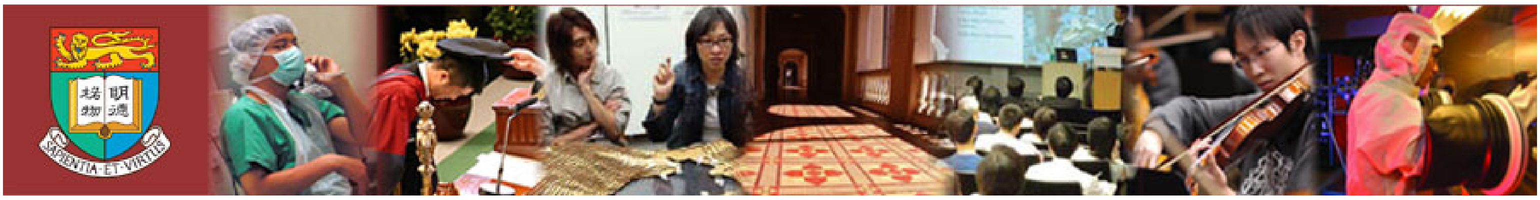

A Calderón Preconditioner for the Electric Field Integral Equation With Layered Medium Green's Function

Yongpin P. Chen, *Member, IEEE*, Sheng Sun, *Senior Member, IEEE*, Lijun Jiang, *Senior Member, IEEE*, and Weng Cho Chew, *Fellow, IEEE*

***Abstract*—A Calderón preconditioner is developed for the analysis of electromagnetic scattering of perfect electrically conducting (PEC) objects embedded in a layered medium. The electric field integral equation (EFIE) is formulated with the kernel of layered medium Green's function to account for the effects from the multilayered background. The Calderón projector is derived based on the general source-field relationship and the extinction theorem for inhomogeneous environment in electromagnetic theory. The Calderón identities can be naturally deduced based on this projector, which is then leveraged to precondition the EFIE with layered kernel. An alternative implementation is then proposed to make the implementation of the preconditioner as efficient as the one in free space. Different numerical examples are designed to show the performance of the preconditioner, where the objects are located in different positions with respect to the layered medium, or different types of excitation are adopted. It is shown that the proposed effective and robust preconditioner makes the EFIE system converge rapidly in all cases, independent of the discretization density.**

***Index Terms*—Calderón preconditioner, Calderón projector, electric field integral equation, layered medium Green's function, method of moments, numerical analysis, surface integral equations.**

I. INTRODUCTION

SURFACE integral equation (SIE) method [1] is widely studied and used in the analysis of electromagnetic radiation and scattering in almost all aspects in electrical engineering where electromagnetic phenomenon is involved. Although the number of unknowns is relatively small in solving the surface integral equation by using the method of moments (MoM) [2], real-world problems are still unaccessible in the bare implementation of the SIE-MoM, such as in the analysis of radiation of large-scale antenna arrays, or the scattering of an aircraft. Fast algorithms for such purpose attracted lots of investigation in the last two decades, and a variety of algorithms [3]–[6] were successfully developed. However, these fast algorithms cannot affect the spectrum properties of the original matrix system. The solution of a particular problem may still fail if the convergence of the iteration is unacceptable, especially in the electric field integral equation (EFIE) for the analysis of perfect electrically conducting (PEC) objects.

Hence, along with the trend of seeking fast integral-equation algorithms, preconditioning techniques also attracted much attention in computational electromagnetic community. Different preconditioners are proposed to improve the spectrum of the matrix system and, hence, improve the convergence in iterative solvers [7]–[9]. Among them, the recently developed Calderón preconditioner turns out to be one of the most promising candidates [9]–[11]. This preconditioner is constructed by leveraging the Calderón identities, which can be naturally deduced from the Calderón projector [12]. The identities state that the square of the EFIE operator (denoted as $\hat{n} \times \mathcal{L}$ or $\mathcal{T}$) equals to $-1/4$ perturbed by a compact MFIE (magnetic field integral equation) operator (denoted as $\hat{n} \times \mathcal{K}$). As we know, the $\hat{n} \times \mathcal{L}$ operator has an undesired spectrum that clusters around the origin and at infinity. Hence, when the operator is discretized, the condition number of the resulting matrix grows rapidly with the discretization density. On the other hand, the $\hat{n} \times \mathcal{K}$ operator, expressed as an ordinary integral after the extraction of the residue term, is compact. Therefore, the spectrum of the $\hat{n} \times \mathcal{L}$ operator can be manipulated to be well behaved by simply operating upon itself again. This self-regularizing property indicates that the $\hat{n} \times \mathcal{L}$ operator is a great preconditioning operator to EFIE, which is also composed of $\hat{n} \times \mathcal{L}$.

Although the good property of the self-regularization is apparently shown in the Calderón identities, the numerical construction of the preconditioner is not trivial, since $\hat{n} \times \mathcal{L}$ has no closed-form solutions and the direct Galerkin discretization of $(\hat{n} \times \mathcal{L})^2$ is impossible. One practical way is to discretize the two $\hat{n} \times \mathcal{L}$ independently and connect them with a Gram matrix. Unfortunately, such discretization strategy in dual finite-element space requires the basis and testing functions satisfying specific mathematical properties [13]. The direct utilization of the standard Rao–Wilton–Glisson (RWG) basis [14] leads to a singular

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Gram matrix [11]. After some initial work such as the splitting of singular term and hypersingular term of $\hat{n} \times \mathcal{L}$ to enforce the vanishing nature of the square of the hypersingular term [10], [15], a purely multiplicative Calderón preconditioner was successfully developed in [9]. This preconditioner succeeds relying on the introduction of a delicately designed Buffa–Christiansen (BC) basis function [16], a subset of the Chen–Wilton (CW) dual basis function [17], in addition to the use of the traditional RWG basis function.

This novel multiplicative Calderón preconditioner then attracted much study and was extended to most aspects in integral equation method. Such an extension includes, but is not limited to, preconditioned combined field integral equation (CFIE) for bounded spectrum and resonant-free solution [18], remedies of low-frequency breakdown or inaccuracy of EFIE [19]–[21], modeling of dielectric objects via Poggio–Miller–Chang–Harrington–Wu–Tsai (PMCHWT) or N-Müller equations [22]–[24], large-scale computation free from spurious internal resonance corruption [25], high-order basis functions [26], etc. Besides time-harmonic problems, transient problems were also studied in [13], where the time-domain Calderón identities were derived and applied to precondition the time-domain (TD) EFIE.

