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Content-Aware Photo Collage Using Circle Packing

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Abstract—In this paper, we present a novel approach for automatically creating the photo collage that assembles the interest regions of a given group of images naturally. Previous methods on photo collage are generally built upon a well-defined optimization framework, which computes all the geometric parameters and layer indices for input photos on the given canvas by optimizing a unified objective function. The complex nonlinear form of optimization function limits their scalability and efficiency. From the geometric point of view, we recast the generation of collage as a region partition problem such that each image is displayed in its corresponding region partitioned from the canvas. The core of this is an efficient power-diagram-based circle packing algorithm that arranges a series of circles assigned to input photos compactly in the given canvas. To favor important photos, the circles are associated with image importances determined by an image ranking process. A heuristic search process is developed to ensure that salient information of each photo is displayed in the polygonal area resulting from circle packing. With our new formulation, each factor influencing the state of a photo is optimized in an independent stage, and computation of the optimal states for neighboring photos are completely decoupled. This improves the scalability of collage results and ensures their diversity. We also devise a saliency-based image fusion scheme to generate seamless compositive collage. Our approach can generate the collages on nonrectangular canvases and supports interactive collage that allows the user to refine collage results according to his/her personal preferences. We conduct extensive experiments and show the superiority of our algorithm by comparing against previous methods.

Index Terms—Photo collage, image saliency, circle packing



1 INTRODUCTION

WITH the prevalence of smartphones equipped with high resolution cameras and the emergence of widely popular photo management and sharing websites, like Flickr and Photobucket, people have access to digital photo collections more often than before. Photo collage, as an important means for summarizing and exhibiting a collection of photos, has received considerable attention recently. It aims to create a compact, informative, and visually pleasant single-image representation by sticking together the pictures from a photo collection. Since, manually creating such a collage is time-consuming and generally requires professional image editing skills, automatic solutions have been intensively studied in the research community.

The existing photo collage approaches can be roughly classified into two categories according to different rendering styles of collages they produce. A typical style is to mimic how real pictures are arranged on a given canvas

with limited size [1], [2], [3], [4]. For each photo, its state parameters including its position on the canvas, orientation, scale as well as layer index are solved by optimizing an objective function, which integrates certain criteria such as salience maximization, blank space minimization, orientation diversity, and so on. Since image overlay is allowed, salient information of some photos can be occluded by less important regions of other photos, wasting precious canvas space. On the other hand, digital photomontage pioneered another style of collage where irregular salient regions detected from photos are assembled in the collage in a topic-based way [5]. To achieve seamless composites, the neighboring regions coming from different images are often blended. Typical methods that fall into this category include digital tapestry [6], autocollage [7], and the most recent puzzle-like collage [8], and so on.

Generally, most traditional approaches to photo collage are built upon a well-defined optimization framework. The objective function quantifies the criteria for a visually pleasing collage, and usually has a complex nonlinear form. Since each photo's state is determined by several geometric parameters together with a layer index, overall a few dozens to up to hundreds of parameters are to be optimized according to the number of input photos. Due to the nonlinear form of energy function, optimization in such a huge solution space is computationally inefficient and may easily get stuck in local minima, thus, providing suboptimal solutions. Although efficient approximate optimization techniques [3] are developed to greatly expedite this process, previous optimization-based methods have intrinsic limitations. On one hand, optimal parameters for

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different photos are tightly coupled, making the update of a photo's state affect other photos globally or locally. This limits scalability of the collage procedure as well as collage results. On the other hand, each photo is often uniformly treated, and photo scale is seldom regarded as an important factor. Taking the importance of photos into account will better summarize the photos and facilitate user control over the collage results with respect to his/her preferences.

Our observation is that photo collage essentially seeks a spatial partition on the given canvas for the input photos to display. The procedure can be completed in two independent steps, partitioning the canvas and displaying important image contents in the partitioned canvas. Rather than optimizing a complex nonlinear energy function as most previous methods have done, we compute the partition directly from the geometric point of view, and assign each photo a resulting subregion for display. To this end, the key is to find a highly efficient region division algorithm. We use a circle to approximate the salient region each image expects to display, and photo collage is formulated as a circle packing problem that aims at tightly arranging multiple circles with given radii in a fixed container. A new variational approach is thus developed to solve this problem based on power diagrams. The circle packing result provides the canvas partition such that each photo can be displayed in the subregion of the corresponding circle. To favor important images, the circle radii are associated with photo importances yielded by an image ranking process. The circles thus obtained are fed into the packing algorithm, producing a content-aware photo collage result.

We solve the problem of circle packing based on power diagram, which is a kind of weighted Voronoi diagram. It is noted that Voronoi diagram has been used in [9] successfully to partition the thumbnail area into regions for browsing large image data sets effectively. In such an application, thumbnails are dynamically packed inside a thumbnail area with an emphasized center and the whole thumbnail area needs to be smoothly re-rendered when the focus is changed. Different from their objective, we seek to produce a static image representation with high visual quality for a set of photos. Display region optimization after circle packing is necessary to achieve a nice and visually pleasant collage.

Our main contribution lies in that different from previous high-level layout constraints-based, optimization-driven photo collage methods, we reformulate photo collage as a content-aware region partition problem directly from the geometric point of view. Photo collage is achieved by several independent steps: photo importance computation, canvas partition, and collage assembly which solve the parameters involved in photo collage in turn. A circle-packing-based photo collage method allowing for fast and scalable photo collage is thus proposed. It has the following benefits.

- *Nonrectangular shapes of the canvases.* The canvas can be nonrectangular as the circle packing algorithm supports. Picture collage [2] supports arbitrary shaped canvas with zigzag boundaries since the boundary of a resultant collage is only approximated by photo boundaries. To the best of our knowledge,

most other methods do not support nonrectangular canvas shapes.

