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Fluctuation effects on the transport properties of unitary Fermi gases

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In this Rapid Communication we investigate the fluctuation effects on the transport properties of unitary Fermi gases in the vicinity of the superfluid transition temperature T_c. Based on the time-dependent Ginzburg-Landau formalism of the Bose-Einstein condensate–BCS crossover, we investigate both the residual resistivity below T_c induced by phase slips and the paraconductivity above T_c due to pair fluctuations. These two effects have been well studied in the weak-coupling BCS superconductor and here we generalize them to the unitary regime of ultracold Fermi gases. We find that while the residual resistivity below T_c increases as one approaches the unitary limit, consistent with recent experiments, the paraconductivity exhibits nonmonotonic behavior. Our results can be verified with the recently developed transport apparatus using mesoscopic channels.

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In the past decade, one of the most exciting topics in cold-atom physics is the unitary Fermi gas characterized by the absence of a small perturbation parameter and strong pairing fluctuations [1,2]. Thermodynamic properties of the unitary Fermi gases have been well studied [3–5] and are shown to be universal [6]. Several experiments have also started to investigate the transport properties of the unitary Fermi gases, including the first and second sound [7], shear viscosity [8], and spin diffusion [9–11]. In the latter two cases, apparent lower quantum limits have been observed in experiments. Recently, a mesoscopic channel between two bulk unitary Fermi gases has been constructed and a drop of resistance below superfluid transition temperature T_c has been seen [12]. With the same setup, contact resistance [13], quantized conductance [14], and the thermoelectric effect [15] have also been observed. These experimental developments offer opportunities to study mesoscopic transport phenomena with the flexibility of cold atoms.

Historically, fluctuation effects on transport properties have been studied extensively in classical superconductors [16] as well as unconventional superconductors [17,18]. Two well-known examples in the vicinity of superconducting transition temperature T_c are that (a) below T_c, a finite resistance appears due to phase slips induced by thermal fluctuations, known as the Langer-Ambegaokar-McCumber-Halperin (LAMH) effect [19,20], and (b) above T_c, conductivity is enhanced due to Cooper pair fluctuations, often called paraconductivity as was first studied by Aslamazov and Larkin [21,22]. In this Rapid Communication we extend the above calculations to the unitary regime and show how the enhanced pair fluctuation modifies the above two effects. Our main conclusions are the following.

(i) For the appearance of resistance below T_c, we find that in the unitary regime, the resistivity drops much slower than in the BCS limit as temperature decreases.

(ii) For the enhancement of conductivity above T_c, we find that this paraconductivity changes *nonmonotonically* from the BCS limit to the unitary regime and a minimum exists in between.

Time-dependent Ginzburg-Landau theory. Our derivation is based on the time-dependent Ginzburg-Landau (TDGL) theory of Bose-Einstein condensate (BEC)–BCS crossover [23]. The partition function of the unitary Fermi gas can be written as $\mathcal{Z} = \int D[\bar{\psi}_\sigma, \psi_\sigma] \exp(-S[\bar{\psi}_\sigma, \psi_\sigma])$, where

$$S[\bar{\psi}_\sigma, \psi_\sigma] = \int d\tau\, d^3\mathbf{x} \left\{ \bar{\psi}_\sigma \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) \psi_\sigma - g \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow \right\}.$$

Here τ is the imaginary time and μ is chemical potential. As usual, g is related to the s-wave scattering length a_s by $1/g = -m/4\pi a_s + \sum_{\mathbf{k}} 1/2\epsilon_{\mathbf{k}}$ with $\epsilon_{\mathbf{k}} = \mathbf{k}^2/2m$. Introducing Hubbard-Stratonovich fields $\Delta(\tau,\mathbf{x})$ to decouple the interaction term in the Cooper channel and then integrating out the fermions, we obtain an effective theory for the bosonic field $\Delta(\tau,\mathbf{x})$ representing the bosonic Cooper pair field. In the vicinity of T_c where Δ is small, we can expand the action in powers of Δ, as well as its spatial and time derivatives (after Wick rotation),

$$S[\bar{\Delta}, \Delta] = \int dt\, d^3\mathbf{x} \left\{ \bar{\Delta} \left(\gamma \partial_t - \frac{\nabla^2}{2m^*} - r \right) \Delta + \frac{b}{2} \bar{\Delta}\bar{\Delta}\Delta\Delta \right\}, \tag{1}$$

where $\gamma = \gamma_1 + i\gamma_2$ is complex in general. All the parameters γ, m^*, r, and b can be expressed in terms of μ, T, and $\zeta \equiv 1/k_F a_s$ [24]. In the following we will focus in the vicinity of the superfluid transition temperature $T \approx T_c$ and as a result $\mu(T) \approx \mu(T_c)$. We determine both T_c and $\mu(T_c)$ within the Nozières-Schmitt-Rink (NSR) scheme [25].