However, the work mentioned above only considers the radiation or scattering in an unbounded homogeneous background, where no scattering effects from the inhomogeneous background is considered and, hence, the integration kernel is relatively simple. There are other applications where the wave phenomenon occurs in an inhomogeneous environment. One of the most useful models of such inhomogeneity is the planarly layered medium [27], [28], and the application includes microstrip antennas and microwave circuits, geophysical exploration, and remote sensing. However, the extensibility of this novel preconditioner to the EFIE with a layered medium Green's function is still in question.

In this paper, we investigate the Calderón preconditioner for the EFIE analysis with a layered medium Green's function. The rest of the paper is organized as follows. Section II first reviews the general formulations of the integral equations with a layered medium Green's function. The Calderón projector and the corresponding Calderón identities are derived in general where scattering effects from the inhomogeneous background are involved in Section III. After that, in Section IV, the implementation of the Calderón preconditioner in a layered medium is introduced, and an alternative implementation is suggested to make the construction of the preconditioner as efficient as the one in free space. Finally, in Section V, several numerical results are presented to validate the great performance of the preconditioner, regardless of the discretization density, the types of excitations, or the relative position of the objects in the layered medium.

II. General Formulation of Integral Equations With Layered Medium Green's Function

Consider that a PEC object with a closed boundary Γ is located in the background of a layered medium. The object can be in the outermost layer, embedded inside one internal layer,

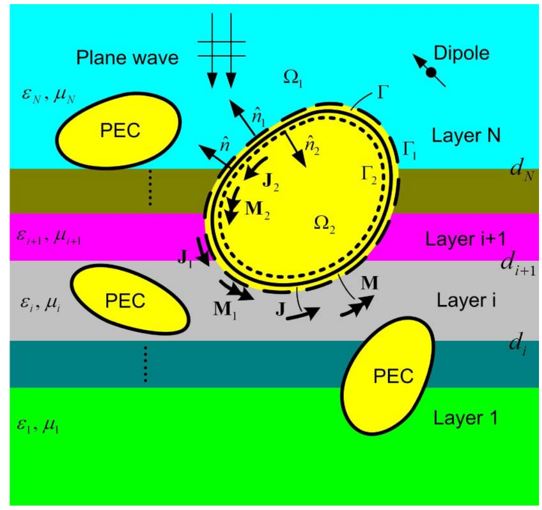

Fig. 1. A PEC object is located in the background of a layered medium. It can be in the outermost layer, embedded inside an internal layer, or straddling different layers. The quantities in the extinction theorem are also shown.

or straddling different layers, as is shown in Fig. 1. The material of each layer is denoted as the relative permittivity ϵ_i and relative permeability μ_i where losses have been accounted for. The outward pointing unit normal vector of Γ is denoted as $\hat{n}$. This object is illuminated by an incident plane wave, or excited by a Hertzian dipole. Surface current $\mathbf{J}$ is then induced to counteract this electromagnetic disturbance. Such current can be determined via the EFIE by enforcing the boundary condition on Γ, namely the tangential component of the total field vanishes:

$$0 = \hat{n} \times \mathcal{L}_{\mathrm{E}}(\mathbf{J}) + \hat{n} \times \mathbf{E}^i \tag{1}$$

where the total field is the summation of the field generated by $\mathbf{J}$ (scattered field) and the incident field $\mathbf{E}^i$. The integration operator $\mathcal{L}_{\mathrm{E}}$ maps the electric current $\mathbf{J}$ at source position $\mathbf{r}'$ to the electric field $\mathbf{E}$ at observation point $\mathbf{r}$ via the Green's function [1], [28]:

$$\mathcal{L}_{\mathrm{E}}(\mathbf{J}) = i\omega \int d\mathbf{r}' \bar{\mathbf{G}}_e(\mathbf{r}, \mathbf{r}')\mu(\mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') \tag{2}$$

where $\bar{\mathbf{G}}_e(\mathbf{r}, \mathbf{r}')$ is the e-type dyadic Green's function in a layered medium, which can be expressed as

$$\begin{aligned}\bar{\mathbf{G}}_e(\mathbf{r}, \mathbf{r}') &= (\nabla \times \hat{z})(\nabla' \times \hat{z})g^{\mathrm{TE}}(\mathbf{r}, \mathbf{r}') \\ &\quad + \frac{1}{k_{nm}^2}(\nabla \times \nabla \times \hat{z})(\nabla' \times \nabla' \times \hat{z})\, g^{\mathrm{TM}}(\mathbf{r}, \mathbf{r}')\end{aligned} \tag{3}$$

where $k_{nm}^2 = \omega^2 \epsilon_n \mu_m$ (the subscripts m, n are the indices of source and observation layers). The scalar function $g^{\mathrm{TE/TM}}$ is composed of a Sommerfeld integral [29]

$$g^{\mathrm{TE/TM}}(\mathbf{r}, \mathbf{r}') = \frac{i}{4\pi} \int_0^{+\infty} \frac{dk_\rho}{k_{mz} k_\rho} J_0(k_\rho \rho) F^{\mathrm{TE/TM}}(k_\rho, z, z') \tag{4}$$

where $\mathbf{r} = \rho\hat{\rho} + z\hat{z}$ is the position vector in the cylindrical coordinate system, $k_m^2 = k_\rho^2 + k_{mz}^2$ is the dispersion relation in the source layer, and $J_0(k_\rho \rho)$ is the Bessel function of order 0. The propagation factor $F^{\mathrm{TE/TM}}(k_\rho, z, z')$ characterizes the propagation (reflection and transmission) of a plane wave with TE and TM modes in a layered medium. The expression depends on the relative position of the source point and observation point [29]–[31]. When they are in the same layer, namely $m = n$, we have

$$F(k_\rho, z, z') = F^{\mathrm{P}}(k_\rho, z, z') + F^{\mathrm{S}}(k_\rho, z, z') \tag{5}$$

with

$$F^{\mathrm{P}}(k_\rho, z, z') = e^{ik_{mz}|z-z'|} \tag{6}$$

$$\begin{aligned} F^{\mathrm{S}}&(k_\rho, z, z') \\ &= \tilde{M}_m \tilde{R}_{m,m-1} e^{ik_{mz}(-2d_m+z'+z)} \\ &\quad + \tilde{M}_m \tilde{R}_{m,m+1} e^{ik_{mz}(2d_{m+1}-z'-z)} \\ &\quad + \tilde{M}_m \tilde{R}_{m,m+1} \tilde{R}_{m,m-1} e^{ik_{mz}(2d_{m+1}-2d_m+z'-z)} \\ &\quad + \tilde{M}_m \tilde{R}_{m,m-1} \tilde{R}_{m,m+1} e^{ik_{mz}(2d_{m+1}-2d_m-z'+z)} \end{aligned} \tag{7}$$

where $\tilde{R}_{i,i\pm1}$ is the generalized reflection coefficients recursively defined based on the Fresnel reflection coefficients $R_{i,i\pm1}$ to account for the multiple reflections and transmissions in a multilayered medium [29], and

$$\tilde{M}_m = \left[1 - \tilde{R}_{m,m-1}\tilde{R}_{m,m+1} e^{2ik_{mz}(d_{m+1}-d_m)}\right]^{-1}. \tag{8}$$