- *High scalability of the collage algorithm and interactive collage.* The reason is two-fold. First, for each photo, optimization of the four state factors, including position, orientation, scale, and layer index, are uncorrelated. Each factor is optimized in an independent stage of the collage process. Second, computation of the optimal positions for neighboring photos is completely decoupled and the display region for each photo is determined by the region division result of circle packing. As a result, the collage result can be adjusted flexibly. For instance, to adjust the display region for each photo, we only need to reoptimize its optimal display region locally and do not need to re-execute circle packing. Adjusting a photo only has effect on its neighboring regions, and will not change the collage result dramatically. Besides, our approach allows the user to refine the collage results according to his/her personal preferences. The interactive operations we supported include adjusting the visible region of a photo, exchanging any two specified photos, replacing a photo with a new one, zooming in a photo, and adding a new photo or removing an old one without modifying relative photo positions too much.

The remainder of this paper is organized as follows: The related work is briefly introduced in Section 2. In Section 3, we present a high-level overview of our method. The key components of our method, image ranking, circle packing, and collage assembly are described in Sections 4, 5, and 6 separately. Experiments and the user study are shown in Section 7 and we conclude the whole paper finally.

2 RELATED WORK

We mainly review here the relevant methods on photo and video collages.

2.1 Photo Collage

Photo collage aims to create a visual and informative summary of a group of images on a given canvas so that visible salient information is maximized. A typical collage is Picture collage which imitates physical arrangement and layout fashion of real pictures by allowing overlay on the limited space. Wang et al. first presented a Bayesian framework [1], which incorporates the requirements such as salience maximization and blank space minimization a nice collage should meet. An efficient Markov chain Monte Carlo method is designed for the optimization. Liu et al. [2] proposed to accelerate collage using a quick initialization algorithm. User interaction is integrated with the formulation and optimization, allowing for interactive collage that reflects personalized preferences. In [3], [4], belief propagation is used to optimize photo collage which is formulated on Markov random fields. Interactive collage refinement and dynamic collage are supported for effective photo browsing.

Photomontage pioneered another style of collage where a new image is synthesized from a collection of images [5]. In the montage, the input set of images are assumed to be of

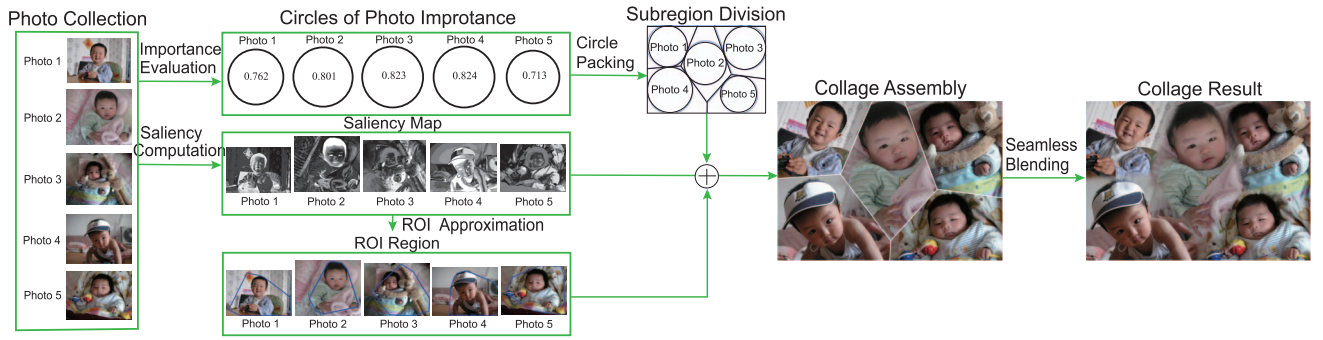


Fig. 1. The overall pipeline of our framework. We first compute for each photo an importance value according to which a circle is assigned to the photo. The circles of all photos are then packed into the given canvas using circle packing. For each photo, the display region in the polygonal area resulting from circle packing is optimized by considering image saliency in the process of collage assembly. This yields the style of collage which reserves white polygon boundaries. The collage with soft boundary between neighboring photos is produced through blending the overlapping regions of photos.

the same scene and roughly registered. More generally, digital tapestry takes a large collection of different images as input [6]. Creating tapestry is formulated as a multiclass labeling problem over a Markov random field, which is optimized via the graph-cut-based expansion move algorithm. Autocollage employs graph-cut, Poisson blending of alpha-masks to hide the joints between input images [7]. The authors developed a sequence of steps to optimize the collage energy that encourages the selection of representative images, special object classes, and an optimal layout. It is challenging to collage those images whose interest regions have arbitrary shapes rather than rectangles. To resolve this issue, the puzzle-like collage [8] assembles regions of interest in a puzzle-like manner.

Except for the summary of photo collections, it has been shown that fantastic and entertaining results can be generated through collage. This can be dated back to the Jigsaw image mosaics [13], where image tiles of arbitrary shapes are used to compose a picture and video mosaics [14], a 2D arrangement of small source videos that suggests a larger, unified target video. The method introduced in [15] creates the Arcimboldo-like collage which represents an image with multiple thematically-related cutouts from filtered Internet images. In [16], given a freehand sketch annotated with text labels, a realistic picture is synthesized by seamlessly combining semantic components originating from different Internet photographs in a collage-like manner.

2.2 Video Collage

Some researchers generalized photo collage to video sequences for fast browsing of video content. Interest regions, with rectangular [17] or arbitrary shapes [18], [19], extracted from keyframes in a video sequence are arranged on a template according to certain rules. The seams of interest regions coming from different frames are often blended for avoiding visual artifacts. The space-time video montage [20] extracts visually informative space-time portions of the input videos and packs them into a short video volume, facilitating efficient browse of video content.

Our work is also inspired by the 3D collage [21] which is constructed from 3D elements that roughly approximate the target shape.

3 OVERVIEW

We formulate photo collage as a region partition problem directly from the geometric point of view. The salient region of each input image is displayed in the corresponding subregion resulted from a newly developed circle packing algorithm. Previous methods on photo collage generally pay little attention to the relative importance of different photos. All input photos are often uniformly scaled to fit the given canvas first. In contrast, to emphasize important images, we first assign each image an importance value which is computed by combining image complexity and its distinctness compared with the other photos. Thereafter, each photo is treated as a circle with given radius, and the problem is thus transformed into a circle packing problem which aims at achieving the optimal layout of the photos in the canvas. We have developed a power-diagram-based efficient variational approach to solve circle packing. This leads to a set of polygonal subregions that envelop the circles for input photos and they are left for displaying corresponding photos. We further use an efficient heuristic search scheme to determine the optimal region for each photo such that most salient information is visible in its corresponding polygon. Finally, to deal with the joins between adjacent regions coming from two different photos and to make the collage visually pleasant, a saliency-based blending operation is applied.