The real part γ_1 describes the damping of Cooper pairs due to coupling to fermionic quasiparticles. It can be shown that γ_1 is proportional to $\sqrt{\mu}\Theta(\mu)$ [23], where $\Theta(\mu)$ is the Heaviside step function. As a result, around unitarity and in the BCS side where $\mu > 0$, the Cooper pairs have a finite lifetime, while in the BEC limit where $\mu < 0$, $\gamma_1 = 0$ and the Cooper pairs (molecules) are infinitely long lived within the NSR scheme. The imaginary part γ_2 represents a propagating behavior and is given by

$$\gamma_2 = -\mathrm{P} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1 - 2N(\xi_{\mathbf{k}})}{4\xi_{\mathbf{k}}^2}, \tag{2}$$

where P denotes principal value, $N(\xi_{\mathbf{k}}) = [\exp(\beta\xi_{\mathbf{k}}) + 1]^{-1}$ is the Fermi distribution function, and $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$. In the

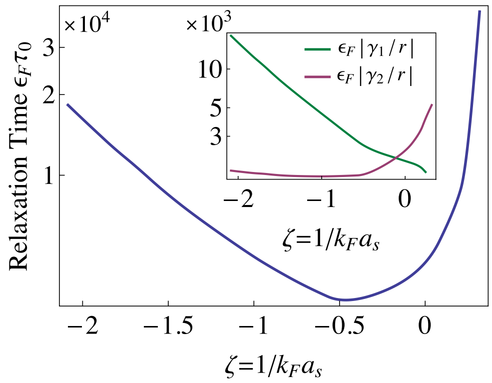

FIG. 1. (Color online) Relaxation time τ_0 as a function of $\zeta \equiv 1/k_F a_s$, in units of $1/\epsilon_F$. The inset shows the parameters γ_1/r and γ_2/r as a function of ζ. The temperature of the system is fixed at $1 - T/T_c = 10^{-3}$.

BCS limit $\zeta \to -\infty$, $\mu \gg \Delta$, the integrand is roughly antisymmetric with respect to the Fermi surface $\epsilon_{\mathbf{k}} = \mu$, a manifestation of particle-hole symmetry of the BCS state. Consequently, $\gamma_2 \simeq 0$. As ζ increases towards the unitarity and the BEC side, γ_2 gradually increases from zero, due to increasing violation of particle-hole symmetry. The behaviors of γ_1 and γ_2 as a function of ζ are shown in the inset of Fig. 1.

Relaxation time. As will be shown later, the relaxation time of the pairing field $\Delta(t,\mathbf{x})$ plays an important role in both the LAMH effect and paraconductivity. In the following, we derive an expression for the relaxation time that is valid close to unitarity. As is known [16], to maintain a nonzero thermal average of the pairing fluctuation, it is necessary to introduce the so-called Langevin force $\eta(t,\mathbf{x})$ into the TDGL equation

$$-\gamma\frac{\partial}{\partial t}\Delta = -\frac{\nabla^2}{2m^*}\Delta - r\Delta + b|\Delta|^2\Delta + \eta(t,\mathbf{x}), \quad (3)$$

where the Langevin force represents the driving force of the environment and is characterized by the white-noise correlations [16,17]

$$\langle\eta^*(t,\mathbf{x})\eta(t',\mathbf{x}')\rangle = 2\gamma_1 k_B T\delta(t-t')\delta^3(\mathbf{x}-\mathbf{x}'). \quad (4)$$

With a straightforward calculation, one finds the correlation function for the order parameter [24]

$$\langle\bar{\Delta}_{\mathbf{k}}(t)\Delta_{\mathbf{k}}(0)\rangle = \frac{k_B T}{k^2/2m + |r|}\exp\left[-\left(\frac{1}{\tau_k} + \frac{1}{i\tau_k'}\right)t\right], \quad (5)$$

where

$$\tau_k = \frac{\gamma_1^2+\gamma_2^2}{\gamma_1(k^2/2m+|r|)}, \quad \tau_k' = \frac{\gamma_1^2+\gamma_2^2}{\gamma_2(k^2/2m+|r|)}. \quad (6)$$

