 road projects in Denmark and England to evaluate the accuracy of travel demand forecasts for do-nothing alternatives.

Given the inaccuracy of many traffic forecasts, we suspect that ***an intrinsic selection bias may exist in the government's competitive transportation project appraisal process,*** in addition to the many factors contributing to inaccurate traffic forecasts identified by previous empirical studies. This selection bias may occur because the observable traffic forecasts are generated by non-randomly selected samples in the transportation system, given that only winning projects will be built and observed and losing projects are never built or appear in the system (Heckman, 1979). Therefore, we hypothesize that the selection bias associated with the appraisal criterion is one of the factors contributing to the inaccuracy of traffic forecasts. This can be analytically derived by quantifying the level and chance of inaccuracy caused by selection bias. In particular, a BOT project bidding and appraisal process may involve a selection bias when some particular criterion (e.g., the highest flow estimate, the lowest cost estimate, or the highest financial benefit estimate) is set as the primary criterion for selecting the successful bid. Recently, Eliasson and Fosgerau (2013) also considered selection bias as a possible source of systematic cost overruns and demand shortfalls. However, these authors focused on a project selection scenario in which a subset of projects are selected to implement within the candidate projects pool according to the relationship between the predicted payoff and the specified threshold of individual projects. Our paper complements Eliasson and Fosgerau's study in terms of both selection scenario (i.e., the selection of which project from what pool of proposals) and justification methodology. This paper deals with a bidder selection scenario in which a single bidder is selected from the multiple bidders to win a single BOT project and the bias occurs between the benefit (or cost) predicted by the successful bidder and the actual project benefit (or cost) after implementation. In reference to the justification methodology, Eliasson and Fosgerau (2013) quantified the mean relative cost (or benefit) error of selected projects using simulation, whereas this paper analytically quantifies both the extent and chance of inaccuracy caused by the selection bias.

As identified by many previous empirical studies, there are many factors and practical considerations other than selection bias that contribute to the inaccuracy of traffic forecasts. For example, Mackie and Preston (1998) identified 21 sources of error and bias in transport project appraisals. The errors they identified were related to the project objectives being unclear, incompletely specified, or inconsistent with the appraisal criteria, definitions of the study areas, and scheme options; multifarious sources of data and model errors; and evaluation errors, such as double counting, inappropriate values, and a failure to balance quantified and non-quantified items. Flyvbjerg *et al.* (2005) analyzed the stated causes of inaccuracies in traffic forecasts for 26 rail projects and 208 road projects. They found that the reasons for these inaccuracies are highly different for rail and road projects. For rail projects, uncertainty about trip distribution and deliberately slanted forecasts are the two most important stated causes; for road projects, uncertainties about trip generation and land-use development are the two most frequent stated causes of forecast inaccuracies. Bain (2009) used the Traffic Risk Index (TRI) to summarize the principal reasons for forecast inaccuracies and to offer investors and financial analysts a systematic way of evaluating forecasting risk. The project attributes in the TRI include tolling culture, tariff escalation, forecast horizon, toll facility details, data collection, private/commercial users, micro-economics, and traffic growth. Lemp and Kockelman (2009) reviewed the literature on the sources of risk and uncertainty in traffic forecasts and how these relate to project financing. Parthasarathi and Levinson (2010) identified errors in model inputs (such as demographic forecasts, trip making characteristics, and network differences between the assumed network and the actual in-place network) as possible sources of inaccuracy in traffic forecasts. For more discussions of demand forecast inaccuracy, interested readers can refer to a literature review by Nicolaisen and Driscoll (2014).