Fig. 1 illustrates the processing pipeline of our approach.

4 PHOTO IMPORTANCE COMPUTATION

Previous methods on photo collage seldom consider photo importance as an important factor influencing collage results. The input photos are often uniformly treated. We argue that important photos should be emphasized. In fact, evaluating photo content for the tasks of photo synthesis and scene completion has been addressed by previous works [22], [23]. For photo collage, important photo content should get larger space to display. In fact, the input photos generally show different relative importances. Important photos, either evaluated automatically or specified by the users, usually convey more information compared with those less important ones. Leaving them more space to display will make the collage result more informative.

Since the subjective criteria for judging whether an image is important or not may vary from person to person, we do not wish to bridge the gap between subjective cognition and objective measurement. Instead, we compute the importance for each photo by combining image complexity and its distinctness compared with other images. Both aspects can be measured quantitatively using some objective criteria.

4.1 Image Complexity Measure

It makes sense that a good collage should show less plainer contents and pay more attention to those informative images. To achieve this, those complex photos should earn more space to show their details. Image complexity has been measured by previous methods using an information theoretic framework [10], Wavelet Transform [11], and Independent Component Analysis [12]. Here we mainly consider two simple, yet easy to compute features, color and edge. Color complexity and edge complexity are computed separately.

Color complexity. Color diversity in an image is an essential characteristic that leads to visual complexity. We define image complexity using color statistics built upon *HSV* color histogram. Color complexity on *H* channel is computed as,

$$S_c^h = 1 - \frac{\sqrt{\sum_{i=1}^m (h_i^h - 1/m)^2}}{\sigma_{\max}^h}, \quad (1)$$

where m represents the number of bins the histogram for *H* channel has and we set it to 16 in our experiments. h_i^h denotes the frequency of pixels that fall into bin- i . σ_{\max}^h , used as a regularization term, is the maximum variance of frequency. When the image has a constant hue value in which case one bin of the histogram is fully occupied and the other bins are empty, σ_{\max}^h is calculated as

$$\sigma_{\max}^h = \sqrt{(1 - 1/m)^2 + (m - 1)(1/m)^2}. \quad (2)$$

S_c^s and S_c^v are computed in a similar way.

Color complexity finally is set to the mean of S_c^h , S_c^s , and S_c^v ,

$$S_c = (S_c^h + S_c^s + S_c^v)/3. \quad (3)$$

Edge complexity. Edges are important visual cues for understanding image content. Normally speaking, the number of edges in an image and the diversity of their orientations reflect image complexity to some extent. To account for this, we measure edge complexity by using gradient statistics built upon the gradient orientation histogram.

The 1D centered, point discrete derivative operators with a horizontal gradient mask $[-1, 0, 1]$ and a vertical one $[-1, 0, 1]^T$ are first applied to the given image. This yields two edge maps by exploiting which we can easily compute a gradient magnitude and an orientation for each edge pixel. A gradient orientation histogram with nine bins in 0-180 degrees can thus be built, and we use it to compute edge complexity as follows:

$$S_e = \lambda \cdot \left(1 - \frac{\sqrt{\sum_{i=1}^9 (h_i^o - 1/9)^2}}{\sigma_{\max}^o} \right), \quad (4)$$

where h_i^o denotes the number of gradient orientations that fall into bin- i . σ_{\max}^o is similarly defined as σ_{\max}^h in (2). λ here is the ratio of pixels with nonzero gradient magnitude to the number of image pixels.

Image complexity S_I is taken as the mean of color and edge complexities.

4.2 Image Distinctness Computation

A photo collection often contains some photographs with similar content. To make the photos with distinct content distinguishable from those similar images in the collage, it is reasonable to assign them higher importance. We employ the Earth Mover's Distance (EMD) [24] to evaluate dissimilarity between two images. The EMD is a cross-bin distance function that addresses the bin-to-bin alignment problem when comparing two color histograms. It defines the distance between two histograms as the solution of the transportation problem which is a special case of linear programming.

Let \mathbf{H}_I and $\mathbf{H}_{I'}$ denote the *HSV* histograms of two images *I* and *I'* separately. The hue channel consists of 16 bins, and both saturation and value channels comprise 4 bins. We concatenate the bins of three channels into a 1D histogram with 24 bins for \mathbf{H}_I and $\mathbf{H}_{I'}$, separately. The histograms with normalized bin values are expressed as

$$\mathbf{H}_I = \{h_i, i = 1, \dots, 24\}, \quad \mathbf{H}_{I'} = \{h'_j, j = 1, \dots, 24\}. \quad (5)$$

The EMD measure first needs to find a flow set $\{f_{ij}\}$, with f_{ij} the flow between h_i and h'_j , that minimizes the following cost,

$$E(\mathbf{H}_I, \mathbf{H}_{I'}, \{f_{ij}\}) = \sum_{i=1}^{24} \sum_{j=1}^{24} f_{ij} d_{ij}, \quad (6)$$

$$s.t. \quad f_{ij} \geq 0 \quad i \in [1, 24] \quad j \in [1, 24],$$

$$\sum_j f_{ij} \leq h_i \quad i \in [1, 24],$$

$$\sum_i f_{ij} \leq h'_j \quad j \in [1, 24], \quad (7)$$

$$\sum_{i=1}^{24} \sum_{j=1}^{24} f_{ij} = \min \left(\sum_{i=1}^{24} h_i, \sum_{j=1}^{24} h'_j \right),$$

where d_{ij} is the L_1 ground distance between bin- i and bin- j .

The EMD between \mathbf{H}_I and $\mathbf{H}_{I'}$ is

$$\text{EMD}(\mathbf{H}_I, \mathbf{H}_{I'}) = \frac{\sum_{i=1}^{24} \sum_{j=1}^{24} f_{ij} d_{ij}}{\sum_{i=1}^{24} \sum_{j=1}^{24} f_{ij}}. \quad (8)$$

To expedite the computation of the EMD, we adopt the fast and robust EMD algorithm [25]. We define content distinctness S_D of an image as the minimum EMD between it and all the other images. In other words, the distinctive images are treated as relatively important ones.