Here τ_k represents the temporal decay of the **k**-th Fourier component of the order parameter, while τ_k' characterizes its propagating behavior. In the limit $k \to 0$, we obtain

$$\tau_0 = \frac{\gamma_1^2+\gamma_2^2}{\gamma_1|r|}. \quad (7)$$

In the BCS limit $\zeta \to -\infty$, $\gamma_2 \approx 0$ and the relaxation time τ_0 only depends on γ_1 and can be reduced to $\tau_{\rm BCS} = \gamma_1/r$. Furthermore, in the same limit, $\gamma_1 \approx m\beta k_F/16\pi$, $r \approx mk_F(T_c - T)/2\pi^2 T_c$ [24], and as a result $\tau_{\rm BCS} = \pi/8k_B(T_c - T)$, consistent with the weak-coupling results [20]. Away from the BCS limit, τ_0 depends on both γ_1 and γ_2. As shown in Fig. 1, as ζ increases from the BCS limit toward the unitary regime, τ_0 first decrease as $\gamma_1/|r|$ and then increases as $\gamma_2^2/\gamma_1|r|$. A minimum of τ_0 occurs between the BCS limit and the unitary regime when $\gamma_1 \approx \gamma_2$. In the BEC side when $\mu < 0$, $\tau_k \to \infty$, indicating an undamped bosonic mode. To capture the effect of damping, it is necessary to go beyond the NSR scheme, which we will not attempt here. Rather we focus around unitarity, where our calculation applies.

Residual resistance below T_c. To simplify our investigation, let us consider the residual resistance of a quasi-one-dimensional unitary Fermi gases of cross-section area A and linear dimension L. The residual resistance below T_c is due to the thermally activated phase slips. The net effect of these events is to lower the current of the state and, as a result, a voltage drop must be sustained in order to maintain a steady current [16]. In other words, a finite resistance appears below T_c. Such a theory is developed by Langer and Ambegaokar and by McCumber and Halperin and later confirmed by experiments on BCS superconductors [26].

Within LAMH theory, residual resistivity due to the phase slips is given by [16,19,20,24]

$$\rho(T \lesssim T_c) = \frac{2\pi A\Omega}{Lk_B T}\exp\left[-\frac{\Delta F_0}{k_B T}\right]. \quad (8)$$

Here ΔF_0 is the lowest free-energy barrier to create one phase slip. Its analytic expression was derived by Langer and Ambegaokar: $\Delta F_0 = \frac{8\sqrt{2}}{3}\frac{r^2}{2b}A\xi$, where $r^2/2b$ is the condensation energy density and $\xi = 1/\sqrt{2m^*|r|}$ is the Ginzburg-Landau coherence length; ΔF_0 is roughly the condensation energy in a volume $A\xi$. Here Ω is the so-called attempt frequency, originally derived by McCumber and Halperin [20] as $\Omega = \frac{L}{\xi}\sqrt{\Delta F_0/k_B T}\frac{1}{\tau_{\rm BCS}}$. In our case the relaxation time $\tau_{\rm BCS}$ has to be replaced by τ_0 derived above.

Let us first investigate the dependence of ρ on the interaction parameter ζ. To do that we first fix the temperature below T_c by $1 - T/T_c = 5 \times 10^{-3}$. Then it is clear from Eq. (8) that the resistivity depends on several ratios $\Delta F_0/k_B T$, $v_F^{-1}T\xi$, and $\epsilon_F\tau_0$. Within our calculation, $\epsilon_F\tau_0$ changes only by a factor of 3–4 from the BCS to the unitary regime. On the other hand, if one uses the weak-coupling expression for $\xi = v_F/\pi\Delta$ and the fact that $T \approx T_c \sim \Delta$ the parameter $v_F^{-1}T\xi$ then remains almost a constant. Numerical calculation shows that in the regime of ζ considered, $v_F^{-1}T\xi$ changes only a few percent [see the inset of Fig. 3(a)]. Now the most important dependence is on $\Delta F_0/k_B T$ since it appears on the exponential factor. Detailed calculation shows that $\Delta F_0/k_B T$ changes by a factor about 3–4 in the relevant regime [see the inset of Fig. 2(a)]. Taking into account all these dependences, we find that from the BCS side to unitarity, the fluctuation-induced residual resistivity increases rapidly by several orders of magnitude, as shown in Fig. 2.