In (6), F^{P} represents the primary interaction at the absence of the inhomogeneous background, which when substituted back into (3), has the simple form of

$$\bar{\mathbf{G}}_e(\mathbf{r}, \mathbf{r}') = \left(\bar{\mathbf{I}} + \frac{\nabla\nabla}{k_m^2}\right) g(\mathbf{r}, \mathbf{r}'). \tag{9}$$

It is actually the spatial dyadic Green's function in an unbounded homogeneous medium (or free space), where the scalar Green's function is $g(\mathbf{r}, \mathbf{r}') = e^{ik_m|\mathbf{r}-\mathbf{r}'|}/4\pi|\mathbf{r}-\mathbf{r}'|$. The secondary term F^{S} in (7) contains four subterms accounting for the scattering effects from the inhomogeneous layered medium. Particularly, if the source and observation points are in the top layer (outermost layer), $\tilde{R}_{i,i+1} = 0$, hence only one subterm in (7) is left, which shows that the scattering comes only from the lower layers. In general, the Sommerfeld integrals for the secondary term cannot be cast into a closed form, which is different from the primary term.

A direct Galerkin discretization scheme with the standard RWG basis function to the EFIE in (1) usually leads to an ill-conditioned system, as was introduced in the previous section. Conventionally, the CFIE is suggested in radar applications in a layered medium [32], [33], as is also normally done in free space. However, as has been mentioned in [18], the CFIE inherits the unbounded nature of the spectrum of EFIE and may deteriorate or fail when an extremely dense mesh is required in certain multiscale problems.

In the following sections, we will investigate the possible extensibility of the Calderón preconditioner to the layered medium problems.

III. Calderón Projector and Calderón Identities in Layered Medium

The Calderón projector and the Calderón identities in an inhomogeneous medium can be derived from the general source-field relationship and the extinction theorem [28], [29], following that of [12]. Due to the much complex kernels involved in the layered medium, the notations here are slightly different from those in the literature for free-space case [9], [12], in order to keep the physical meanings of each operation clear.

The general source-field relationship forming the basis of integral equations reads

$$\mathbf{E} = \mathcal{L}_{\mathrm{E}}(\mathbf{J}) + \mathcal{K}_{\mathrm{E}}(\mathbf{M}) \tag{10}$$

$$\mathbf{H} = \mathcal{L}_{\mathrm{H}}(\mathbf{M}) + \mathcal{K}_{\mathrm{H}}(\mathbf{J}). \tag{11}$$

In free space, we have [1], [34]

$$\mathcal{L}_{\mathrm{E}}^{\mathrm{free}} = \mathcal{L} \tag{12}$$

$$\mathcal{K}_{\mathrm{E}}^{\mathrm{free}} = \mathcal{K} \pm \frac{1}{2}\hat{n}\times \tag{13}$$

$$\mathcal{L}_{\mathrm{H}}^{\mathrm{free}} = \frac{1}{\eta^2}\mathcal{L}_{\mathrm{E}}^{\mathrm{free}} \tag{14}$$

$$\mathcal{K}_{\mathrm{H}}^{\mathrm{free}} = -\mathcal{K}_{\mathrm{E}}^{\mathrm{free}} \tag{15}$$

where

$$\mathcal{L}(\mathbf{X}) = \int d\mathbf{r}' \left[i\omega\mu\mathbf{X}(\mathbf{r}') - \frac{1}{i\omega\epsilon}\nabla\nabla'\cdot\mathbf{X}(\mathbf{r}')\right]\cdot g(\mathbf{r}, \mathbf{r}') \tag{16}$$

$$\mathcal{K}(\mathbf{X}) = \mathrm{P.V.}\int d\mathbf{r}'\mathbf{X}(\mathbf{r}')\times\nabla g(\mathbf{r}, \mathbf{r}'). \tag{17}$$

In (17), P.V. stands for the Cauchy principal value integration, and $\mathbf{X}$ is either the electric current $\mathbf{J}$ or magnetic current $\mathbf{M}$. The "$-$" in (13) corresponds to the case when the observation point $\mathbf{r}$ approaches the boundary along the direction of outward pointing unit normal vector and "+" from the opposite direction. In (14), $\eta = \sqrt{\mu_0/\epsilon_0} = 120\pi$ is the wave impedance in free space. The Calderón identities can be derived following the procedure in [12]:

$$\frac{1}{\eta^2}(\hat{n}\times\mathcal{L})^2 = (\hat{n}\times\mathcal{K})^2 - \frac{1}{4} \tag{18}$$

$$(\hat{n}\times\mathcal{K})(\hat{n}\times\mathcal{L}) + (\hat{n}\times\mathcal{L})(\hat{n}\times\mathcal{K}) = 0. \tag{19}$$

For the layered medium, three other integral operators are needed in addition to $\mathcal{L}_{\mathrm{E}}$ in (2):

$$\mathcal{K}_{\mathrm{H}}\cdot = \mu^{-1}(\mathbf{r})\int d\mathbf{r}'\nabla\times\bar{\mathbf{G}}_e(\mathbf{r}, \mathbf{r}')\mu(\mathbf{r}')\cdot \tag{20}$$

$$\mathcal{L}_{\mathrm{H}}\cdot = i\omega\int d\mathbf{r}'\bar{\mathbf{G}}_m(\mathbf{r}, \mathbf{r}')\epsilon(\mathbf{r}')\cdot \tag{21}$$

$$\mathcal{K}_{\mathrm{E}}\cdot = -\epsilon^{-1}(\mathbf{r})\int d\mathbf{r}'\nabla\times\bar{\mathbf{G}}_m(\mathbf{r}, \mathbf{r}')\epsilon(\mathbf{r}')\cdot \tag{22}$$

where the m-type Green's function in (21) and (22) is

$$\begin{aligned} \bar{\mathbf{G}}_m(\mathbf{r}, \mathbf{r}') &= (\nabla\times\hat{z})(\nabla'\times\hat{z})g^{\mathrm{TM}}(\mathbf{r}, \mathbf{r}') \\ &\quad + \frac{1}{k_{mn}^2}(\nabla\times\nabla\times\hat{z})(\nabla'\times\nabla'\times\hat{z})g^{\mathrm{TE}}(\mathbf{r}, \mathbf{r}') \end{aligned} \tag{23}$$

with $k_{mn}^2 = \omega^2 \epsilon_m \mu_n$. The detailed expressions of the kernels can be found in [28] and will not be repeated here. As is shown in (3) and (23), the e-type Green's function and the m-type counterpart are dual with each other in a general layered medium. Therefore, the operators with m-type and e-type kernels have no simple direct relations as in (14) and (15). The relation can only be obtained implicitly by invoking the duality principle [28].