In this paper, we examine the potential influence of selection bias on the BOT transportation project appraisal process. Although many criteria can be considered simultaneously in a realistic BOT project appraisal process, the evaluation usually depends on the traffic flow estimate. For the purposes of this paper, we consider that the bidding consortium with the highest traffic flow estimate will have the highest chance to win the contract because, everything else being equal, it offers a lower toll and a higher benefit for the BOT project. To quantify the selection bias associated with the highest flow estimate criterion, we develop two approaches (the expected value and probability approaches) to analytically derive the level and chance of inaccuracy. The expected value approach addresses the question "*to what **extent** is inaccuracy caused by selection bias?*" The probability approach addresses the question "*what is the **chance** of inaccuracy due to selection bias?*" Two representative symmetric and asymmetric distributions (i.e., normal and lognormal distributions) are adopted to characterize the traffic forecast variability in both the expected value and probability approaches. The quantification of the extent and probability of selection bias provided in this paper has the potential to assist in the transportation infrastructure financial planning process.

The remainder of this paper is organized as follows. Section 2 quantifies the selection bias associated with the highest flow estimate criterion, including analyses using both the expected value and probability approaches. In Section 3, we summarize our concluding remarks and future research directions.

2 SELECTION BIAS QUANTIFICATION

In a transportation BOT project bidding process, each consultant (or consortium) prepares an estimate of the traffic demand in the target year. However, these traffic forecast estimates may deviate significantly from the long-term, true value due to factors unforeseen during the construction of the models and the selection of the project participants. In this section, we consider the highest flow estimate criterion. In other words, we presume that the consortium offering the plan with the highest flow estimate will have a higher chance of winning the contract, because, everything else being equal, this plan offers a lower toll and a higher benefit for the BOT project.

To construct a mathematical analysis of flow estimates, we consider N consultants (consortiums) in the project bidding process. We also assume that there is no systematic bias in the traffic forecast estimates V_i made by each consultant i. Given the assumption of no systematic bias, $\{V_1, V_2,\ldots, V_N\}$ should be independently and identically distributed (or IID). We let V denote the true (random) traffic forecast estimate of the examined project. The traffic forecast estimate made by the winner is $\max\{V_1, V_2,\ldots, V_N\}$, which is also a random variable. Based on this, we may then raise a straightforward question: "*What is the relationship between the winner's estimate* $\max\{V_1, V_2,\ldots, V_N\}$ *and the true value of* V?"

2.1 Expected Value Approach

The expected value approach is the most widely used statistical method to evaluate random variables in probability theory. Hence, the deviation between the winners' traffic forecast estimates and the true traffic flow value can be characterized by the difference in their expected values. In this paper, we use the following ratio to quantify the deviation:

$$E\left[\max\left\{V_1, V_2, \cdots, V_N\right\}\right] \big/ E\left[V\right] - 1, \tag{1}$$

where $E[.]$ is the expectation operator. When the ratio is positive, the traffic forecast estimate on the examined project is overestimated; otherwise, it is underestimated.

Because the maximum function is convex, Jensen's inequality (Jensen, 1906) gives

$$E\left[\max\left\{V_1, V_2, \cdots, V_N\right\}\right] \geq \max_i \left\{E\left[V_i\right]\right\} = E\left[V\right], \tag{2}$$

where the equality uses the identical assumption of V_i. Thus, Eq. (1) is always non-negative, indicating that the traffic forecast is overestimated. However, the *exact extent* of the overestimation is unknown from Jensen's inequality. In the following series of calculations, we derive the level of this overestimation.

We define U as max{V_1, V_2,…, V_N}. The cumulative distribution function (CDF) of U is as follows:

$$F_U\left(z\right) = \Pr\left(\max\left\{V_1, V_2, \cdots, V_N\right\} \leq z\right) = \Pr\left(V_1 \leq z, V_2 \leq z, \cdots, V_N \leq z\right). \tag{3}$$

Using the IID assumptions of V_i, we obtain:

$$F_U\left(z\right) = \left(\Pr\left(V \leq z\right)\right)^N = \left(F_V\left(z\right)\right)^N, \tag{4}$$

where $F_V\left(z\right)$ is the CDF of V. In this case, the expected value of U is:

$$E\left[U\right] = \int_{-\infty}^{\infty} z dF_U\left(z\right) = \int_{-\infty}^{\infty} z d\left(\left(F_V\left(z\right)\right)^N\right). \tag{5}$$