With the computed image complexity S_I and content distinctness S_D , the importance S_I of an image *I* is calculated as

$$S_I = S_I + \omega \cdot S_D, \quad (9)$$

where ω is a parameter that is used to control the influence of two terms. In implementation, we set it to 0.5 for emphasizing the effect of image complexity. It is noted that people's photos such as portraits are usually important in a photo collection. To account for this, the users can set a high value to those photos with high-level semantic objects such as faces that can be detected easily.

5 CIRCLE PACKING FOR CANVAS PARTITION

Circle packing is a classical mathematical problem, which aims at arranging given circles, identical or nonidentical, without overlap into the smallest container with a fixed shape [26]. For this photo collage application, we transform the circle packing problem into an equivalent form. That is, each input image is treated as a circle whose radius is its image importance, and then the goal is to pursue the tightest packing in the given canvas, while keeping the radius ratios. In the process of optimizing the positions of these nonidentical circles, they are uniformly scaled to fit the canvas.

5.1 Problem Formulation

Given a region $\Omega \in \mathbb{R}^2$ and n circles $\{C_i\}_{i=1}^n$ with known radii $\{r_i\}_{i=1}^n$ ($r_i > 0$) assigned to the images, we focus on the circle packing problem of determining the tightest configuration for all the circles encompassed in Ω without overlap, and meanwhile keeping the given radius ratios. We introduce a scale factor $k \in \mathbb{R}$, $k > 0$ for all circles, such that each circle C_i becomes kC_i . The circle packing problem can be stated as the following optimization problem:

$$\begin{aligned} & \text{Maximize } k \\ & \text{Subject to } kC_i \subseteq \Omega, \quad i \in \{1, \dots, n\} \\ & \quad kC_i \cap kC_j = \emptyset, \quad i, j \in \{1, \dots, n\}, i \neq j. \end{aligned} \quad (10)$$

An arrangement of a set of circles C_i in \mathbb{R}^2 can be represented by $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$, where \mathbf{x}_i is the coordinate of the center of C_i . We name \mathbf{X} a *configuration*. If all circles satisfy the two above constraints, we say the configuration is *valid*. Note that a circle here is considered as a closed disk, and thus the second constraint can be written as $\|\mathbf{x}_i - \mathbf{x}_j\| - kr_i - kr_j \geq 0$.

5.2 Variational Circle Packing Algorithm

We then present our geometric and variational algorithm based on power diagram for solving the circle packing problem.

The power diagram is a kind of weighted Voronoi diagram introduced by Aurenhammer [27]. Let $P = \{p_1, \dots, p_n\}$ be a set of distinct points in \mathbb{R}^m and each point p_i be associated with a weight $w_i \geq 0$, $i = 1, \dots, n$. The power distance $d_w(p, p_i)$ from a point p to p_i is defined as

$$d_w(p, p_i) = \|p - p_i\|^2 - w_i.$$

The power distance induces a partition of \mathbb{R}^m by P . Let $V(p_i)$ be a region associated with p_i so that

$$V(p_i) = \{p \in \mathbb{R}^m \mid d_w(p, p_i) \leq d_w(p, p_j), \forall p_j \in P\}.$$

The set of all $V(p_i)$ is then the *power diagram* of P . It is easy to see that if all points carry the same weight, the power diagram is essentially the same as the Voronoi diagram.

Given a bounded region $\Omega \subset \mathbb{R}^m$, let Ω_i be the intersection of $V(p_i)$ and Ω , i.e.,

$$\Omega_i = V(p_i) \cap \Omega.$$

The set of all Ω_i constitutes the *bounded power diagram* of P in Ω , where Ω_i is called the *cell* for p_i .

In 2D, a point p_i of weight w_i is treated as a circle centered at p_i with radius $\sqrt{w_i}$. Then we can see that a circle packing configuration is well associated with a bounded power diagram, where each circle centered at \mathbf{x}_i with radius r_i is treated as a point \mathbf{x}_i of weight $w_i = r_i^2$. Since the circles in a valid configuration do not overlap, every circle is completely contained within its cell.

In analogy to computing a centroidal Voronoi diagram [28] using Lloyd's method [29] to minimize a certain cost function, our idea is to iteratively update a bounded power diagram to maximize the scale factor k while maintaining nonoverlap among the circles. Hence, the essence of our algorithm is

- to keep increasing k at each iteration, and at the same time; and
- to ensure the update of the circle locations does not result in any overlap.

Our framework for solving this optimization problem is given in Algorithm LCP1.0 as follows:

Algorithm LCP1.0: Local Circle Packing Algorithm.

Input : Circles $\{C_i\}_{i=1}^n$ with radii $\{r_i\}_{i=1}^n$, a container Ω , an initial value for k , and an initial *valid* configuration $\mathbf{X} = \{\mathbf{x}_i\}_{i=1}^n$ of the scaled circles $\{kC_i\}_{i=1}^n$.

Output: A resulting configuration of $\{kC_i\}_{i=1}^n$ and k .

Steps:

- 1) Assign a radius of kr_i to each circle C_i , $i = 1, \dots, n$.
- 2) Construct the bounded power diagram $\{\Omega_i\}_{i=1}^n$ associated with $\{C_i\}_{i=1}^n$ and \mathbf{X} in Ω .
- 3) For each region Ω_i , $i = 1, \dots, n$, compute its maximum inscribed circle (MIC), \tilde{C}_i , with center $\tilde{\mathbf{x}}_i$ and radius $\tilde{r}_i = \max_{q \in \Omega_i} \min_{p \in \partial \Omega_i} \|p - q\|$, where $\partial \Omega_i$ denotes the boundary of Ω_i .
- 4) $k' \leftarrow k$, $k \leftarrow \min_i \{\tilde{r}_i / r_i\}$.
- 5) If $k = k'$, go to step 6; otherwise, $\mathbf{X} \leftarrow \{\tilde{\mathbf{x}}_i\}_{i=1}^n$ and go to step 1.
- 6) Return \mathbf{X} and k .