For simplicity, we first assume that the source and observation points are in the same layer $(m = n)$. In this case, we can always separate the integration kernel as well as the operator into two parts: the primary term and the secondary term, as is also shown in (5)

$$\bar{\mathbf{G}}_{e,m} = \bar{\mathbf{G}}_{e,m}^{\mathrm{P}} + \bar{\mathbf{G}}_{e,m}^{\mathrm{S}} \tag{24}$$

$$\{\mathcal{L}, \mathcal{K}\}_{\mathrm{E,H}} = \{\mathcal{L}, \mathcal{K}\}_{\mathrm{E,H}}^{\mathrm{P}} + \{\mathcal{L}, \mathcal{K}\}_{\mathrm{E,H}}^{\mathrm{S}} \tag{25}$$

where $\{\mathcal{L}, \mathcal{K}\}_{\mathrm{E,H}}^{\mathrm{P}}$ are essentially those in free space, as are shown in (12)–(15).

Consider an object filled with material ϵ_2 and μ_2 immersed in a layered medium, as is shown in Fig. 1. It has a closed boundary Γ, and the outward pointing unit normal vector of Γ is denoted as $\hat{n}$. Two mathematical surfaces Γ_1 and Γ_2 are defined with an infinitesimal distance from the real surface Γ in region Ω_1 and Ω_2, respectively. The corresponding "outward pointing" unit normal vectors $\hat{n}_1$ and $\hat{n}_2$ are defined on Γ_1 and Γ_2. Apparently, we have $\hat{n} = \hat{n}_1 = -\hat{n}_2$. Two sets of electromagnetic sources $[\mathbf{J}_1, \mathbf{M}_1]$, $[\mathbf{J}_2, \mathbf{M}_2]$ are also defined on Γ_1 and Γ_2, and similar relations of $\mathbf{J} = \mathbf{J}_1 = -\mathbf{J}_2$, $\mathbf{M} = \mathbf{M}_1 = -\mathbf{M}_2$ hold. In order to activate the layered medium Green's function, we invoke the exterior part of the extinction theorem and assume that no excitation exists in region Ω_1. If we put the source point on Γ_1 and the observation point on Γ_2, namely, $\mathbf{r} \in \Gamma_2$, $\mathbf{r}' \in \Gamma_1$, the extinction theorem reads [29]

$$\begin{aligned} 0 &= \hat{n}_1 \times \mathcal{L}_{\mathrm{E}}(\mathbf{J}_1) + \hat{n}_1 \times \mathcal{K}_{\mathrm{E}}(\mathbf{M}_1) \\ &= \hat{n}_1 \times \mathcal{L}_{\mathrm{E}}(\mathbf{J}_1) + \hat{n}_1 \times (\mathcal{K}_{\mathrm{E}}^{\mathrm{P}} + \mathcal{K}_{\mathrm{E}}^{\mathrm{S}})(\mathbf{M}_1) \\ &= \hat{n}_1 \times \mathcal{L}_{\mathrm{E}}(\mathbf{J}_1) + \hat{n}_1 \times (\mathcal{K} - \frac{1}{2}\hat{n}_1 \times + \mathcal{K}_{\mathrm{E}}^{\mathrm{S}})(\mathbf{M}_1) \\ &= \hat{n} \times \mathcal{L}_{\mathrm{E}}(\mathbf{J}) + \hat{n} \times \mathcal{K}_{\mathrm{E}}(\mathbf{M}) + \frac{1}{2}\mathbf{M}. \end{aligned} \tag{26}$$

Here, $\mathcal{K}_{\mathrm{E}}$ in the last line does not include the residue term anymore and the subscript has been skipped. Consequently

$$\mathbf{M} = -\hat{n} \times \mathcal{L}_{\mathrm{E}}(\mathbf{J}) - \hat{n} \times \mathcal{K}_{\mathrm{E}}(\mathbf{M}) + \frac{1}{2}\mathbf{M}. \tag{27}$$

Similarly, for the $\mathbf{H}$ field

$$\begin{aligned} 0 &= \hat{n}_1 \times \mathcal{L}_{\mathrm{H}}(\mathbf{M}_1) + \hat{n}_1 \times \mathcal{K}_{\mathrm{H}}(\mathbf{J}_1) \\ &= \hat{n}_1 \times \mathcal{L}_{\mathrm{H}}(\mathbf{M}_1) + \hat{n}_1 \times (-\mathcal{K}_{\mathrm{E}}^{\mathrm{P}} + \mathcal{K}_{\mathrm{H}}^{\mathrm{S}})(\mathbf{J}_1) \\ &= \hat{n}_1 \times \mathcal{L}_{\mathrm{H}}(\mathbf{M}_1) + \hat{n}_1 \times \left[-(\mathcal{K} - \frac{1}{2}\hat{n}_1 \times) + \mathcal{K}_{\mathrm{H}}^{\mathrm{S}}\right](\mathbf{J}_1) \\ &= \hat{n} \times \mathcal{L}_{\mathrm{H}}(\mathbf{M}) + \hat{n} \times \mathcal{K}_{\mathrm{H}}(\mathbf{J}) - \frac{1}{2}\mathbf{J}. \end{aligned} \tag{28}$$

Hence,

$$\mathbf{J} = \hat{n} \times \mathcal{L}_{\mathrm{H}}(\mathbf{M}) + \hat{n} \times \mathcal{K}_{\mathrm{H}}(\mathbf{J}) + \frac{1}{2}\mathbf{J}. \tag{29}$$

Again, the residue term is excluded from $\mathcal{K}_{\mathrm{H}}$. In matrix notation, (27) and (29) become

$$\begin{bmatrix} \hat{n} \times \mathcal{K}_{\mathrm{H}} + \frac{1}{2} & \hat{n} \times \mathcal{L}_{\mathrm{H}} \\ -\hat{n} \times \mathcal{L}_{\mathrm{E}} & -\hat{n} \times \mathcal{K}_{\mathrm{E}} + \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{J} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{J} \\ \mathbf{M} \end{bmatrix}. \tag{30}$$