Let $t = \left(F_V\left(z\right)\right)^N$, so $z = F_V^{-1}\left(t^{1/N}\right)$, where $F_V^{-1}\left(\cdot\right)$ is the inverse CDF of V. In this case, Eq. (5) can be rewritten as:

$$E\left[U\right] = \int_0^1 F_V^{-1}\left(t^{1/N}\right) dt. \tag{6}$$

Note that $E[U]$ depends on the inverse CDF of V. In the following analysis, we examine two representative symmetric and asymmetric distributions (i.e., normal and lognormal distributions) for the true traffic flow (V). A lognormal distribution is a non-negative and asymmetric distribution extensively used in general reliability applications to model failure times (e.g., Blischke and Murthy, 2000). In the field of transportation, it has been adopted to model travel demand uncertainty (Zhao and Kockelman, 2002; Zhou and Chen, 2008) and network performance uncertainty (Xu et al., 2014). Table 1 summarizes the *level* of overestimation (i.e., Eq. (1)) caused by selection bias under the above distributions.

Table 1 The selection bias quantification due to the highest flow estimate criterion

Distribution	**Normal**	**Lognormal**
Level of overestimation	$\mathrm{CoV} \cdot \int_0^1 \Phi^{-1}\left(t^{1/N}\right)dt$	$\dfrac{\int_0^1 \exp\left[\Phi^{-1}\left(t^{1/N}\right)\sqrt{\ln\left(1+(\mathrm{CoV})^2\right)}\right]dt}{\sqrt{1+(\mathrm{CoV})^2}}-1$
Chance of overestimation	$1-0.5^N$	$1-\left[\Phi\left(\frac{1}{2}\sqrt{\ln\left(1+(\mathrm{CoV})^2\right)}\right)\right]^N$

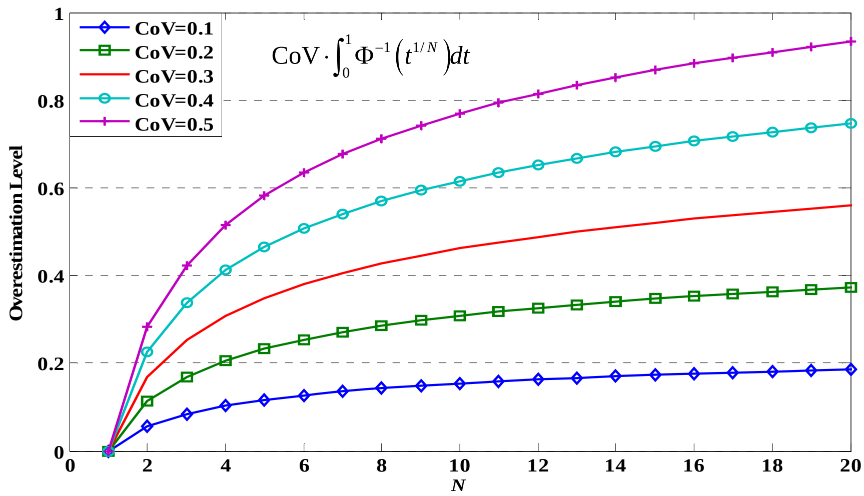
Note: CoV = the coefficient of variation

Figure 1 illustrates the level of overestimation shown in Table 1. The following observations can be drawn from this figure:

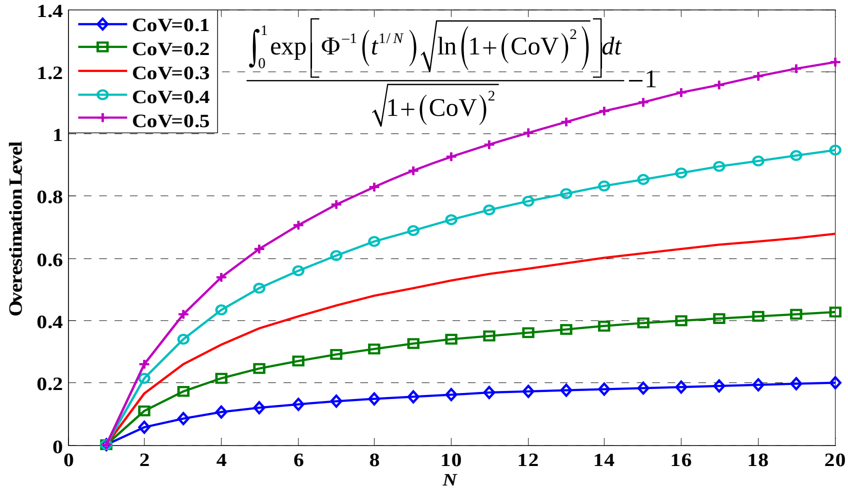
(1) The level of overestimation is always non-negative for both normal and lognormal distributions, which is consistent with Eq. (2) (i.e., Jensen's inequality). This means that, everything else being equal, using the expected value of the maximum flow estimate as the project selection criterion will always lead to overestimation compared to the true mean value. This confirms the existence of selection bias in the BOT project appraisal process when using the highest flow estimate criterion.

(2) The level of overestimation increases with the number of consortiums in the BOT project bidding process. More consortium bids will result in a larger overestimation.

(3) The level of overestimation in both normal and lognormal distribution cases also depends on the coefficient of variation (CoV) in the true traffic flow distribution. From Figure 1, we can see that the level of overestimation increases with respect to the CoV. When the true traffic flow is more randomly distributed (i.e., has a larger CoV), the overestimation due to the selection bias becomes more significant.



(a) Normal distribution



(b) Lognormal distribution

Figure 1 The level of overestimation due to the highest flow estimate criterion

2.2 Probability Approach

From Section 2.1, we see that $E[U]/E[V]$-1 in Eq. (1) quantifies the expected **level/extent** of overestimation. However, this does not tell us **how often** this overestimation occurs. *Probability* is another commonly used statistical method to evaluate a random variable. In this section, we provide a **probability approach** to estimate the *chance of overestimation* when the government uses the maximum traffic forecast estimate (i.e., $U = \max\{V_1, V_2, \ldots, V_N\}$) as the BOT project selection criterion. The formula for this is as follows:

$$\Pr\left(\max\left\{V_1, V_2, \cdots, V_N\right\} \geq E\left[V\right]\right). \tag{7}$$

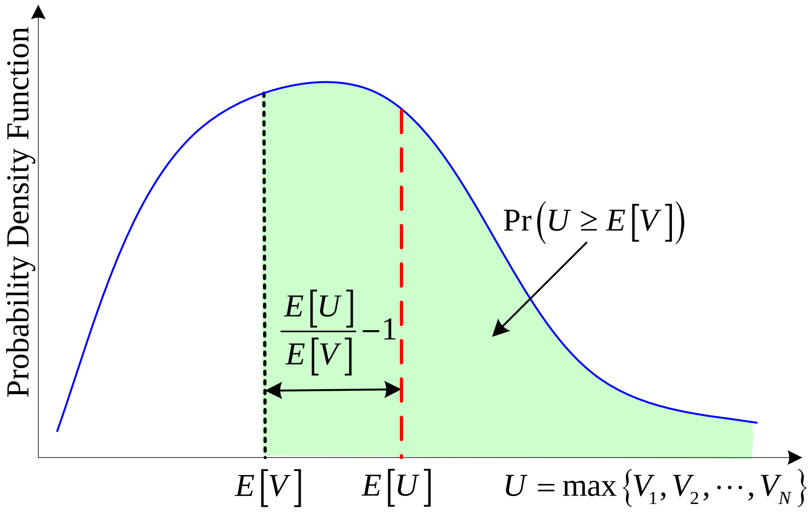


Figure 2 A comparison of the expected value and probability approaches

By substituting Eq. (4) into Eq. (7), the chance of overestimation due to selection bias can be expressed as follows:

$$\Pr\left(U \geq E\left[V\right]\right) = 1 - F_U\left(E\left[V\right]\right) = 1 - \left[\Pr\left(V \leq E\left[V\right]\right)\right]^N. \tag{8}$$