The above algorithm requires an initial scale factor k and an initial position \mathbf{X} of the scaled circles kC_i , $i = 1, \dots, n$, which give a valid configuration, i.e., $kC_i \subset \Omega$ for all i , and $kC_i \cap kC_j = \emptyset$ for all $i \neq j$. By choosing a very small value of k , say $k = 10^{-6} / \max_i \{r_i\}$, which depends on the size of the input circles, and a set of random initial positions for the scaled circles, a valid configuration is easy to achieve. In each iteration, the algorithm constructs the bounded power diagram for the scaled circles, updates the circle configuration, and computes a new scale factor k for the next round. The algorithm ends when no further improvement can be made to k . Fig. 2 demonstrates the process of the algorithm.

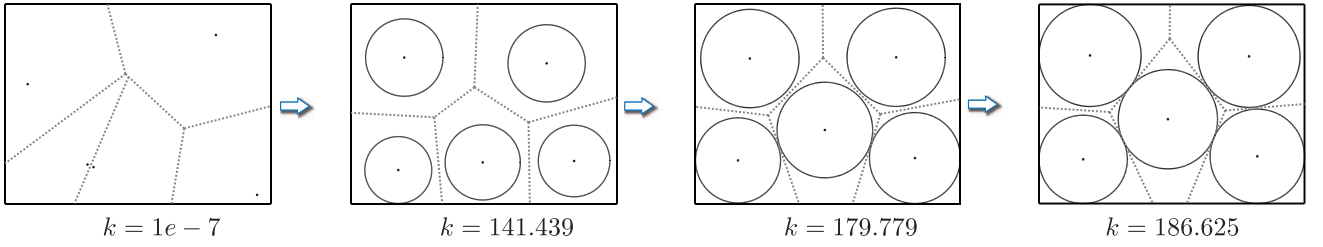


Fig. 2. The process of packing five circles in a rectangular canvas, $r_1 = 0.762$, $r_2 = 0.801$, $r_3 = 0.823$, $r_4 = 0.824$, and $r_5 = 0.713$. The canvas is of size 800×600 .

Given a cell Ω_i , the largest circle that it can contain is given by the maximum inscribed circle (MIC). Inspired by Lloyd's method which updates the sites to the centroids of the Voronoi cells iteratively in computing a CVT, we update the circle position in each cell by moving the circle to the center of the MIC. We also choose a new scale factor $k = \min_i \{\tilde{r}_i / r_i\}$, where \tilde{r}_i and r_i are the radii of the MIC and the circle in Ω_i , respectively. The factor k is the maximum enlargement possible so that the scaled circles kC_i are all contained within their cells, and hence are nonoverlapped. Moreover, the choice of k in relation to the MIC guarantees that k is nondecreasing. The termination condition for the iteration $k = k'$ is realized only when a circle is the MIC of its cell. Hence, we have the following statement:

Statement 1: The algorithm LCP1.0 optimizes the objective function in (10) monotonically by increasing the value of k in each iteration. When the algorithm terminates, the resulting configuration always gives a valid circle packing. Also, at least one of the circles coincides with the MIC of its power diagram cell.

There is a difficulty in handling a power diagram cell whose MIC is not unique. For example, if the cell is rectangular, there are infinite MICs that slide along the two long parallel edges. In this case, we shall simply pick an arbitrary MIC. Fortunately, this problem has little chance to happen in practice due to the numerical accuracy. So we just assume that each power diagram cell has a unique MIC. Also note that the circle packing problem is known to be NP-hard [30] and the algorithm LCP1.0 could only achieve the local extrema of the objective function. For the improvement of local extrema, please see [31].

6 COLLAGE ASSEMBLY

6.1 Display Region Optimization

Circle packing divides the canvas into a set of polygons which enclose the input circles. Each polygon is assigned to its corresponding photo for display. To generate an informative collage, it is important, for each photo, to compute a saliency map which indicates the importance of each pixel. Saliency detection has been extensively explored in the past several years [32], [33], [34]. We use the global contrast-based method developed in [34] to compute a normalized saliency value for each image pixel. Besides, we employ the Viola-Jones face detector implemented by OpenCV to detect faces because faces are expected to be visible, especially in family photos. The user can also specify an important region in an image by drawing a closed contour enclosing it. The saliency values for those

pixels in face and user specified regions are set to the highest saliency value. With the detected saliency maps, the key issue is to display salient content as much as possible for each input photo. The geometric parameters associated with each photo include its position, scale, and orientation. To ensure orientation diversity of the displayed photos, we specify photo orientations in advance. Then, we optimize the position and scale for each photo independently.

Orientation. The collage would look very stiff if all the images are arranged in the same orientation. To make the collage visually attractive, it is expected that the images are arranged with diverse orientations. We set the orientation angle of each photo to a random value generated from a uniform distribution in $[-\theta, \theta]$. θ is specified by the user, and a maximum of 30 degrees is suggested to avoid too cluttered result.

Position and scale. Given the rotation angle of each photo, the solution space for position and scale is still very large, although optimization for different photos is completely decoupled and independent. Brute-force search is in general computationally intractable. To solve this problem, we first extract the region-of-interest (ROI) of the photo. Then, we develop an efficient heuristic algorithm which pursues the appropriate values of position and scale alternatively and recursively such that salient information in ROI is displayed in the polygonal area as much as possible.

Previous methods often use a rectangle to represent ROI roughly. We instead approximate it with a polygon for saving space. The saliency map is first converted into a binary map via thresholding. Then we execute a few steps of dilation and erosion for merging adjacent areas. We convert the envelop of the largest one-third salient regions into a polygon and regard it as a compact representation of ROI.