The operator in (30) is the Calderón projector, which maps the vector $[\mathbf{J}, \mathbf{M}]$ into itself. The following identities can then be derived based on the procedure in [12]:

$$(\hat{n} \times \mathcal{L}_{\mathrm{H}})(\hat{n} \times \mathcal{L}_{\mathrm{E}}) = (\hat{n} \times \mathcal{K}_{\mathrm{H}})^2 - \frac{1}{4} \tag{31}$$

$$(\hat{n} \times \mathcal{K}_{\mathrm{H}})(\hat{n} \times \mathcal{L}_{\mathrm{H}}) - (\hat{n} \times \mathcal{L}_{\mathrm{H}})(\hat{n} \times \mathcal{K}_{\mathrm{E}}) = 0 \tag{32}$$

$$-(\hat{n} \times \mathcal{L}_{\mathrm{E}})(\hat{n} \times \mathcal{K}_{\mathrm{H}}) + (\hat{n} \times \mathcal{K}_{\mathrm{E}})(\hat{n} \times \mathcal{L}_{\mathrm{E}}) = 0 \tag{33}$$

$$(\hat{n} \times \mathcal{L}_{\mathrm{E}})(\hat{n} \times \mathcal{L}_{\mathrm{H}}) = (\hat{n} \times \mathcal{K}_{\mathrm{E}})^2 - \frac{1}{4}. \tag{34}$$

It is shown in (31) and (34) that $\hat{n} \times \mathcal{L}_{\mathrm{E}}$ and $\hat{n} \times \mathcal{L}_{\mathrm{H}}$ in inhomogeneous medium are not commutable, which is different from the one in free space.

These identities can be derived due to the fact that the operators can be naturally decomposed into the primary term and the secondary term, shown in (24) and (25). Physically, the primary term corresponds to the direct interaction between the source and the field, while the secondary term describes the scattering caused by the inhomogeneous background. Apparently, the latter is a secondary contribution to the field and is usually weaker than the former. Hence, the secondary term can be viewed as a perturbation to the primary term. From a mathematical point of view, the primary term usually contains strong singularities. As the discretization density increases, the singularity becomes more severe. The eigenvalues of the two branches of the singular and hypersingular terms in discretized $\hat{n} \times \mathcal{L}^{\mathrm{P}}$ deviates more, leading to a larger condition number. This operator is unbounded and also ill-posed. For $\hat{n} \times \mathcal{K}^{\mathrm{P}}$, however, due to the extraction of the residue term, the remaining term involving ordinary integral is compact. On the other hand, no similar singularity issues occur in the weaker secondary term since the field has to propagate to the inhomogeneous boundary and then reflect back. This operator is supposed to be compact. Therefore, (31) can be utilized to construct a Calderón preconditioner for the EFIE in a layered medium.

The Calderón identities in (31)–(34) are true for object confined in one single layer (which is the most common situation in layered medium applications) where $m = n$ always holds. For object-straddling different layers, we have $m \neq n$ in certain source–field interactions. In this situation, the identities are not naturally derivable. However, it can be shown that the spectrum of $\hat{n} \times \mathcal{L}_{\mathrm{E}}$ can also be improved by $\hat{n} \times \mathcal{L}_{\mathrm{H}}$ for general straddling cases. Consider that an object straddles N_s layers in an N-layered medium. The operator (discretized) can always be divided into an $N_s \times N_s$ block system. It can be easily found that the blocks of the $m = n$ case are always the diagonal blocks, and those of $m \neq n$ are off-diagonal. For $m \neq n$, only the secondary term exists. It corresponds to the transmitted field in different layers, and usually falls into the category of well-separated interactions. These weak interactions have no similar singularity issues either. Hence, a "dual-squared" system of $(\hat{n}\times\mathcal{L}_{\rm H})(\hat{n}\times\mathcal{L}_{\rm E})$ leads to a block system with its diagonal blocks filled with $-1/4$ perturbed by compact operators, and off-diagonal blocks filled with compact operators. Such a system is well behaved. Therefore, $\hat{n}\times\mathcal{L}_{\rm H}$ is still a good preconditioning operator for straddling objects. Note that this argument is very similar to the case in the preconditioned system for dielectric objects in free space, where 2×2 systems are always set up [22].

IV. Calderón Preconditioner in Layered Medium

The Calderón identity in (31) can be applied to construct a Calderón preconditioner to the EFIE in (1). The preconditioned system is formulated as

$$\hat{n}\times\mathcal{L}_{\rm H}\left[\hat{n}\times\mathcal{L}_{\rm E}(\mathbf{J})\right] = -\hat{n}\times\mathcal{L}_{\rm H}(\hat{n}\times\mathbf{E}^i). \tag{35}$$

The inner $\hat{n}\times\mathcal{L}_{\rm E}$ is discretized by using the div-conforming RWG basis functions $\mathbf{f}_{\rm RWG}$ as the expansion function in the domain space, and the curl-conforming RWGs $\hat{n}\times\mathbf{f}_{\rm RWG}$ as the testing function in the range space; while the outer $\hat{n}\times\mathcal{L}_{\rm H}$ is discretized by the div- and quasi-curl-conforming BC basis functions $\mathbf{f}_{\rm BC}$ and the curl- and quasi-div-conforming BCs $\hat{n}\times\mathbf{f}_{\rm BC}$ for expansion and testing, respectively. After discretization, the matrix system is

$$\left[(\hat{n}\times\mathcal{L}_{\rm H})(\hat{n}\times\mathcal{L}_{\rm E})\right]_{\rm dis} = \bar{\mathbf{Z}}^{\rm H}_{\rm BC}\bar{\mathbf{G}}_m^{-1}\bar{\mathbf{Z}}^{\rm E}_{\rm RWG}. \tag{36}$$

As a consequence of this delicate discretization scheme, the Gram matrix linking the domain of $\hat{n}\times\mathcal{L}_{\rm H}$ and the range of $\hat{n}\times\mathcal{L}_{\rm E}$ can be made well conditioned

$$\left[\bar{\mathbf{G}}_m\right]_{ji} = \langle n\times\mathbf{f}_{{\rm RWG}j}, \mathbf{f}_{{\rm BC}i}\rangle. \tag{37}$$

The impedance matrices in (36) are

$$\left[\bar{\mathbf{Z}}^{\rm E}_{\rm RWG}\right]_{ji} = \langle n\times\mathbf{f}_{{\rm RWG}j}, \hat{n}\times\mathcal{L}_{\rm E}, \mathbf{f}_{{\rm RWG}i}\rangle \tag{38}$$

$$\left[\bar{\mathbf{Z}}^{\rm H}_{\rm BC}\right]_{ji} = \langle n\times\mathbf{f}_{{\rm BC}j}, \hat{n}\times\mathcal{L}_{\rm H}, \mathbf{f}_{{\rm BC}i}\rangle. \tag{39}$$