Now let us look at the temperature dependences of the residual resistivity. In Fig. 2(b) we plot the resistivity ρ in the BCS limit ($\zeta = -3$) and at unitarity ($\zeta = 0$), normalized to their respective values ρ^* at $1 - T/T_c = 1.5 \times 10^{-3}$. We observe that as temperature decreases, the resistivity drops much slower at unitarity than in the BCS limit. We also note

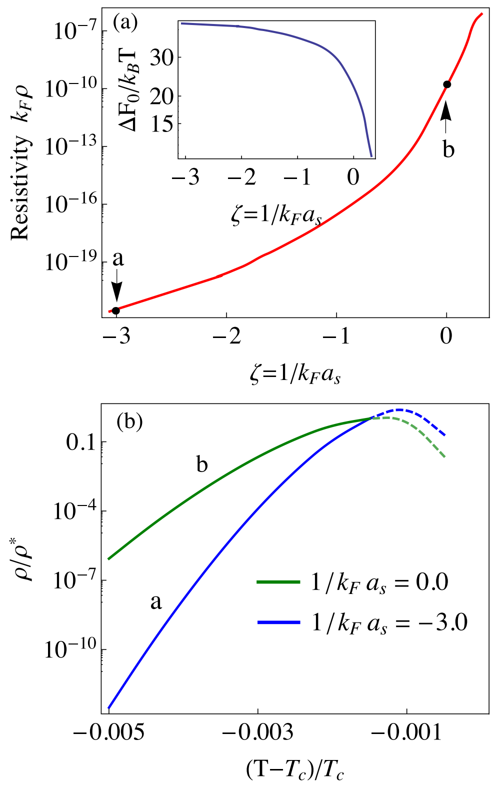

FIG. 2. (Color online) (a) Resistivity (and in the inset $\Delta F_0/k_BT$) as a function of the coupling $1/k_Fa_s$ with the temperature below T_c by $(T_c-T)/T_c=5\times10^{-3}$. (b) Resistivity as a function of temperature. Curves a and b correspond to two different scattering lengths marked in (a). Here we take $k_F^2A=10^6$.

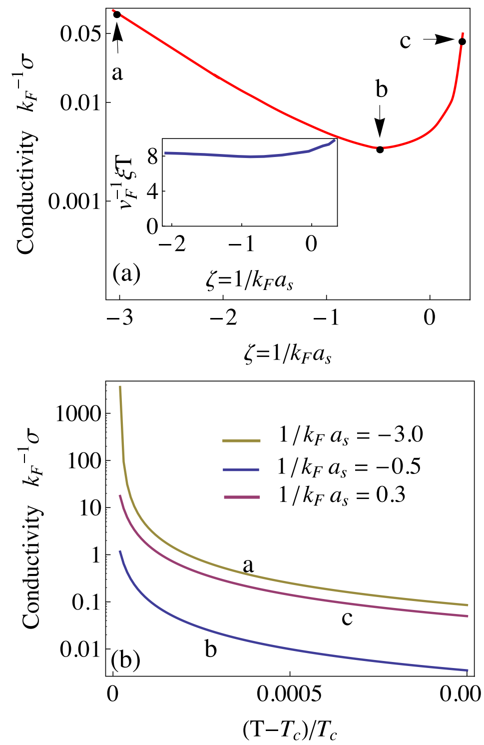

FIG. 3. (Color online) (a) Fluctuation-induced paraconductivity as a function of the coupling $1/k_Fa_s$ with the temperature above T_c by $(T-T_c)/T_c=10^{-3}$. (b) Fluctuation-induced paraconductivity as a function of temperature. Curves marked by a, b, and c correspond to three different scattering lengths marked in (a). Here we take $k_F^2A=10^6$.

that in Fig. 2(b) there is an unusual drop of resistivity (marked by the dashed lines) when temperature is very close to T_c. This is because the LAMH theory fails very close to T_c [22].

The above two observations at unitarity, the increased residual resistivity and its slower decrease as a function of temperature, suggest the more pronounced role of superconducting fluctuations below T_c at unitarity in comparison with the BCS limit. The increase of resistivity is monotonic as one approaches unitarity from the BCS side, in accord with our general expectations. In fact, as was discovered recently, close to unitarity when $A/\xi^2\gg1$, the energetically more favorable defects is a solitonic vortex [27–29], instead of the phase soliton in the BCS regime where $A/\xi^2\lesssim1$. Thus our estimation of ΔF_0 is an overestimate of the defect energy and the residual resistivity should in fact increase more rapidly close to unitarity and decrease even slower as the temperature is lowered. However, when we turn to the fluctuation-induced conductivity above T_c, as we will show shortly, the effect is not monotonic and in fact exhibits a minimum in between.