Without loss of generality, we assume that Q_R and Q_p represent the ROI and polygonal area on the canvas for a photo to be displayed, separately. We place p_R , the barycenter of Q_R , on the center of Q_p . Since orientation of the photo is determined beforehand, the scale factor can be computed by requiring the rotated photo to cover Q_p completely. We then build a Cartesian coordinate system whose origin is located at p_R on the canvas. We define the saliency loss of Q_p in each quadrant as the means of saliency values of those pixels that belong to Q_R , but out of Q_p . Let s_{tl} , s_{tr} , s_{br} , and s_{bl} denote the saliency loss in the top left, top right, bottom right, and bottom left quadrants, respectively. A heuristic proposal indicating the moving direction of Q_R is calculated as

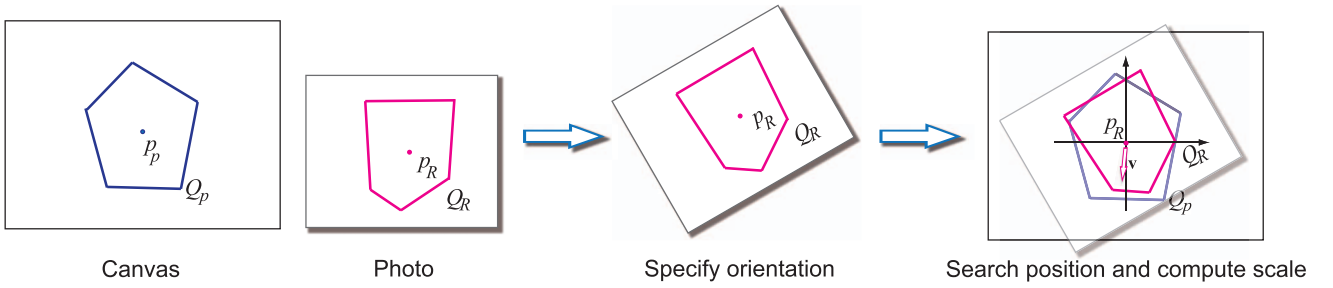


Fig. 3. The workflow of display region optimization. Given Q_R , ROI of the photo, and Q_p , the polygonal area assigned to the photo on the canvas, orientation of the photo is first specified. The optimal position and scale are further determined through a heuristic searching process.

$$\mathbf{v} = ((s_{tl} + s_{bl}) - (s_{tr} + s_{br}), (s_{tl} + s_{tr}) - (s_{bl} + s_{br})). \quad (11)$$

We further normalize \mathbf{v} to $\hat{\mathbf{v}}$. Suppose that the maximum step-size each time Q_R moves is r . We sample the circle with radius of $r/2$ around $p_R + r/2 \cdot \hat{\mathbf{v}}$ using the Gaussian distribution $N(p_R + r/2 \cdot \hat{\mathbf{v}}, r)$. Q_R will move to the sample point where saliency gains are maximized. With the new position of Q_R , we recompute an appropriate scale of the photo.

The above procedure is recursively executed for a certain time until it converges or the number of iterations exceeds a maximum number of iterations. Generally, the initial iterations reduce the saliency loss fast and it remains nearly stable after several iterations. Fig. 3 shows the workflow of display region optimization.

6.2 Saliency-Based Blending

To generate a visually pleasing collage, the transitions between input images that are adjacent in final collage need to be specifically dealt with. Previous methods mainly focus on the seamless color transitions by using various blending schemes in α -channel or color channels. We propose to incorporate visual saliency into the blending operation. With our blending scheme, natural transitions between adjacent images are ensured. Furthermore, salient features from the overlapping regions of adjacent images are visible.

For each pixel p on the collage, a list of labels $\{l_1(p), l_2(p), \dots, l_N(p)\}$ are assigned to it. N is the number of images. The probabilities are $\{\text{Prob}_1(p), \text{Prob}_2(p), \dots, \text{Prob}_N(p)\}$, where $\text{Prob}_i(p)$ is the probability image I_i contributes to p . Let Q_{pi} and Q_{Ri} denote the polygonal area assigned to I_i , and the transformed ROI on the canvas after display region optimization, separately. The goal is to make the salient features in Q_{Ri} visible even though they are on the outside of Q_{pi} . This is achieved by computing the probabilities as follows:

$$\text{Prob}_i(p) = \begin{cases} 1 & p \in Q_{pi}, p \in Q_{Ri} \\ e^{-\frac{d(p, Q_{Ri})}{2\sigma}} & p \in Q_{pi}, p \notin Q_{Ri} \\ e^{-\frac{d(p, Q_{pi})}{2\sigma}} & p \notin Q_{pi}, p \in Q_{Ri} \\ 0 & \text{otherwise,} \end{cases} \quad (12)$$

where $d(\cdot)$ is the euclidian distance from p to its closet point in the polygon.

Each pixel's probabilities computed using the above formula are further propagated equally to its four-connected neighboring pixels, so that neighboring pixels tend to have similar probabilities after several iterations. Afterward the probabilities are normalized, and used as the alpha values for image blending, to generate the

collage picture finally. Our saliency-based blending enhances the visibility of overlapped images without bringing visual artifacts.

7 EXPERIMENTS

We implemented and tested our framework using the C++ programming language on an Intel Dual Core 3.2-GHz Desktop PC with 4-GB RAM. Figs. 4, 7, and 8 show some of our results.

Fig. 4 shows the collage results for 21 photos of a little child who is now three-year old. The polygons enclosing the circles for their corresponding photos are generated by circle packing. We then give the assembly result as the upper right image, where white polygon boundaries between different display regions are reserved. The collage assembly results with polygon boundaries reserved are also given in Fig. 7. Such style provides an alternative to the traditional collage style. In contrast, the images shown in the bottom row of Fig. 4 are the results with soft boundaries between neighboring photos.

Fig. 5 shows the collage result for seven photos of dogs. The importance values of those complex photos, for instance, the photo of a dog sitting on the boat deck and the photos of dogs in the grassland, are bigger than the importances of the bottom right two photos with simple background. As a result, the complex photos are assigned more space to display.

Fig. 6 shows our collage results for assembling photos of different categories. In Fig. 6a, the photos of different people, football players, and animals are taken as the input and Fig. 6b is another collage result for the same photo group as Fig. 6a. In Fig. 6c the images of a baby, animals, the statue of Liberty, flowers, butterflies, and so on, are used to create a collage.

Fig. 7 shows two collage results on the canvases with nonrectangular shapes. Our approach intrinsically supports the collages with nonrectangular shapes as the circle packing algorithm supports.

7.1 Interactive Collage

The user may wish to adjust the initial collage result with respect to his/her preferences, for instance, adjusting the visible region of a photo and exchanging the positions of any two photos. Since we solve the collage problem directly from the viewpoint of region division and images are displayed in the polygonal area resulting from circle packing, optimizing the visible regions of different photos



Fig. 4. The collage results for the photos of a three-year-old boy. (a) The polygonal regions resulting from circling packing. (b) The collage style with white polygon boundaries shown. (c) The corresponding collage result with soft boundary. (d) Another collage result for the same group of photos.