Finally, the preconditioned EFIE system takes the form of

$$\bar{\mathbf{Z}}^{\rm H}_{\rm BC}\bar{\mathbf{G}}_m^{-1}\bar{\mathbf{Z}}^{\rm E}_{\rm RWG}\mathbf{I}_{\rm RWG} = \bar{\mathbf{Z}}^{\rm H}_{\rm BC}\bar{\mathbf{G}}_m^{-1}\mathbf{V}_{\rm RWG} \tag{40}$$

where the excitation $\mathbf{V}_{\rm RWG}$ is obtained by testing the incident field $\hat{n}\times\mathbf{E}^i$ with $\hat{n}\times\mathbf{f}_{\rm RWG}$. To define the BC basis function, a barycentric mesh is required in addition to the original mesh [16]. As the BCs as well as the RWGs in the original mesh can be expressed as a linear superposition of the RWGs in the barycentric mesh, the impedance matrices $\bar{\mathbf{Z}}^{\rm H}_{\rm BC}$ and $\bar{\mathbf{Z}}^{\rm E}_{\rm RWG}$ can be obtained by the impedance matrices assembled with RWGs in the barycentric mesh. Such operations can be achieved via the linear transformation matrices $\bar{\mathbf{P}}$ and $\bar{\mathbf{R}}$[9]. For the same reason, the Gram matrix can also be obtained from RWGs in the barycentric mesh. Detailed information can be found in [9].

Compared with the Calderón preconditioner in free space, it is observed that another full impedance matrix involving $\hat{n}\times\mathcal{L}_{\rm H}$ is required in layered medium, as is shown in (40). This may lead to extra complexity and computational cost. To make the construction of the preconditioner as convenient as in free space, one can heuristically apply $\hat{n}\times\mathcal{L}_{\rm E}$ as the preconditioning operator

$$\left[\hat{n}\times\mathcal{L}_{\rm E}(\mathbf{J})\right]^2 = -\hat{n}\times\mathcal{L}_{\rm E}(\hat{n}\times\mathbf{E}^i). \tag{41}$$

Although not guaranteed by the Calderón identities, this preconditioned system may still succeed due to the dominant primary term, where

$$\mathcal{L}^{\rm P}_{\rm E} = \eta_m^2\mathcal{L}^{\rm P}_{\rm H}. \tag{42}$$

This is essentially the case in (14), which can be deduced from (2) and (21), since $\bar{\mathbf{G}}^{\rm P}_e = \bar{\mathbf{G}}^{\rm P}_m$ [28]. The secondary term, according to our previous argument, is a perturbation to the dominant primary contribution. Hence, although $\bar{\mathbf{G}}^{\rm S}_e \neq \bar{\mathbf{G}}^{\rm S}_m$, we may weakly have the following approximate relation for "preconditioning purpose":

$$\mathcal{L}_{\rm E} \simeq \eta_m^2\mathcal{L}_{\rm H}. \tag{43}$$

Based on this argument, it is reasonable to utilize $\hat{n}\times\mathcal{L}_{\rm E}$ as an alternative preconditioning operator in the layered medium. In this case, the preconditioned system can easily be obtained based on the existing EFIE code, without the necessity to invoke the extra m-type Green's function.

The preconditioned system in (41) is exactly the same as the one in free-space problems [9]. Although the use of $\hat{n}\times\mathcal{L}_{\rm E}$ is justified rigorously from the Calderón identities in free space, it is only a heuristic suggestion with approximations in layered medium. However, numerical tests show that this alternative choice works very well in different situations.

V. Numerical Results

Several numerical results are presented to validate the effectiveness of the Calderón preconditioner in this section. The generalized minimal residual (GMRes) algorithm [35] is adopted as the iterative solver. The targeted relative residual error is set to be 10^{-6} in all cases. Both preconditioned systems in (35) and (41) are investigated.

A. Sphere at Top Layer of a Three-Layer Medium

First, the scattering of a PEC sphere with $r = 1$ m situated at the top layer of a three-layer medium is analyzed. The configuration as well as the material parameters of the layered medium are shown in Fig. 2. The sphere is illuminated by a y-polarized plane wave of $f = 150$ MHz with normal incidence. Fig. 3 shows the number of iterations required to achieve the targeted relative residual error, with respect to the discretization density (wavelength λ /mesh size δ). It is shown that the number of iterations in EFIE increases rapidly as the mesh becomes denser. However, the Calderón preconditioned EFIE has a very low and stable number of iterations. It is also shown that the alternative implementation in (41) (denoted as $\mathcal{L}_{\rm E}$ preconditioner) performs almost the same as the original one in (35) (denoted as $\mathcal{L}_{\rm H}$ preconditioner) in this case. For instance, at the finest mesh ($\lambda/\delta = 16$), the number of iterations is both 14 in the two preconditioned EFIEs, while the number is 555 in the EFIE without preconditioner.

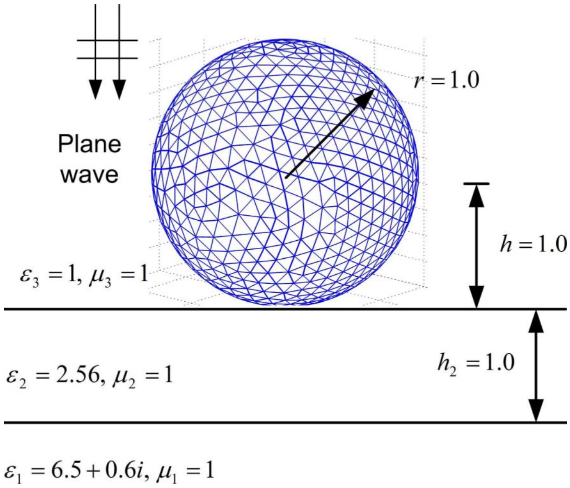

Fig. 2. A PEC sphere located above a three-layer medium is excited by a plane wave of $f = 150$ MHz, where $r = 1$, $h_2 = 1$ (unit: m). The material parameters of each layer are shown in the figure.

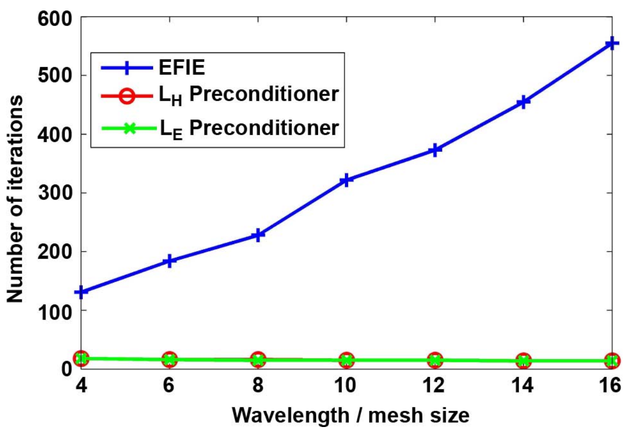

Fig. 3. Analysis of scattering from the sphere. Number of iterations required in GMRes to achieve a relative residual error of 10^{-6} versus discretization density (λ/δ).