Enhanced conductivity above T_c. Above T_c, in addition to the usual conductivity given by normal fermions, there will be an extra contribution to conductivity due to thermal fluctuation of the Cooper pair field $\Delta(x,t)$, known as paraconductivity. We introduce the fluctuating supercurrent $J(t)$ along one of spatial direction, say, $\hat{x}$, where $J_x(t)$ is given by $J_x(t)=\frac{1}{m^*}\sum_{\mathbf{k}}k_x|\Delta_{\mathbf{k}}(t)|^2$. The fluctuation-induced paraconductivity can be directly calculated using the Kubo formalism as

$$\sigma_{xx}(\omega)=\frac{1}{k_BT}\int_0^\infty dt\langle J_x(t)J_x(0)\rangle\cos(\omega t). \quad (9)$$

A straightforward calculation yields the current-current correlation function as [16,17]

$$\langle J_x(t)J_x(0)\rangle=\left(\frac{1}{m^*}\right)^2\sum_{\mathbf{k}}k_x^2|\langle\bar{\Delta}_{\mathbf{k}}(t)\Delta_{\mathbf{k}}(0)\rangle|^2. \quad (10)$$

While $\langle\Delta\rangle=0$ for $T>T_c$, the thermal fluctuation of the Cooper pair field $\Delta(t,\mathbf{x})$ renders a nonzero value of the time-correlation function $\langle\bar{\Delta}_{\mathbf{k}}(t)\Delta_{\mathbf{k}}(0)\rangle$, as found previously in Eq. (5). This yields a nonzero contribution to the conductivity above T_c,

$$\sigma_{xx}(\omega)=\frac{1}{k_BTm^{*2}}\sum_{\mathbf{k}}\left[\frac{k_xk_BT}{k^2/2m^*+|r|}\right]^2\frac{\tau_k/2}{1+(\tau_k\omega/2)^2}. \quad (11)$$

We note that only τ_k, which characterizes the temporary decay of the order parameter correlation function, contributes to the conductivity. Specializing to the quasi-one-dimensional case and considering the dc component $\sigma_0\equiv\sigma(\omega=0)$, the paraconductivity can be written as

$$\sigma_0=2k_BT\tau_0\int_{-\infty}^{+\infty}\frac{dk_x}{2\pi}\frac{k_x^2\xi^4}{A(k^2\xi^2+1)^3}=\frac{k_BT\xi}{8A}\tau_0, \quad (12)$$

with τ_0 given by Eq. (7). To see how σ_0 changes as a function of interaction strength ζ, let us fix the temperature slightly above T_c, $1 - T/T_c = 10^{-3}$. As we show in Fig. 3(a), as one goes from the BCS limit to the unitary regime, fluctuation-induced paraconductivity first decreases and then increases, in comparison with the monotonic behavior of the phase-slip-induced resistivity below T_c. The similar dependence on ζ of σ_0 and the relaxation time τ_0 can be understood in the following way. According to Eq. (12), σ_0 is proportional to τ_0 with the coefficient $T\xi$. Now, as we have shown before, since again $T \sim T_c \sim \Delta$, $T\xi$ remains approximately a constant and as a result, σ_0 exhibits qualitatively the same dependence on ζ as τ_0.

Now let us look at the temperature dependences of σ_0 for various values of ζ. In Fig. 3(b), we show σ_0 at three interaction strengths: $\zeta = -3$ (marked by a), $\zeta = -0.5$ (marked by b), which is at the minimal of σ_0, and $\zeta = 0.3$ (marked by c). They all show a rapid increase as one approaches T_c from above.

Discussion. In this work we have discussed fluctuation effects on the transport properties of the unitary Fermi gas based on TDGL theory. At present, our results cannot be directly applied to the BEC limit since we have not taken properly into account the interactions between molecules. This leads to the infinite lifetime in the BEC side of the crossover where $\mu < 0$. Furthermore, in the BEC limit, it is also important to take into account the correction to chemical potential arising from the molecular interaction. We shall leave this to a future investigation.

In a recent experiment Brantut *et al.* observed a drop of the resistance for unitary Fermi gas below T_c, but the drop is much slower compared with typical BCS superconductor [12], consistent with our findings, although their experimental situation is much complicated than what is discussed here. Namely, the finite resistance observed below T_c is due to the thermally activated phase slips, which becomes much easier when close to unitarity, reflecting its enhanced superconducting fluctuations. Furthermore, we find that fluctuation-induced conductivity (paraconductivity) above T_c exhibits nonmonotonic behavior as one approaches unitarity from the BCS side. This can be verified in the same experimental setup used in Ref. [12].

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