Fig. 5. The collage result for seven photos of dogs. The importance values of those complex photos are bigger than the importances of those photos with simple background. As a result, they are assigned more space to display.



Fig. 6. Our collage results for assembling photos of different categories.

are decoupled. Such interactions are conveniently supported by our framework. Overall, we support the following interactive operations.

Adjust the visible region of a photo. This is made possible by directly changing the geometric parameters of the specified image. The user can adjust the position of the image in the polygonal area, rotate it, and modify its scale factor. We do not need to re-execute the whole

algorithm, and only need to reprocess the joint regions between the image and its neighbors.

Exchange any two specified photos. This is realized by simply exchanging the mapping between the specified images and the polygonal area. The optimal display regions of the two photos need to be reoptimized.

Replace a photo with a new one. We only need to compute the saliency for the new photo, and to optimize its display



Fig. 7. Our collage results on canvases with nonrectangular shapes.

region in the polygonal area locally such that salient content can be displayed.

Zoom in a photo without modifying relative photo positions too much. Given the initial collage, the user may want to achieve focus+context visualization of the collage, by zooming in the display region of a specified photo and shrinking the spaces of rest of the photos. This is realized by re-executing the circle packing algorithm, with the adjusted circle radius of the specified photo, initial radii of the other photos, and original positions of all the circles as input.

Add a new photo or remove an old one without modifying relative photo positions. This is achieved by re-executing the circle packing algorithm. The optimal display regions of most photos need to be reoptimized as the polygonal regions are changed after circle packing. Note that, removing an old image runs very fast since circle packing can take the final state of previous packing result as input in this situation.

Fig. 8 demonstrates an example where the above operations are sequentially applied to an initial collage result.

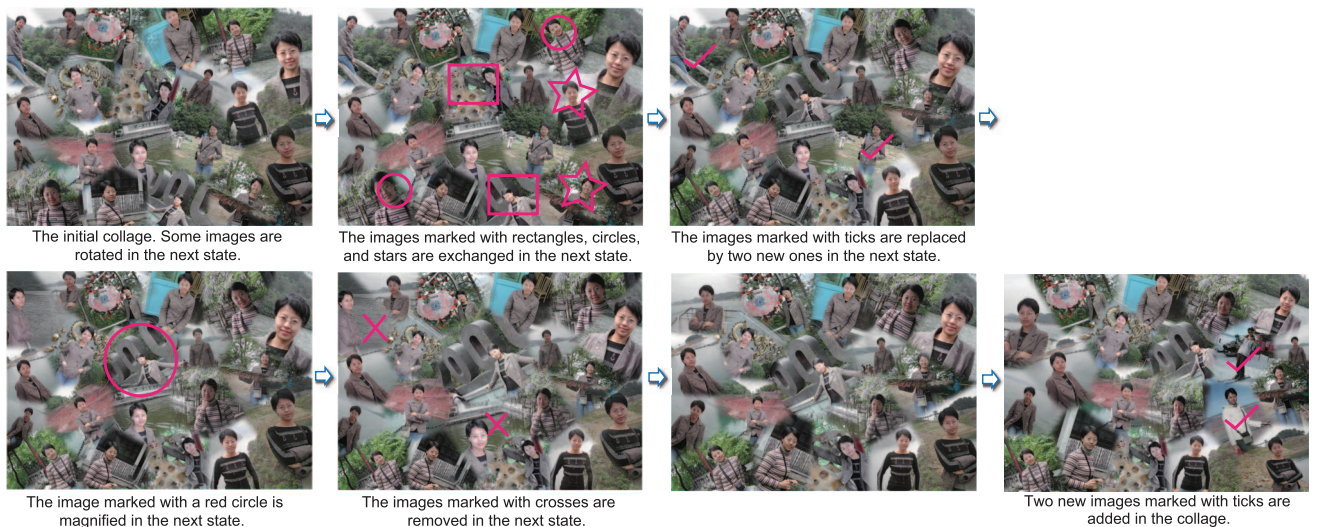


Fig. 8. A sequence of collage results generated by sequentially applying the interactive operations we supplied to an initial collage result. Temporally coherent image browsing experience is achieved since image transitions are smoothly rendered.



Fig. 9. The collage result generated by Microsoft Autocollage.

Image transitions among the collages in the resulting sequence are smoothly rendered, without abrupt visual discontinuities. This brings temporally coherent browsing experience.

7.2 Comparison with Autocollage

Our collage results resemble the style of Autocollage which also aims to seamlessly combine the representative elements from a set of images to produce eye-catching collages. The user study conducted in [7] has shown that such style is useful as a summary of a set of photos for browsing photo content rapidly and sharing them with friends. A software implementation of Autocollage is Microsoft Autocollage which is a plug-in of Windows live.¹ We, thus, compare our results to those produced by the Autocollage software.

Fig. 9 is the collage result of Autocollage for the same group of child photos used in Fig. 4. Note that, the Autocollage software supports face detection and it successfully detects all the faces in these photos, even for the two faces with closed eyes. The child's faces in the two input photos are, however, severely occluded by their neighbors. We analyze the reason for this. Autocollage solves an objective function quantifying the criteria for a visually pleasing collage. Although region-of-interest and high-level semantic objects such as faces detected are subject to meeting some constraints in their objective, the optimization procedure cannot always yield a solution which simultaneously satisfies all hard constraints in their function. In comparison, we first partition the canvas for getting a set of polygons for the photos to display. Important contents such as faces are rendered after an explicit process of display region optimization. They are guaranteed to be visible in the collage, as shown in Fig. 4.

Another comparison on the collage results of the photos of a group of sports players is shown in the first row of Fig. 10. The second row of Fig. 10 further compares our approach with Autocollage using a group of toy photos. For the collage result of toy photos, the assembled images in the result of Autocollage have fixed orientations. In contrast, the displayed regions for different photos in our collage have diverse orientations, making the result seem more interesting. Another comparison on the collage

results of marine life is given in the third row. Our result is comparable to the collage produced by Autocollage.