B. Cuboid Confined in the Middle Layer of a 3-Layer Medium

Next, a cuboid with $a = b = 2$ m, $c = 1$ m is totally confined in the middle layer of a three-layer medium, shown in Fig. 4. It is illuminated by a y-polarized plane wave of $f = 75$ MHz with normal incidence. The number of iterations to achieve the targeted relative residual error versus discretization density is shown in Fig. 5. Again, the preconditioned EFIEs converge rapidly to the targeted error and are independent with respect to the discretization density. When $\lambda/\delta = 14$, the number of iterations in $\mathcal{L}_{\rm H}$ preconditioner is 22, in $\mathcal{L}_{\rm E}$ preconditioner is 25, and the number in EFIE is 1759.

C. Multi-Scale Composite Cylinder Structure Confined in One Internal Layer of a Four-Layer Medium

In the following, a multiscale composite cylinder structure is confined in one internal layer of a four-layer medium, with the parameters shown in Fig. 6. The radius and the height of the bottom cylinder are $r_1 = 0.5$ m and $d_1 = 0.2$ m, while the numbers of the top cylinder are $r_2 = 0.05$ m and $d_2 = 0.3$ m. To test the performance of the preconditioner for different types of excitation, the structure is excited by a Hertzian dipole located at $(x = 0.3, y = 0.1, z = 0.6)$ m, with polarization of

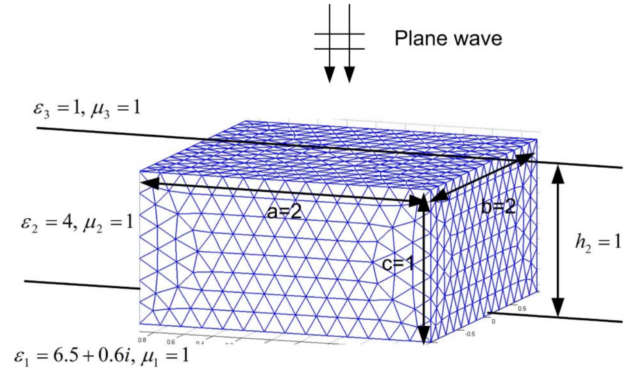

Fig. 4. A PEC cuboid is totally confined in the middle layer of a three-layer medium, where $a = b = 2$, $c = 1$ (unit: m). It is excited by a plane wave of $f = 75$ MHz. The material parameters of each layer are shown in the figure.

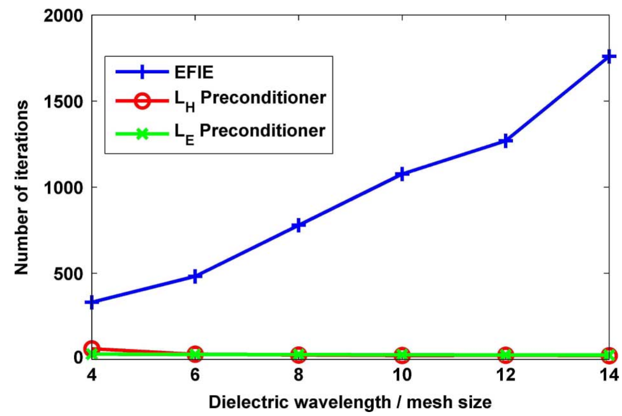

Fig. 5. Analysis of scattering from the cuboid. Number of iterations required in GMRes to achieve a relative residual error of 10^{-6} versus discretization density (λ/δ).

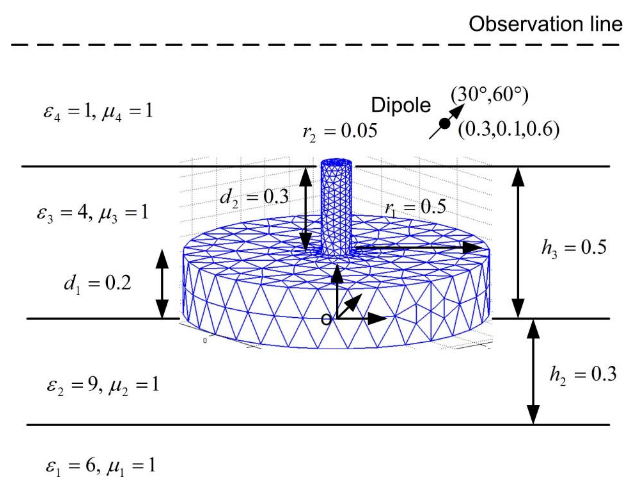

Fig. 6. Multiscale composite cylinder structure is totally confined in one middle layer of a four-layer medium, where $r_1 = 0.5$, $d_1 = 0.2$, $r_2 = 0.05$, $d_2 = 0.3$ (unit: m). It is excited by a Hertzian dipole with $f = 300$ MHz. The material parameters of each layer are shown in the figure.

$(\theta = 30^\circ, \phi = 60^\circ)$. The working frequency is $f = 300$ MHz. The convergence history is shown in Fig. 7, where the preconditioned EFIEs converge quickly. However, the convergence rate of the EFIE is extremely low. The number of iterations in $\mathcal{L}_{\rm H}$ preconditioner is 112, in $\mathcal{L}_{\rm E}$ preconditioner is 115, and the number in EFIE is 15 483. To show the accuracy of our method,

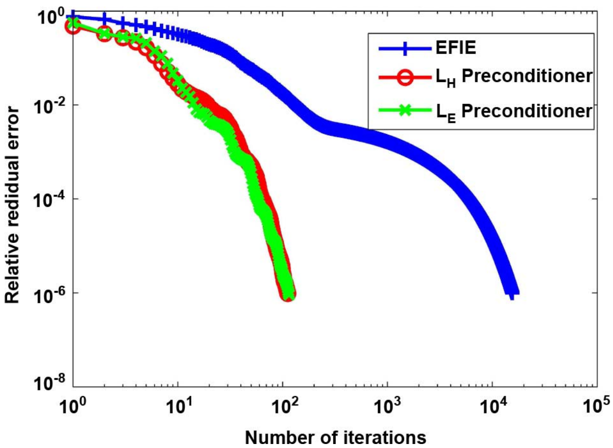

Fig. 7. Analysis of scattering from the composite cylinder structure. Iteration history recorded in GMRes to achieve a relative residual error of 10^{-6}.