Table 1 also shows the *chance* of overestimation caused by selection bias when using the highest flow estimate criterion. Figure 3 illustrates the chance of overestimation due to the highest flow estimate selection criterion for both the normal and lognormal distributions. Clearly, the chance of overestimation is always larger than 50% when $N > 1$. If there is no selection bias when using the maximum flow estimate as the project selection criterion, then the probability of overestimation should be equal to 50%. Therefore, the calculated chance of overestimation confirms the existence of selection bias from the probability perspective. Similar to the level of overestimation presented in Section 2.1, the probability approach indicates that including more consortiums in the BOT project bidding process will result in a greater chance of overestimation. However, the chance of overestimation manifests in a different way than the level of overestimation does (as shown in Figure 1). For a normal distribution, the chance of overestimation is independent of the traffic flow characteristics (i.e., CoV). However, for the lognormal distribution, the chance of overestimation depends on the CoV of traffic flows. A larger CoV corresponds to a more dispersed state of traffic flow, which may have a lesser chance of exceeding the expected value. Therefore, as the CoV increases the chance of overestimation decreases.

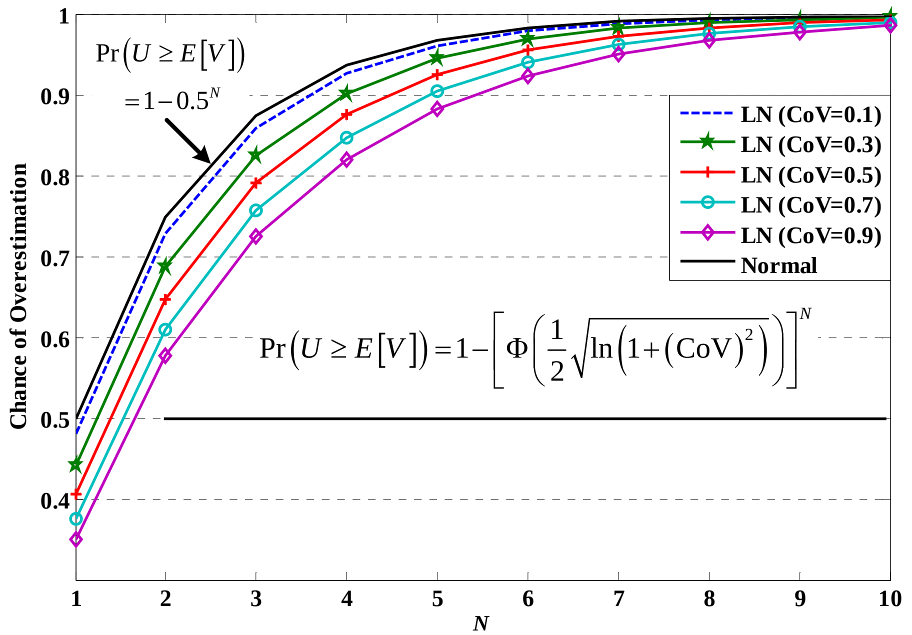


Figure 3 The chance of overestimation due to the highest flow estimate criterion

3 DISCUSSIONS AND FUTURE RESEARCH

In this paper, we examined the possible existence of selection bias in a government's BOT transportation project appraisal process. This bias could be one of the factors contributing to the inaccuracy of traffic forecasts. For the purposes of this paper, we focused on the selection bias associated with the highest flow estimate criterion. The expected value approach and the probability approach were developed to quantify the *level* and *chance* of inaccuracy caused by selection bias, respectively. In addition, two representative symmetric and asymmetric distributions (i.e., the normal and lognormal distributions) were adopted to characterize traffic forecast variability in both approaches. The quantification of the extent and chance of selection bias provided in this paper has the potential to assist in transportation infrastructure financial planning.