A typical problem of Microsoft Autocollage is that it produces only a specific result for a given group of images. That is, the collage result remains the same even if we apply the software to the same group of images for many times. It is possibly due to the intrinsic characteristic of energy minimization of the original Autocollage framework. Furthermore, user interactions are not allowed by Autocollage to fine-tune the initial collage result. By comparison, different collage results for the same group of images can be generated by using our approach, as executing our algorithm two times can yield different circle positions and polygonal areas for the images to display. Therefore, the user can select his or her favorite results among the different collages. As aforementioned, interactive optimization on the initial collage result is supported by our approach.

Computational complexity of our approach mainly depends on the number of input photos. The major computation is spent on image complexity computation, circle packing, and display region optimization for each image. All the three aspects have nearly linear complexity with the photo number. Thus, time complexity of the whole algorithm is nearly linear. Since optimizing the display regions for different photos are completely decoupled, we parallelize this process for acceleration. Our approach takes less than 10s to create the collage for 20 photos. It is however difficult to accurately record the running time of Microsoft Autocollage, since it will pop up a dialog asking the user to select the directory and folder where the collage result is to be saved during the collage procedure. We believe that the program is running in the background during this process. For the same number of input photos, Autocollage generally takes about 10 s.

7.3 User Study

To further evaluate the effectiveness of our approach, we conducted a preliminary user study. The objective is to determine whether the results produced by our method are preferred by the users to those of Microsoft Autocollage.

Ten pairs of collages, one created by our system and the other by Autocollage, are used in the experiments. The photo collections used for creating the collages have several different types such as traveling and home photos, animal photos, toy photos, photos for a group of cocktail glasses, and photos of football players. Fig. 11 shows six pairs of collages used in the user study, and two of the rest four pairs are the collage results for the little child shown in Figs. 4c and 9, and the results of toys in Fig. 10. The results on a group of football players and cartoon characters are not shown here.

We posted the advertisements for inviting volunteers in our department to take part in our user study through email. One hundred and twenty four subjects who did not know about our work volunteered to participate in the study. Each subject was shown all the ten pairs of collages. For each pair, whether our result was on the left or right was randomized. The participant was asked to select one from the pair they preferred, and to write the answer on the answer sheet.

We counted the total number of times that our result was preferred by participants. Overall, participants selected our

1. <http://plugins.live.com/photo-gallery/detail/autocollage>.



Fig. 10. Comparison between the collages produced by Microsoft Autocollage (left) and our results (right) for a group of images of the athletes (up), toys (middle), and marine life (bottom).

results over the collage results of Autocollage 813 out of 1,240 times (65.56 percent). Fig. 12 shows the statistics of the user study on each pair of collages. It has been shown that the users preferred most of our results, except our result of the girl in a white coat shown as the last image in the middle row of Fig. 11.

We also asked some participants the reasons why our results or the results of Autocollage were selected after the study has finished. Some participants said that the assembled images in our collage have diverse orientations, making some results more interesting and vivid. In contrast, all displayed images of Autocollage remain fixed orientations, and the results for some photos look stiff. Some participants pointed out that salient photo contents, especially human faces, are occluded in a few results of Autocollage. They, thus, selected our results as a result. Besides, for some results, a few participants said that the

transitions between the neighboring display regions of different photos in the results of Autocollage are more smooth than ours. Actually, we did not take color consistency of neighboring photos into account during collage assembly and this is a limitation of our approach.

8 CONCLUSIONS AND FUTURE WORK

We have given a new approach to generate photo collages by assembling the salient contents of photos directly onto the partitioned subregions of the given canvas. Our approach provides a novel insight into the photo collage problem from the geometric point of view, and the core is a variational circle packing algorithm for efficient region division. Circle packing determines the optimal region for each image to display. A heuristic search process is developed to ensure that salient information of each photo



Fig. 11. The collage results used in the user study. The order and even columns are the results produced by Autocollage and by our approach separately for a group of images of animals, cocktail glasses, dogs, a girl in white coat, flowers, and a girl in gray coat (from the first pair to the last one).

is displayed as much as possible. Under our framework, optimizations of the states for different photos are decoupled and independent. This endows the collage process with more flexibility and facilitates the realization of several interactive operations on the collage results. Comparisons with commercial software and the user study further verify the effectiveness of our approach.

The current implementation of circle packing easily causes the polygonal regions around the edges of the canvas, especially around the corners, to be much bigger. The main reason is that a corner is often shared by two neighboring circles. This leads to narrow angle area which is difficult to be used. It is a limitation of our algorithm. Our current implementation supports the operations of zooming in a specified photo and adding a new photo into an initial collage through re-executing circle packing. Although the circle packing algorithm runs very fast, this consumes a lot of unnecessary computation and is another limitation of our approach. As the future work, to expedite the response to such operations we intend to investigate more efficient scheme for locally optimizing the circle packing result.

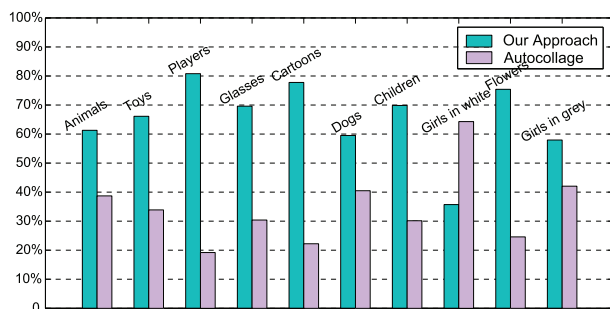


Fig. 12. Statistics of the user study.

Circle packing in fact provides a nonuniform distribution of points representing the centers of given photos with some distance properties. It has been shown in [35] that the blue noise point set with nonuniform density can be generated directly. We will explore the possibility of applying blue noise point set to the creation of photo collage in future. Furthermore, we would like to generalize our framework to the assembly of more general arbitrarily shaped pattern images with the aim of assisting the design and creation of wallpaper and cloth. This is another interesting work for the future.

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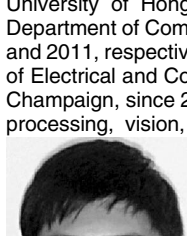
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