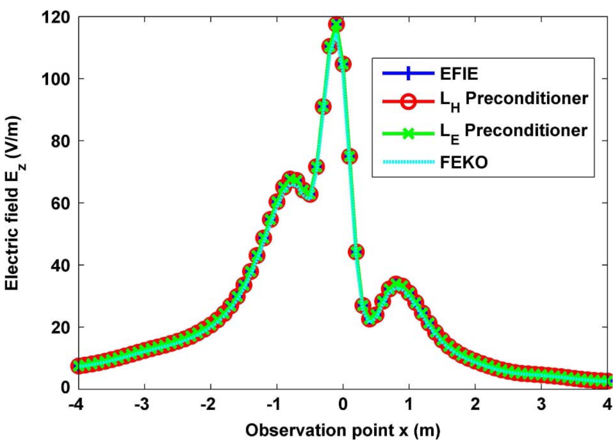

Fig. 8. Analysis of scattering from the composite cylinder structure. Scattered near field distribution validated by FEKO.

the scattered near field is further collected along the observation line ($-4 \leq x \leq 4, y = 0, z = 0.8$) m. It can be seen that the results agree well with the reference data calculated by the commercial software FEKO, as is shown in Fig. 8.

D. Block Straddling Different Layers

The last example is designed to show the effectiveness of the preconditioner for straddling objects. The dimension of the block is $a = b = 0.5$, $c = 1$ m. It is penetrating the three-layer medium, shown in Fig. 9. The structure is excited by a dipole with $f = 150$ MHz located either at $(x = 0.5, y = 0.2, z = 0.4)$ (denoted as dipole 1 in the top layer) or at $(x = 0.5, y = 0.2, z = 0.0)$ (denoted as dipole 2 in the middle layer), with polarization of $(\theta = 20^\circ, \phi = 30^\circ)$. It is observed from Fig. 10 that the preconditioners also work well for this case. For dipole 1, the number of iterations in $\mathcal{L}_\mathrm{H}$ preconditioner is 13, in $\mathcal{L}_\mathrm{E}$ preconditioner is 20, while the number in EFIE is 298. The performance for dipole 2 case is similar: 12 in $\mathcal{L}_\mathrm{H}$ preconditioner, 22 in $\mathcal{L}_\mathrm{E}$ preconditioner, and 355 in EFIE. Although in both cases, $\mathcal{L}_\mathrm{E}$ deteriorates slightly compared to $\mathcal{L}_\mathrm{H}$ in this situation, the difference is trivial if compared with the number of iterations of the original system. To show the accuracy, the scattered near field is also collected along the observation line

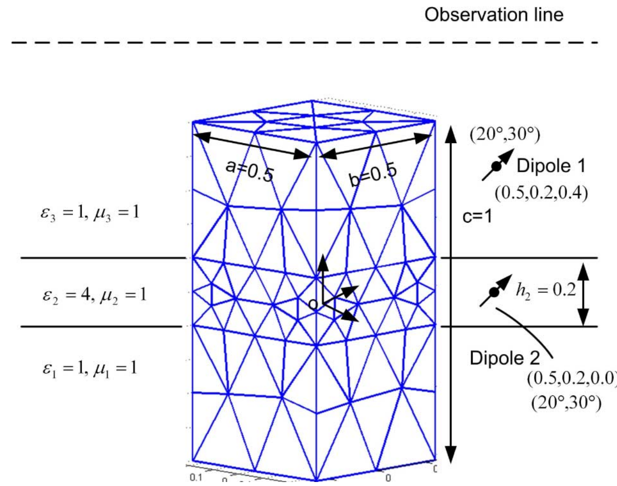

Fig. 9. A block with $a = b = 0.5$, $c = 1$ (unit: m) is straddling different layers of a three-layer medium. It is excited by a Hertzian dipole with $f = 150$ MHz. The material parameters of each layer are shown in the figure.

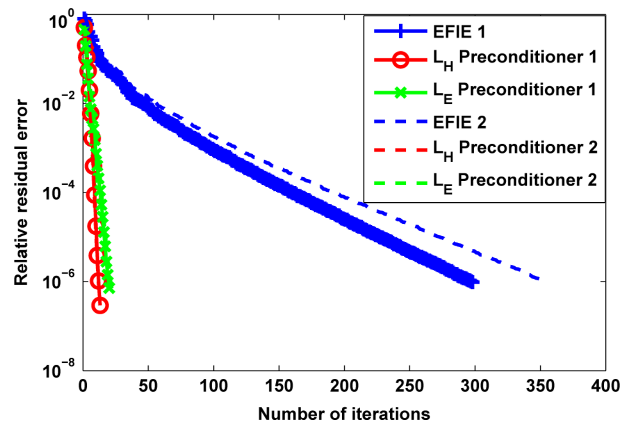

Fig. 10. Analysis of scattering from the straddling block. Iteration history recorded in GMRes to achieve a relative residual error of 10^{-6}. The structure is excited by either dipole 1 or dipole 2.

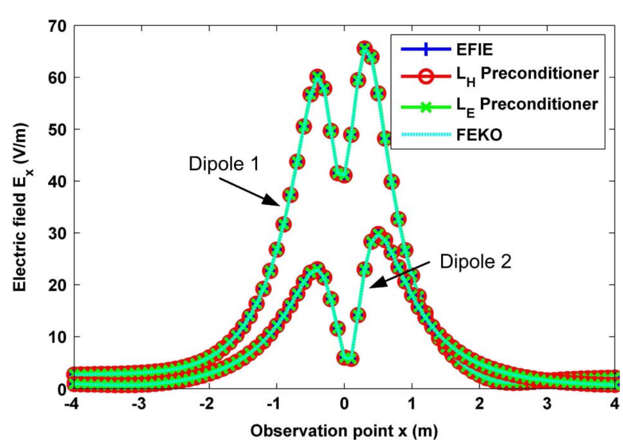

Fig. 11. Analysis of scattering from the straddling block. Scattered near field distribution validated by FEKO. The structure is excited by either dipole 1 or dipole 2.

($-4 \leq x \leq 4, y = 0, z = 0.8$) m, which is further validated by FEKO, as is shown in Fig. 11. Again, good agreement can be achieved for both cases.

VI. Conclusion

A Calderón multiplicative preconditioned electric field integral equation with a layered medium Green's function is developed for the analysis of electromagnetic scattering of perfect electrically conducting objects in the layered medium. Motivated by the Calderón projector and Calderón identities in free space, their counterparts in the layered medium are carefully studied. By applying the Rao–Wilton–Glisson and Buffa–Christiansen basis functions, a multiplicative preconditioner based on Calderón identities is constructed. To make the construction of the preconditioner as convenient as the one in free space, an alternative implementation is suggested. It is shown through different numerical examples that excellent convergence can always be achieved for different types of excitations, or different positions of the scattering objects in the layered medium. It is also shown that the convergence of the iteration is independent of the discretization density.

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