In this paper, we use the highest flow estimate criterion to demonstrate the possible existence of selection bias. In addition to the highest flow estimate criterion, the lowest cost estimate (e.g., construction and operation costs) is another widely used criterion in BOT project appraisals. When using the lowest cost estimate criterion, we are also able to demonstrate the possible existence of selection bias by using the same approach outlined above.

We found that, everything else being equal, using either the expected value of the maximum traffic forecast estimate (or the minimum cost estimate) as the primary BOT project selection criterion always led to overestimation (or underestimation), compared to the true mean value. Using the maximum flow estimate (or the minimum cost estimate) as the project selection criterion always had at least a 50% chance of overestimation (or underestimation). These results verified our hypothesis concerning the existence of selection bias in the BOT project appraisal process from both the expected value and probability perspectives. Both the level and chance of inaccuracy caused by selection bias increased with the number of consortiums in the BOT project bidding process. More bidders (i.e., a more competitive selection process) result in a greater extent and chance of selection bias. In addition, the selection bias coexists with uncertainty. A larger coefficient of variation (CoV) increased the level of inaccuracy caused by the selection bias.

This does not mean that selection bias is the only reason for inaccurate traffic forecasts in BOT projects, but it does appear that it could be an intrinsic factor worthy of further attention and investigation. Additional considerations and investigations might include the following:

(1) Forensic studies exploring empirical evidence concerning the existence of selection bias in existing BOT projects.

(2) The relaxation of the assumption of an independent and identical distribution (IID) for all of the consultants to enhance the realism of the analyses. For example, the collaboration of multiple consultants on a single large BOT project or bidding for

multiple BOT projects as a bundle under the same consortium would typically violate the IID assumptions.

(3) The consideration of more appraisal criteria for a more realistic evaluation of BOT project bids, which would rely on a more comprehensive analysis of existing selection biases.

Given the existence of selection bias and its contribution to traffic forecast inaccuracies, it would be important to develop practical and meaningful ways to avoid or reduce the adverse effects of this bias. One possibility would be to carry out a two-stage tendering process. After receiving the first-stage bids, a shortlist of bidders would be created, based on all of the significant considerations in practice. At the same time, the traffic flow forecasts of all of the bidders (including the shortlisted ones) would be collected and an envelope of forecasts (e.g., low, average, and high) would be constructed. This envelope information would be provided to the shortlisted bidders for a second-stage of submissions. These submissions would not include any more traffic forecasts but would instead focus on other technical and financial aspects of the proposed project. To achieve a lower selection bias, it would be important to explore which criterion of selecting project/bidder (e.g., cost-benefit analysis or multi-criterion decision-making) can produce a lower selection bias, given that the highest flow estimate criterion leads to overestimation and the lowest cost estimate criterion leads to underestimation (Section 2). To this end, we need to explore which benefit metric (e.g., in terms of total travel time, total social welfare, total revenue, etc.) is more robust or less affected by selection bias. Section 2 showed that both the level and chance of inaccuracy caused by selection bias increase with the number of consortiums in the BOT project bidding process. A more competitive selection process with more bidders results in a greater extent and chance of selection bias. With this observation, it is also meaningful to impose a tighter requirement on the bidding consortiums admittance. Besides, with the information on the level and chance of inaccuracy caused by selection bias, correction/discount factors could be applied in the bidding selection process based on tendering inputs from various consultants (e.g., discounting the level of overestimation).

Although we use BOT projects to demonstrate the possible existence of selection bias in this paper, we believe that this discussion may also be useful in explaining the widespread traffic forecast inaccuracy of other types of transportation project. We hope that this short communication serves as a useful analytical tool for examining the issues related to inaccurate traffic forecasts and provides a different perspective to stimulate further discussions on this important topic.

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